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Dynamical Modeling and Distributed Control of Connected and Automated Vehicles: Challenges and Opportunities

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Abstract—The platooning of connected and automated vehicles (CAVs) is expected to have a transformative impact on road transportation, *e.g.*, enhancing highway safety, improving traffic utility, and reducing fuel consumption. Requiring only local information, distributed control schemes are scalable approaches to the coordination of multiple CAVs without using centralized communication and computation. From the perspective of multi-agent consensus control, this paper introduces a decomposition framework to model, analyze, and design the platoon system. In this framework, a platoon is naturally decomposed into four interrelated components, *i.e.*, 1) node dynamics, 2) information flow network, 3) distributed controller, and 4) geometry formation. The classic model of each component is summarized according to the results of the literature survey; four main performance metrics, *i.e.*, internal stability, stability margin, string stability, and coherence behavior, are discussed in the same fashion. Also, the basis of typical distributed control techniques is presented, including linear consensus control, distributed robust control, distributed sliding mode control, and distributed model predictive control.

I. Introduction

The platooning of connected and automated vehicles (CAVs) on the highway has attracted extensive interest due to its potential to significantly impact road transportation. The control of a platoon aims to ensure all the vehicles in the same lane move at a consistent speed while maintaining the desired spacing between adjacent vehicles. To our best knowledge, the earliest implementations date back to the PATH program in the 1980s, where many topics were studied such as division of control tasks, the layout of control architecture, as well as control laws for headway control [1]. Recently, some demos were performed in the real world, including the GCDC in the Netherlands [2], SARTRE in Europe [3], and Energy-ITS in Japan [4].

Earlier studies on platooning often only consider radar-based sensing systems, in which the types of information exchange topologies are quite limited. However, the rapid deployment of vehicle-to-vehicle (V2V) communications, such as dedicated short range communication (DSRC)[5], [6], can generate a variety of new topologies for platoons, *e.g.,* two-predecessor following type and multiple-predecessor following type [7], [8]. New challenges naturally arise due to the topological variety, which is even critical when considering time delay, packet loss, and quantization error in the communications. In such cases, it is preferable to view a platoon as a network of dynamical systems, and to employ multi-agent consensus control schemes to design distributed controllers [7], [23]. For example, Wang *et al.* introduced a weighted and constrained consensus seeking framework to study the influence of time-varying network structures on the platoon dynamics by using a discrete-time Markov chain [10]. Bernardo *et al.* analyzed vehicle platoons from the viewpoint of consensus control of a dynamic network [11]. Zheng *et al.* introduced two fundamental methods to improve the stability margin of platoons via topological selection and control adjustment [22]. Gao *et al.* proposed an $\mathcal{H}_\infty$ control method to address uncertain vehicle dynamics and time-delays [42]. More recently, both robustness analysis and distributed $\mathcal{H}_\infty$ control synthesis have been discussed for a platoon of connected vehicles with undirected topologies in [12].

From the perspective of multi-agent consensus control, a platoon of CAVs is actually a one-dimensional network of dynamical systems, where the vehicles only use their neighboring information for feedback. This perspective naturally decomposes a platoon of CAVs into four interrelated components, *i.e.*, node dynamics (ND), information flow network (IFN), distributed controller (DC), and formation geometry (FG), which is originally proposed in [22], [23]. This decomposition is able to provide a unified framework to analyze, design, and synthesize the platoon system, as well as further on-road implementations [23], [24]. The main contributions of this paper are: 1) we summarize the modeling techniques of each component in a platoon according to the proposed four-component framework; 2) we present a detailed discussion of techniques on four performance metrics in a platoon, *i.e.*, internal stability, stability margin, string stability, and coherence behavior; 3) we introduce basic distributed synthesis methods, including linear consensus control, distributed robust control, distributed sliding mode control, and distributed model predictive control.

The rest of this paper is organized as follows: Section II introduces the four-component framework, and presents the modeling techniques of each component. Section III reviews four main performance metrics, followed by discussions on distributed controllers in Section IV. Section V concludes this paper.

II. Modeling of a platoon of CAVs: the Four-component Framework

This paper considers a platoon of CAVs on a flat road, which aims to move at the same speed while maintaining desired spacing among vehicles. The platoon has a leading vehicle and other following vehicles. As shown in Fig. 1, the platoon system can be viewed as a combination of four main components [22], [23]:

1) Node dynamics (ND), which describes the behavior of each involved CAV;

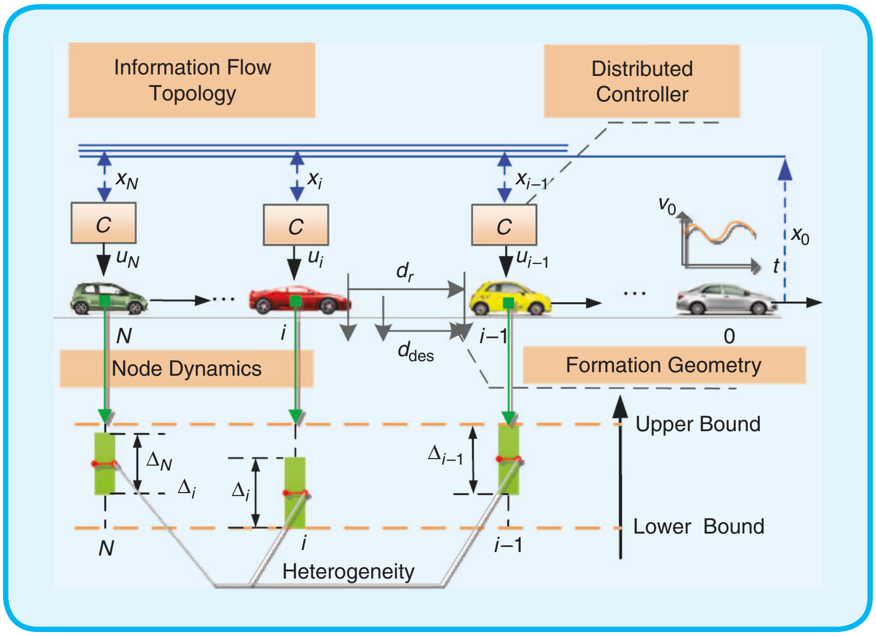

FIG 1 Four major components of a platoon [22], [23]: 1) node dynamics, 2) information flow network, 3) distributed controller, 4) geometry formation; where d_r is the actual relative distance, d_{des} is the desired distance, u_i is the the control signal, x_i is the state, Δ_i denote the dynamical uncertainty, and C denotes the controller.

2) Information flow network (IFN), which defines how the nodes exchange information with each other, including the topology and quality of information flow;
3) Distributed controller (DC), which implements the feedback control only using neighboring information;
4) Formation geometry (FG), which dictates the desired inter-vehicle distance when platooning.

Each component in Fig. 1 has significant influence on the collective behavior of a platoon. According to the four-component framework, a categorization of existing literature can be found in [23], [24]. In this section, we present a detailed discussion of the modeling techniques for each component. For completeness, we first introduce the definitions of some performance measures that are widely used in platoon control.

Definition 1 (Internal Stability). A linear platoon is said to be internally stable if and only if the closed-loop system has eigenvalues with strictly negative real parts [5], [7];

Definition 2 (Stability Margin). The stability margin of a platoon is defined as the absolute value of the real part of the least stable eigenvalue, which characterizes the convergence speed of initial errors [22], [25];

Definition 3 (String Stability). A platoon is said to be string stable if the disturbances are not amplified when propagated downstream along the vehicle string [27], [33];

Definition 4 (Coherence Behavior). The coherence behavior describes how well the formation resembles a rigid body subject to exogenous disturbances [39], [54], which is quantified as a certain $\mathcal{H}_2$ norm of the closed-loop system.

Note that we only present the basic descriptions of the performance metrics, and the exact mathematical definitions might be slightly different in the literature.

A. Node Dynamics (ND)

Many previous studies on platoon control only emphasize on the longitudinal dynamical behaviors. Only a few studies discussed the integrated longitudinal and lateral control [44], [45]. Bicycle model is usually used to describe the lateral dynamics for control design (see [44], [45] for details). Here, we only review the modeling of longitudinal dynamics. In addition, we mainly focus on continuous models, and there is an alternative class of modeling techniques using timed automata or hybrid models; see [65]–[67] for details.

Vehicle longitudinal dynamics are inherently nonlinear, consisting of drive line, brake system, aerodynamics drag, rolling resistance, gravitational force [15], [29]. The following nonlinear equations and its variants are widely employed to model the nonlinear longitudinal dynamics:

$$\begin{cases} \dot{p}_i(t) = v_i(t) \\ \dfrac{\eta_{T,i}}{r_{w,i}} T_i(t) = m_i \dot{v}_i(t) + C_{A,i} v_i^2(t) + m_i g f, \; i \in \mathcal{N}, \\ \tau_i \dot{T}_i(t) + T_i(t) = T_{\mathrm{des},i}(t) \end{cases} \quad (1)$$

where, $\mathcal{N} = \{1, 2, \ldots, N\}$; $p_i(t)$ and $v_i(t)$ denote the position and velocity of vehicle i; m_i is the vehicle mass; $C_{A,i}$ is the lumped aerodynamic drag coefficient; g is the acceleration due to gravity; f is the coefficient of rolling resistance; $T_i(t)$ denotes the actual driving/braking torque; $T_{\mathrm{des},i}(t)$ is the desired driving/braking torque; τ_i is the inertial delay of vehicle longitudinal dynamics; $r_{w,i}$ denotes the wheel radius and $\eta_{T,i}$ is the mechanical efficiency of driveline. The position and velocity of the leading vehicle are denoted by $p_0(t)$ and $v_0(t)$, respectively.

Some studies directly use nonlinear models for platoon control (see [19], [21], [27], [34], [45]). The asymptotic stability and string stability can be guaranteed by carefully selecting the control parameters, but explicit performance limits are rather difficult to analyze with given spacing policy and communication topology. Actually, linear models are more frequently used for tractable issues. In the literature, the commonly used models include 1) single integrator model, 2) second-order model, 3) third-order model, and 4) single-input-single-output (SISO) model.

The single integrator model is the simplest case, which takes the vehicle speed as control input and position as the exclusive state, *i.e.*,

$$\dot{p}_i(t) = u_i(t). \tag{2}$$

where the control input $u_i(t)$ is the velocity of each vehicle. This single integrator model (2) can significantly simplify the theoretical analysis on controller design. For instance, the structured optimal control of platoons can be transformed into a convex problem under the single integrator assumption, but this problem is challenging for other models [39]. However, in addition to largely departing from actual vehicle dynamics, the single integrator model fails to reproduce string instability [35]. An improvement is to assume ND as a point mass, resulting in the double-integrator model [18], [35], [36]:

$$\begin{cases} \dot{p}_i(t) = v_i(t) \\ \dot{v}_i(t) = u_i(t) \end{cases}. \tag{3}$$

where $u_i(t)$ is the acceleration of each vehicle. Many important theoretical results, like decentralized optimal control [39], stability margin analysis [18], [25], [26], and coherence behavior [54], rely on the assumption of second-order dynamics. This assumption still does not catch many features of real vehicle dynamics, *e.g.*, inertial delay in powertrain dynamics. One modeling trend is to further increase one state and yield so-called third-order model. The added state is often to approximate the input/output behaviors of powertrain dynamics, which equivalently degrades the control input to engine torque and/or braking torque [15], [17], [20]. Most approximations use either feedback linearization technique [5], [15], [37] or lower-layer control technique [1], [45], resulting in a state space model as:

$$\begin{aligned} &\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) \\ &x_i(t) = \begin{bmatrix} p_i \\ v_i \\ a_i \end{bmatrix}, A_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau_i} \end{bmatrix}, B_i = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\tau_i} \end{bmatrix}, \end{aligned} \tag{4}$$

where $x_i(t) = [p_i, v_i, a_i]^T$ is the state, $u_i(t)$ is the desired acceleration of each vehicle, and τ_i is the time constant of approximated powertrain dynamics. An alternative of the abovementioned models is the transfer function model, which is often used to analyze string stability in frequency domain.

$$\begin{aligned} &p_i(s) = \frac{1}{s} v_i(s),\ v_i(s) = \frac{1}{s} a_i(s), \\ &a_i(s) = H_i(s) u_i(s), \end{aligned} \tag{5}$$

Many important theoretical results rely on the assumption of second-order dynamics which still does not catch many features of real vehicle dynamics, e.g., inertial delay in powertrain dynamics.

where $p_i(s), v_i(s), a_i(s)$ are the Laplace transforms of node i's position, velocity and acceleration, respectively; $u_i(s)$ is the control input, and $H_i(s)$ is a linear single-input-single-output (SISO) strictly proper transfer function. This model has two integrators and a lower order inertial delay, which leads to some fundamental limitations for certain platoons [33]. The pioneer work on this model started from Seiler, Pant, and Hedrick [33], and later widely employed in many other studies; see *e.g.*, [50] and [53].

B. Information Flow Network (IFN)

The control of the collective behavior of multiple CAVs is based on vehicles' mutual awareness of their states (*e.g.*, inter-vehicle distance and vehicle speed), which is achieved by inter-vehicle sensing and communication. As shown in Fig. 1, the information provided by inter-vehicle sensing and communication serves as an important input to each local controller, thus having a significant impact on the collective behavior. Here, we briefly discuss the topology and quality of information flow, as well as a graph-based modeling approach.

1) **Information Flow Topology (IFT)**. The IFT captures the connectivity of information exchange between vehicles, and affects the platoon behavior such as string stability [33], stability margin [22], [25], and coherence behavior [39], [54]. Early-stage platoon control is mainly based on radar-sensing, and a vehicle can only obtain information about its nearest neighbors, *i.e.*, immediately preceding and following vehicles. In this case, feasible IFTs include the predecessor following (PF) and bidirectional (BD) topologies, as shown in Fig. 2(a) and (b) respectively. Nowadays, with V2V communication via technologies such as IEEE 802.11p-based DSRC and the emerging 5G solutions, various IFTs become feasible since a vehicle can communicate with vehicles beyond its immediate surroundings [5], [7]. Typical topologies include the predecessor-following leader (PFL) type, bidirectional leader (BDL) type, two predecessor-following (TPF) type, and two predecessor-following leader (TPFL) type, as shown in Fig. 2(c)–(f) respectively. Since IFT has a significant impact on the behavior of a platoon, it is important to adapt IFT (*e.g.*, by controlling the transmission power of communications) based on the need of platoon control [49].

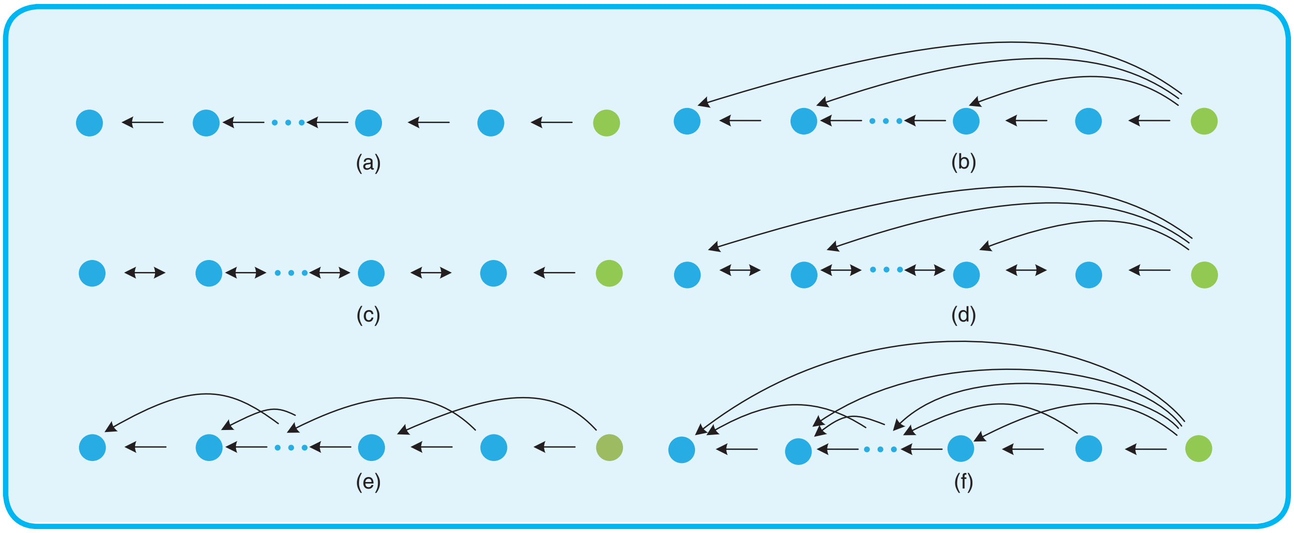

FIG 2 Typical IFTs: (a) PF, (b) BD, (c) PFL, (d) BDL, (e) TPF, (f) TPFL.

2) **Quality of information exchange.** Given a specific IFT, the quality of inter-vehicle sensing and communication also impacts the platoon behavior. For instance, the accuracy of radar sensing impacts the robustness and performance of platoon control. Meanwhile, wireless communication quality significantly impacts the safety and performance of platoon control [47], [48]. Therefore, it is important to take the sensing and wireless communication quality into account in platoon control. For wireless communication whose quality can be controlled by using mechanisms such as scheduling, power control, and rate control, it is important to consider the joint control of wireless communication and vehicle platoon [63]. Existing IEEE 802.11p-based inter-vehicle wireless communication does not enable predictable control of co-channel wireless interference and thus unable to ensure predictable control of communication quality. With recent breakthroughs in wireless control networking, wireless communication quality can be controlled in a predictable manner [49], thus having opened the door to the co-design of vehicular wireless networking and platoon control [63].
3) **Graph-based topological modeling.** Directed graphs can be used to model allowable information flow between the vehicles in a platoon [7], [22]. More descriptions on graph theory can be found in [59] and the references therein.

The information flow among followers is described by a directed graph $\mathcal{G}_N = \{\mathcal{V}_N, \mathcal{E}_N, \mathcal{A}_N\}$ with a set of nodes $\mathcal{V}_N = \{1, 2, \ldots, N\}$, a set of edges $\mathcal{E}_N \subseteq \mathcal{V}_N \times \mathcal{V}_N$ and the adjacency matrix $\mathcal{A}_N = [a_{ij}] \in \mathbb{R}^{N\times N}$. Each edge (j, i) represents a directed information flow from j to i. An edge (j, i) belongs to $\mathcal{E}_N$, if and only if $a_{ij} = 1$; otherwise $a_{ij} = 0$. It is assumed that there are no self-edges. The neighbor set of node i is denoted by $\mathbb{N}_i = \{j \,|\, a_{ij} = 1\}$. The in-degree of i-th node is $\deg_i = \sum_{j=1}^{N} a_{ij}$. Denote $\mathcal{D}_N = diag\{\deg_1, \deg_2, \ldots, \deg_n\}$, and the Laplacian matrix $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N\times N}$ of $\mathcal{G}_N$ is defined as $\mathcal{L} = \mathcal{D}_N - \mathcal{A}_N$.

To model the information flow from the leader to the followers, we define an augmented graph as $\mathcal{G}_{N+1}$ with a set of nodes $\mathcal{V}_{N+1} = \{0, 1, 2, \ldots, N\}$ and a set of edges $\mathcal{E}_{N+1} \subseteq \mathcal{V}_{N+1} \times \mathcal{V}_{N+1}$. A pinning matrix represents how each follower connects the leader, which is defined as $\mathcal{P} = diag\{\wp_1, \wp_2, \ldots, \wp_N\}$, where $\wp_i = 1$ if edge $\{0, i\} \in \mathcal{E}_{N+1}$; otherwise $\wp_i = 0$. The leader accessible set of node i is

$$\mathbb{P}_i = \begin{cases} \{0\}, & \text{if } \wp_i = 1 \\ \emptyset, & \text{if } \wp_i = 0 \end{cases}.$$

A spanning tree is a directed path connecting all the nodes in the graph [59]. The augmented graph $\mathcal{G}_{N+1}$ should contain one spanning tree rooted at the leader for controllability. It is easy to see that all the IFTs demonstrated in Fig. 2 contain at least a spanning tree. To illustrate some notations, considering PF and BD topologies, we have

$$\mathcal{L}_{\text{PF}} = \begin{bmatrix} 0 & & & \\ -1 & 1 & & \\ & \ddots & \ddots & \\ & & -1 & 1 \end{bmatrix}, \mathcal{P}_{\text{PF}} = \begin{bmatrix} 1 & & & \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{bmatrix},$$

$$\mathcal{L}_{\text{BD}} = \begin{bmatrix} 1 & -1 & & \\ -1 & 2 & \ddots & \\ & \ddots & \ddots & -1 \\ & & -1 & 1 \end{bmatrix}, \mathcal{P}_{\text{BD}} = \begin{bmatrix} 1 & & & \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{bmatrix}.$$

The matrices $\mathcal{L}, \mathcal{P}$ encapsulate the topological connections of the information flow in a platoon. In the proposed framework, $\mathcal{L} + \mathcal{P}$ plays an important role in the closed-loop dynamics (see (15)). The eigenvalues of $\mathcal{L} + \mathcal{P}$ have a key impact on the stability margin for platoons with linear node dynamics (see Lemma 5 in [7]). It is proved that all the eigenvalues of $\mathcal{L}+\mathcal{P}$ locate in the open right-half plane when $\mathcal{G}_{N+1}$ contains one spanning tree rooted at the leader [7], [59]. Further, it is shown that for undirected topologies, the minimum eigenvalue of $\mathcal{L}+\mathcal{P}$ has a close relationship with the number of followers that are pinned to the leader (see Theorem 1 in [22]). The discussion in this paper mainly focuses on the impact of IFT, but the graph-based modeling approach can be extended to model the quality of information flow by introducing weights to each edge of the graph [59].

C. Distributed Controller (DC)

The DC implements the feedback control using neighbors' information, specified by $\mathbb{I}_i = \mathbb{N}_i \cup \mathbb{P}_i$, to enable the global coordination. An unstructured DC is one that corresponds to a complete graph which requires all-to-all communications. Many existing studies consider structured control laws either in an explicit or implicit way; see [14], [15], [37] and [39].

The commonly used DC is linear for comprehensive results on theoretical analysis, and convenience in hardware implementations [7]. The general form of linear controller is:

$$\begin{aligned} u_i(t) = -\sum_{j \in \mathbb{I}_i} [&k_{ij,p}(p_i(t-\gamma_{ii}) - p_j(t-\gamma_{ij}) - d_{i,j}) \\ &+ k_{ij,v}(v_i(t-\gamma_{ii}) - v_j(t-\gamma_{ij})) \\ &+ k_{ij,a}(a_i(t-\gamma_{ij}) - a_j(t-\gamma_{ij}))], \end{aligned} \tag{6}$$

where $k_{ij,\#}$ $(\# = p, v, a)$ is the local controller gain, γ_{ii} is the time delay corresponding to obtain its own state, and γ_{ij} is the time delay corresponding to receive the state of node j via a communication channel. Many previous work only employed specific types of (6). The internal stability of a platoon with a linear controller largely depends on the structure of IFT. For example, the stabilizing region of linear control gains was explicitly derived in [7] for a class of topologies, and string stability requirements for PF topology were established in [15]. The optimization methods, either numerical or analytical, were also used to optimize the localized gains [36], [39]. There are also some studies employing sliding mode control (SMC) to design a string-stable platoon [15]. For SMC, the internal stability and string stability of platoons are usually realized through a posterior controller tuning.

There are two main drawbacks in the design methods above: 1) they are unable to explicitly address string stability, and 2) they are unable to handle the state or control constraints. Recently, $\mathcal{H}_\infty$ controller synthesis has been proposed to include the string stability requirement in the design specification [20]. Model predictive control (MPC) has been introduced into platoon control to forecast system dynamics, explicitly handling actuator/state constraints by optimizing given objectives [2], [19], [21]. In Section IV, we will introduce more details on how to design linear consensus controller, distributed robust controller, sliding mode controller, and model predictive controller for platoons.

D. Formation Geometry (FG)

The objective of platoon control is to track the speed of the leader and to maintain a desired formation governed by an inter-vehicle spacing policy, *i.e.*,

$$\begin{cases} \lim_{t\to\infty} \| v_i(t) - v_0(t) \| = 0 \\ \lim_{t\to\infty} \| p_{i-1}(t) - p_i(t) - d_{i-1,i} \| = 0 \end{cases}, \ i \in \mathcal{N}, \tag{7}$$

where $d_{i-1,i}$ is the desired space between $i-1$ and i, which determines the formation geometry of a platoon.

There are three major policies of FG: 1) constant distance (CD) policy, 2) constant time headway (CTH) policy, and 3) nonlinear distance (NLD) policy [13]. For the CD policy, the desired distance between two consecutive vehicles is independent of vehicle velocity, which can lead to a high traffic capacity. In this case, $d_{i-1,i}$ is a given constant number,

$$d_{i-1,i} = d_0, \ i \in \mathcal{N}, \tag{8}$$

where d_0 is a positive number. For CTH policy, the desired inter-vehicle range varies with the velocity, which is in accord with driver behaviors to some extent but limits achievable traffic capacity. One commonly used formulation is:

$$d_{i-1,i} = t_h v_i + d_0, \ i \in \mathcal{N}, \tag{9}$$

where t_h is the time headway. For NLD policy, the desired inter-vehicle is a nonlinear function of velocity, *i.e.*,

$$d_{i-1,i} = g(v_i), \quad i \in \mathcal{N}. \tag{10}$$

Note that NLD policy has the potential to improve both the traffic flow stability and traffic capacity compared with CD and CTH policies [58].

III. Performance of a Platoon of CAVs: Stability and Robustness

In this section, we discuss the performance of a platoon of CAVs, with a special focus on stability and robustness. Some practical benefits, such as reducing fuel consumption and improving traffic efficiency, are not covered here (we refer the interested reader to [30]–[32]). For example, a special design for fuel optimized platoon control was designed in [32], which uses a distributed

Pulse-and-Glide (PnG) controller to switch the engine operation point between two optimum positions to achieve lower average fuel.

In the field of platoon control, there are four important and commonly discussed performance metrics, namely, internal stability, stability margin, string stability, and coherence behavior. In general, the first two metrics focus on stability of a platoon, while the other two metrics focus on the robustness of a platoon considering external disturbances. We note that a detailed categorization can be found in [24]. In addition to these performance measures, some other metrics, such as fast convergence and safety, are also important when designing a practical platoon system.

A. Internal Stability

No matter what kind of topology is employed, internal stability must be guaranteed in a platoon. Two main approaches have been proposed to ensure the internal stability: 1) global approach [17], [57], and 2) local approach [9], [15], [21].

The first approach is to straightforwardly take the overall platoon as a structured system and then design a controller in a centralized way, in which IFT becomes less important in the design process. For example, a linear matrix inequality (LMI) was obtained based on the global platoon dynamics to guarantee internal stability [17], [57]. One major drawback of this approach is that the computation efficiency quickly worsens with increasing the platoon size. Therefore, most studies decomposed a platoon into sub-systems and applied decentralized control methods, leading to the second approach. For instance, under PF topology, a platoon can be naturally viewed as unidirectional cascade systems, which only needs to study any two successive vehicles to guarantee stability; see [9], [15], [20] and [28]. Besides, the inclusion principle was used to decompose such kind of platoon into local subsystems, where an overlapping controller was designed [37]. This technique is not suitable for a platoon with BD topology since its spacing errors propagate from both forward and backward directions. Partial differential equation (PDE) techniques were applied to approximate the dynamics of platoons with BD topologies [18], [25], [26], which could avoid the analysis of high dimensional dynamics. In additional, Lyapunov method based energy function was used to prove both longitudinal car following stability and latitudinal lane keeping stability [44]. As an extension, Lasalle's invariance principle can also be utilized to prove asymptotic stability for time invariant platoon systems in the case where the derivative of a Lyapunov candidate is only negative semi-definite.

B. Stability Margin

Stability margin is used to characterize the convergence speed of the spacing errors in a platoon [22]. Most of current research on stability margin focus on the CD policy, which has revealed that stability margin is a function of 1) platoon size (N), 2) ND, 3) IFN, 4) DC structure [18], [22], [25], [26].

By considering ND as a point mass, Barooah *et. al* proved that the stability margin approached zero as $O(1/N^2)$ under symmetric bidirectional control, and asymptotic behavior of stability margin could be improved to $O(1/N)$ by introducing small amounts of "mistuning" [18]. This result was extended to linear third-order dynamics, which covers the inertial delay of powertrain dynamics in [7]. Using the PDE approximation, Hao *et al.* showed that the scaling law of stability margin could be improved to $O(1/N^{2/D})$ under D-dimensional IFTs [25]. Recently, it was shown that employing asymmetric control, the stability margin could be bounded away from zero, which is independent of the platoon size [26]. Zheng *et al.* further pointed out two basic methods to improve the stability margin via topology selection and control adjustment [22].

C. String Stability

The achievability of string stability has a tight relationship with the FG and IFN in a platoon. Seiler *et al.* showed that string stability cannot be guaranteed for any linear identical controllers under PF topology and CD policy [33]. Barooah *et al.* further pointed out that for a homogeneous platoon with BD topology, linear identical controllers also suffered fundamental limitations on the string stability [50]. Middleton *et al.* extended the work in [33] by considering heterogeneous ND, limited communication range, nonzero time headway policy, which showed that both forward communication range and small time headway cannot alter the string instability [53]. Some solutions have been proposed to improve string stability, including: 1) relaxing formation rigidity, *i.e.*, introducing enough time headway in the spacing policy [14], [15], [36], or using nonlinear policy [58]; 2) using non-identical controllers for different vehicles [18], [38]; 3) extending the information flow by using more complex IFTs [16], [37].

Recently, some advanced controllers have been proposed to ensure string stability, including sliding mode control [15], model predictive control [2], [19] and $\mathcal{H}_\infty$ control [17], [20]. Note that all of them either employ CTH policy or use the leader's information. Current research results usually focus on string stability caused by disturbances or maneuvers of the lead vehicle. However, following vehicles in a platoon also have the same probability encountering external disturbances. There are a few research results that consider the effects of disturbances from any vehicle in a platoon. For example, Seiler *et al.* [33] investigated multiple disturbances propagate in the platoon, where string instability was addressed based on a constant spacing. This work has been extended to a more practical situation with a variable spacing strategy in [52].

D. Coherence Behavior

The coherence behavior is a scalar metric adopting $\mathcal{H}_2$-norm of the closed-loop system, which characterizes the robustness of a platoon driven by exogenous disturbances. This captures the notion of coherence [40], [54]. Bamieh *et al.* investigated the asymptotic scaling of upper bounds on coherence behavior with respect to the platoon size, and indicated that the IFN may play a more important role than DC [54]. Several recent research used coherence behavior as the cost function to optimize the local control gains using augmented Lagrangian approach [39]. In addition, alternative direction method of multipliers (ADMM) was used to optimize the communication structure of IFTs in [40], where the cost function was the measure of coherence behavior. Recently, chordal decomposition has been applied in the design of structured controllers, which has the potential to address the coherence behavior of platoons efficiently [55]; also see one recent result in semidefinite programming [56].

IV. Controller Design of a platoon of CAVs: Distributed Methods

This section introduces four types of distributed controller design methods for a platoon of CAVs, *i.e.*, linear consensus control, robust control, distributed sliding mode control, and distributed model predictive control. For the first two methods, linear vehicle models are used, while nonlinear vehicle models can be employed for the last two methods. The main strategy of these methods is to decouple the platoon dynamics into several subsystems depending on the eigenvalues of $\mathcal{L}+\mathcal{P}$. After decoupling, many synthesis methods can be used to solve the required distributed controller using LMIs.

A. Linear Consensus Control

Linear control is one of the most commonly used methods for platoon control [13], [16], [18], since it can not only facilitate theoretical analysis but is also suitable for hardware implementations. Many existing results on stability region, stability margin, and string stability requirements [16], [17], [37] are based on linear controllers.

Here, we introduce a generic approach to analyze the collective behavior of platoons when employing linear consensus controllers (see [7], [22] for more details). The 3rd-order state space model is adopted to describe node dynamics, as shown in (4). The platoon is assumed to be homogeneous (*i.e.*, $A_i = A,\ i \in \mathcal{N}$) and the CD policy is employed. The general linear consensus controller is given in (6). It is assumed that controller gains are identical, *i.e.*, $k_{i,\#} = k_\#\ (\# = p, v, a),\ i \in \mathcal{N}$. Besides, we assume that there are no time-delays, *i.e.*, $\gamma_{ij} = 0,\ i,j \in \mathcal{N}$. To write (6) into a compact form, we define a new tracking error

$$\tilde{x}_i(t) = x_i(t) - x_0(t) - \tilde{d}_i, \tag{11}$$

where $\tilde{d}_i = [d_{i,0}, 0, 0]^T$. For CD policy, the desired distance between the i-th follower and the leader is $d_{i,0} = -i \times d_0$. Then, (6) is rewritten into

$$u_i(t) = -\sum_{j \in \mathbb{I}_i} k^T(\tilde{x}_i(t) - \tilde{x}_j(t)), \tag{12}$$

where $k = [k_p, k_v, k_a]^T$ is a vector of local feedback gains. To derive the collective dynamics of a platoon, we define the collective state vector $X = [\tilde{x}_1^T, \tilde{x}_2^T, \ldots, \tilde{x}_N^T]^T \in \mathbb{R}^{3N\times 1}$ and the collective control input vector $U = [u_1, u_2, \ldots, u_N]^T \in \mathbb{R}^{N\times 1}$. The collective dynamics of nodes from 1 to N are

$$\dot{X}(t) = I_N \otimes A \cdot X(t) + I_N \otimes B \cdot U(t) \tag{13}$$

with $I_N \otimes A \in \mathbb{R}^{3N\times 3N}$, $I_N \otimes B \in \mathbb{R}^{3N\times N}$, where $\otimes$ denotes the Kronecker product. Based on (12), the collective form of the distributed control law is written into

$$U(t) = -(\mathcal{L}+\mathcal{P}) \otimes k^T \cdot X(t). \tag{14}$$

Substituting (14) into (13), the closed-loop dynamics of a homogeneous platoon become

$$\dot{X}(t) = [I_N \otimes A - (\mathcal{L}+\mathcal{P}) \otimes B k^T] \cdot X(t). \tag{15}$$

Using the eigenvalue decomposition of $\mathcal{L}+\mathcal{P}$, it is proved that (15) is asymptotically stable if and only if

$$A - \lambda_i B k^T, \forall i \in \mathcal{N}. \tag{16}$$

are all Hurwitz, where $\lambda_i, i \in \mathcal{N}$ are the eigenvalues of $\mathcal{L}+\mathcal{P}$ [7]. By this way, the collective dynamics of a platoon are reduced into the behavior of multiple subsystems. Thus, the design of feedback gains is decoupled from the IFT, leading to scalable solutions for large-scale platoons [7]. Note that the eigenvalue decomposition of $\mathcal{L}+\mathcal{P}$ plays a key role in decoupling the dynamics from IFTs, which is also used in the design of distributed robust control (see (19)).

As shown in (15), the closed-loop dynamics of a vehicular platoon is a function of four components, namely 1) ND, denoted by A, B; 2) IFT, denoted by $\mathcal{L}+\mathcal{P}$; 3) FG, included in X as the desired distance (see (11)); and 4) DC, denoted by k. In addition, it is easy to see that the performance measures of platoons must have a tight relationship with the four main components. However, it is usually rather difficult to explicitly obtain the relationship between the performance metrics and the decomposed components. Most of existing research on string stability focuses on specific cases only; see [9], [16], [17] and [37] for example.

B. Distributed Robust Control

The robustness of platoon control systems is an important topic. One practical way to handle model mismatches in vehicle dynamics is to use the consistent and accurate input-output behavior of node dynamics [41]. But it is not easy to accommodate the heterogeneity in node dynamics.

Considering the requirements of string stability, robustness, and tracking performance, Gao *et al.* proposed an $\boldsymbol{H}_\infty$ control method for a heterogeneous platoon with uncertain dynamics and uniform time delays [42]. In this study, all nodes were combined as a big system. One disadvantage is that the designed controller only works for a specific platoon and it needs to be redesigned when the scale or interaction topology changes. Similar to the case of linear control, the decoupling strategy of robust control is also an effective way to overcome this problem. This strategy is motivated by the eigenvalue decomposition of $\mathcal{L}+\mathcal{P}$, and can balance the performances of robustness and disturbance attenuation. As shown in Fig. 3, the coupling arising from information topology can be successfully transferred to the uncertain parts of nodes $\varDelta$ by applying a linear transformation to the platoon.

In Fig. 3, the dynamics of a platoon in frequency domain are derived by adding the model uncertainties to (5):

$$E(s)=\frac{H(s)}{s^2}U(s)+\frac{1}{s^2}W(s)-\mathbf{1}_N p_0(s)-\frac{1}{s}\varGamma_0,$$
$$Z(s)=\Omega(s)P(s)U(s),\; W(s)=\varDelta Z(s), \tag{17}$$

where $E(s)=\begin{bmatrix} e_1(s)\\ \vdots \\ e_N(s)\end{bmatrix}=\begin{bmatrix} p_1(s)-p_0(s)-d_{1,0}/s\\ \vdots \\ p_N(s)-p_0(s)-d_{N,0}/s\end{bmatrix},\ \mathbf{1}_N=\begin{bmatrix}1\\ \vdots \\ 1\end{bmatrix},$ $\varGamma_0=\begin{bmatrix} d_{1,0}\\ \vdots \\ d_{N,0}\end{bmatrix}$, $\varDelta=diag(\Delta_1,\ldots,\Delta_N)$ is the normalized model uncertainty satisfying $\|\Delta_i\|_\infty\le 1$; $\Omega(s)$ is its weighting function; $W(s)$ is the disturbance arising from $\varDelta$, whose input is $Z(s)$. Note that this model (17) includes additional uncertainty part Δ_i compared to the 3rd-order state space model (4). The CD policy is used here. Each node is controlled by the distributed state-feedback control logic (12). Here, we present its expression in frequency domain:

$$U(s)=K(s)(\mathcal{L}+\mathcal{P})E(s), \tag{18}$$

where $K(s)=k_p+k_v s+k_a s^2$. The nodes are interacting by the information topology $\mathcal{L}+\mathcal{P}$. For undirected topologies, $\mathcal{L}+\mathcal{P}$ has an eigenvalue decomposition as

$$\mathcal{L}+\mathcal{P}=\varPsi diag(\lambda_1,\ldots,\lambda_N)\varPsi^{-1}. \tag{19}$$

where λ_i is the eigenvalue of $\mathcal{L}+\mathcal{P}$, and $\varPsi$ is the eigenvector matrix of $\mathcal{L}+\mathcal{P}$. Using the linear transformation in Fig. 3, the certain parts are then decoupled to a diagonal structure:

$$\bar{E}(s)=\frac{H(s)}{s^2}\bar{U}(s)+\frac{1}{s^2}\bar{W}(s)-\varPsi^{-1}\mathbf{1}_N p_0(s)-\frac{1}{s}\bar{\varGamma}_0,$$
$$\bar{U}(s)=K(s)\,diag(\lambda_1,\ldots,\lambda_N)\bar{E}(s),$$
$$\bar{Z}(s)=\Omega(s)P(s)\bar{U}(s),\; \bar{W}(s)=\bar{\varDelta}\bar{Z}(s). \tag{20}$$

The variables in (20) are linear transformation of its original counterparts:

$$\bar{Z}(s)=\varPsi^{-1}Z(s),\; \bar{U}(s)=\varPsi^{-1}U(s),$$
$$\bar{E}(s)=\varPsi^{-1}E(s),\; \bar{W}(s)=\varPsi^{-1}W(s),$$
$$\bar{\varGamma}_0=\varPsi^{-1}\Gamma_0,\; \bar{\varDelta}=\varPsi^{-1}\varDelta\varPsi. \tag{21}$$

Based on (20), the synthesis approach of $\boldsymbol{H}_\infty$ control can be used to numerically solve $K(s)$ offline, which ensures the requirements of robustness and disturbance.

C. Distributed Sliding Mode Control

The sliding mode control (SMC) is a promising method for platooning of multiple vehicles to handle nonlinear dynamics and actuator saturations. In [27], an adaptive SMC was proposed for equilibrium-stable interconnected systems to guarantee string stability. In this study, however, applicable topologies are limited to unidirectional topologies, where one node can only obtain the information from its predecessors. In [64], an urban scenario platoon control method was introduced with two basic modes: cruise mode and collision avoidance mode.

In this section, we introduce a distributed SMC scheme for platoons with homogeneous linear dynamics (4) and undirected (symmetric) IFTs.

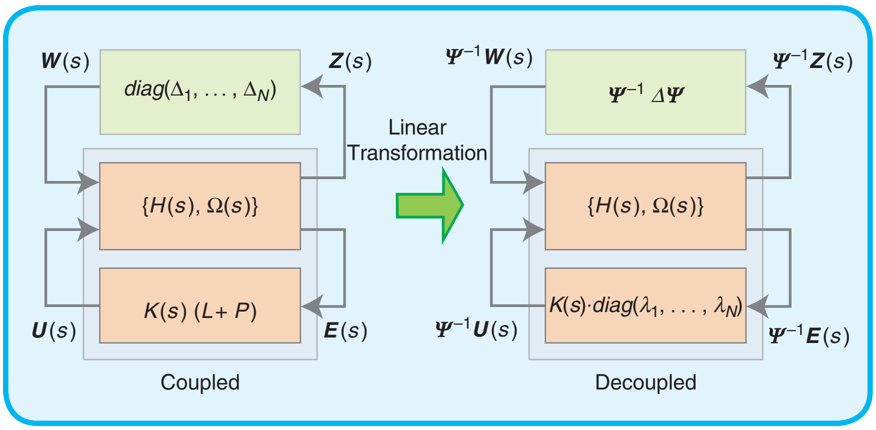

FIG 3 Decoupling of vehicular platoon systems.

If there exists a spanning tree in $\mathcal{G}_{N+1}$ and information flow between followers is undirected, $\mathcal{L}+\mathcal{P}$ is positive definite [22]. The design of distributed SMC is divided into two stages, *i.e.*, 1) topological sliding surface design and 2) topological reaching law design. The design of sliding surface and reaching law highly depends on the following topological structured function:

$$\Gamma_i(Z)=\sum_{j=1,\, j\neq i}^{N} a_{ij}(z_i-z_j)+p_i z_i,$$

where $Z\triangleq[z_1, z_2, \dots, z_N]$, $i, j\in\mathcal{N}$, a_{ij} and P_i are elements of the adjacency matrix and pinning matrix. Using topological structured function, the distributed sliding surface is designed as

$$\begin{aligned} s_i(t) &= \Gamma_i(c_P^T X)-p_i c_P^T x_o \\ &= \sum_{j=1, j\neq i}^{N} a_{ij}(c_P^T x_i - c_P^T x_j)+p_i(c_P^T x_i - c_P^T x_o), \end{aligned}$$

where $c_P^T X = c_P^T[x_1, x_2, \dots, x_N]$ and $c_p\in\mathbb{R}^{3\times 1}$ is the common vector coefficient of distributed sliding mode, satisfying that $c_P^T B_i$ is invertible. Note that (4) is a single input model, and the invertibility of $c_P^T B_i$ means that $c_P^T B_i$ is a nonzero constant. The topological sliding surface for the whole system is defined as

$$S\triangleq\begin{bmatrix} s_1\\ \vdots \\ s_N\end{bmatrix}=(\mathcal{L}+\mathcal{P})\begin{bmatrix} c_P^T(x_1-x_0)\\ \vdots \\ c_P^T(x_N-x_0)\end{bmatrix}.$$

The topological reaching law is designed by substituting $-\lambda s_i$ to topological structured function

$$\dot{s}_i\triangleq\Gamma_i(-\lambda s_i)=-\lambda\left(\sum_{j=1, j\neq i}^{N} a_{ij}(s_i-s_j)+p_i s_i\right),$$

where $\lambda>0$ is the common coefficient for the distributed reaching law. The topological reaching law for the whole system is

$$\dot{S}(t)=-\lambda(\mathcal{L}+\mathcal{P})S.$$

Comparing the derivative of sliding error and reaching law, we can cancel $\mathcal{L}+\mathcal{P}$ if it is positive definite,

$$(\mathcal{L}+\mathcal{P})[c_p^T(\dot{x}_1-\dot{x}_0),\dots,c_p^T(\dot{x}_N-\dot{x}_0)]^T=-\lambda(\mathcal{L}+\mathcal{P})S. \tag{22}$$

The positive definiteness of $\mathcal{L}+\mathcal{P}$ is essential since it guarantees the cancellation. Then, we have

$$c_p^T A_i x_i + c_p^T B_i u_i = -\lambda s_i. \tag{23}$$

The control law is obtained by solving the upper equation. The stability proof of distributed SMC is also divided into two parts, *i.e.*, reaching phase analysis and sliding phase analysis. The stability of the reaching phase is analyzed with respect to the whole platoon by choosing a Lyapunov candidate

$$V(t)=\frac{1}{2}S^T(\mathcal{L}+\mathcal{P})^{-1}S. \tag{24}$$

By taking the derivative of Lyapunov function, it can be shown that sliding surface is reached asymptotically. During the sliding phase, a proper selection of c_p is necessary to achieve stable sliding dynamics. Details can be found in [46].

D. Distributed Model Predictive Control

Model predictive control (MPC) is an optimization-based control technique to anticipate future behavior of plants and take control actions accordingly. Using MPC techniques, the control input is obtained by numerically optimizing a finite horizon optimal control problem where both nonlinearity and constraints can be explicitly handled. This technique has been embraced by many industrial applications, for instance, collision avoidance and vehicle stability [29], [60].

Currently, most MPCs are implemented in a centralized way, where all the control inputs are computed by assuming all the states are known. When considering a platoon system involving multiple vehicles, the centralized implementation is not suitable due to the challenges of gathering the information of all vehicles and solving a large-scale optimization problem. Most existing work for platoon control relies on the problem formulation of adaptive cruise control (ACC) [61], [62], which only involves two vehicles in the problem formulation. There exist some extensions to the cooperative ACC which considers multiple vehicles [2], [19]. The treatments in [2] and [19], however, also directly consider two consecutive vehicles in the problem formulation, which are only applicable to limited types of communication topologies.

Here, a synthesis method of distributed MPC is presented for a heterogeneous platoon, where each vehicle is assigned a local optimal control problem only relying on its neighboring vehicles' information [21]. This method is suitable for any type of IFTs, and the asymptotical stability of the closed-loop platoon system can be derived for unidirectional topologies [21]. The discrete version of nonlinear equations (1) are used to model the longitudinal dynamics of each vehicle, *i.e.*,

$$\begin{aligned} x_i(t+1) &= \phi_i(x_i(t))+\psi_i\cdot u_i(t),\\ y_i(t) &= \gamma x_i(t) \end{aligned} \tag{25}$$

where $\psi_i=[0,0,(1/\tau_i)\Delta t]^T\in\mathbb{R}^{3\times 1}$, $\gamma=\begin{bmatrix}1&0&0\\0&1&0\end{bmatrix}\in\mathbb{R}^{2\times 3}$, $\phi_i(x_i)\in\mathbb{R}^{3\times 1}$ is defined as

$$\phi_i = \begin{bmatrix} s_i(t) + v_i(t)\Delta t \\ v_i(t) + \frac{\Delta t}{m_{\mathrm{veh},i}}\left(\frac{\eta_{T,i}}{R_i} T_i(t) - C_{A,i} v_i^2(t) - m_{\mathrm{veh},i} g f_i\right) \\ T_i(t) - \frac{1}{\tau_i} T_i(t)\Delta t \end{bmatrix}.$$

Now, we define the local open-loop optimal control problem for each node i:

Problem $\mathcal{F}_i$: For $i \in \{1, 2, \ldots, N\}$ at time t

$$\min_{u_i^p(0|t),\ldots,u_i^p(N_p-1|t)} J_i(y_i^p, u_i^p, y_i^a, y_{-i}^a)$$
$$= \sum_{k=0}^{N_p-1} l_i\left(y_i^p(k|t), u_i^p(k|t), y_i^a(k|t), y_{-i}^a(k|t)\right), \qquad (26)$$

subject to

$$\begin{aligned} x_i^p(k+1|t) &= \phi_i(x_i^p(k|t)) + \psi_i \cdot u_i^p(k|t), \\ y_i^p(k|t) &= \gamma \cdot x_i^p(k|t), x_i^p(0|t) = x_i(t), u_i^p(k|t) \in \mathcal{U}_i, \\ y_i^p(N_p|t) &= \frac{1}{|\mathbb{I}_i|}\sum_{j\in\mathbb{I}_i}(y_j^a(N_p|t) + \tilde{d}_{i,j}), \\ T_i^p(N_p|t) &= h_i(v_i^p(N_p|t)), \end{aligned} \qquad (27)$$

where $[u_i^p(0|t), \ldots, u_i^p(N_p-1|t)]$ denotes the unknown variables, N_p is the predictive time horizon, $\mathcal{U}_i$ denotes the convex set of input constraints, $|\mathbb{I}_i|$ is the cardinality of $\mathbb{I}_i$, and $\tilde{d}_{i,j} = [d_{i,j}, 0]^T$ denotes the desired distance vector between i and j, the function l_i in (26) is the cost associated with node i, $y_i^a(k|t)$ is the assumed trajectory of node i, and $y_{-i}^a(k|t)$ is the assumed trajectory of the neighbors.

The formulation of problem $\mathcal{F}_i$ only needs the information from its neighbors, and thus it is suitable for various communication topologies, including all of those shown in Fig. 2. The key idea in the design of each local optimal control problem $\mathcal{F}_i$ is to construct and transmit assumed trajectories: each vehicle solves a local optimal control problem to obtain its own control input, and then sends its assumed output trajectory to its neighbors. Here, the assumed variable is a shifted optimal result of the last-step problem $\mathcal{F}_i$, synthesized by disposing the first value and adding a last value. The last added value ensures that the vehicle moves at a constant speed. Details of this design and the asymptotical stability of the closed-loop system can be found in [21].

V. Conclusion

This paper has presented a four-component framework to model, analyze, and synthesize a platoon of CAVs from the perspective of multi-agent consensus control. This framework is well-suited for designing distributed control schemes of CAV platooning. With the four-component framework, this paper has introduced the modeling techniques, discussed the major performance metrics, and presented the design of four types of distributed controllers.

There are some open questions, especially considering the emerging V2V and V2I communications. Two of them are briefly discussed here: 1) How to analyze and synthesize a platoon in a systematic fashion, considering the nonlinearity of node dynamics, the variety of topologies, and the need for low-cost controllers? Most existing work only considers simplified models and specific topologies for controller synthesis. Communication issues, such as time delays, quantization errors, and packet loss pose a significant challenge to platoon control as well. 2) How to balance different performance metrics in platoon control when considering practical requirements from highway operations? The balance of stability (*e.g.*, internal stability and stability margin) and robustness (*e.g.*, string stability and coherence behavior) for a platoon is attracting research interest. The ultimate objectives of platooning are to enhance highway safety, improve traffic utility, and reduce fuel consumption. How to explicitly take practical performance requirements into account is rather challenging for platoon control.

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