Comparison of Liquefaction Constitutive Models for a Hypothetical Sand

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Abstract

This paper presents numerical liquefaction simulations of a hypothetical sand in cyclic direct simple shear tests for four different constitutive models. The four models are PM4Sand in FLAC, UBCSand in FLAC, Pressure Dependent Multi Yield 02 (PDMY02) in OpenSees, and the Manzari-Dafalias04 model in OpenSees. Parameters published by others for various sands were used for this work to avoid possible introduction of our bias into the calibration process. The material properties were determined by others for different uniformly graded sands, all with a relative density of approximately 50%. The simulations do not pertain to one sand or one set of laboratory data, so therefore, there is no right or wrong answer. Instead, the goal of this paper is compare a consistent set of results that show the implementations of each model behave as expected, and to illustrate basic differences in behavior of the different models. Cyclic strength curves (cyclic stress as a function of number of cycles) illustrate the behavior of the models over a range of cyclic stresses. Each model displays pore pressure build up, softening, and cyclic contraction-dilation cycles associated with cyclic mobility. Two of the models soften to a point, but then stabilize in a repeated hysteresis loop with no additional growth in the cyclic strain amplitude after some number of cycles.

INTRODUCTION

With the increasing use of numerical analysis methods for solving geomechanical problems, a verification of the constitutive models used would be very valuable. According to Oberkampf et al. (2004) verification is the process of showing that the implementation of a model accurately represents the underlying mathematical model and its solution. However, since numerical solutions are discrete representations of exact mathematical models, the verification decision depends on the required accuracy of the physical system modeled. A clear understanding of the physics that the developer intends to model is therefore needed before passing judgement on accuracy. Kutter et al. (2014) makes the case that verification should provide evidence that the model is being solved correctly, and that the evidence is able to be evaluated by decision makers who, for example, decide what software should be purchased to analyze a particular problem. Therefore, the evidence describing the behavior of the implemented models must be in a format understood by engineers who are not experts in numerical analysis. Ultimately, the decision maker on an engineering project must be able to judge that a numerical solution accurately predicts the important physics of the problem considered.

Positive verification of a numerical solution requires an accurate set of solutions that fully assesses the range of intended uses of the developer's conceptual model. However, due to the severe nonlinearity of the liquefaction mechanisms, a rigorous exact solution is not available; this precludes complete positive verification at present. However, one required step in verification process is to illustrate the behavior of the implemented model so that it "could be" compared to the developer's conceptual model. This paper illustrates and compares the behavior of four implemented liquefaction models. In the absence of rigorous solutions of the developer's model for comparison purposes, at present it would be necessary for the developers to tell us that the implementations match their concept. On the other hand, the types of simulations performed here can be sufficient for negative verification – to show that a particular implementation is not correct.

The selected constitutive models, PM4Sand, Manzari-Dafalias04, UBCSand, and Pressure Dependent Multi Yield 02 (PDMY02), models are currently used for liquefaction analysis by practitioners and in research. PM4Sand and UBCSand are implemented in FLAC (Fast Lagrangian Analysis of Continua) version 7 (Itasca 2011), whereas Manzari-Dafalias04 and PDMY02 are implemented in OpenSees (Open Systems for Earthquake Engineering Simulation) version 2.50 (Rev 6248) (McKenna et al. 2009).

The comparison of the models is performed for plane-strain, cyclic, direct simple shear (DSS) idealized test path simulations. The cyclic resistance ratio (CRR), which is the ratio of shear to normal stress a sand can resist before liquefaction is initiated, will be compared for the different initial conditions for the models over a range of cyclic shear stress magnitudes. It is not the intent of this work to speculate which model correctly predicts soil behavior during undrained cyclic loading, but illustrate model behavioral trends to reasonably expected loading conditions.

To illustrate the quality and the achieved response of the simulations, a set of figures for each model demonstrating the effective stress, shear stress, and shear strain evolution from a typical simulation is presented. This combination of figures was selected to be consistent with the types of data typically produced in physical DSS experiments.

CONSTITUTIVE AND NUMERICAL MODELS

A brief description of the selected models is provided below with a reference to more complete descriptions.

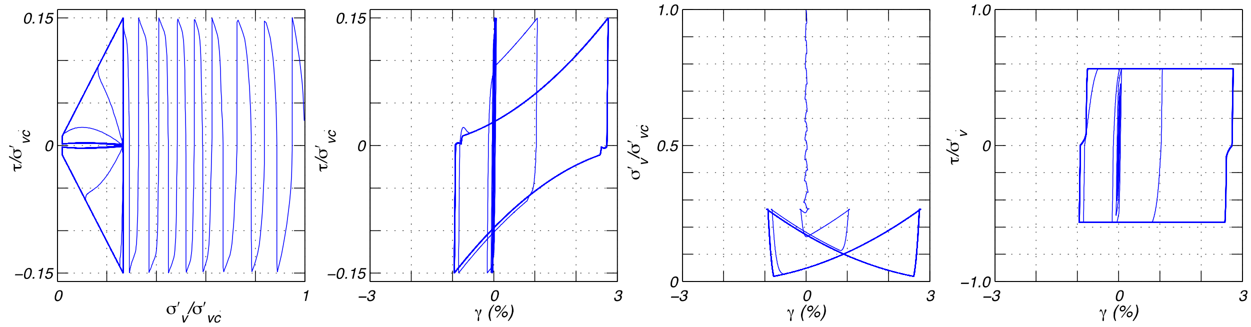
- PDMY02: Pressure Dependent Multi Yield 02 is an elastic-plastic material model for simulating the pressure sensitive response of granular material during general loading, described by Yang et al. (2003). The PDMY02 material model uses a series of nested yield surfaces to model cyclic loading. It is not formulated from critical state soil mechanics framework, so different parameters are required for different initial void ratio states. The DSS driver uses a single SSPQuad element with strain controlled loading.
- UBCSAND Version 904aR: UBCSAND is an elastic-plastic material model developed for sand-like granular materials that have the potential to liquefy during cyclic loading (Beaty and Byrne 2011). The yield surface follows a Mohr-Coulomb formulation that has the ability to kinematically and isotropically harden through a hyperbolic relationship. The implementation of UBCSAND in FLAC is described by (Beaty and Byrne 2011). The DSS driver is a single quadrilateral zone, analogous to elements in FEM, with strain controlled loading.
- Manzari-Dafalias04: This model is a generalized stress state, elastic-plastic material model developed from critical state and stress-ratio controlled framework (Dafalias and Manzari 2004). Additionally, the model includes a fabric-dilatancy tensor that accounts for enhanced contraction upon stress reversal associated with fabric changes during dilative behavior of the material. The model was implemented in OpenSees by A. Ghofrani and P. Ardunio in 2013. The simulations presented in this paper were performed using a gauss point driver provided by Arduino (2016). With the gauss point driver, the six states of stress or strain can be controlled independently. Cyclic loading was applied to the model with a stress controlled sine wave.
- PM4Sand: PM4Sand was formulated with the critical state and stress-ratio controlled framework developed by Dafalias and Manzari (2004), but was modified to better model stress-dilatancy relationships, empirical modulus reduction curves, and the characterization of fabric development during liquefaction (Boulanger and Ziotopoulou 2015). PM4Sand can only simulate a 2-D state of stress and hence the model is not intended nor able to model applications requiring a 3-D state of stress. The DSS driver is a single quadrilateral zone, analogous to elements in FEM, with strain controlled loading.

SOIL PROPERTIES, SIMULATION PARAMETERS AND TYPICAL TEST PATHS

The simulations presented in this paper model different sands with a relative density of about 50%. The input parameters at 50% relative density were chosen from suggested parameters by provided by the model developers (i.e. Yang et al. 2003, Beaty and Byrne 2011, Dafalias and Manzari 2004, and Boulanger and Ziotopoulou 2015). Using parameters recommended by the developers was thought to most likely illustrate the intended behaviors of the developers.

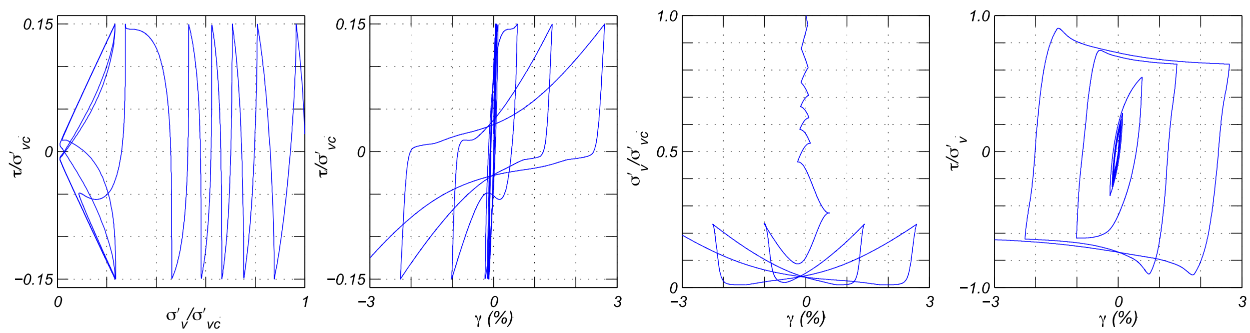
The DSS testing plan for the selected models consisted of applying an initial vertical effective stress of 100 kPa and then followed by a cyclic stress. The cyclic stress ratios (CSR's) ranged from 0.05 to 0.55 in increments of 0.05 for 11 simulations per model.

Following each simulation, the data was processed and figures generated to qualitatively check that the computed response looked suitably smooth and reasonable. Figures 1 through 4 show example of these simulations for each model with a cyclic stress ratio (CSR) of 0.15.



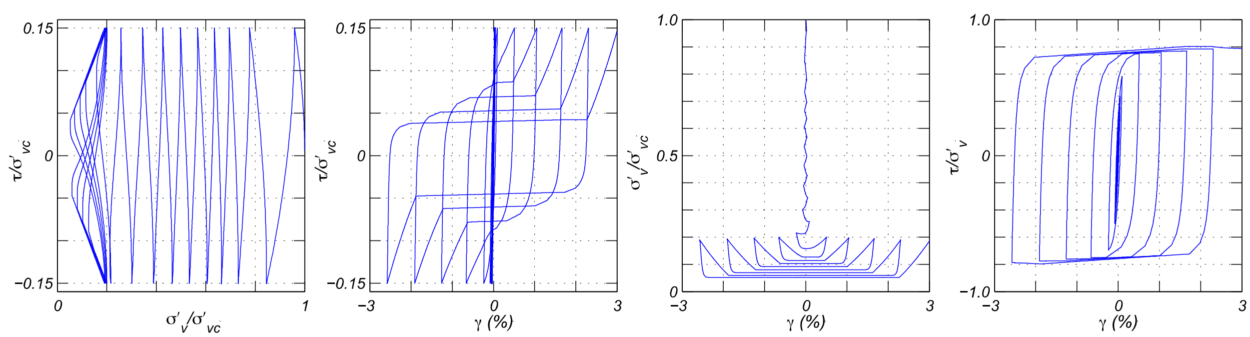
a. b. c. d.

Figure 1: UBCSAND stress path for undrained DSS loading with σ'_{vc}=100 kPa, CSR= 0.15



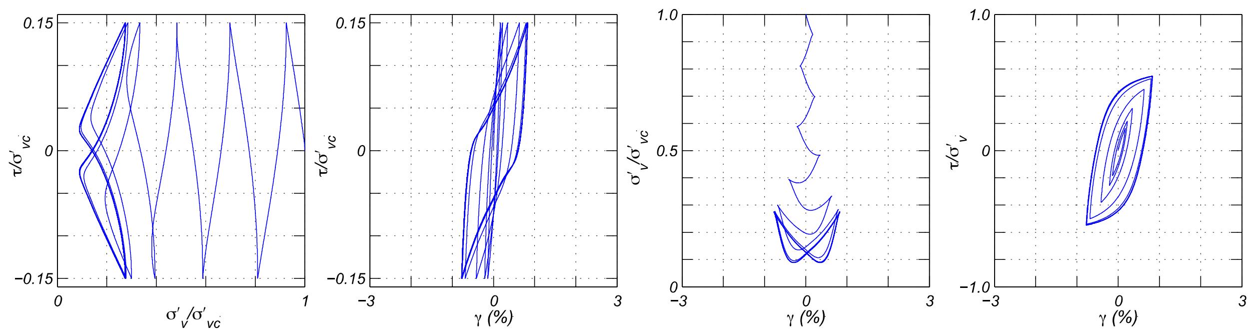
a. b. c. d.

Figure 2: PM4Sand stress path for undrained DSS loading with σ'_{vc}=100 kPa, CSR= 0.15



a. b. c. d.

Figure 3: PDMY02 stress path for undrained DSS loading with σ'_{vc}=100 kPa, CSR= 0.15

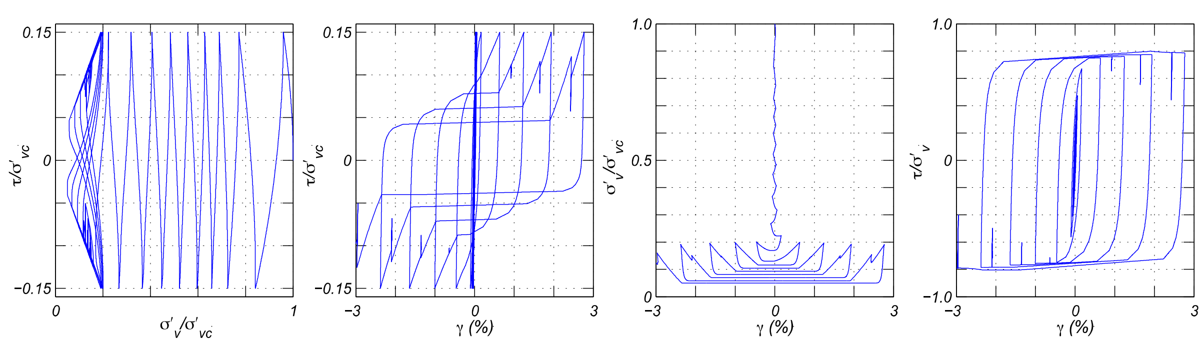


a. **b.** **c.** **d.**

Figure 4: Manzari-Dafalias04 stress path for undrained DSS loading with σ'$_{vc}$=100 kPa, CSR= 0.15

Figure 5 shows an example of a simulation using PDMY02 with a stress-controlled DSS driver, in contrast to the strain-controlled driver used to produce Figure 3. The spikes in the response apparent in Figure 5b and 5d show that the stress-controlled solution scheme did not accurately solve the constitutive equations for this particular numerical solution scheme in OpenSees. As the shear stress approaches the maximum stress ratio (τ/σ'_v) in Fig. 5d, a spike in the constitutive response due to over prediction and a numerical correction. While a positive verification technically requires known accurate solutions, a negative verification judgement can be made by showing evidence of numerical errors that are obvious in Figure 5. The stress-controlled solution scheme used in OpenSees that produced the results in Figure 5 was negatively verified for the present problem.

For Figures. 1-3, the DSS driver was run under displacement or strain control and these figures show a smooth response consistent with the equations developed and/or the response envisioned by the developers of the constitutive models.



a. **b.** **c.** **d.**

Figure 5: PDMY02 stress path for undrained DSS loading with σ'$_{vc}$=100 kPa, CSR= 0.15

The simulations performed to produce Figures 1 through 4 were terminated when the shear strain reached 3% or after 40 loading cycles, whichever came first. It has been previously shown that UBCSand and Manzari-Dafalias04 can reach a stabilized hysteretic behavior during undrained cyclic loading without continued decrease in effective stress or growth in the amplitude of the cyclic shear strains with added cycles of loading (Dafalias and Manzari 2004 & Beaty and Byrne 2011). A stabilized hysteretic behavior occurred for both UBCSand (Figure 1) and Manzari-Dafalias04 (Figure 4). For these two models, many of the 40 loading cycles applied are indistinguishable since they fall precisely on the previous cycle. It is also interesting to note that PDMY02 and Manzari-Dafalias04 (Figs 3c and 4c) predict a minimum effective stress of 5% or 10% of the initial effective stress; they do not approach an effective stress very close to zero.

PM4Sand initially has a stiff response during loading. In Figures 2b and 2c very little shear strain accumulates until the vertical effective stress is reduced to 25% of the original 100 kPa initial effective stress. Once the effective stress was reduced, an additional 2 ½ cycles caused the model to reach 3% shear strain. Although PM4Sand builds upon the framework established by the Manzari-Dafalias04 model, modifications introduced by Boulanger and Ziotopoulou (2015) allow the strains to continue to grow without stabilization in a repetitive stress-strain loop. In Figure 2a, PM4Sand nearly produces repetitive ‘butterfly loops" in the path of the shear stress and effective stress.

PDMY02, similar to PM4Sand has a very stiff initial response during loading. In the last 4 cycles of the 13.25 cycles it took to accumulate 3% shear strain, 97% of the shear deformation occurred (Figure 3b). During the unloading to reloading phase, the shear stress remains constant as it passes through zero shear strain (Figure 3b) and once the strain of the current cycle exceeds the strain of the previous cyclic, the material stiffens and the shear stress increases linearly. Subsequently, the amplitude of the shear strain increases by an almost equal increment per cycle.

Figures 1d, 2d, 3d, and 4d present the results of the simulations in an atypical but illuminating plot, with the effective stress ratio (shear stress normalized by the current vertical effective stress, σ'_v) instead of the initial effective stress, σ'_{vc} as shown in Figures 1b, 2b, 3b, and 4b. PM4Sand shows a reduction of the effective stress ratio as strain increases, but all of the other models show continuously increasing effective stress ratio as the shear strain increases.

CYCLIC RESITANCE RATIO (CRR) CURVES

The number of cycles at each CSR to reach 3% shear strain was recorded and presented as cyclic resistance ratio (CRR) curves in Figure 6. The Manzari-Dafalias04 model is not shown in Figure 6 because it did not reach 3% shear strain at any of the CSRs tested for the selected constitutive model parameters; the stabilized hysteresis loop had a shear strain amplitude less than 3% (e.g., see Fig 4b). UBCSand reaches 3% shear strain only for CSRs larger than 0.35, below this CSR, the hysteresis loops of the model stabilize; hence the simulated CRR curve for UBCSand becomes a horizontal line at CSR = 0.35 in Fig. 6. The CRR of PDMY02 is greater than PM4Sand at CSRs greater than 0.1. Although stiffer than PM4Sand, the PDMY02 material still achieves the typical curve with increasing number of cycles to accumulate 3% shear strain with reducing CSRs. PM4Sand shows a weaker cyclic strength at high CSRs than PDMY02, but stiffer at lower CSRs. PM4Sand follows a reasonably smooth CRR curve. The shapes of each curve is, of course, dependent on the constitutive model parameters chosen for the sand. An additional set of CRR curves is provided in Figure 7 to show the number of cycles to reach 1% shear strain.

Note in Figs. 6 and 7 that a fractional number of cycles plotted is counted by determining the shear stress in relation to the shear stress limits (±CSR). For example, if the sample reaches 3% strain at the peak stress in the first quarter of the 11th cycle, the number of cycles would be plotted at 10.25 cycles. As another example, if during the first loading portion of the first loading cycle, the sample reaches 3% strain at half of the peak shear stress, then the number of cycles to 3% strain is taken as 0.125 cycles. In Figure 7, at high shear stress ratios, many of the simulations reached 1% shear strain near 0.25 or 0.75 cycles (near the peak of the first loading and the first reverse loading cycle). For CSRs lower then 0.25, the Manzari-Dafalias04 model reaches a steady-state, consistent with Figure 4. UBCSand predicts roughly 9.5 cycles to cause 1% shear strain from CSRs of 0.1 to 0.3. Similar trends can be observed for the PDMY02 and PM4Sand models for the 1% threshold shear stain as the 3% threshold.

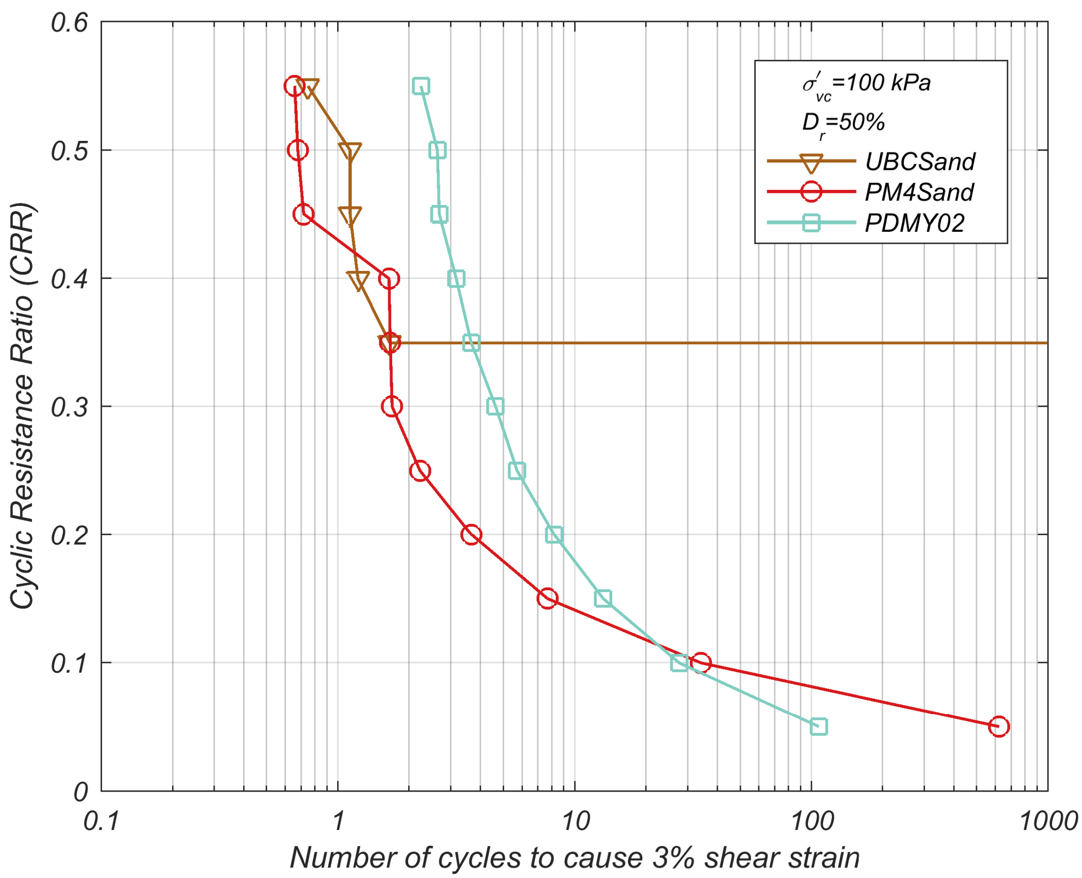


Figure 6: Cyclic resistance ratio as a function of the number of cycles to reach 3% shear strain.

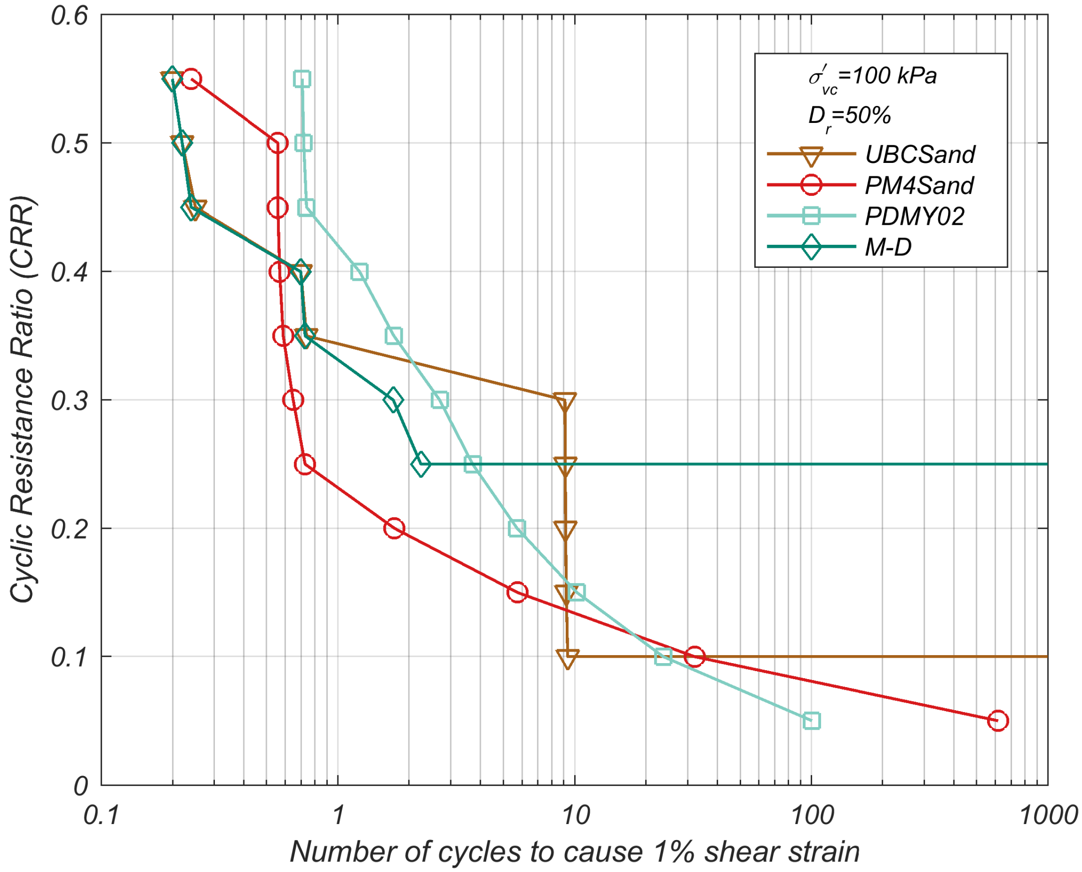


Figure 7: Cyclic resistance ratio as a function of the number of cycles to reach 1% shear strain.

DATABASE

The simulations herein will be included with a larger data set that considers three initial relative densities, different initial confinement stresses, and cyclic loads superimposed on static shear stresses. The suite of analysis will be included in a NHERI database for future comparison among the models, or models not considered for this work. The OpenSees and FLAC DSS driver files will be also included with the database. Including the drivers will serve dual purpose in; 1) it will allow users of the numerical frameworks to quickly use the driver to compare their simulations with similar simulations in the database, 2) it will allow for a transparent review of the numerics used for the DSS drivers.

CONCLUSIONS

According to Oberkampf et al. (2004) verification is the process of showing that a numerical solution accurately solves a set of equations. However, the acceptability of inevitable errors depends on the required accuracy of the numerical solution. A clear understanding and demonstration (e.g., graph) of the physics for the underlying constitutive model enhances our ability to qualitatively choose the best constitutive model for the considered application and help users of the model to understand the expected accuracy of the solution. In the absence of rigorous exact solutions of complex nonlinear liquefaction models, this paper cannot positively verify the numerical implementations used. However, it was possible, to "negatively verify" some implementations; that is, it is possible to show that some implementations of the models do not work well.

Four constitutive models used for liquefaction analysis in current research and/or practice were implemented and unique aspects of the models were shown herein. Of the models, UBCSand, and PM4Sand were implemented in FLAC and PDMY02, and Manzari-Dafalias04 implemented in OpenSees. The input parameters selected for the models are for different sands with a 50% relative density and were chosen from recommendations of the developers. In some cases, however, (e.g., Manzari-Dafalias04), the parameters were obtained from triaxial simulations, and hence our application for use in DSS simulations may require recalibration of the material properties. Nevertheless, the simulations presented clearly illustrate some of the nuances of each model. The UBCSand and Manzari-Dafalias04 models showed stabilization of hysteresis loops in some cases where shear strain amplitudes remain constant in successive cycles. The cyclic resistance of the models was compared on a CRR curve to 1% and 3% shear strains. Some models produced smooth CRR curves while others produced curves with kinks. To investigate fully if the models have undiscovered nuances, a testing plan that considers different generic soil densities, static shear stresses, and low/high vertical consolidation stresses will be considered in future work. This suite of analysis will help decision makers for engineering projects better understand which models are appropriate for which applications.

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