Optimum UAV Positioning for Better Coverage-Connectivity Tradeoff

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***Abstract*—Unmanned aerial vehicle (UAV) plays prominent role in enhancing backhaul connectivity and providing extended coverage areas due to its mobility and flexible deployment. To realize these objectives *simultaneously*, we present a new framework for positioning the UAV to maximize the small-cells backhaul network connectivity characterized by its Fiedler value, the second smallest eigenvalue of the Laplacian matrix representing the network graph, while maintaining particular signal-to-noise ratio constraint for each individual user equipment. Moreover, we show that the localization problem can be approximated by a low complexity convex semi-definite programming optimization problem. Finally, our extensive simulations verify the approximation validity and demonstrate the potential gain of UAV deployment.**

***Index terms*—** connectivity, coverage, 3-D positioning, semi-definite programming, unmanned aerial vehicle.

I. Introduction

Deploying unmanned aerial vehicles (UAVs) has recently gained an increasing interest in both the academic and the industrial circles to enhance wireless services. In emergency network breakdown, they are efficient and reliable alternatives to restore the backhaul network connectivity among small-cells (SCs) due to their mobility. Moreover, they can be used as aerial base stations to extend the coverage areas and boost quality of service (QoS) at user equipment (UEs).

Scanning the open literature, optimizing the UAV's position to enhance the *coverage* has been widely considered. In [1], the authors derive a closed-form expression for the UAV position to maximize the coverage radius in the presence of Rician fading channel. Moreover, UAV positioning for cellular system is discussed in [2], [3]. The authors of [2] investigate the network coverage in terms of outage probability, which is used to define the coverage, while in [3] signal-to-interference-plus-noise ratio (SINR) is used to present the coverage.

On the other hand, positioning the UAV to restore and enhance the network *connectivity* has also been studied aiming to avoid isolated nodes scenario and reduce networks congestion. In [4], the authors deliver a broadband wireless connectivity for destructed communication infrastructure during temporary events at hotspot areas or after disasters. The random graph theory concepts are used in [5] to illustrate the connectivity of the sensor network under Rayleigh fading channels.

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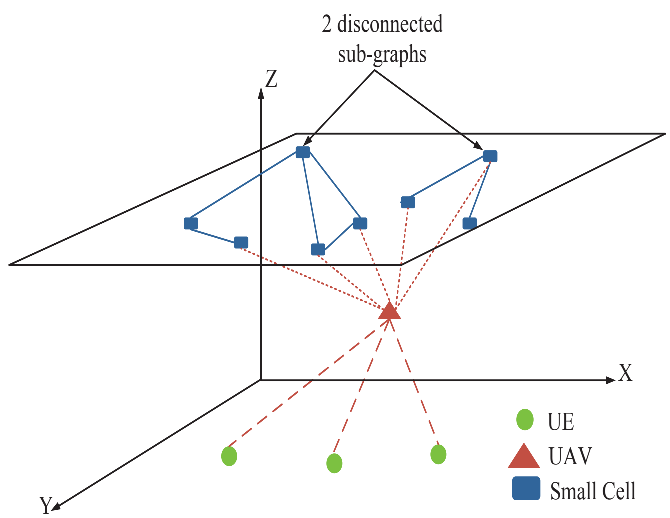

Fig. 1. System model.

Moreover, Fiedler value is a widely used metric to accurately quantify the network connectivity and it is defined as the second smallest eigenvalue of the graph Laplacian matrix. The network algebraic connectivity is addressed in [6] for network maintenance and repair purposes in wireless sensor network. In [7], a coverage-based and a connectivity-based mobility models are introduced for the same set of nodes toward a UAV network monitoring followed by a comparison between both models to clarify the tradeoff between achievable area coverage for the connectivity-based model and achievable connectivity for the coverage-based model. The models in [7] aim at enhancing the network coverage or its connectivity separately, however in this work we propose an inter-layer model in which we consider enhancing coverage for some nodes, UEs, and enhancing connectivity for other nodes, SCs.

None of the aforementioned works has optimized the UAV position to *jointly* extend the network coverage and enhance the backhaul connectivity. To the best of our knowledge, this work is the first to address the tradeoff between coverage and connectivity due to the UAV positioning. Our main contributions in this paper are summarized as follows. We jointly enhance the SCs network connectivity, while maintaining UEs QoS greater than particular threshold. In particular, we constrain the UEs SNR levels to be greater than particular threshold. To this end, we formulate this problem as a non-convex optimization problem and we accurately approximate it as convex semi-definite programming (SDP) problem. Finally, we quantify the accuracy of the presented approximation through numerical experiments.

The rest of the paper is organized as follows. First, the system model is described in Section II. The optimization problem and its SDP approximation are presented in Section III. Finally, numerical results and concluding remarks are provided in Sections IV and V, respectively.

Notation: Lower- and upper-case bold letters denote vectors and matrices, respectively, also $\mathbf{I}_M$ denotes the identity matrix of size M. The operations $(\cdot)^T$, $\mathbb{E}[\cdot]$, and $|\cdot|$ denote the transpose, statistical expectation and absolute value, respectively. The $\mathbf{A} \preceq \mathbf{B}$ denotes that $\mathbf{B}-\mathbf{A}$ is a positive semi-definite matrix. Finally, $\otimes$ denotes the Kronecker product operation.

II. System Model

We consider a SC network consisting of M SCs and N UEs suffering from severe path loss with no links to any SC as shown in Fig. 1. For connectivity among SCs, we use a simple, yet accurate, and widely used disc model [3], [7] with its center at particular SC and all other small SCs inside this disc are assumed to be connected with the center SC. The disc radius, denoted by R_{SC} which is a predetermined parameter that depends on the SCs transmit power.

We model SCs' connectivity using an undirected weighted simple finite graph $\mathcal{G}(\mathbf{V},\mathbf{E})$ where $\mathbf{V}=\{v_1,\ \ldots,\ v_M\}$ is the vertices set of the M SCs and $\mathbf{E}=\{e_1,\ \ldots,\ e_K\}$ is the set of K edges [6]. For any edge l connecting two vertices v_i and $v_j \in \mathbf{V}$, the edge vector $\mathbf{a}_l \in \mathbf{R}^M$ is all zeros vector except its i^{th} and j^{th} elements are $a_{l,i}=1$ and $a_{l,j}=-1$, respectively. The graph incidence matrix of $\mathbf{A} \in \mathbf{R}^{M\times K}$ is given by $\mathbf{A} \triangleq [\mathbf{a}_1,\ \ldots,\ \mathbf{a}_K]$ and its the $M \times M$ Laplacian matrix can be written as follows [6]

$$\mathbf{L}=\mathbf{A}\,\mathrm{diag}(\mathbf{w})\mathbf{A}^T=\sum_{l=1}^{K} w_l \mathbf{a}_l \mathbf{a}_l^T, \tag{1}$$

where $\mathbf{w}$ denotes the $K \times 1$ weighting vector coefficients for the K edges and is given by $[w_1, w_2, \cdots, w_K]^T$. The Laplacian matrix is a positive semi-definite matrix, i.e., $\mathbf{L} \succeq 0$, with the smallest eigenvalue denoted by $\lambda_1(\mathbf{L})$ is equal to zero [6]. We term $\lambda_2(\mathbf{L})$ as the second smallest eigenvalue, also known as Fiedler value, of the graph Laplacian matrix which represents its algebraic connectivity. The smaller Fiedler value is, the less connected the network is, and vice verse. It is worth mentioning that when $\lambda_2(\mathbf{L})=0$, the graph is disconnected in which at least one of its vertices is unreachable from any other vertices in the graph.

With a UAV deployment, a new graph $\mathcal{G}'$ is obtained with the same number of M nodes, a larger set of edges denoted by $\mathbf{E}'$ with K' edges where $K' \geq K$, i.e., $\mathbf{E} \subseteq \mathbf{E}'$, thanks to the UAV for connecting the SCs within its disc radius R_{UAV}. Particularly, the UAV's impact appears in relaying information between SCs within R_{UAV} distance from it, hence creating additional $K'-K$ edges between the original SCs. Comparing the network graphs pre and post UAV deployment, we can realize the gain by computing $\lambda_2(\mathbf{L}') \geq \lambda_2(\mathbf{L})$.

In addition, the UAV aims to extend the coverage area and serve the N UEs who are in deep fade situation. The downlink communications between the UAV and UEs are assumed to occur over orthogonal resources such as frequency/time division duplexing transmission to avoid interference among the scheduled UEs as the UAV is assumed to be equipped with a single antenna and multi-user beam-forming techniques can not be utilized. Considering the interference among the scheduled UEs is left for future work. Hence, the received signal at the i^{th} UE is given by

$$y_i = d_i^{-\alpha} x_i + n_i \tag{2}$$

where $d_i^{-\alpha}$ is the large scale fading between the UAV and the i^{th} UE with d_i denoting the distance between them and α is the the path loss exponent. The data symbol x_i is the data transmitted to UE i with $\mathbb{E}\left[|x_i|^2\right]=P_i$. Moreover, n_i denotes complex zero-mean circularly-symmetric additive-white-Gaussian noise (AWGN) with variance σ^2 and it is assumed to be independent across the UEs. Note that in our system model, we model only the large scale fading and ignore the small scale fading. This is a well justified assumption whenever the UAV position is calculated during the channel coherence time and it varies from time slot to another which effectively allows us to combine both large and small channel fading coefficients in a single coefficient. Moreover, since no multi-user multiple-input multiple-output (MU-MIMO) can be applied with a single antenna UAV, we find beneficial to forgo the small scaling fading in our model.

To quantify the coverage, we use the instantaneous signal-to-noise ratio (SNR). For the i^{th} UE, it is defined as follows

$$\mathrm{SNR}_i=\frac{d_i^{-\alpha}P_i}{\sigma^2}. \tag{3}$$

The reason of using distinct metrics, Fiedler value and SNR, to quantify the connectivity among SCs and coverage for UEs, respectively, is that in SCs backhaul network, multi-hop communication is allowed. Therefore, we care more about connectivity of the graph which is characterized by its Fiedler value. On the other hand, in the access network, the UAV directly communicates with the UEs, i.e., single hop link, which makes the SNR appropriate to reflect the coverage.

III. Problem Formulation and Solution

In this section, we formulate the UAV positioning optimization problem in which the backhaul network connectivity is maximized subject to particular QoS constraints for each UE. Mathematically, it can be written as follows

$$\begin{aligned} \max_{\mathbf{u}} \quad & \lambda_2(\mathbf{L}'(\mathbf{u})) \\ \text{s. t.} \quad & \mathrm{SNR}_i \geq \gamma_{\text{th}},\ \forall i \in \{1,\ \ldots,\ N\}, \end{aligned} \tag{4}$$

where $\mathbf{u}$ is the 3×1 UAV position coordinates vector in a Cartesian coordinate system and γ_{th} is SNR threshold. As

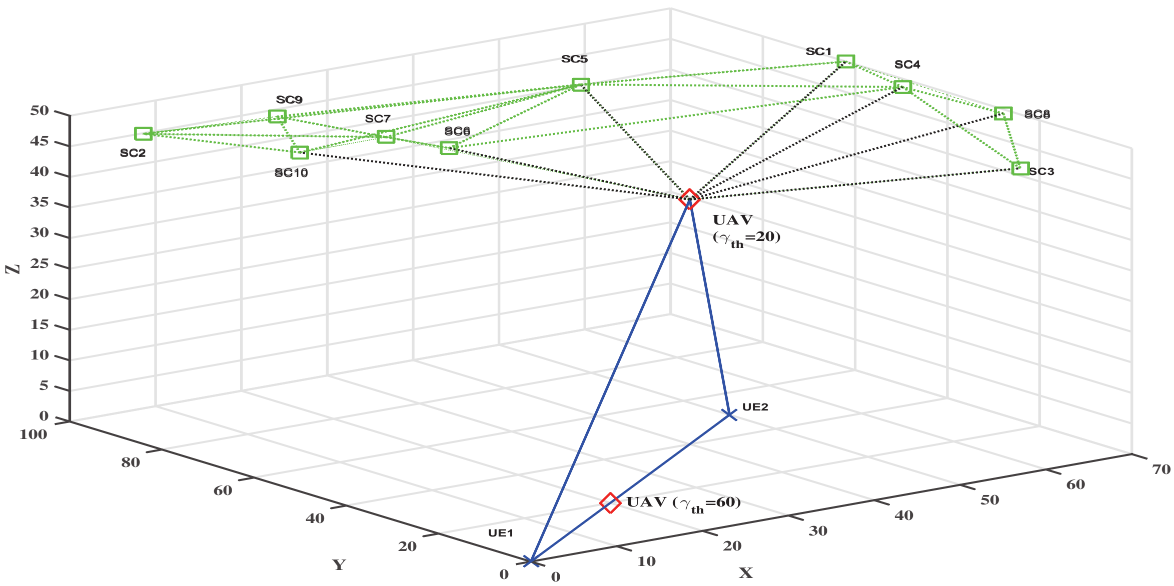

Fig. 2. UAV positioning for different γ_{th}. The square, cross and diamond markers represent SC, UE and UAV, respectively.

illustrated in Section II, the Laplacian matrix $\mathbf{L}'(\mathbf{u})$ depends on the UAV location which determines the new links.

The optimization problem in (4) is a non-convex optimization problem because of two reasons. First, $\lambda_2(\mathbf{L}'(\mathbf{u}))$ is not a concave function. Second, the indirect dependence function between $\mathbf{L}'(\mathbf{u})$ and $\mathbf{u}$ which is described in Section II.

To cope with the non-concavity of $\lambda_2(\mathbf{L}'(\mathbf{u}))$, we exploit the properties of the graph Laplacian matrix. In particular, its smallest eigenvalue is equal to zero and the entries summation of the eigenvector corresponding to $\lambda_2(\mathbf{L})$ is equal to zero [6]. Hence, $\lambda_2(\mathbf{L})$ can be formulated as a convex problem as

$$\begin{aligned} \lambda_2(\mathbf{L}) = \min_{\mathbf{z}\in\mathbf{R}^M} \quad & \mathbf{z}^T\mathbf{L}\mathbf{z} \\ \text{s. t.} \quad & \|\mathbf{z}\| = 1 \;\; \text{and} \;\; \mathbf{1}^T\mathbf{z} = 0, \end{aligned} \tag{5}$$

where $\mathbf{1}$ is the $M \times 1$ all-ones vector.

To address the indirect relation between the graph Laplacian matrix and the UAV position, we assume that SCs and UEs are distributed over $h \times h \times h$ volume. Moreover, the search space over the $x,\ y,$ and z axes is uniformly quantized with a step size δ to get a search grid consisting of β candidate positions for the UAV. This simplifies the Laplacian matrix to be represented by the following formula

$$\mathbf{L}' = \mathbf{L} + \sum_{j=1}^{\beta} x_j \mathbf{A}_j \operatorname{diag}(\mathbf{w}_j)\mathbf{A}_j^T, \tag{6}$$

where $\mathbf{L}$ is the original network graph before UAV deployment, x_j is equal to one if UAV is positioned in the j^{th} grid point, otherwise $x_j = 0$. Moreover, $\mathbf{w}_j$ and $\mathbf{A}_j$ are the weighting coefficients vector and the incidence matrix when the UAV is deployed in this grid point.

Collecting $x_j,\ j \in \{1,\ \dots,\ \beta\}$, in the $\beta \times 1$ vector $\mathbf{x}$, Eqn. (6) can be written as follows

$$\mathbf{L}' = \mathbf{L} + (\mathbf{x} \otimes \mathbf{I}_M)\,\mathbf{\Gamma}, \tag{7}$$

where
$\mathbf{\Gamma} \triangleq \left[\left(\mathbf{A}_1 \operatorname{diag}(\mathbf{w}_1)\mathbf{A}_1^T\right)^T,\ \dots,\ \left(\mathbf{A}_\beta \operatorname{diag}(\mathbf{w}_\beta)\mathbf{A}_\beta^T\right)^T\right]^T.$

Similarly, stacking the SNR levels between the i^{th} UE and the UAV located in any of the candidate positions in the search grid in the $\beta \times 1$ vector denoted by $\mathbf{v}_i$ such that $\text{SNR}_i = \mathbf{x}^T\mathbf{v}_i$. Hence, the optimization problem can be written in terms of the UAV position index vector x rather than its actual physical locations, i.e., x-axis, y-axis and z-axis values, as follows

$$\begin{aligned} \max_{\mathbf{x}} \quad & \lambda_2(\mathbf{L}'(\mathbf{x})) \\ \text{s. t.} \quad & \mathbf{x}^T\mathbf{v}_i \geq \gamma_{\text{th}},\ \forall i \in \{1,\ \dots,\ N\}, \\ & \mathbf{x} \in \{0, 1\}. \end{aligned} \tag{8}$$

The combinatorial optimization problem in (8) is non-deterministic polynomial-time (NP)-hard problem with high complexity. Therefore, we relax the constraint on the entries of $\mathbf{x}$ and allow them to take any value in the interval $[0, 1]$. Hence, approximated optimization problem can be written as standard convex SDP problem as follows [8]

$$\begin{aligned} \max_{\mathbf{x},s} \quad & s \\ \text{s. t.} \quad & s(\mathbf{I} - \frac{1}{\beta}\mathbf{1}\mathbf{1}^T) \preceq \mathbf{L}'(\mathbf{x}), \\ & \mathbf{x}^T\mathbf{v}_i \geq \gamma_{\text{th}},\ \forall i \in \{1,\ \dots,\ N\}, \\ & 0 \leq \mathbf{x} \leq 1, \end{aligned} \tag{9}$$

which can be solved using any SDP solver such as CVX SDPT3 solver [9]. Since the entries of output vector $\mathbf{x}$ are continuous, we choose the maximum entry and set it to 1 while others are set to zero.

Table I. Table of simulation parameters

Parameter	Value
h	100 m
M	10
N	2
R_{SC}	40 m
R_{UAV}	30 m
α	4
σ^2	-130 dBm
P_i	40 dBm

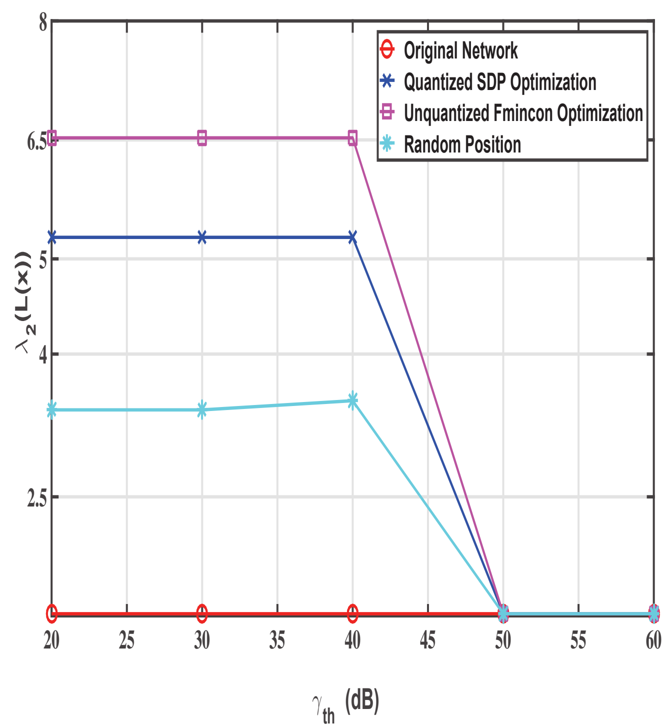

Fig. 3. The connectivity of the SCs versus the UEs SNR constraint for $\beta = 2197$, $\delta \cong 8$ m.

IV. Numerical Results

In this section, we present Monte-Carlo simulation results to demonstrate the accuracy of the proposed convex approximation of the UAV positioning problem and show coverage-connectivity tradeoff. We consider a backhaul network between M SCs in the presence of N UEs in deep fade situations. The simulated network's parameters are listed in Table I and all our results are averaged over 10^4 different backhaul network realizations and UEs dropping. In our numerical

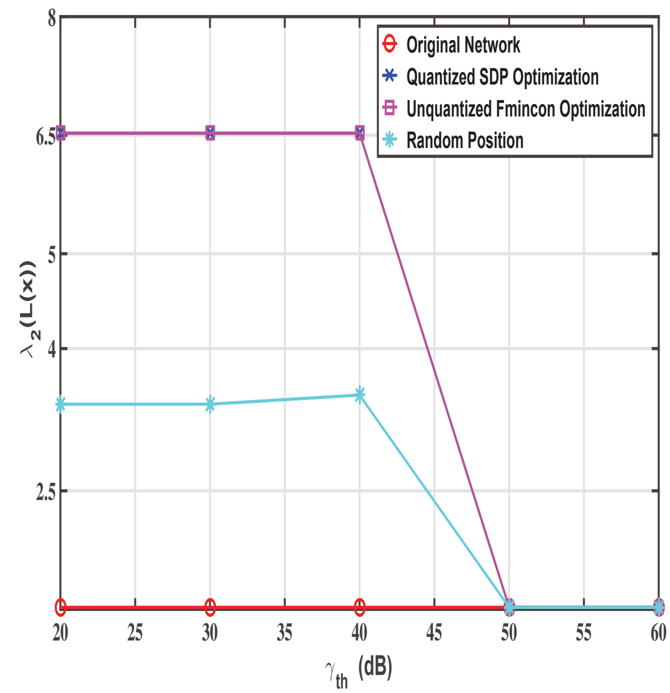

Fig. 4. The connectivity of the SCs versus the UEs SNR constraint for $\beta = 3,375$, $\delta \cong 6$ m.

results, we compare between four different scenarios. First, the solution of the approximated SDP optimization in (9) to find the optimal UAV position. Second, we consider UAV random solution for the UAV positioning in the feasible region in which the SNR threshold constraints are satisfied for all the UEs. This is considered as a lower bound on the achieved performance. Third, unquantized optimization (4), which is the alternative for exhaustive search in the 3-D search grid. This is considered as the performance upper bound because we solve the nonconvex optimization problem in (4) using fmincon solver in MATLAB with multi-initial point searching to avoid getting stuck at local minimas. Fourth, to benchmark the performance gain, we show the original backhaul network connectivity before the UAV deployment.

The impact of γ_{th} on the UAV positioning is shown in Fig. 2, where the UAV position in the 3-D search grid with red diamond markers is plotted for two extreme cases for the UEs constraints. In the first case, $\gamma_{th} = 20$ dB corresponding low QoS constraints. In this case, the UAV gets closer the SCs to enhance the backhaul network connectivity. In the other case, is at $\gamma_{th} = 60$ dB, which represents a high QoS constraints. In this scenario, the UAV gets closer to the UEs to satisfy their constraints with no much improvement for the backhaul network connectivity.

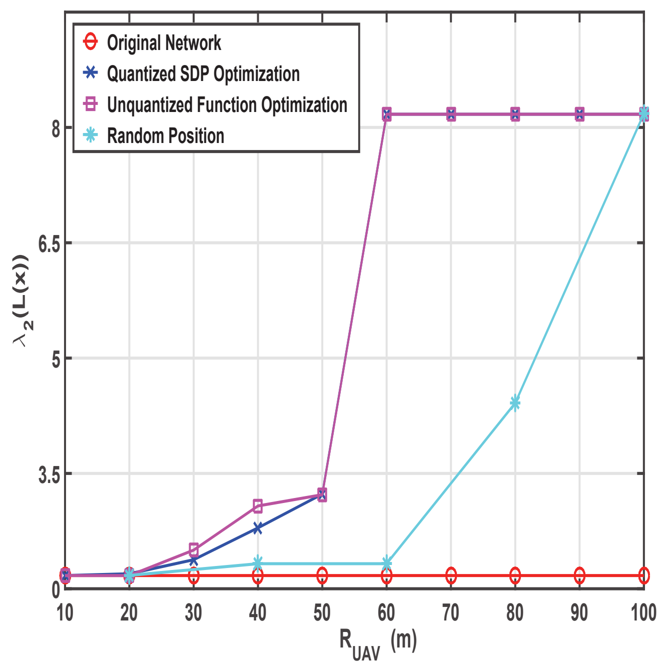

Fig. 5. The connectivity of the SCs versus the UAV transmission range for $\beta = 3,375$, $\delta \cong 6$ m.

To further investigate the tradeoff between the SCs connectivity and the UEs coverage, in Fig. 3, the Fiedler value of the SCs backhaul network graph is plotted versus the UEs SNR threshold γ_{th}. We use $\delta = 8$ m in (9) for the quantized SDP optimization. The figure shows that increasing the γ_{th} decreases the connectivity for the different solving schemes. At high γ_{th} levels, very tight QoS constraints, there is no enhancement in the SCs backhaul network connectivity. We notice that the SDP optimization is closer to the upper bound (unquantized optimization) than the lower bound (random position), with a loss of 25% from the upper bound and nearly 35% gain from the lower bound for $\gamma_{th} = 30$ dB. To reduce the gap between unquantized and the SDP, we reduce the step size to $\delta = 6$ m, which leads to larger feasible set. In Fig. 4, the SCs backhaul network connectivity is plotted versus the UEs SNR threshold γ_{th}. We notice that the output from the SDP optimization is the same as the upper bound (the unquantized function optimization) for all values of γ_{th}, this indicate that the step size $\delta = 6$ m is convenient to introduce the grid for the choosen area. The use of $\delta = 6$ m increases the quantized SDP optimization gain from the random positioning to 60% for SNR threshold $\gamma_{th} = 30$ dB, this is nearly double the gain from using a step size of $\delta = 8$ m.

We demonstrate the SCs backhaul network connectivity versus the UAV range for $\delta = 6$ m in Fig. 5 by changing the UAV transmission range R_{UAV}. As the UAV transmission range increases the connectivity increases until R_{UAV} is large enough to cover the whole nodes so the connectivity saturates at its maximum value. Clearly the unquantized optimization gives a higher performance than the quantized SDP optimizatation due to the quantization loss, however as this graph uses a small step size compared to the total area so the quantization loss is small. We note that the unquantized function optimization and the quantized SDP optimization both gives a better performance than the random positioning.

V. Conclusion and Future Work

In this paper, we proposed utilizing a single UAV to jointly enhance the SCs backhaul network connectivity and achieve better UEs coverage from network access perspective. We converted the UAV positioning non-convex optimization problem to a convex one using 3-D quantized search grid. Moreover, we approximated it as a SDP problem which can be efficiently solved using the widely known SDP solvers. Furthermore, we numerically demonstrated the accuracy of our approximation and showed performance gain from the upper and lower bounds. Compared with random positioning at $\gamma_{th} = 30$ dB, 35 and 60% connectivity gain were achieved for $\delta = 8$ m and $\delta = 6$ m, respectively. At $\delta = 6$ m, the original non-convex and the relaxed SDP solutions gave the same connectivity metrics.

There are several interesting extensions to our work, for instance, we will further study the effects of the small-scale fading reflected in the SNR constraints instead of only considering the large-scale fading. Also, we will investigate the multi-user transmission schemes when the UAV is equipped with multiple antennas and will explore different beam-forming techniques to suppress the interference between the scheduled UEs.

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