Beamforming for Simultaneous Wireless Information and Power Transfer in Two-Way Relay Channels

WEI WANG1, RUI WANG2, HANI MEHRPOUYAN3, (Member, IEEE), NAN ZHAO4, (Senior Member, IEEE), AND GUOAN ZHANG5

1W. Wang is with the Department of Communication Engineering, Nantong University, Nantong 226019, China, and also with the Nantong Research Institute for Advanced Communication Technologies, Nantong 226019, China
2R. Wang is with the Department of Information and Communications, Tongji University, Shanghai 201804, China
3H. Mehrpouyan is with the Department of Electrical and Computer Engineering, Boise State University, Boise, ID 83725, USA
4N. Zhao is with the School of Information and Communication Engineering, Dalian University of Technology, Dalian 116024, China
5G. Zhang is with the Department of Communication Engineering, Nantong University, Nantong 226019, China

Corresponding author: Rui Wang (ruiwang@tongji.edu.cn)

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ABSTRACT This paper studies the beamforming designs for simultaneous wireless information and power transfer systems in two-way relaying (TWR) channels. The system consists of two energy-constrained source nodes which employ the power splitting (PS) to receive the information and the energy simultaneously from the power-supply relay. To maximize the weighted sum energy subject to the constraints of the quality of service and the transmit powers, three well-known relaying protocols, i.e., amplify-and-forward, bit level XOR-based decode-and-forward (DF), and symbol level superposition coding-based DF, are considered. For each relaying protocol, we formulate the joint relay beamforming, the source transmit power, and the PS ratios optimization as a nonconvex quadratically constrained problem. To solve the complex nonconvex problem, we decouple the objective problem into two subproblems in which one is to optimize the beamforming vectors while another is to optimize the remaining parameters. We show that the optimal solution of each subproblem can be obtained in the closed-form expressions. The solution is finally obtained with the proposed convergent iterative algorithm. Extensive numerical results demonstrate the advantage of adapting the different relaying strategies and weighted factors to harvest energy in TWR channels.

INDEX TERMS Beamforming, energy harvesting (EH), simultaneous wireless information and power transfer (SWIPT), two-way relaying (TWR), power splitting (PS).

I. INTRODUCTION

Energy harvesting (EH) from surrounding environments is an emerging solution to prolong the operational time of energy-constrained nodes in wireless networks [1]–[6]. Compared with conventional energy sources, radio frequency (RF) signals can carry both information and energy simultaneously. Simultaneous wireless information and power transfer (SWIPT) has recently drawn significant attention for various wireless channels [7]–[15]. For example, a point-to-point single-antenna additive white Gaussian noise (AWGN) channel was first studied in [7], the fundamental performance tradeoff for simultaneous information and power transfer was studied by using the capacity energy function. Later on, SWIPT was extended to a frequency selective channels in [8]. Liu et al. [9] studied SWIPT for fading channels subject to time-varying co-channel interference. In [10] and [11], SWIPT schemes with multiple-input-multiple-output (MIMO) channels were investigated. The transmit beamforming design was studied for SWIPT in multiple-input-single-output (MISO) broadcast channels in [12] and [13]. Moreover, SWIPT was investigated in other physical layer setups such as the OFDM, and more in [14] and [15].

All the above works consider the one-hop transmission, which, however, is usually impractical in the long distance scenario. To evaluate this issue, the authors
study the potentiality of SWIPT in the wireless relay networks [16]–[23]. Krikidis et al. [16] and Nasir et al. [17] considered a one-way single-antenna amplify-and-forward (AF) relay networks, where the time switching (TS) and power splitting (PS) protocols are proposed. Specifically, SWIPT was extended to a wireless-powered MIMO one-way relay channel in [18], where the relay is able to harvest energy from the destination and simultaneously receive the information signal from the source. Since the two-way relaying (TWR) system can further improve the spectral efficiency, it is potential to use the SWIPT protocols in the TWR scenario. Chen et al. [19] considered a two-way AF relaying networks with SWIPT, where an energy harvesting relay node is used to cooperate in exchanging the information for two source nodes. Tutuncuoglu et al. [20] studied the sum-throughput maximization problem in a two-way AWGN relay channel, where all nodes are powered by EH. Particularly, the authors considered a SWIPT AF TWR channels in [21], where two source nodes harvest energy from multiple relay nodes. Li et al. [22] studied the relay beamforming design problem for SWIPT in a non-regenerative TWR network. Moreover, for the AF relaying strategy, Wang et al. [23] considered a SWIPT TWR network where the two source nodes are powered via wireless energy transfer from the relay.

A. MOTIVATION

Recently, most studies on SWIPT in relay networks focused on energy-constrained relay nodes [16]–[20]. Since the limited batteries are usually used for the terminals, it is difficult to prolong the operation time for the requirement of the increasing traffic. To provide a convenient way for charging the batteries of the terminals, therefore, to employ the EH is an efficient way worth trying. Therefore, in this paper, we consider a SWIPT TWR system with battery-limited source nodes and a relay node which servers as a source of energy (will be described in details in the Section II). Under this setup, the source nodes receive information and energy simultaneously from the signals sent by a relay node. Furthermore, to enhance bandwidth efficiency and power transfer efficiency, we assume that the relay node is equipped with multiple antennas. This setup applies to lots of practical wireless transmission scenarios. Since TS can be regarded as a special case of PS with only binary split power ratios [9], [10], we focus our study on PS receivers instead of TS receivers.

B. RELATED WORKS

To the best of our knowledge, a few works have been done on SWIPT TWR systems with the battery-limited source nodes [21]–[23]. Li et al. in [21] and [22], studied the optimal relay beamforming design to maximize the sum rate subject to the constraints of the transmit power at the relay and the EH at source nodes. However, in [21], the source node is assumed to be able to simultaneously decode information and extract power which is impractical. In contrast to [21], Li et al. in [22] considered a more complicated system with separated EH and information decoding (ID) receivers. In [23], we studied a PS-based SWIPT TWR system where the received signal at the source is split for ID and EH. Nevertheless, it is worth pointing out that all existing works [21]–[23] were only investigated the AF relaying strategy. Furthermore, some related works without EH scenario were also proposed in [24]. Because the EH constraint is non-convex, the proposed resource allocation and beamforming designs are more challenging than the conventional TWR networks. In this paper, we propose joint transceiver design schemes for the wireless-powered TWR channels with SWIPT. In particular, we consider three simple and practical relaying strategies: AF, bit level XOR based decode-and-forward (DF-XOR), symbol level superposition coding based DF (DF-SUP) as [24]. Besides, unlike the SWIPT studied in the existing works, we consider a utility optimization problem, i.e., maximize the weighted sum energy at the two battery-limited source nodes subject to the constraints of the received signal-to-interference-and-noise ratio (SINR), the PS ratios and the transmit powers. Since in some scenarios, a large number of terminals will be operating in close vicinity, SINR is an important metric to valuate the throughput while maximizing energy transferred of the terminals by the relay. The latter maximizes the operational time of the terminals which is particularly important for the energy-constrained scenarios. To the authors’ best knowledge, the joint design of the beamforming, power allocation and PS ratio is still a blank field for our proposed scenario.

C. OUR CONTRIBUTIONS

With above illustrations, a PS-based SWIPT TWR system is considered in this paper. Different from existing works, we consider a utility optimization problem aiming to maximize the weighted sum energy by jointly optimizing design of the transmit power, the PS ratio at the sources and the beamforming matrix at the relay. Moreover, various relaying strategies may result in different transmit signals at the relay node. The impact of various relaying protocols on the amount of harvested energy has not been considered in existing studies. Besides, another challenging doubly-near-far problem [2] could be mitigated effectively by setting different EH priorities for different source nodes.

The main contributions of this work are summarized as follows. Firstly, different from the pure relaying strategies, we propose the optimal relaying protocols that included a new signal, which provides more degrees of freedom (DoF) to optimize the power transfer from the relay to the source nodes. Secondly, to explore the performance limit of the system, we formulate the weighted sum energy maximization problems by considering three practical two-way relaying strategies [24]–[26], i.e., AF, DF-XOR and DF-SUP. Compared with previous result in [23] where only considered the AF protocol, the proposed two DF protocols with the corresponding joint design scheme always yields better EH performance. Thirdly, for three relaying protocols, the
formulated optimization problems are nonconvex. To overcome this issue, the primal optimization problem is decomposed into two subproblems. The first subproblem only optimizes the beamforming vectors. We solve this nonconvex problem by applying the technique of semidefinite programming (SDP) [27]. The second subproblem only includes the source transmit powers and PS ratios. We propose a novel algorithm to find the optimal closed-form solutions of the latter nonconvex subproblem by separating it into eight cases. By this way, the objective problem can be tackled by the proposed convergent iterative algorithm. Finally, we provide numerical results to evaluate the performance of the proposed joint optimal designs. It is shown that when the priority and the distance of two source nodes are symmetric, the DF-XOR joint optimal designs. It is shown that when the priority and the distance of two source nodes are symmetric, the DF-XOR relaying strategy outperforms the other two strategies. On the other hand, while the distances of two source nodes are asymmetric, by using the DF-SUP protocol and applying the proposed joint design scheme, the furthest node can achieve a higher EH efficiency.

**D. ORGANIZATION**

The remainder of this paper is organized as follows. The SWIPT TWR system model is described in Section II. In Section III, the weighted sum energy maximization problems are formulated for different relaying strategies. The solutions for the associated optimization problems by using suitable optimization tools are presented in Section IV. In Section V, numerical simulation results are provided. Finally, the paper is concluded in Section VI.

**Notations:** Boldface lowercase and uppercase letters denote vectors and matrices, respectively. For a square matrix $A$, $A^T$, $A^*$, $A^H$, $\text{Tr}(A)$, $\text{Rank}(A)$ and $||A||$ denote its transpose, conjugate, conjugate transpose, trace, rank, and Frobenius norm, respectively. $A \succeq 0$ indicates that $A$ is a positive semidefinite matrix. $\text{vec}(A)$ denotes the vectorization operation by stacking the columns of $A$ into a single vector $a$. $E(\cdot)$ denotes the statistical expectation. $\otimes$ denotes the Kronecker product. $\mathbf{0}$ and $\mathbf{I}$ denote the zero and identity matrix, respectively. The distribution of a circular symmetric complex Gaussian vector with mean vector $\mathbf{x}$ and covariance matrix $\Sigma$ is denoted by $\mathcal{CN}(\mathbf{x}, \Sigma)$. $\mathbb{C}^{x \times y}$ denotes the $x \times y$ domain of complex matrices.

**II. SYSTEM MODEL**

Consider a half-duplex TWR system consisting of two single-antenna source nodes $S_1$ and $S_2$, where the two sources exchange information via an $N$-antenna relay node $R$, as shown in Fig. 1. The channel vectors from $S_1$ and $S_2$ to the relay are denoted by $\mathbf{h}_1$ and $\mathbf{h}_2$, respectively, and the channels between the relay and $S_1$, $S_2$ are denoted by $\mathbf{g}_1$ and $\mathbf{g}_2$, respectively. In order to improve the spectral efficiency, the two-time slot TWR model is used to realize bidirectional communications. Throughout this paper, the following assumptions are considered:

- Since the source nodes cannot communicate with each other directly due to that the direct link is blocked due to long-distance path loss or obstacles [28], [29]. Therefore, all the messages can only be exchanged with the help of the relay.
- The relay is connected to the power grid, which implies that it has access to reliable power at all times. However, the source nodes are powered by the energy limited batteries, and need to replenish their energy by wireless power transfer.
- Amongst the different relaying protocols, AF, DF-XOR and DF-SUP schemes are applied at the relay node due to their implementation simplicity [24]–[26].
- Quasi-static block fading channels are assumed, i.e., the channels will not be changed in their current time slot, except during other time slot. The use of such channels is motivated by prior researches in the field [9]–[13], [19], [21] and the practical considerations.

As shown in Fig. 2, we propose a two-phase PS-based protocol for the TWR system. In the first phase of duration $T/2$, two source nodes $S_1$ and $S_2$ deliver their information to the relay node $R$ simultaneously. In the second phase with the remaining time duration $T/2$, the received information signal at $R$ is processed by the aforementioned relaying strategies and then forwarded to the source nodes. With the assumption of the PS ratio, i.e., $\rho$, the transmit signal at the relay can be used to complete the transferring the information and the power simultaneously.

With above assumptions, the received signal at the relay at the end of first phase, i.e., the multiple access (MAC) phase, is given by

$$\mathbf{y}_R = \mathbf{h}_1 x_1 + \mathbf{h}_2 x_2 + \mathbf{n}_R,$$  \hspace{1cm} (1)

where $x_i$, for $i \in \{1, 2\}$, represents the transmit signal from node $S_i$ with the power constraint of $E(|x_i|^2) = P_i$, $\mathbf{h}_1$, $\mathbf{h}_2$, $\mathbf{n}_R$, $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{n}_R$. \hspace{1cm} (1)
and \( n_R \) denotes the AWGN vector at the relay following \( CN(0, \sigma_n^2 I_N) \).

At second phase, also referred as broadcast (BC) phase, upon receiving \( y_R \), the relay node performs certain processing and forwards the new signal to the source nodes. The signal transmitted from relay node can be expressed as

\[
x_R = x_{12} + x,
\]

where \( x_{12} \) is the combined signal consisting of the messages from two nodes by using physical-layer network coding (PLNC). Different from the conventional relaying protocols, a new signal \( x \) is also serving as the part of the transmission. If the optimal solution \( x = 0 \), the optimal relay strategy in our considered network is essentially equivalent to the pure TWR. If the optimal solution \( x \neq 0 \), it provides that more DoF to optimize power transfer from relay to the source nodes.

III. RELAYING STRATEGIES AND OPTIMIZATION PROBLEM FORMULATIONS

Based on the channel setup described in Section II, in this section we shall present different transmit signals \( x_R \) for the TWR SWIPT system by considering three practical relaying strategies. Moreover, to explore the system performance limit, we formulate three optimization problems for these strategies in this section.

A. AF RELAY STRATEGY

With the AF relaying strategy, the relay transmits signal \( x_R \) in (2) can be expressed as \( x_R = W_h x_1 + W_h x_2 + W_n R + x \), where \( W \) denotes the precoding matrix at the relay. Specifically, as shown in Fig. 2, letting \( \rho \in (0, 1) \) as the PS ratio, after converting the received signal into baseband and performing self-interference cancelation (SIC), the obtained signal at the source node is denoted as \( y_i = \sqrt{1-\rho} (g_i^T W_h x_1 + g_i^T W_h x_2 + n_i,d) + n_i,c \), where \( i = 2 \) if \( i = 1 \) and \( \bar{g}_i = \alpha g_i \). Using PS, the obtained signal is given by \( y_i = \sqrt{1-\rho} (g_i^T W_h x_1 + g_i^T W_h x_2 + n_i,d) + n_i,c \). Similarly, \( y_i \) and \( y_i \) are additive Gaussian noises due to the antenna imperfection and the signal conversion, respectively. Accordingly, the SINR at the node \( S_i \) is given by \( \text{SINR} = \frac{P_i |g_i^T W_h x_1|^2}{\sigma_n^2 + \sigma_{g_i}^2 + \frac{\sigma_{n_i}^2}{\alpha^2} + \frac{\sigma_{g_i}^2}{\alpha^2}} \). Meanwhile, since the noise terms at the EH receiver is ignored, the harvested energy \( E_i \) is given as \( E_i = \frac{1}{T} \rho \eta |g_i^T W_h x_1|^2 + |g_i^T W_h x_2|^2 P_2 + |g_i^T Q_i g_i|^2 P_1 + |g_i^T Q_i g_i|^2 \), where \( \eta \) is the energy conversion efficiency with \( 0 < \eta < 1 \) which depends on the rectification process and the EH circuitry. It is worth noting that the self-interference is useful to the EH, which is totally different from the conventional ID. Assuming the relay node is with the maximum transmit power \( P_r \), i.e., \( \text{Tr}(E(x_R x_R^H)) \leq P_r \), which is equivalent to \( P_i |W_h x_1|^2 + P_2 |W_h x_2|^2 + \text{Tr}(Q_i) + \sigma_n^2 |W|^2 \leq P_r \), where \( Q_i = \text{E}(x x_R^H) \) is the covariance matrix of \( x \). Then, the weighted sum energy maximization problem can be formulated as

\[
\begin{align*}
\max_{P_1, P_2, \rho, Q_i \geq 0} & \quad (\alpha E_1 - \frac{P_1 T}{2}) + \beta (E_2 - \frac{P_2 T}{2}) \\
\text{s.t.} & \quad \text{SINR} \geq \tau_i, \quad i = 1, 2, \\
& \quad P_i \leq P_{\text{max},i}, \quad i = 1, 2, \\
& \quad \text{Tr}(E(x_R x_R^H)) \leq P_r,
\end{align*}
\]

where \( \alpha \) and \( \beta \) correspond to the given energy weights for the two EH receivers \( S_1 \) and \( S_2 \), with \( \alpha + \beta = 1 \), where the larger weight value indicates a higher priority of transferring energy to the corresponding EH receiver. \( \tau_i \) and \( P_{\text{max},i} \) denote the minimum SINR requirement and the maximum transmit power at node \( S_i \), respectively.

B. DF-XOR RELAY STRATEGY

With DF-XOR relaying strategy, if the relay node can decode successfully the messages sent from both source nodes, the transmit powers \( P_1 \) and \( P_2 \) need to satisfy the following rate region constraints [24], [30]

\[
C_{\text{MAC}}(\bar{R}_1, \bar{R}_2) = \left\{ \begin{array}{ll}
\bar{R}_1 & \leq \log_2(1 + \frac{\bar{P}_1 |h_i|^2}{\sigma_n^2}) \\
\bar{R}_2 & \leq \log_2(1 + \frac{\bar{P}_1 |h_i|^2}{\sigma_n^2}) \\
\bar{R}_1 + \bar{R}_2 & \leq \log_2 \\
d(\bar{I} + \frac{\bar{P}_1 |h_i|^2}{\sigma_n^2} h_i^H h_i^H + \frac{\bar{P}_1 |h_i|^2}{\sigma_n^2} h_i^H h_i^H),
\end{array} \right.
\]

where \( \bar{R}_1 \) and \( \bar{P}_1 \) are the transmit rate and the maximum transmit power at the source nodes \( S_i \), for \( i \in \{1, 2\} \), respectively. We assume that the messages sent from the nodes in the MAC phase can be successfully decoded at the relay node. Let \( b_i \) denote the decoded bit sequence from \( S_i \), for \( i \in \{1, 2\} \). With the DF-XOR relaying strategy, the combined bit sequence is yielded as \( b_{12} = b_1 \oplus b_2 \). Then the transmit signal in the second time interval of \( T/2 \), denoted by \( x_R \), can be expressed as \( x_R = s_{12} + x \), where \( s_{12} \) is the modulated signal of bit sequence \( b_{12} \). Using PS, the obtained signal at the source node is denoted as \( y_i = \sqrt{1-\rho} (g_i^T s_{12} + g_i^T x + n_i,d) + n_i,c \). Subsequently, the SINR at the node \( S_i \) can be determined as \( \text{SINR} = \frac{g_i^T Q_i g_i + \sigma_n^2 + \sigma_{g_i}^2 + \frac{\sigma_{n_i}^2}{\alpha^2} + \frac{\sigma_{g_i}^2}{\alpha^2}}{\sigma_n^2} \). On the other hand, the harvested energy \( E_i \) is given as \( E_i = \frac{1}{T} \rho \eta |g_i^T Q_i g_i|^2 \), for \( i = 1, 2 \). By assuming that the relay power constraint \( \text{Tr}(E(x_R x_R^H)) \leq P_r \), the corresponding optimization problem can be formulated as

\[
\begin{align*}
\max_{P_1, P_2, \rho, Q_i \geq 0} & \quad (\alpha E_1 - \frac{P_1 T}{2}) + \beta (E_2 - \frac{P_2 T}{2}) \\
\text{s.t.} & \quad \text{SINR} \geq \tau_i, \quad i = 1, 2, \\
& \quad \text{Tr}(E(x_R x_R^H)) \leq P_r.
\end{align*}
\]

Note that here different from (3), it is not necessary to optimize \( P_1 \) and \( P_2 \) as they are determined via the constraints presented in (4).
C. DF-SUP RELAY STRATEGY

In this subsection, we consider a case where the relay uses the DF-SUP relaying strategy. Then the transmit signal \( x_R \) can be expressed as \( x_R = s_1 + s_2 + x \), where \( s_i \) is the modulated signal of bit sequence of node \( S_i \). By assuming a PS ratio, \( \rho \), the obtained signal at the source node is given by \( y_i = \sqrt{1 - \rho} g^s_i S_i + x + n_i, d \). Then, the SINR at the node \( S_i \) can be denoted as \( \text{SINR}_{i}^{\text{SUP}} = \frac{g^s_i Q_i g^s_i}{g^s_i Q_i g^s_i + g^s_i Q_i g^s_i} \). Meanwhile, the harvested energy is given as \( E_i = \frac{1}{2} \rho (g^s_i Q_i g^s_i + g^s_i Q_i g^s_i) \). Due to that the relay power constraint \( \text{Tr}(\mathbb{E}(x_R x_R^H)) = \text{Tr}(Q_{s,1}) + \text{Tr}(Q_{s,2}) + \text{Tr}(Q_{d}) \leq P_r \), the corresponding optimization problem can be formulated as

\[
\begin{align*}
\max_{\rho, Q_{s,1} \geq 0, Q_{s,2} \geq 0, Q_{d} \geq 0} & \quad \alpha(E_1 - \frac{P_1 T}{2}) + \beta(E_2 - \frac{P_2 T}{2}) \\
\text{s.t.} & \quad \text{SINR}_{i}^{\text{SUP}} \geq \tau_i, \ i = 1, 2, \\
& \quad \text{Tr}(\mathbb{E}(x_R x_R^H)) \leq P_r. \tag{6}
\end{align*}
\]

Similarly to (5), we are not necessary to optimize \( P_1 \) and \( P_2 \) as they are determined by (4).

For different relaying strategies, we have formulated three weighted sum energy maximization problems in (3), (5) and (6). In the following sections, we will propose three algorithms to solve these optimization problems.

IV. OPTIMAL DESIGN OF THREE MAXIMIZATION PROBLEMS

For the AF relaying strategy, the optimization problem in (3) is nonconvex due to not only the coupled beamforming vectors \( \{W, Q_s\} \) and the remaining parameters \( \{P_r, \rho\} \) in both the SINR and transmitted power constraints, but also all the quadratic terms involving \( W \). In general, it is difficult or even intractable to obtain the global optimal solution of a nonconvex problem [13, 22]. To deal with the nonconvex optimization problem, we can decouple it by solving some subproblems [31]. Thus, our idea is to optimize a portion of variables with the remaining ones fixed to search the local optimal solution [18]. More specifically, in the first step, we fix \( P_r \) and \( \rho \), the resulting beamforming optimization problem reduces to a nonconvex problem with a rank-one constraint. The latter can be efficiently solved by using some rank relaxation techniques [27]. In the second step, when the beamforming vectors \( \{W, Q_s\} \) are fixed, the resulting optimization problem over \( \{P_r, \rho\} \) is still a nonconvex problem. However, as show later the optimal solution can be obtained in closed-form by separating this subproblem into eight cases. In the following, we first decouple problem (3) into two subproblems that can be solved separately, and then propose an iterative algorithm to solve the joint optimization problem (3). Finally, we show the iterative optimization algorithm can converge. Similarly, we decouple problems (5) and (6) into two subproblems. Different from (3), two subproblems from (5) and (6) only involve the beamforming vectors and PS ratios.

A. JOINT DESIGN FOR AF RELAYING STRATEGY

Let us solve the two subproblems stemming from (3). In the first subproblem, we try our best to find the solutions of \( W \) and \( Q_s \) with fixed \( P_r, P_2 \) and \( \rho \) values. In the second one, we update the values of \( P_r, P_2 \) and \( \rho \) by fixing the remaining parameters.

1) Optimize \( W \) and \( Q_s \) for fixed \( P_r, P_2 \) and \( \rho \): Note that when fixing \( P_r, P_2 \) and \( \rho \), the problem of optimizing variables \( W \) and \( Q_s \) is equivalent to

\[
\begin{align*}
\max_{W, Q_s, \rho \geq 0} & \quad \alpha \rho (|g^T_1 W h_{12}^T|^2 P_2 + |g^T_1 W h_{11}^T|^2 P_1 + g^T_1 Q_s g^s_1) \\
& \quad + \beta \rho (|g^T_2 W h_{21}^T|^2 P_1 + |g^T_2 W h_{22}^T|^2 P_2 + g^T_2 Q_s g^s_2) \\
& \quad + \sigma^2 \| W \|_F^2 \\ s.t. & \quad \text{SINR}_{i}^{\text{SUP}} \geq \tau_i, \ i = 1, 2, \\
& \quad P_r \| W h_{11}^T \|^2 + P_r \| W h_{22}^T \|^2 + \text{Tr}(Q_s) + \sigma^2 \| W \|_F^2 \leq P_r. \tag{7}
\end{align*}
\]

To find the optimal solution of problem (7), we conduct some further transformations on (7). To be specific, we transform \( |g^T_1 W h_{12}^T|^2 \) and \( |W h_{11}^T|^2 \) into their equivalent forms as

\[
\begin{align*}
|g^T_1 W h_{12}^T|^2 &= \text{Tr}(g^T_1 W h_{12}^T h_{12}^T W^H g_1^s) \tag{8a} \\
&= \text{Tr}(g^T_1 g^T_1 W h_{12}^T W^H) \tag{8b} \\
&= w^H h_1 h_1^T \otimes g^s_1^T w \tag{8c} \\
&= \text{Tr}(h_1^T h_1^T \otimes g^s_1^T) w w^H, \tag{8d}
\end{align*}
\]

and similarly

\[
\begin{align*}
|W h_{11}^T|^2 &= \text{Tr}(W h_{11}^T W^H) \tag{9a} \\
&= \text{Tr}(W h_{11}^T h_{11}^T W^H) \tag{9b} \\
&= w^H h_1 h_1^T \otimes I w \tag{9c} \\
&= \text{Tr}(h_1^T h_1^T \otimes I) w w^H, \tag{9d}
\end{align*}
\]

where \( w = \text{vec}(W) \). In obtaining (8c) and (9c), we have used the identity

\[
\text{Tr}(A B C D) = (\text{vec}(A^T))^T (C^T \otimes A) \text{vec}(B). \tag{10}
\]

Similar to (8) and (9), we apply the above transformations to other terms in (7). Let \( \overline{W} = w w^H \), (7) can be rewritten as

\[
\begin{align*}
\max_{W, 0 \leq Q_s \leq 0} & \quad \text{Tr}(A_1 \overline{W}) + \text{Tr}(B_1 Q_s) \\
& \quad \text{s.t.} \quad \text{Tr}(C_1 \overline{W}) - \text{Tr}(\tau_i g^s_i^T Q_s) \geq D_i^1, \ i = 1, 2, \\
& \quad \text{Tr}(E_1 \overline{W}) + \text{Tr}(Q_s) \leq P_r, \\
& \quad \text{Rank} (\overline{W}) = 1. \tag{11}
\end{align*}
\]

where \( A_1 \triangleq (P_r h_1 h_1^T + P_r h_1 h_1^T) \otimes (\alpha p g^s_1 g^s_1 + \beta p g^s_2 g^s_2), \)

\( B_1 \triangleq \alpha p g^s_1 g^s_1 + \beta p g^s_2 g^s_2, \)

\( C_1 \triangleq (P_r h_1 h_1^T - \tau \sigma_1^2 I) \otimes g^s_1^T, \)

\( D_i^1 \triangleq (\alpha^2 r_{1d} + \frac{\sigma_1^2}{1 - r_{1d}} \tau_i), \)

\( E_1 \triangleq (P_r h_1 h_1^T + P_r h_1 h_1^T + \sigma_2 I) \otimes I. \)

Since the optimal solution of (11) is difficult to be obtained, we construct an equivalent SDP problem as follows

\[
\begin{align*}
\max_{W, 0 \leq Q_s \leq 0} & \quad \text{Tr}(A_1 \overline{W}) + \text{Tr}(B_1 Q_s) \\
& \quad \text{s.t.} \quad \text{Tr}(C_1 \overline{W}) - \text{Tr}(\tau_i g^s_i^T Q_s) \geq D_i^1, \ i = 1, 2, \\
& \quad \text{Tr}(E_1 \overline{W}) + \text{Tr}(Q_s) \leq P_r. \tag{12}
\end{align*}
\]
Even that problem (12) is convex which can be efficiently solved by CVX [32], however, it only holds for the existing rank-one optimal solution of \( \mathbf{W} \). Consequently, we have the following lemma.

Lemma 1: The rank-one optimal solution of the problem in (12) always exists.

Proof: See Appendix A.

After acquiring the optimal rank-one solution of (12), we can get the optimal solution of (11) and then the optimal solution of (7).

2) Optimize \( P_1, P_2 \) and \( \rho \) for fixed \( \mathbf{W} \) and \( \mathbf{Q}_c \): In the second step, we need to optimize the power \( P_1, P_2 \) and the power ratio \( \rho \) with the remaining variables fixed. The corresponding optimization problem can be reformulated as

\[
\begin{align*}
\max_{P_1, P_2, \rho} & \quad \alpha(E_1 - \frac{P_1 T}{2}) + \beta(E_2 - \frac{P_2 T}{2}) \\
\text{s.t.} & \quad \text{SINR}_{AF}^k \geq \tau_i, \quad i = 1, 2, \\
& \quad P_1||\mathbf{W}_1||_F^2 + P_2||\mathbf{W}_2||_F^2 + \text{Tr}(\mathbf{Q}_c) + \sigma_r^2||\mathbf{W}||_F^2 \leq P_r, \\
& \quad 0 < P_i \leq P_{\text{max},i}, \quad i = 1, 2, \\
& \quad 0 < \rho < 1.
\end{align*}
\]

(13)

Similar to (8) and (9), we apply the transformations in (13). As a result, the problem of optimizing the variables \( P_1, P_2 \) and \( \rho \) is equivalent to

\[
\begin{align*}
\max_{P_1, P_2, \rho} & \quad A_2 \rho P_2 + B_2 \rho P_1 - \alpha P_1 - \beta P_2 + C_2 \rho \\
\text{s.t.} & \quad (E_2 P_2 - D_2)(1 - \rho) \geq \tau_1 \sigma_{1,c}^2, \\
& \quad (G_2 P_1 - F_2)(1 - \rho) \geq \tau_2 \sigma_{2,c}^2, \\
& \quad P_1 J_2 + P_2 K_2 \leq P_r - L_2, \\
& \quad 0 < P_1 \leq P_{\text{max},1}, \\
& \quad 0 < P_2 \leq P_{\text{max},2}, \\
& \quad 0 < \rho < 1.
\end{align*}
\]

(14a)

where

\[
\begin{align*}
A_2 & \triangleq \frac{\alpha T}{2} ||\mathbf{g}_1||_F^2 ||\mathbf{W}_1||_2^2 + \frac{\beta T}{2} ||\mathbf{g}_2||_F^2 ||\mathbf{W}_2||_2^2, \\
B_2 & \triangleq \frac{\alpha T}{2} ||\mathbf{g}_3||_F^2 ||\mathbf{W}_1||_2^2 + \frac{\beta T}{2} ||\mathbf{g}_4||_F^2 ||\mathbf{W}_2||_2^2, \\
C_2 & \triangleq \frac{\alpha T}{2} ||\mathbf{Q}_c||_F ||\mathbf{g}_1||_F ||\mathbf{g}_2||_F + \frac{\beta T}{2} ||\mathbf{Q}_c||_F ||\mathbf{g}_3||_F ||\mathbf{g}_4||_F, \\
D_2 & \triangleq ||\mathbf{g}_1||_F ||\mathbf{W}_1||_2^2, \\
E_2 & \triangleq ||\mathbf{g}_2||_F ||\mathbf{W}_2||_2^2, \\
F_2 & \triangleq ||\mathbf{g}_3||_F ||\mathbf{W}_1||_2^2 + ||\mathbf{g}_4||_F ||\mathbf{W}_2||_2^2 + \sigma_{2,c}^2, \\
G_2 & \triangleq ||\mathbf{g}_1||_F ||\mathbf{W}_1||_2^2, \\
J_2 & \triangleq ||\mathbf{W}_1||_2^2, \\
K_2 & \triangleq ||\mathbf{W}_2||_2^2 \quad \text{and} \quad L_2 \triangleq \text{Tr}(\mathbf{Q}_c) + \sigma_r^2||\mathbf{W}||_F^2.
\end{align*}
\]

Problem (14) is still quite complicated as variables \( P_1, P_2 \) and \( \rho \) are coupled. To solve problem (14), we give the following theorem.

Theorem 1: The optimal solution of problem (14) can be obtained in closed-form by comparing the following eight cases:

- When the constraints (14b) and (14c) hold with equality, the optimal solutions \( \{P_1^*, P_2^*, \rho^*\} \) are given by

\[
\begin{align*}
P_1^* &= \frac{\tau_1 \sigma_{1,c}^2}{1 - \rho} + F_2, \\
P_2^* &= \frac{\tau_2 \sigma_{2,c}^2}{1 - \rho} + D_2, \\
\rho^* &= 1 - \sqrt{\frac{d_1}{a_1}}.
\end{align*}
\]

(15)

where \( a_1 \triangleq -(A_2 D_2 G_2 + B_2 E_2 F_2 + C_2 E_2 G_2), a_2 \triangleq A_2 G_2 B_2 \sigma_{1,c}^2 + B_2 E_2 \sigma_{2,c}^2 + A_2 D_2 G_2 + B_2 E_2 F_2 + C_2 E_2 G_2 \) and \( a_3 \triangleq A_2 E_2 \sigma_{1,c}^2 + B_2 G_2 \sigma_{1,c}^2 \).

- When the constraints (14b) and (14d) hold with equality, the optimal solutions \( \{P_1^*, P_2^*, \rho^*\} \) are given by

\[
P_1^* = \frac{P_r - L_2 - P_2^* K_2}{J_2}, \\
P_2^* = \frac{\tau_1 \sigma_{1,c}^2}{1 - \rho} + D_2, \\
\rho^* = 1 - \sqrt{\frac{b_1 + b_2}{J_2 E_2 b_3}}.
\]

(16)

where \( b_1 \triangleq (A_2 J_2 - B_2 K_2) \tau_1 \sigma_{1,c}^2, b_2 \triangleq (a K_2 - \beta J_2) \tau_1 \sigma_{1,c}^2 \) and \( b_3 \triangleq (P_1 E_2 - L_2 - D_2 - G_2) B_2 + (A_2 D_2 + C_2 E_2) J_2 \).

- When the constraints (14b) and (14e) hold with equality, the optimal solutions \( \{P_1^*, P_2^*, \rho^*\} \) are given by

\[
P_1^* = P_{\text{max},1}, \\
P_2^* = \frac{\tau_1 \sigma_{1,c}^2}{1 - \rho} + D_2, \\
\rho^* = 1 - \sqrt{\frac{c_1 - e_1}{E_2 \gamma c_3}}.
\]

(17)

where \( c_1 \triangleq A_2 \tau_1 \sigma_{2,c}^2, c_2 \triangleq \beta \tau_1 \sigma_{2,c}^2 \) and \( c_3 \triangleq A_2 D_2 + B_2 \rho P_{\text{max},1} + C_2 E_2 G_2 \).

- When the constraints (14c) and (14d) hold with equality, the optimal solutions \( \{P_1^*, P_2^*, \rho^*\} \) are given by

\[
P_1^* = \frac{\tau_2 \sigma_{2,c}^2}{1 - \rho} + F_2, \\
P_2^* = \frac{P_r - L_2 - P_1^* J_2}{K_2}, \\
\rho^* = 1 - \sqrt{\frac{d_1 + d_2}{K_2 G_2 D_3}}.
\]

(18)

where \( d_1 \triangleq (B_2 K_2 - A_2 J_2) \tau_2 \sigma_{2,c}^2, d_2 \triangleq (\beta J_2 - a K_2) \tau_2 \sigma_{2,c}^2 \) and \( d_3 \triangleq (P_1 E_2 - G_2 F_2 - F_2 J_2) A_2 (P_1 E_2 - G_2 F_2 - F_2 J_2) B_2 + (A_2 D_2 + C_2 E_2) K_2 \).

- When the constraints (14c) and (14f) hold with equality, the optimal solutions \( \{P_1^*, P_2^*, \rho^*\} \) are given by

\[
P_1^* = \frac{\tau_2 \sigma_{2,c}^2}{1 - \rho} + F_2, \\
P_2^* = P_{\text{max},2}, \\
\rho^* = 1 - \sqrt{\frac{e_2 - e_1}{G_2 c_3}},
\]

(19)

where \( e_1 \triangleq B_2 \tau_2 \sigma_{2,c}^2, e_2 \triangleq \alpha \tau_2 \sigma_{2,c}^2 \) and \( e_3 \triangleq B_2 \tau_2 \sigma_{2,c}^2 + C_2 E_2 G_2 \).

- When the constraints (14d) and (14e) hold with equality, the optimal solutions \( \{P_1^*, P_2^*, \rho^*\} \) are given by

\[
P_1^* = P_{\text{max},1}, \\
P_2^* = \frac{P_r - L_2 - J_2 P_{\text{max},1}}{K_2}, \\
\rho^* = \min\{1 - \frac{\tau_1 \sigma_{1,c}^2}{E_2 P_2^* - D_2}, 1 - \frac{\tau_2 \sigma_{2,c}^2}{G_2 P_1^* - F_2}\}.
\]

(20)
The proposed iterative algorithm 1.

1. Set $L_{\text{max}} = 1000$ (maximum number of iterations); $l = 0$;
   $\epsilon = 10^{-6}$ (convergence tolerance); $E_{\text{d,eff}} = 1000$. $E_0^* = 0$.
2. Initialize $P_1 = P_{\text{max},1}$, $P_2 = P_{\text{max},2}$, and $\rho = 0.5$.
3. While $E_{\text{d,eff}}^l \geq \epsilon$ and $l < L_{\text{max}}$ do:
   4. Calculate $\mathbf{W}$ and $\mathbf{Q}_2$ of problem (12) by CVX [32], then get the optimal $\{\mathbf{W}, \mathbf{Q}_2\}$ of (7) by using eigenvalue decomposition (EVD).
   5. Calculate $P_1$, $P_2$ and $\rho$ of problem (14) by substituting (15) into (14a).
   6. Calculate the corresponding harvested energy $E_1^l$ and $E_2^l$, and let the weighted sum energy $E_{\text{d,eff}}^l = \alpha(E_1^l - E_2^l) + \beta(E_2^l - E_2^l)$.
   7. $E_{\text{d,eff}}^l = |E_1^l - E_2^l|$: $E_{\text{d,eff}}^l$.
   8. $|E_1^l - E_2^l|$
   9. $l = l + 1$.
10. Until convergence.

- When the constraints (14d) and (14f) hold with equality, the optimal solutions $\{P_1^*, P_2^*, \rho^*\}$ are given by
  \[
  P_1^* = \frac{P_r - L - K_2 P_{\text{max},2}}{J_2}, \quad P_2^* = P_{\text{max},2},
  \]
  \[
  \rho^* = \min\{1 - \frac{\tau_1\sigma_{1,c}^2}{E_2 P_2^* - D_2}, \frac{1 - \frac{\tau_2\sigma_{2,c}^2}{4}}{G_2 P_1^* - F_2}\}. \tag{21}
  \]
- When the constraints (14e) and (14f) hold with equality, the optimal solutions $\{P_1^*, P_2^*, \rho^*\}$ are given by
  \[
  P_1^* = P_{\text{max},1}, \quad P_2^* = P_{\text{max},2},
  \]
  \[
  \rho^* = \min\{1 - \frac{\tau_1\sigma_{1,c}^2}{E_2 P_2^* - D_2}, \frac{1 - \frac{\tau_2\sigma_{2,c}^2}{4}}{G_2 P_1^* - F_2}\}. \tag{22}
  \]

Proof: See Appendix B.

We compare the objective function values by substituting (15) into (14a) and select one with the highest objective function value as the optimal solution.

3) Iterative optimization algorithm

By combining the solution processes in steps 1) and 2), the optimal design for AF relaying strategy can be achieved. For clarity, the detailed procedure of the iterative optimization algorithm is listed in Table 1.

Lemma 2: The proposed iterative algorithm listed in Table 1 converges.

Proof: See Appendix C.

B. JOINT DESIGN FOR DF-XOR RELAYING STRATEGY

In this subsection, we consider optimization problem (5) to determine the relay node adopts the DF-XOR TWR strategy. Similar to problem (3), we decouple problem (5) into two subproblems. It is worth noting that here different from (3), two subproblems from (5) only involve the beamforming vectors and PS ratios, where $P_1$ and $P_2$ are not necessary to be optimized as they are determined via the constraints included in (4).

1) Optimize $\mathbf{Q}_2$ and $\mathbf{Q}_1$ for fixed $\rho$: Note that when fixing $\rho$, the problem of optimizing variables $\mathbf{Q}_2$ and $\mathbf{Q}_1$ can be equivalent to

\[
\max_{\mathbf{Q}_1 \geq 0, \mathbf{Q}_2 \geq 0} \alpha(g_1^T \mathbf{Q}_1 g_1^T + g_2^T \mathbf{Q}_2 g_2^T)
+ \beta(g_1^T \mathbf{Q}_1 g_2^T + g_2^T \mathbf{Q}_2 g_1^T),
\]

s.t. $\text{SINR}^\text{XOR}_i \geq \tau_i, i = 1, 2$.

$\text{Tr}(\mathbf{Q}_1) + \text{Tr}(\mathbf{Q}_2) \leq P_r,$ \tag{23}

which is rewritten as

\[
\max_{\mathbf{Q}_1 \geq 0, \mathbf{Q}_2 \geq 0} \text{Tr}(\mathbf{A}_3 \mathbf{Q}_1) + \text{Tr}(\mathbf{A}_3 \mathbf{Q}_2),
\]

s.t. $\text{Tr}(\mathbf{B}_3 \mathbf{Q}_1) - \text{Tr}(\tau_1 \mathbf{B}_3 \mathbf{Q}_1) \geq D_3$,

$\text{Tr}(\mathbf{C}_3 \mathbf{Q}_1) - \text{Tr}(\tau_2 \mathbf{C}_3 \mathbf{Q}_1) \geq E_3$,

$\text{Tr}(\mathbf{Q}_1) + \text{Tr}(\mathbf{Q}_2) \leq P_r,$ \tag{24}

where $A_3 \triangleq \alpha g_1^T g_1^T + \beta g_1^T g_2^T, B_3 \triangleq g_1^T g_2^T, C_3 \triangleq g_2^T g_2^T, D_3 \triangleq (\sigma_{1,d}^2 + \sigma_{1,c}^2)\tau_1$ and $E_3 \triangleq (\sigma_{2,d}^2 + \sigma_{2,c}^2)\tau_2$. It is easy to verify that (24) is a standard SDP problem. Thus, its optimal solution $\{\mathbf{Q}^*_1, \mathbf{Q}^*_2\}$ can be easily obtained using existing software, e.g., CVX [32].

2) Optimize $\rho$ for fixed $\mathbf{Q}_2$ and $\mathbf{Q}_1$: In the second step, we need to maximize the PS ratio $\rho$ with the remaining variables fixed. The corresponding optimization problem can be reformulated as

\[
\max_{\rho} \frac{\alpha \eta T}{2} \rho(\mathbf{g}_1^T \mathbf{Q}_1 g_1^T + \mathbf{g}_2^T \mathbf{Q}_2 g_2^T) - \frac{\alpha T}{2} P_1
+ \frac{\beta \eta T}{2} \rho(\mathbf{g}_1^T \mathbf{Q}_1 g_2^T + \mathbf{g}_2^T \mathbf{Q}_2 g_1^T) - \frac{\beta T}{2} P_2,
\]

s.t. $\text{SINR}^\text{XOR}_i \geq \tau_i, i = 1, 2$.

\[
0 < \rho < 1, \tag{25}
\]

which is equivalent to

\[
\max_{\rho} (A_4 + B_4)\rho - \frac{T}{2}(\alpha P_1 + \beta P_2)
\]

s.t. $C_4(1 - \rho) \geq \tau_1 \sigma_{1,c}^2$,

$D_4(1 - \rho) \geq \tau_2 \sigma_{2,c}^2$,

\[
0 < \rho < 1, \tag{26}
\]

where $A_4 \triangleq \frac{\alpha \eta T}{2}(\mathbf{g}_1^T \mathbf{Q}_1 g_1^T + \mathbf{g}_2^T \mathbf{Q}_2 g_2^T), B_4 \triangleq \frac{\beta \eta T}{2}(\mathbf{g}_1^T \mathbf{Q}_1 g_2^T + \mathbf{g}_2^T \mathbf{Q}_2 g_1^T), C_4 \triangleq \mathbf{g}_1^T \mathbf{Q}_1 g_1^T - (\mathbf{g}_1^T \mathbf{Q}_1 g_2^T + \sigma_{1,d}^2)\tau_1$ and $D_4 \triangleq \mathbf{g}_2^T \mathbf{Q}_2 g_2^T - (\mathbf{g}_2^T \mathbf{Q}_2 g_1^T + \sigma_{2,d}^2)\tau_2$. According to the definition of the minimum transmit powers in (4), the simplified PS design problem yields the following problem

\[
\max_{\rho} (A_4 + B_4)\rho
\]

s.t. $\rho \leq \frac{\tau_1 \sigma_{1,c}^2}{C_4}$,

$\rho \leq \frac{\tau_2 \sigma_{2,c}^2}{D_4}$,

\[
0 < \rho < 1. \tag{27}
\]

It can be observed that the objective function in (27) achieves a higher value when one of the SINR constraints holds with equality. Hence, the optimal solution $\rho^* = \min\{1 - \frac{\tau_1 \sigma_{1,c}^2}{C_4}, 1 - \frac{\tau_2 \sigma_{2,c}^2}{D_4}\}$ can be obtained from problem (27).

3) Iterative optimization algorithm

Table 2 summarizes the overall algorithm to find the final solution of (5). Note that Algorithm 2 differs from Algorithm 1 in two main aspects: First, in step 1), the optimal beamforming matrices $\{\mathbf{Q}^*_1, \mathbf{Q}^*_2\}$ can be easily obtained due to the absence of rank-one constraint; and second,
TABLE 2. The proposed iterative algorithm 2.

1. Set $L_{\text{max}} = 1000$ (maximum number of iterations); $l = 0$; $\varepsilon = 10^{-5}$ (convergence tolerance); $E_{\text{diff}}^l = 1000$; $E_{\text{diff}}^0 = 0$.
2. Initialize $\rho = 0.5$.
3. While $E_{\text{diff}}^l \geq \varepsilon$ and $l < L_{\text{max}}$ do:
   4. Calculate $Q_i$ and $Q_2$ of problem (24) by CVX [32].
   5. Calculate $\rho$ using (27) for fixed $(Q_i, Q_2)$.
   6. Calculate the corresponding harvested energy $E_i^l$ and $E_2^l$ and let the weighted sum energy $E_{\text{XOR}}^l = \alpha(E_i^l - D_2^l) + \beta(E_2^l - D_2^l)$.
   7. $E_{\text{diff}}^{l+1} = E_{\text{diff}}^l - E_{\text{XOR}}^l$.
   8. $E_{\text{diff}}^{l+1} = E_{\text{diff}}^l$.
   9. $l = l + 1$.
10. Until convergence.

TABLE 3. The proposed iterative algorithm 3.

1. Set $L_{\text{max}} = 1000$ (maximum number of iterations); $l = 0$; $\varepsilon = 10^{-5}$ (convergence tolerance); $E_{\text{diff}}^l = 1000$; $E_{\text{diff}}^0 = 0$.
2. Initialize $\rho = 0.5$.
3. While $E_{\text{diff}}^l \geq \varepsilon$ and $l < L_{\text{max}}$ do:
   4. Calculate $(Q_i, Q_2, Q_3)$ of problem (29) by CVX [32].
   5. Calculate $\rho$ using (32) for fixed $(Q_i, Q_2, Q_3)$ and $(Q_4)$.
   6. Calculate the corresponding harvested energy $E_1^l$ and $E_2^l$ and let the weighted sum energy $E_{\text{SUP}}^l = \alpha(E_1^l - D_2^l) + \beta(E_2^l - D_2^l)$.
   7. $E_{\text{diff}}^{l+1} = |E_0^l - E_{\text{SUP}}^l|$.
   8. $E_{\text{diff}}^{l+1} = E_{\text{diff}}^l$.
   9. $l = l + 1$.
10. Until convergence.

0 $< \rho < 1$, (30)

which is equivalent to

$$\max_\rho \quad (A_6 + B_6)\rho - \frac{T}{2}(\alpha P_1 + \beta P_2)$$

s.t. $C_6(1 - \rho) \geq 1\tau_1\sigma_{i,c}^2$, $D_6(1 - \rho) \geq 1\tau_2\sigma_{i,c}^2$, $0 < \rho < 1$. (31)

where $A_6 \triangleq \frac{\alpha\rho T}{2}(g_1^T Q_i g_i^* + g_2^T Q_i g_2^* + g_1^T Q_2 g_1^* + g_2^T Q_2 g_2^*)$, $B_6 \triangleq \frac{\beta\rho T}{2}(g_1^T Q_3 g_1^* + g_2^T Q_3 g_2^* + g_1^T Q_4 g_1^* + g_2^T Q_4 g_2^*)$, $C_6 \triangleq g_i^T Q_i g_i^* - (g_1^T Q_3 g_1^* + \sigma_{i,d}^2)\tau_1$ and $D_6 \triangleq g_i^T Q_3 g_i^* - (g_1^T Q_4 g_1^* + \sigma_{i,d}^2)\tau_2$. Since $P_1$ and $P_2$ are determined based on the first phase, problem (31) is simplified as

$$\max_\rho \quad (A_6 + B_6)\rho$$

s.t. $\rho \leq 1 - \frac{\tau_1\sigma_{i,c}^2}{C_6}$, $\rho \leq 1 - \frac{\tau_2\sigma_{i,c}^2}{D_6}$, $0 < \rho < 1$. (32)

Similar to the problem (27), the optimal PS solution $\rho^* = \min\{1 - \frac{\tau_1\sigma_{i,c}^2}{C_6}, 1 - \frac{\tau_2\sigma_{i,c}^2}{D_6}\}$ can be obtained from problem (32).

3) Iterative optimization algorithm

The proposed iterative algorithm for DF-SUP is summarized in Table 3.

V. SIMULATION RESULTS

In this section, we numerically evaluate the performance of the proposed energy harvesting schemes. The channel vectors $h_i$ and $g_i$ are set to be Rayleigh fading, i.e., the elements of each channel matrix or vector are complex Gaussian random variables with zero mean and unit variance. The channel gain is assumed to be the distance path loss model [24], which can be estimates as $g_{i,j} = c \cdot d_{i,j}^{-n}$, where $c$ and $n$ denote the attenuation constant and the path loss exponent with the fixed values as 1 for $c$ and 3 for $n$, respectively. Moreover, $d_{i,j}$ denotes the distance between nodes $i$ and $j$. We further assume that the noise variances are equivalent, i.e., $\sigma_{i,c}^2 = \sigma_{i,d}^2$.

step 2) only involves PS ratios $\rho$, $P_1$ and $P_2$ are not necessary to be optimized.

C. JOINT DESIGN FOR DF-SUP RELAYING STRATEGY

In this subsection, we consider that the DF-SUP relaying strategy is adopted at the relay node. To find the optimal solution of problem (6), we similarly decouple problem (6) into two subproblems, and then propose an iterative algorithm to obtain a finally solution of the original optimization problem.

1) Optimize $Q_i$, $Q_2$, and $Q_4$, with $Q_3$ fixed $\rho$: In the first step, we need to optimize the beamforming matrices $Q_i$, $Q_2$, and $Q_4$ with the PS ratio $\rho$ fixed. The corresponding optimization problem can be formulated as

$$\max_{Q_i, Q_2, Q_4, \rho} \quad \alpha(g_1^T Q_1 g_1^* + g_2^T Q_2 g_2^*)$$

$$+ \beta(g_1^T Q_1 g_1^* + g_2^T Q_2 g_2^*)$$

s.t. $\text{SINR}_{i}^{\text{SUP}} \geq \tau_i$, $i = 1, 2$.

$$\text{Tr}(Q_i) + \text{Tr}(Q_2) + \text{Tr}(Q_4) \leq P_r$$

which is rewritten as

$$\max_{Q_i, Q_2, Q_4} \quad \text{Tr}(A_5(Q_i, Q_2, Q_4))$$

s.t. $\text{Tr}(B_5 Q_4) - \text{Tr}(1\rho B_5 Q_4) \geq D_5$$

$$\text{Tr}(C_5 Q_5) - \text{Tr}(1\rho C_5 Q_5) \geq E_5$$

$$\text{Tr}(Q_i) + \text{Tr}(Q_2) + \text{Tr}(Q_4) \leq P_r$$

where $A_5 \triangleq \alpha g_1^T g_1^* + \beta g_2^T g_2^*$, $B_5 \triangleq g_1^T g_1^* + \sigma_{i,d}^2$, $C_5 \triangleq g_2^T g_2^*$, $D_5 \triangleq (\sigma_{i,d}^2 + \frac{\tau_1^2}{\sigma_{i,c}^2})\tau_1$ and $E_5 \triangleq (\sigma_{i,d}^2 + \frac{\tau_2^2}{\sigma_{i,c}^2})\tau_2$. Note that (29) is a standard SDP problem. Thus, its optimal solution can be easily obtained via CVX [32].

2) Optimize $\rho$ for fixed $Q_i$, $Q_2$, and $Q_4$: In the second step, we need to optimize the PS ratio $\rho$ with the remaining variables fixed. The corresponding optimization problem can be formulated as

$$\max_\rho \quad \frac{\alpha T}{2}(g_1^T Q_1 g_1^* + g_2^T Q_2 g_2^*) - \frac{\alpha T}{2}P_1$$

$$+ \frac{\beta T}{2}(g_1^T Q_3 g_1^* + g_2^T Q_3 g_2^*) - \frac{\beta T}{2}P_2$$

s.t. $\text{SINR}_{i}^{\text{SUP}} \geq \tau_i$, $i = 1, 2$.
In addition, the maximum transmit powers at the two sources, if not specified, are fixed with $P_{\text{max},1} = P_{\text{max},2} = 1.25$ W. Moreover, the results given in the following examples are obtained by using 1000 independent channel realizations.

In Fig. 3, the harvested energy for AF relaying strategy with different priority at source nodes is shown where the relay node is assumed to be with $N = 4$ transmit antennas and the distances are with $d_{R,S_1} = d_{R,S_2} = 5$ meters and $d_{R,S_1} = 5$ meters, $d_{R,S_2} = 10$ meters. From simulation results illustrated in Fig. 3(a), for the case of equivalent distances between the two source nodes and the relay, it is easy to see that if $S_1$ and $S_2$ are with the same priority, i.e., $\alpha = \beta = 0.5$, the two nodes can obtain a fair EH efficiency. When $S_1$ and $S_2$ have different priorities, i.e., $\alpha = 0.8$ and $\beta = 0.2$, the node $S_1$ can harvest more energy since its energy weight factor is set to be a larger value. However, it is noted that for the asymmetric case as illustrated in Fig. 3(b), although $S_1$ and $S_2$ are with the same priority, the node $S_2$ still harvests much lower energy. This is because that the location of $S_2$ is far away from the relay node $R$, which result in a very small channel gain as compared to the near node. Nevertheless, when with higher priority, i.e., $\beta = 0.9$, it is easy to see that node $S_2$ can share more energy for the harvested total energy, which can provide an effective solution to the doubly-near-far problem [2].

Secondly, in Fig. 4, for DF-XOR and DF-SUP relaying strategy, we illustrate the weighted sum energy of the two source nodes, i.e., $E_{\text{XOR}}$ and $E_{\text{SUP}}$, and the shared energy ratio at the $S_2$ node, i.e., $\frac{E_{S_2}}{E_{S_1} + E_{S_2}}$, in different distances and different priorities. It is noted that in Fig. 4(a), compared with the symmetric scenario, although $S_1$ and $S_2$ have same priority, i.e., $\alpha = \beta = 0.5$, the two source nodes still obtains much lower the weighted sum energy in the asymmetric scenario. Moreover, note that when the relay transmit power, $P_r$, is low, the weighted sum energy of the two source nodes is negative, which implies that the harvested energy from the relay is smaller than the consumed energy for signal transmission.
However, in the asymmetric case, when with higher priority, i.e., $\beta = 0.7$ and $\beta = 0.9$, in Fig. 4(b), we find that the node $S_2$ all can share more energy for the harvested total energy in different relaying strategies. This indicates that under the asymmetric scenario, the doubly-near-far problem [2] could be mitigated effectively by setting different EH priorities for different source nodes. In addition, from Fig. 4(b), we also find that when the distances of the two source nodes are asymmetric, by using the DF-SUP strategy and applying the proposed optimal energy harvesting scheme, the node $S_2$ can achieve a higher EH efficiency.

In Fig. 5, for three relay strategies, we compare the proposed joint optimization scheme with the other two schemes, i.e., only precoding scheme and only power allocation scheme, respectively. For fair comparison, the priorities and the distances of two source nodes are set to be the same, i.e., $\alpha = \beta = 0.5$, $d_{R,S_1} = d_{R,S_2} = 5$ meters and the number of antennas at relay is $N = 4$. In only precoding scheme, the sources transmit power and PS ratio are $P_1 = P_{\text{max},1}$, $P_2 = P_{\text{max},2}$ and $\rho = 0.5$. In only power allocation scheme, besides above the setting, the beamforming matrixes are identity matrices. From simulation results, for three considered two-way relaying strategies, we find that the joint optimization scheme achieves the optimal performance as it uses the DoF of both power, PS ratio allocation and precoding. Nevertheless, it is worth noting that when the relay transmit power is small, the proposed joint design scheme achieves lower the weighted sum energy than the only power allocation scheme, and then outperforms the latter as $P_r$ increases. The main reason is that the joint design scheme can always use the maximum available relay transmit power to improve the total harvested energy. Moreover, Fig. 5 also shows that the only precoding scheme can improve the system performance and it performs much better than the only power allocation scheme. In addition, we find that the DF-XOR relaying strategy achieves the best performance, and the DF-SUP relaying strategy outperforms the AF relaying strategy. This indicates that DF relaying strategy has a higher EH efficiency due to the assumption that the relay has enough processing ability to correctly decode the received signals. Moreover, combining the information using XOR is better than using superposition since the power of the relay node can be used more efficiently in the DF-XOR relaying strategy.

The impact of the maximum source transmit power on the weighted sum energy with three different relay strategies is shown in Fig. 6. For fairness, we set $\alpha = \beta = 0.5$, $d_{R,S_1} = d_{R,S_2} = 5$ meters and $P_r = 10$ W. In this case, with all considered three relay strategies, we find that the weighted sum energy is not improved as the maximum source transmit power increases. The main reason is that unlike the relay, two sources need to adjust its transmit power rather than using full power. Thus, increasing the power budget of the source nodes does not necessarily improve the performance.

Finally, in Fig. 7, we consider the impact of the number of antennas at relay on the weighted sum energy. Here, the setting of each node is the same with the one in Fig. 6. From simulation results, it is observed that the weighted sum energy steadily increases as more antennas are equipped at the
VI. CONCLUSION

This paper studied the joint energy transmit beamforming and power splitting design for a multi-antenna TWR system with SWIPT. The weighted sum energy at two source nodes was maximized subject to the constraints of the SINR and the transmit powers. Considering three different relaying strategies, the objective problems were first decomposed into two subproblems and then be tackled by the proposed convergent iterative algorithm. At each iteration, the optimal solution of each subproblem can be found. The performance of three relay strategies were compared and some practical implementation issues were also discussed. Simulation results show that, when the priority and the distance of two source nodes are asymmetric, the far node can achieve a higher EH efficiency when it uses the DF-SUP strategy and applied the proposed optimal scheme.

Appendix A

Proof of Lemma 1

Note that, problem (12) is a convex optimization problem, its optimal solution can be easily obtained [32]. Let us denote the optimal solution of $\mathbf{W}$ and $\mathbf{Q}_i$ in (12) by $\mathbf{W}^\ast$ and $\mathbf{Q}_i^\ast$, respectively. It is easy to verify that $\mathbf{W}^\ast$ is an optimal solution of the following optimization problem

$$\begin{align*}
\max_{\mathbf{W} \succeq 0} & \quad \text{Tr}(\mathbf{A}_i \mathbf{W}) \\
\text{s.t.} & \quad \text{Tr}(\mathbf{C}_i \mathbf{W}) \geq D_i^f + \text{Tr}(\mathbf{g}_i^g \mathbf{g}_i^g \mathbf{Q}_i^f), \quad i = 1, 2. \\
& \quad \text{Tr}(\mathbf{E}_i \mathbf{W}) \leq P_r - \text{Tr}(\mathbf{Q}_i^f). 
\end{align*}$$

(33)

According to Lemma 3.1 in [27], there exists an optimal solution $\mathbf{W}^\ast$ for the problem (33) such that

$$\text{Rank}(\mathbf{W}^\ast)^2 \leq 3. \quad (34)$$

It can be verified that $\mathbf{W}^\ast \neq \mathbf{0}$. Thus, from (34), we have $\text{Rank}(\mathbf{W}^\ast) = 1$. The proof of Lemma 1 is thus completed.

Appendix B

Proof of Theorem 1

Suppose that problem (14) is feasible and let $\{P_1^\ast, P_2^\ast, \rho^\ast\}$ and $f(\cdot)$ denote the optimal solution and the objective function, respectively. Next, we show that for problem (14), with the optimal solution $\{P_1^\ast, P_2^\ast, \rho^\ast\}$, either the SINR constraints or the transmit power constraints must hold with equality. We prove this result by contradiction. Namely, if the above conditions are not satisfied, we can find another solution of (14), denoted by $\{P_1^f, P_2^f, \rho^f\}$, which achieves a higher objective function value. First, suppose that the two SINR constraints do not hold with equality. In this case, note that in problem (14), the two SINR constraints are equivalent to

$$\rho^f \leq 1 - \frac{\tau_1 \sigma_{1,c}^2}{EP_2 - D}, \quad (35)$$

and

$$\rho^f \leq 1 - \frac{\tau_2 \sigma_{2,c}^2}{GP_1 - F}. \quad (36)$$

It can be observed that when PS solution $\rho^f = \min\{1 - \frac{\tau_1 \sigma_{1,c}^2}{EP_2 - D}, 1 - \frac{\tau_2 \sigma_{2,c}^2}{GP_1 - F}\}$, i.e., one of the SINR constraints holds with equality, the objective function $f(P_1^f, P_2^f, \rho^f) = (A_2 \rho^f - \beta)P_2^f + (B_2 \rho^f - \alpha)P_1^f + C_2 \rho^f$ achieves a higher value with the same transmit power constraints. Hence, this assumption is not true. Next, consider the case that the transmit power constraints do not hold with equality. In this case, if we want the PS solution $\rho$ to increase to achieve a higher value of objective function, the transmit power constraint $P_i$ increases, which leads to the conclusion that at least a power constraint of (14d), (14e) and (14f) holds with equality. Thus, the assumption that the transmit power constraints do not hold with equality cannot be true. Hence, we conclude that the optimal solution $\{P_1^\ast, P_2^\ast, \rho^\ast\}$ makes either the SINR or transmit power constraint must hold with equality.

Moreover, given the optimal solution $\{P_1^\ast, P_2^\ast, \rho^\ast\}$, we can prove that at least two constraints of problem (14) are achieved with equality. First, suppose that the optimal solution $\{P_1^\ast, P_2^\ast, \rho^\ast\}$ can be obtained if and only if the SINR constraint (14b) holds with equality. In this case, we can easily find another solution of $P_1$ for (14) while two transmit power constraints (14d) or (14e) hold with equality. We denote by $\tilde{P}_1^\ast$ as the optimal solution of $P_1$, it is easy to verify that the value of the objective function under $[\tilde{P}_1^\ast, P_2^\ast, \rho^\ast]$ is larger than that under $[P_1^\ast, P_2^\ast, \rho^\ast]$. Hence, this assumption cannot be true. Similarly, for all the other assumptions where if and only if a constraint holds with equality, we can easily prove these assumptions cannot be true too. In conclusion, for the optimal solution $\{P_1^\ast, P_2^\ast, \rho^\ast\}$, there exist at least two constraints of problem (14) holding with equality. Based on the above observations, we can separate the problem (14) into ten cases by setting the two SINR constraints (14b), (14c) and the three transmit power constraints (14d), (14e), (14f) hold with equality. However, in the ten cases, when the constraints (14b) and (14f) hold with equality, if we want the objective function $f(\cdot)$ to increase, the transmit power solution $P_1$ increases, which leads to the conclusion that the constraints (14d) or (14e) are active. Hence, this combination is included in (14d) and (14f) or (14e) and (14f) implies that this case can be removed. Similarly, constraints (14c) and (14e) combination will also be removed since this case was contained in (14d) and (14e) or (14e) and (14f). Therefore, the optimal solution $\{P_1^\ast, P_2^\ast, \rho^\ast\}$ of problem (14) is able to be obtained in closed-form by comparing following eight cases.
When the two SINR constraints (14b) and (14c) hold with equality, we obtain the following two equations

\[ P_2^* = \frac{\tau_1\sigma_1^2 + D_2}{E_2}, \quad P_1^* = \frac{\tau_2\sigma_2^2 + F_2}{G_2}. \]  

(37)

By substituting (37) into (14a), the objective function \( f(\rho^*) \) can be equivalently written as

\[ f(\rho^*) = A_2\rho^* + B_2\rho^* + F_2 \]

\[ -\alpha \frac{\tau_1\sigma_1^2 + D_2}{G_2} - \beta \frac{\tau_1\sigma_1^2 + D_2}{G_2} + C_2 \rho^*. \]  

(38)

which is further equivalent to

\[ (\rho^*) = \frac{a_1(\rho^*)^2 + a_2\rho^* - a_3}{E_2G_2(1 - \rho^*)^2} - a_4, \]  

(39)

where \( a_1, a_2, a_3 \) are defined as in (15) and \( a_4 \triangleq \alpha F_2/G_2 + \beta D_2/E_2 \). Hence, problem (14) is simplified as

\[
\max_{\rho^*} f(\rho^*) \\
\text{s.t. } P_1^*J_2 + P_2^*K_2 \leq P_r - L_2, \\
0 < P_1^* \leq P_{\text{max},1}, \\
0 < P_2^* \leq P_{\text{max},2}, \\
0 < \rho^* < 1.
\]  

(40)

To proceed to solve (40), we have the following lemma.

**Lemma 3:** The optimal solution \( \rho^* = 1 - \sqrt{\frac{a_1 + a_2 - a_3}{a_1}} \) can be obtained in problem (40) while \( 0 < a_2 - a_3 < a_1 \).

**Proof:** Please refer to Appendix D. Then we obtain the optimal solution in (15).

When the constraints (14b) and (14d) hold with equality, we obtain the following two equations

\[ P_2^* = \frac{\tau_1\sigma_1^2 + D_2}{E_2}, \quad P_1^* = \frac{P_r - L_2 - P_2^*K_2}{J_2}. \]  

(41)

By substituting (41) into (14a), the objective function \( f(\rho^*) \) can be equivalently written as

\[ f(\rho^*) = \frac{-b_3J_2E_2(\rho^*)^2 + (b_3J_2E_2 + b_1)\rho^* + b_2}{J_2E_2(1 - \rho^*)} \]  

(42)

where \( b_1, b_2, b_3 \) are defined as in (16) and \( b_4 \triangleq \frac{(\tau_1\sigma_1^2 + D_2 - \tau_2\sigma_2^2 + F_2)\rho^*}{J_2E_2} \). (42) is equivalent to

\[ f(\rho^*) = \frac{-b_3(1 - \rho^*)^2 + b_1 + b_2}{J_2E_2(1 - \rho^*)^2} - b_1 + b_3 - b_4. \]  

(43)

Then problem (14) is equivalent to the following problem

\[
\max_{\rho^*} f(\rho^*) \\
\text{s.t. } \frac{\tau_1\sigma_1^2 + F_2}{G_2} \leq P_1^* \leq P_{\text{max},1}, \\
0 < P_1^* \leq P_{\text{max},2}, \\
0 < \rho^* < 1.
\]  

(44)

Similar to Lemma 3, when \( b_3 > 0 \) and \( b_1 + b_2 < 0 \), the objective function \( f(\rho^*) \) must have a maximum value, which can be further derived from \( -b_3(1 - \rho^*) = \frac{b_1 + b_2}{J_2E_2(1 - \rho^*)} \). On the other hand, to guarantee the optimal solution \( \rho^* \) satisfying \( 0 < \rho^* < 1 \), we have \( b_1 + b_3 + J_2E_2b_3 > 0 \), which results in an optimal solution \( \rho^* \) of problem (44) given as

\[ \rho^* = 1 - \sqrt{\frac{b_1 + b_3}{J_2E_2b_3}}. \]  

(45)

We thus obtain the solution given in (16).

When the constraints (14b) and (14e) hold with equality, we obtain the following two equations

\[ P_2^* = \frac{\tau_1\sigma_1^2 + D_2}{E_2}, \quad P_1^* = P_{\text{max},1}. \]  

(46)

By substituting (46) into (14a), the objective function \( f(\rho^*) \) can be written as

\[ f(\rho^*) = \frac{-c_3E_2(\rho^*)^2 + (c_3E_2 + c_1)\rho^* - c_2}{E_2(1 - \rho^*)} - c_4. \]  

(47)

where \( c_1, c_2, c_3 \) are defined as in (17) and \( c_4 \triangleq \frac{aE_2P_{\text{max},1} + \beta D_2}{E_2} \). (47) is equivalent to

\[ f(\rho^*) = \frac{-c_3(1 - \rho^*)^2 + c_1 - c_2}{E_2(1 - \rho^*)} - c_1 + c_3 - c_4. \]  

(48)

Then problem (14) is simplified as

\[
\max_{\rho^*} f(\rho^*) \\
\text{s.t. } P_1^* \geq \frac{\tau_1\sigma_1^2 + F_2}{G_2}, \quad P_1^*J_2 + P_2^*K_2 \leq P_r - L_2, \\
0 < P_1^* \leq P_{\text{max},1}, \\
0 < \rho^* < 1.
\]  

(49)

Due to the fact \( c_3 > 0 \), i.e., \( -c_3 < 0 \). Similar to Lemma 3, if \( \frac{c_1 - c_2}{E_2} < 0 \), i.e., \( c_2 > c_1 \), the objective function \( f(\rho^*) \) must have a maximum value, which can be inferred from a fact \( -c_3(1 - \rho^*) = \frac{c_1 - c_2}{E_2(1 - \rho^*)} \). Note that, to guarantee the optimal solution \( \rho^* \) satisfying \( 0 < \rho^* < 1 \), \( c_2 \) must satisfy \( c_2 < c_1 + E_2c_3 \). As a result, the optimal solution \( \rho^* \) of problem (49) can be derived as

\[ \rho^* = 1 - \sqrt{\frac{c_2 - c_1}{E_2c_3}}. \]  

(50)

Then we obtain the optimal solution in (17).

When the constraints (14c) and (14d) hold with equality, we obtain the following two equations

\[
\frac{\tau_1\sigma_1^2 + F_2}{G_2} \leq P_1^* \leq P_{\text{max},1}, \quad P_2^* = \frac{P_r - L_2 - P_1^*J_2}{K_2}. \]  

(51)
By substituting (51) into (14a), the objective function $f(\rho^*)$ can be equivalently written as

$$
\begin{align*}
    f(\rho^*) &= -d_3 K_2 G_2 (\rho^*)^2 + (d_2 K_2 G_2 + d_1) \rho^* + d_2 \over K_2 G_2 (1 - \rho^*) - d_4,
\end{align*}
$$

where $d_1$, $d_2$, $d_3$, and $d_4$ are defined as in (18) and $d_4 \triangleq (P \rho - L_2 - F J_2 + F K_2 a).$ (52) is equivalent to

$$
\begin{align*}
    f(\rho^*) &= -d_3 (1 - \rho^*) + \frac{d_1 + d_2}{K_2 G_2} - d_3 - d_4.
\end{align*}
$$

Then problem (14) can be rewritten as

$$
\begin{align*}
    \max_{\rho^*} & \quad f(\rho^*) \\
    \text{s.t.} & \quad \frac{\tau_1 \sigma_{1,c}^2}{1 - \rho^*} + D_2 \leq P^\ast_2 \leq P_{\max,2}^\ast, \\
                   & \quad 0 < P_1^\ast \leq P_{\max,1}, \\
                   & \quad 0 < \rho^* < 1.
\end{align*}
$$

Then we obtain the optimal solution in (18).

Similar to Lemma 3, when $d_2 > 0$ and $d_1 + d_2 < 0$, the objective function $f(\rho^*)$ must have a maximum value, which can be further derived from $-d_3 (1 - \rho^*) = \frac{d_1 + d_2}{K_2 G_2 (1 - \rho^*)}$. When $d_1 + d_2 + K_2 G_2 d_3 > 0$, the optimal solution $\rho^*$ of problem (54) can be derived as

$$
\rho^* = 1 - \frac{d_1 + d_2}{K_2 G_2 d_3}.
$$

Then we obtain the optimal solution in (18).

When the constraints (14c) and (14f) hold with equality, we obtain the following two equations

$$
\begin{align*}
    P_1^\ast &= \frac{\tau_1 \sigma_{1,c}^2}{1 - \rho^*} + F_2, \\
    P_2^\ast &= P_{\max,2}.
\end{align*}
$$

By substituting (56) into (14a), the objective function $f(\rho^*)$ can be equivalently written as

$$
\begin{align*}
    f(\rho^*) &= -e_3 G_2 (\rho^*)^2 + (e_2 G_2 + e_1) \rho^* - e_2 \over G_2 (1 - \rho^*) - e_4,
\end{align*}
$$

where $e_1$, $e_2$, $e_3$ are defined as in (19) and $e_4 \triangleq \beta G_2 P_{\max,2} + \alpha F_2$. (57) is further equivalent to

$$
\begin{align*}
    f(\rho^*) &= -e_3 (1 - \rho^*) + \frac{e_1 - e_2}{G_2 (1 - \rho^*)} - e_1 \over G_2 + e_3 - e_4.
\end{align*}
$$

Hence, problem (14) is simplified as

$$
\begin{align*}
    \max_{\rho^*} & \quad f(\rho^*) \\
    \text{s.t.} & \quad P_2^\ast \geq \frac{\tau_1 \sigma_{1,c}^2}{1 - \rho^*} + D_2 \over E_2, \\
                   & \quad P_1^\ast J_2 + P_2^\ast K_2 \leq P_r - L_2, \\
                   & \quad 0 < P_1^\ast \leq P_{\max,1}^\ast, \\
                   & \quad 0 < \rho^* < 1.
\end{align*}
$$

Similar to Lemma 3, due to the fact that $e_3 > 0$, if $\frac{e_1 - e_2}{G_2} < 0$, i.e., $e_2 > e_1$, the objective function $f(\rho^*)$ must have a maximum value, which can be inferred from $-e_3 (1 - \rho^*) = \frac{e_1 - e_2}{G_2 (1 - \rho^*)}$. On the other hand, to guarantee the optimal solution $\rho^*$ satisfying $0 < \rho^* < 1$, we must have $e_2 < e_1 + G_2 e_3$, which implies that the optimal solution $\rho^*$ of problem (59) can be derived as

$$
\rho^* = 1 - \frac{\sqrt{e_2 - e_1}}{G_2 e_3}.
$$

Then we obtain the optimal solution in (19).

When the two transmit power constraints (14d) and (14e) hold with equality, we obtain the following two equations

$$
\begin{align*}
    P_1^\ast &= P_{\max,1}, \\
    P_2^\ast &= P - L_2 - J_2 P_{\max,1} \over K_2.
\end{align*}
$$

Based on (61), the two SINR constraints (14b) and (14c) can be equivalently written as

$$
\rho^* \leq 1 - \frac{\tau_1 \sigma_{1,c}^2}{E_2 P_2^\ast - D_2}, \quad (62)
$$

and

$$
\rho^* \leq 1 - \frac{\tau_2 \sigma_{2,c}^2}{G_2 P_1^\ast - F_2}. \quad (63)
$$

Note that, to guarantee the optimal solution $\rho^*$ satisfying $0 < \rho^* < 1$, we must have $0 < \frac{\tau_1 \sigma_{1,c}^2}{E_2 P_2^\ast - D_2} < 1$ and $0 < \frac{\tau_2 \sigma_{2,c}^2}{G_2 P_1^\ast - F_2} < 1$, which implies that the optimal solution $\rho^*$ of problem (14) can be derived as

$$
\rho^* = \min\{1 - \frac{\tau_1 \sigma_{1,c}^2}{E_2 P_2^\ast - D_2}, 1 - \frac{\tau_2 \sigma_{2,c}^2}{G_2 P_1^\ast - F_2}\}. \quad (64)
$$

Then we obtain the optimal solution in (20).

When the constraints (14d) and (14f) hold with equality, we obtain the following two equations

$$
\begin{align*}
    P_2^\ast &= P_{\max,2}, \\
    P_1^\ast &= P - L_2 - K_2 P_{\max,2} \over J_2.
\end{align*}
$$

Then, substituting (65) into (14b) and (14c), respectively, the two SINR constraints can be equivalently written as

$$
\rho^* \leq 1 - \frac{\tau_1 \sigma_{1,c}^2}{E_2 P_2^\ast - D_2}, \quad (66)
$$

and

$$
\rho^* \leq 1 - \frac{\tau_2 \sigma_{2,c}^2}{G_2 P_1^\ast - F_2}. \quad (67)
$$

To guarantee the optimal solution $\rho^*$ satisfying $0 < \rho^* < 1$, we must have $0 < \frac{\tau_1 \sigma_{1,c}^2}{E_2 P_2^\ast - D_2} < 1$ and $0 < \frac{\tau_2 \sigma_{2,c}^2}{G_2 P_1^\ast - F_2} < 1$, which implies that the optimal solution $\rho^*$ of problem (14) can be derived as

$$
\rho^* = \min\{1 - \frac{\tau_1 \sigma_{1,c}^2}{E_2 P_2^\ast - D_2}, 1 - \frac{\tau_2 \sigma_{2,c}^2}{G_2 P_1^\ast - F_2}\}. \quad (68)
$$
Then we obtain the optimal solution in (21).

When the constraints (14e) and (14f) hold with equality, we obtain the following two equations

\[ P_1^* = P_{\text{max},1}, \quad P_2^* = P_{\text{max},2}. \tag{69} \]

Hence, the two SINR constraints (14b) and (14c) can be equivalently written as

\[ \rho^* \leq 1 - \frac{\tau_1 \sigma_{1,c}^2}{E_2 P_2^* - D_2}, \tag{70} \]

and

\[ \rho^* \leq 1 - \frac{\tau_2 \sigma_{2,c}^2}{G_2 P_1^* - F_2}. \tag{71} \]

Similar to above discussion, the optimal solution \( \rho^* \) of problem (14) can be derived as

\[ \rho^* = \min \{ 1 - \frac{\tau_1 \sigma_{1,c}^2}{E_2 P_2^* - D_2}, 1 - \frac{\tau_2 \sigma_{2,c}^2}{G_2 P_1^* - F_2} \}. \tag{72} \]

Then we obtain the optimal solution in (22). The proof of Theorem 1 is thus completed.

Appendix C
Proof of Lemma 2
At the \( t \)th iteration, since the two subproblems can be optimally solved by steps 4 and 5 in Table 1, the objective function value of problem (3) must monotonically non-decreasing for this step. Because if the objective value \( E_{AF}^t \) will decrease, we can keep the optimal solutions \( \{W^{l-1}, Q_s^{l-1}\} \) or \( \{P_1^{l-1}, P_2^{l-1}, \rho^{l-1}\} \) unchanged. In addition, the constraints of problem (3) are bounded. Therefore, the objective value \( E_{AF} \) is bounded as well. Hence, we conclude that the proposed iterative algorithm can converge. The proof of Lemma 2 is thus completed.

Appendix D
Proof of Lemma 3
First, the objective function \( f(\rho^*) \) in problem (40) is equivalently written as

\[ f(\rho^*) = \frac{a_1((\rho^*)^2 - 1)}{E_2 G_2 (1 - \rho^*)} + \frac{a_2 (\rho^* - 1)}{E_2 G_2 (1 - \rho^*)} + \frac{a_1 + a_2 - a_3}{E_2 G_2} - a_4. \tag{73} \]

According to the property of the function \( f(x) = ax + \frac{b}{x} \), the objective function \( f(\rho^*) \) has a maximum value when \( \frac{a_1}{E_2 G_2} < 0 \) and \( \frac{a_1 + a_2 - a_3}{E_2 G_2} < 0 \). Due to the fact that \( a_1 < 0 \) and \( E_2 G_2 > 0 \), we have \( \frac{a_1}{E_2 G_2} < 0 \). Hence, if \( a_1 + a_2 - a_3 \) is i.e., \( a_2 - a_3 < -a_1 \), the objective function \( f(\rho^*) \) must exist the maximum value, which can be inferred from the fact \( \frac{a_1 ((1 - \rho^*)^2)}{E_2 G_2} = \frac{a_1 + a_2 - a_3}{E_2 G_2 (1 - \rho^*)} \). Note that, to guarantee the optimal solution \( \rho^* \) satisfying \( 0 < \rho^* < 1 \), we also let \( a_2 - a_3 < 0 \).

As a result, the optimal solution \( \rho^* \) of problem (40) can be derived as

\[ \rho^* = 1 - \sqrt{\frac{a_1 + a_2 - a_3}{a_1}}. \tag{74} \]

Next, we show that \( a_1 + a_2 - a_3 > 0 \) cannot happen at the optimal solution \( \rho^* \). We prove this result by contradiction. In this case, if we want the objective function \( f(\rho^*) \) to increase in (73), the optimal solution \( \rho^* \) will be \( \rho^* \rightarrow 1 \), which implies that the transmit power solution \( P_1^* \rightarrow \infty \). It is easy to verify that the above case cannot happen due to the transmit power constraints in problem (40). In conclusion, the optimal solution \( \rho^* \) of (40) can be obtained while \( 0 < a_2 - a_3 < -a_1 \). The proof of Lemma 3 is thus completed.

REFERENCES
WEI WANG received the B.S. degree in electronic and information engineering from China West Normal University, China, in 2005, the M.S. degree in signal and information processing from Chengdu University of Technology, China, in 2008, and the Ph.D. degree in communication and information system from Shanghai University, China, in 2011. From 2011 to 2014, he was a Lecturer with the School of Electronics and Information, Nantong University. In 2016, he was a Visiting Scholar with the Department of Electrical and Computer Engineering, Boise State University, Boise, ID, USA. He is currently an Associate Professor with the School of Electronics and Information, Nantong University. His current research interests include wireless information and power transfer, cooperative communication, full-duplex communication, and digital image processing.

HANI MEHRPOUYAN (S’05–M’10) received the B.Sc. degree (Hons.) from Simon Fraser University, Burnaby, BC, Canada, in 2004, and the Ph.D. degree in electrical engineering from Queen’s University, Kingston, ON, Canada, in 2010. From 2010 to 2012, he held a postdoctoral position with Chalmers University of Technology, Gothenburg, Sweden, where he led the MIMO aspects of the microwave backhauling for next generation wireless networks project. In 2012, he joined the University of Luxembourg, Luxembourg, as a Research Associate, where he was responsible for new interference cancellation and synchronization schemes for satellite communication links. From 2012 to 2015, he was an Assistant Professor with California State University, Bakersfield, CA, USA. Since 2015, he has been an Assistant Professor with Boise State University, Boise, ID, USA. His research interests include applied signal processing and physical layer of wireless communication systems, including millimeter-wave systems, reconfigurable antennas, energy harvesting systems, synchronization, and channel estimation.

Dr. Mehrpouyan has close to 50 publications in prestigious IEEE journals and conferences. He served as a TPC Member for the IEEE Globecom, ICC, and VTC. He is an Associate Editor of the IEEE COMMUNICATION LETTERS. He has been involved with industry leaders, such as Ericsson AB and Blackberry. He has received many scholarships and awards, including the IEEE Globecom Early Bird Student Award, NSERC-IRDF, NSERC PGS-D, NSERC CGS-M Alexander Graham Bell, and B.C. Wireless Innovation.
NAN ZHAO (S’08–M’11–SM’16) received the B.S. degree in electronics and information engineering, the M.E. degree in signal and information processing, and the Ph.D. degree in information and communication engineering from Harbin Institute of Technology, Harbin, China, in 2005, 2007, and 2011, respectively.

From 2011 to 2013, he was a Post-Doctoral Researcher with Dalian University of Technology, Dalian, China. He is currently an Associate Professor with the School of Information and Communication Engineering, Dalian University of Technology. He has authored more than 80 papers in refereed journals and international conferences. His current research interests include interference alignment, cognitive radio, wireless power transfer, and physical layer security.

Dr. Zhao is a Senior Member of the Chinese Institute of Electronics. He serves on the editorial boards of several journals, including the Journal of Network and Computer Applications, the IEEE Access, the Wireless Networks, the Physical Communication, the AEU-International Journal of Electronics and Communications, the Ad Hoc & Sensor Wireless Networks, and the KSII Transactions on Internet and Information Systems. He served as a Technical Program Committee member for many conferences, including Globecom, VTC, and WCSP.

GUOAN ZHANG received the B.S. degree in precision instruments, the M.S. degree in automatic instruments and equipment, and the Ph.D. degree in communication and information systems from Southeast University, Nanjing, China, in 1986, 1989, and 2001, respectively. He is currently a Full Professor with the School of Electronics and Information, Nantong University, Nantong, China. His current research interests include cognitive wireless networks and vehicular ad hoc networks.

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