Metrics for the Comparison of Acceleration Time Histories

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Abstract

A decomposition is used to express the mean squared deviation, quantifying the dissimilarities between time histories of input (or response) quantities of multiple replicas of a soil system centrifuge test, as a unique aggregate of three discrepancy measures associated with shape, phase and frequency-shift. The shape measure quantifies the deviations associated with dissimilarities in form and amplitude. The phase measure estimates the deviations associated with differences in phase angle. The frequency-shift measure quantifies the deviations associated with differences in frequency components. These measures are illustrated using simple synthetic motions and used to assess the discrepancies among six replicas of centrifuge input motion achieved at six different facilities. The conducted analysis shows that the proposed decomposition accurately quantifies the different types of discrepancies between time histories.

INTRODUCTION

The Liquefaction Experiments and Analysis Projects (LEAP) is an international effort to produce high quality trusted-experimental data sets and undertake a systematic exercise to validate existing computational models of saturated granular soil response and liquefaction (Manzari, et al., 2014). Such an assessment sheds light on the strengths and shortcomings of these models and also provides valuable insight into the mechanisms of soil liquefaction that would eventually lead to further developments and refinement in soil dynamic response modeling. The availability of adequate experimental data is essential in any validation exercise. In this regard, a centrifuge test of a sloping deposit was lately repeated at six different facilities and the corresponding results were used in a validation exercise (Manzari, et al., 2016).

The repeatability of tests at different centrifuge facilities is aimed at addressing experimental uncertainties and biases, and requires that the input parameters and input motions (for these experiments) be similar to a great extent. Nevertheless, different experimental facilities produce input motions with some dissimilarities due to variability in setup and procedures, along with other uncertainties. It is therefore necessary to assess and quantify the consistency of these motions before comparing the outcome of experiment replicates.

A number of metrics have been used by researchers to assess discrepancies among dynamic time histories (e.g., accelerations), including vector norms, average residual and standard deviation, coefficient of correlation and cross-correlation, Sprague and Geers metric (Geers, 1984), Russell's error measure (Russel, 1997), normalized integral square error, root mean square error and the goodness-of-fit score (Anderson, 2004). Dissimilarities were also assessed using discrepancy slopes and Dynamic Time Warping (Sarin, Kokkolarass, Hulbert, Papalambros, Barbat, & Yang, 2010). Root mean square errors, goodness-of-fit and vector norms do not provide information to differentiate discrepancies due to phase and magnitude of time histories. The average residual and standard deviation also have the same limitation. The discrepancies negate each other when using an average residual. The coefficient of cross-correlation may be used to determine discrepancies associated with dissimilar phase angles. However, correlation parameters do not provide a reasonably accurate measure of discrepancy magnitudes. Sprague and Geers metric (Geers, 1984) can isolate magnitude and phase discrepancies, but does not include information on discrepancies associated with (differences in) shape (shape discrepancy is a measure of dissimilarities between two signals irrespective of phase lag and frequency shift between them, while the magnitude discrepancy refers to differences in signal amplitudes). Also, the normalized integral square error (Donnelly, Morgan, & Eppinger, 1983) does not account for the shape discrepancy. Russell's error measure (Russel, 1997) does not quantify the magnitude discrepancies.

This paper proposes a new approach to identify and quantify the phase, shape and frequency-shift discrepancies among time histories of input or response quantities, such as accelerations, velocities, and displacements during experiments and centrifuge tests. Herein, this approach is used to assess the differences and similarities between input accelerations achieved at the six different centrifuge facilities and the corresponding target motion.

QUANTIFICATION OF DISCREPANCIES

The discrepancy between the dynamic responses of two replicates of a centrifuge test of a soil model subjected to a base excitation can stem from numerous sources. First, the input motions generated by actuators can be different and affected by the experimental setup. Then, the dynamic response of soil deposits is modified further by variability in deposit properties leading to amplification, de-amplification, frequency lengthening, phase difference, etc. Analysis and comparison of the input and response time histories can shed light on the nature, source and significance of discrepancies.

The discrepancy d_{ij} between two time histories, referred to a signal $a_i = a_i(t)$ and $a_j = a_j(t)$ in which t is time, over a time window of length W may be quantified using a normalized mean square deviation (MSD):

$$d_{ij} = \frac{\int_0^W (a_i - a_j)^2 dt}{2(\int_0^W a_i^2 dt + \int_0^W a_j^2 dt)} \tag{1}$$

The measure d_{ij} may be decomposed in terms of three specific fundamental components; namely phase, shape and frequency-shift discrepancies. The phase component d_{ij}^{phase} reflects discrepancies due to difference in signal phase angles. The shape component d_{ij}^{shape} quantifies the discrepancy associated with the geometrical shape (i.e., wave form and amplitude). The frequency shift component

d_{ij}^{Fshift} evaluates the discrepancy dealing with differences in frequency components. Quantitatively, these different discrepancy measures may be evaluated using

$$d_{ij} = d_{ij}^{phase} + d_{ij}^{shape} + d_{ij}^{Fshift} \quad (2)$$

with:

$$d_{ij}^{phase} = \frac{\int_{-\infty}^{+\infty}\left[2|A_i||A_j| - \left(A_i A_j^* + A_i^* A_j\right)\right]df}{2\left(\int_{-\infty}^{+\infty} A_i^2 df + \int_{-\infty}^{+\infty} A_j^2 df\right)} \quad (3)$$

$$d_{ij}^{shape} = \frac{DFW(|A_i|,|A_j|)}{2\left(\int_{-\infty}^{+\infty} A_i^2 df + \int_{-\infty}^{+\infty} A_j^2 df\right)} \quad (4)$$

$$d_{ij}^{Fshift} = \frac{\int_{-\infty}^{+\infty}\left(|A_i| - |A_j|\right)^2 df}{2\left(\int_{-\infty}^{+\infty} A_i^2 df + \int_{-\infty}^{+\infty} A_j^2 df\right)} - d_{ij}^{shape} \quad (5)$$

In which A_i and A_j are the Fourier transforms of a_i and a_j respectively and A_i^* refers to the complex conjugate of A_i. DFW refers to Dynamic Frequency Warping. DFW is similar to the dynamic time warping (DTW) used in speech recognition (Rabiner & Huang, 1993). Briefly, the DFW computes the optimal (minimum) distance (i.e., dissimilarity) between two time histories by employing a non-linear frequency mapping between the two Fourier amplitude spectra of the histories. The use of DFW enables the isolation of the magnitude discrepancies associated with (slight) shifts in signal frequencies. The discrepancies defined above are normalized so that they vary between 0 and 1. A discrepancy metric of zero means that the two signals are essentially the same whereas a discrepancy metric of 1 refers to two signals that are 180 degrees out of phase with each other. The relative values of the different metrics d_{ij}^{phase}, d_{ij}^{shape} and d_{ij}^{Fshift} can be used an indicator to ascertain the discrepancy that prevails.

VERIFCATION

The proposed technique was first applied to synthetic time histories that have specific discrepancies associated with shape, phase, and frequency-shift. A target or base time history, $T = T(t)$, was first selected:

$$T = (1 - e^{-3t})(sin(2\pi t) + sin(6\pi t))e^{-0.1t} \quad (6)$$

This signal may represent, for instance, part of an input motion for a certain centrifuge test. Signals a_1 to a_4 were obtained by modifying $\boldsymbol{T}$ (Fig. 1); a_1 has only shape (amplitude) discrepancy (with respect to $\boldsymbol{T}$) including two spikes, a_2 corresponds to the original signal corrupted by (high frequency) noise with an amplitude of about 20 % (of that of a_1), a_3 has phase discrepancy, and a_4 has frequency difference with respect to $\boldsymbol{T}$:

$$a_1 = 1.5 * a1 + e^{-10|t-4.1|} + e^{-12|t-6.1|} + e^{-13|t-10.1|} \quad (7)$$

$$a_2 = a_1 + rand(t) \quad (8)$$

$$a_3 = (1 - e^{-3t})\big(\sin\big(2\pi(t - 0.1)\big) + \sin\big(6\pi(t - 0.1)\big)\big)e^{-0.1t} \quad (9)$$

$$a_4 = (1 - e^{-3t})(\sin(2.2\pi t) + \sin(6.3\pi t))e^{-0.1t} \quad (10)$$

in which $rand(t)$ is random noise which have an amplitude in the range -0.2 to 0.2.

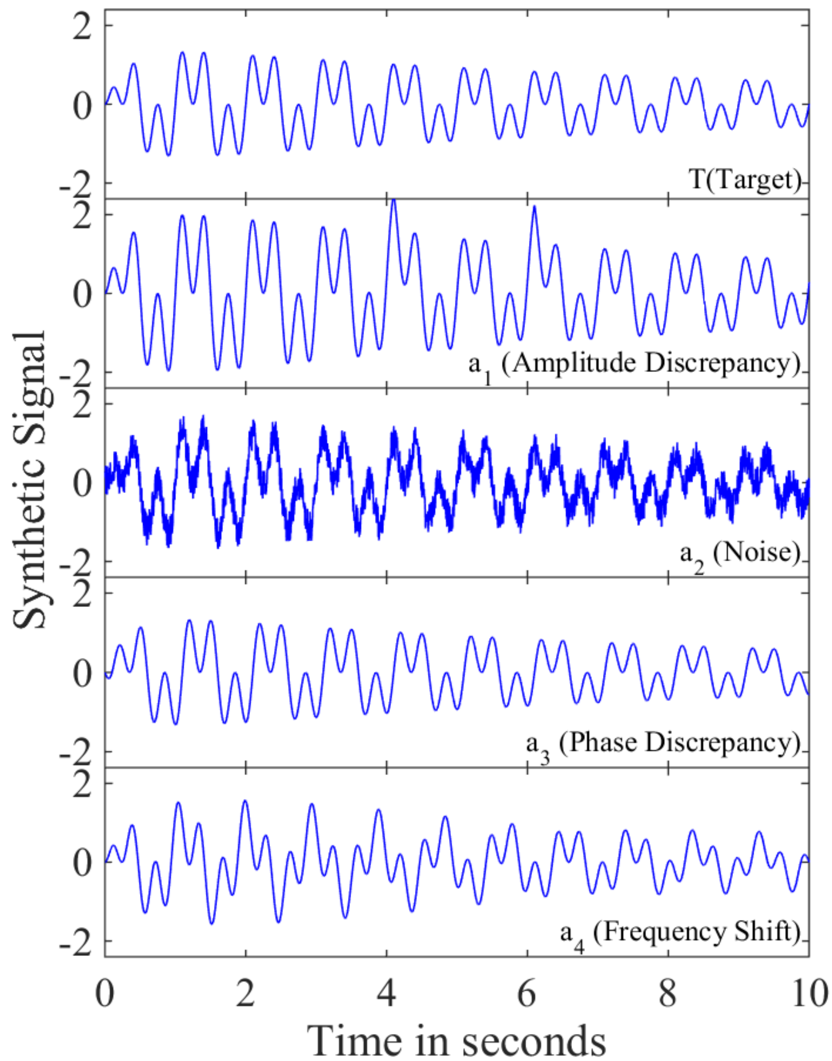

Figure 1 Synthetic Motions used in the Verification Analysis.

Analysis of Discrepancy. The discrepancies between each pair of the synthetic time histories, $\boldsymbol{T}$ and $\boldsymbol{a_1}$ to $\boldsymbol{a_4}$, were quantified using the procedure described above and are presented in Fig. 2 using three dimensional bar graphs. In this figure, the vertical axis represents the measures of discrepancy and is color-coded according to (discrepancy) magnitude. Half of the discrepancy pairs are shown in Fig. 2 in view of the discrepancy measure symmetry (i.e., discrepancy between $\boldsymbol{a_i}$ and $\boldsymbol{a_j}$ equals the one between $\boldsymbol{a_j}$ and $\boldsymbol{a_i}$).

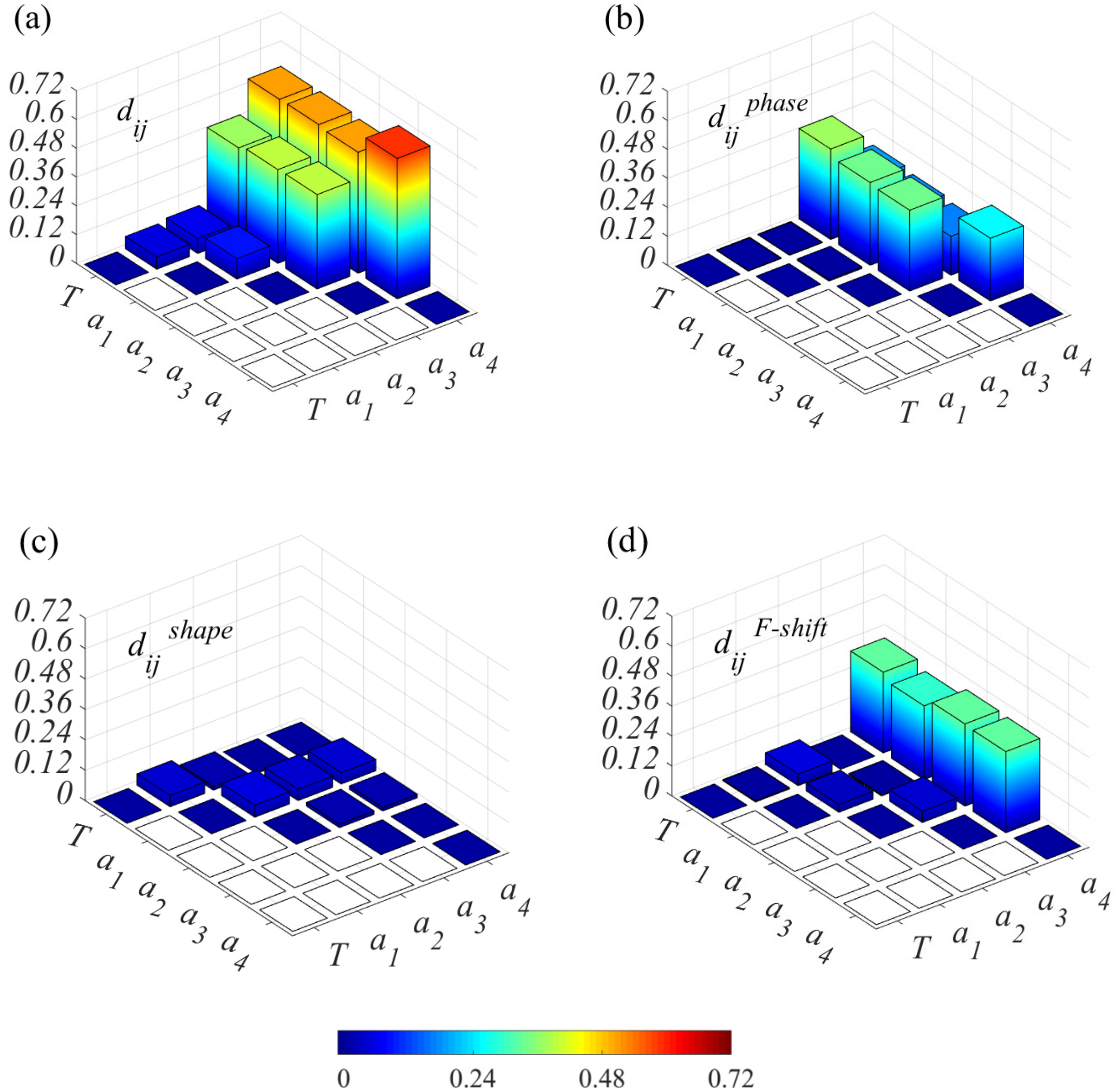

Figure 2 Discrepancy Measures of Analyzed Synthetic Time Histories

The discrepancies between the target and other synthetic signals had the largest values for the cases of phase and frequency-shift ($T - a_3$ and $T - a_4$, Fig. 2a). A qualitative visual assessment of the synthetic motion time histories (Fig. 1) confirms the result for a_4. The differences in shape (signal amplitude) between T and a_1 and the noise in a_2 led to significantly lower values of the discrepancy measure (Fig. 2a). The total discrepancy was effectively decomposed in terms of phase, shape and frequency-shift components, as shown in Figs. 2b, c and d respectively. The obtained results were in full agreement with the characteristics of the used synthetic signals (as defined by Eqs. 7,8, 9 and 10). For instance, a_1 has only a shape discrepancy with the target signal T ($d_{Ta_1} = d_{Ta_1}^{shape}$). The signal a_2 appears visually (Fig. 1) to have a shape discrepancy with T. In fact, the noise introduces only a frequency-shift discrepancy (Fig. 2d), in agreement with the added higher frequency noise (Eq. 8). The signal a_3 has a phase discrepancy with respect to T leading to

$d_{Ta_3} = d_{Ta_3}^{phase}$. The shifting of frequencies in $a4$, leads to a dominant $d_{Ta_4}^{Fshift}$, but also a significant $d_{Ta_4}^{phase}$ component. Investigation of this component revealed that this is associated with the combined effects of frequency shifting and the ramp-up and ramp-down of the signals.

Overall, the evaluated discrepancies provided quantitative measures indicating that a_1 and a_2 are quite similar to T, while a_3 and a_4 are significantly different from T and each other.

CASE STUDY

An analysis was conducted to assess the discrepancies associated with the input motions that were recorded at six different experimental LEAP centrifuge facilities. These motions, termed F_1 to F_6, were aimed at achieving the same target motion T shown in Fig. 3. Assessment of the similarities and differences in achieved motions is fundamental to address the LEAP objectives (Manzari, et al., 2016). The discrepancy measures described above provide tools that were used to help address this repeatability issue.

Analysis of Discrepancy. A cross correlation analysis was first used to ensure a zero global phase lag between the input motions of the different facilities with the target motions. Broadly, the six input motions had different levels of similarities and differences. A qualitative comparison of these motions (Fig. 3) indicates that $\boldsymbol{F_1}$, $\boldsymbol{F_2}$ and $\boldsymbol{F_3}$ were closer to the target than other three motions. The computed total discrepancies $\boldsymbol{d_{ij}}$ (Fig. 4) provided quantitative measures with numerical values varying from 0.02 to 0.05 (for $\boldsymbol{F_1}$, $\boldsymbol{F_2}$ and $\boldsymbol{F_3}$) that are consistent with the basic qualitative assessment. Overall, $\boldsymbol{F_4}, \boldsymbol{F_5}$ and $\boldsymbol{F_6}$ had noticeably higher discrepancy than $\boldsymbol{F_1}, \boldsymbol{F_2}$ and $\boldsymbol{F_3}$ with values of $\boldsymbol{d_{ij}}$ exceeding 0.2. The total discrepancy was decomposed into phase, shape and frequency shift components to assess the nature and reasons of the associated differences.

The computed d_{ij}^{phase} between the target motion and F_1 to F_6 motions were vanishingly small in view of the synchronization mentioned above. The signal F_6 had a significant frequency-shift discrepancy, d_{ij}^{Fshift}, with respect to the target. This discrepancy was also noticeable for F_5. Nevertheless, the shape term d_{ij}^{shape} had the largest contribution to the overall measure of discrepancy with respect to the target for all analyzed input motions. The largest d_{ij}^{shape} was for F_5 followed by F_6 and F_4 which were marked by the presence of an extraneous frequency component and also a significant difference in amplitude for the 1 Hz dominant frequency of the target motion. F_1 had vanishingly small (phase, shape and frequency shift) discrepancies and may be considered as the closest to the target motion. F_5 and F_6 had large discrepancies with the other input motions with the largest discrepancy being for $F_4 - F_6$ and $F_2 - F_6$. These discrepancies were associated with significant contributions from the phase, shape and frequency shift components. In contrast, F_1, F_2 and F_3 had small discrepancies in phase and frequency-shift with most of the contribution associated with shape. F_2 has a high phase discrepancy when compared to F_3, F_4, F_5 and F_6.

Work is currently under way to assess the sensitivity of soil response to the different discrepancies. The outcome of this assessment will then be employed to develop criteria to quantify the level of repeatability of time histories.

CONCLUSIONS

This article presented a new approach to assess the discrepancies among time histories of input and response quantities of soil systems. The mean squared deviation of two specific signals is decomposed in terms of phase, shape and frequency components. The new discrepancy quantification tools were verified using simple synthetic signals with prescribed discrepancies. These tools were also employed to assess and quantify the discrepancies among the input motions of six different LEAP replicates of the same centrifuge test. The conducted assessment showed that three of the analyzed motions were close and had low values of the three discrepancy measures. In contrast, the three other ones were marked by large discrepancy values associated mainly with significant differences in shape along with sizeable phase and frequency discrepancies. Additional work is under way to develop repeatability criteria of input and response quantities of centrifuge test employing the developed discrepancy measures.

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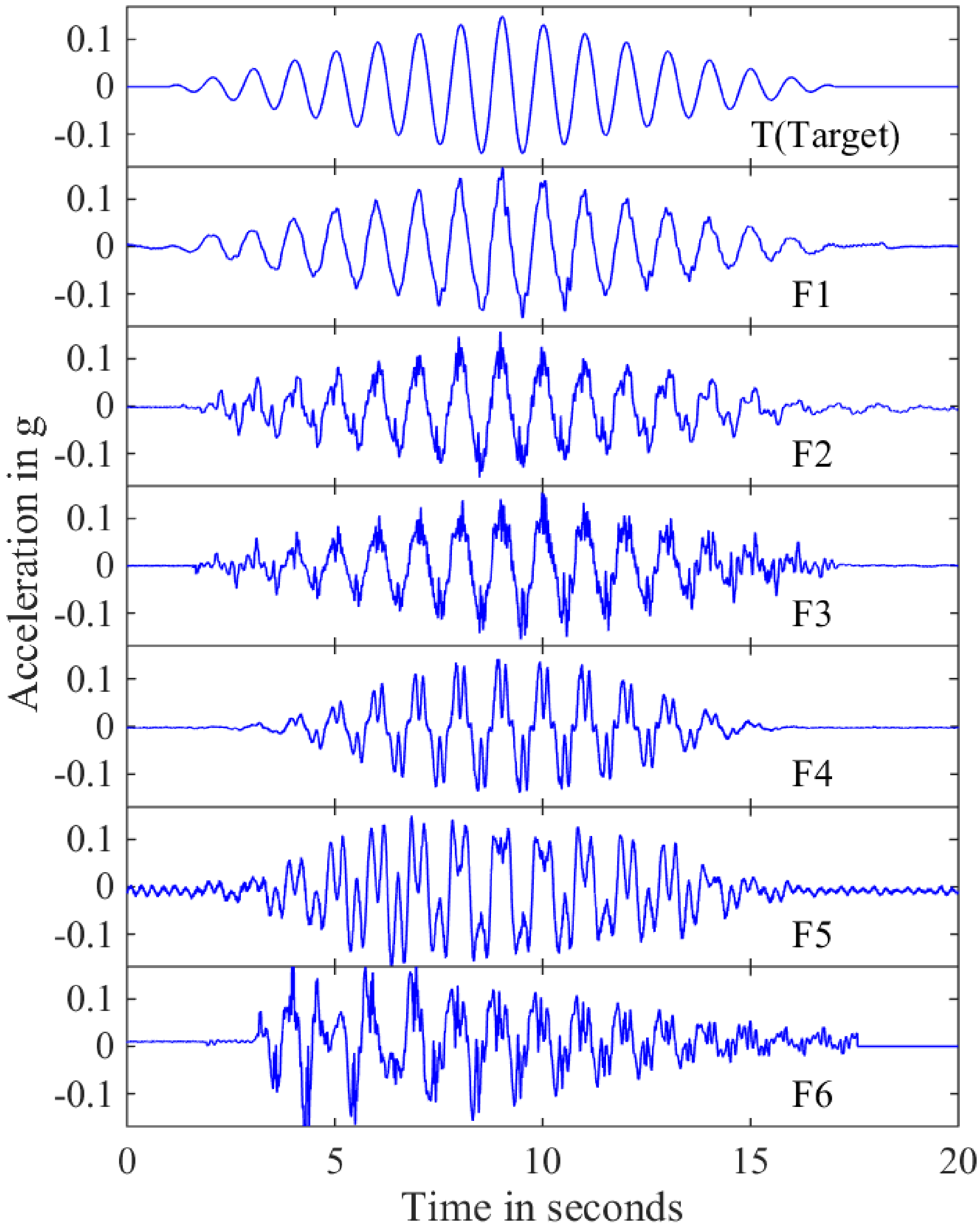

Figure 3 Target and Input Motions of Six Centrifuge Tests

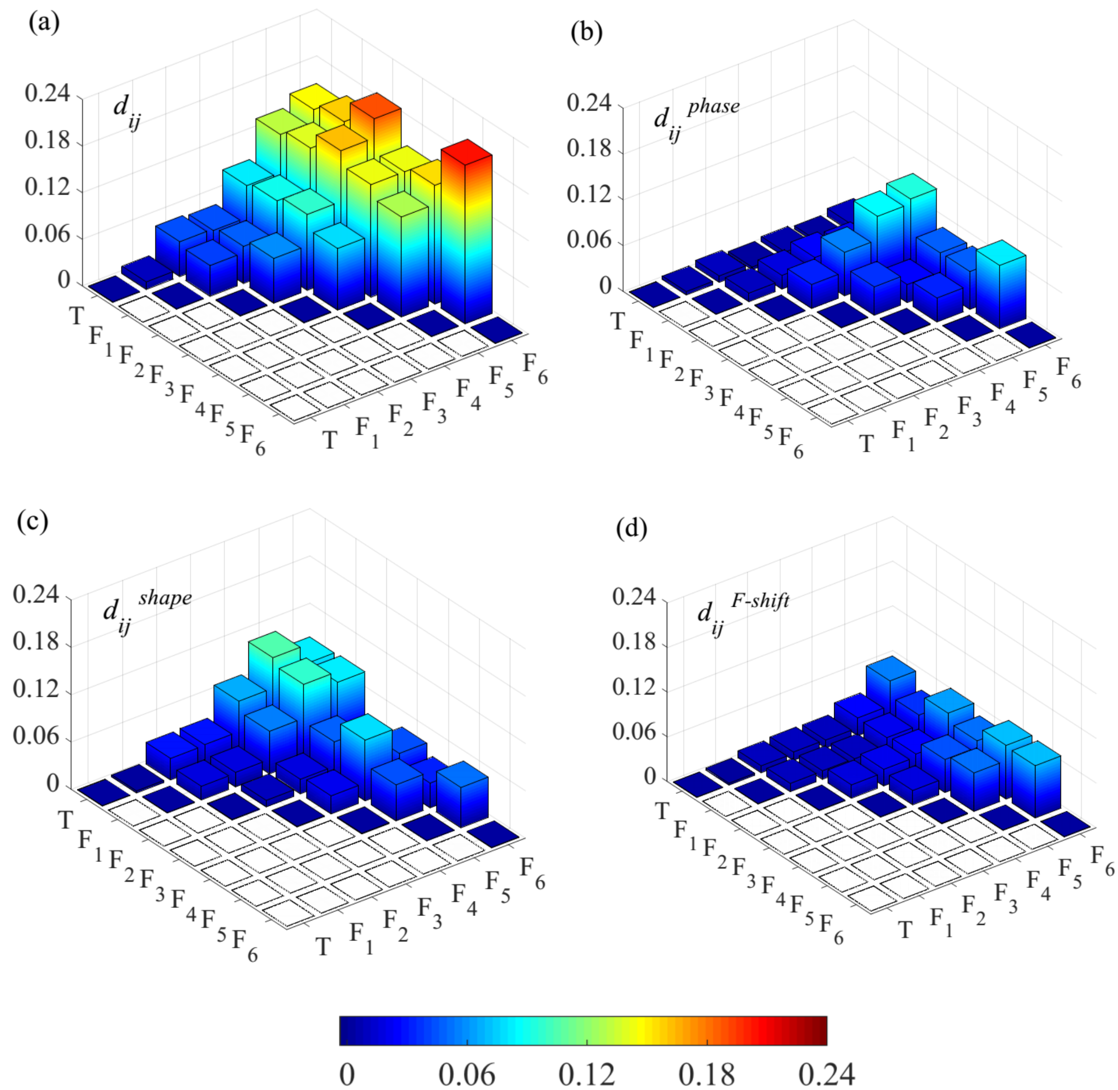

Figure 4 Discrepancies of Analyzed Centrifuge Input Motions

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