High-Rate and Low-Complexity Space-Time Block Codes for 2 × 2 MIMO Systems

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***Abstract*—The main design criteria for space-time block codes (STBCs) are the code rate, diversity order, coding gain, and low decoder complexity. In this letter, we propose a full-rate full-diversity STBC for 2 × 2 multiple-input multiple-output (MIMO) systems with a substantially lower maximum likelihood (ML) detection complexity than that of existing schemes. This makes the implementation of high-performance full-rate codes feasible for practical systems. Our numerical evaluation shows that the proposed code achieves significantly lower decoding complexity while maintaining a similar performance compared to that of existing rate-2 STBCs.**

***Index Terms*—Multiple-input multiple-output (MIMO), space-time coding, decoding complexity.**

I. Introduction

Space-time block codes (STBCs) mitigate the effect of fading in wireless channels by introducing spatial and temporal diversity [1]–[3]. There are numerous studies on designing STBCs for multiple-input multiple-output (MIMO) systems [4]–[7]. Many of these STBCs suffer from rate loss, e.g. the Alamouti scheme transmits 1 symbol per channel use which is only half the maximum rate possible with two transmit antennas. In order to compensate the low transmission rate of the Alamouti code, a rate-2 STBC referred to as *Matrix C* was developed in [8]. The Matrix C code is a threaded algebraic space-time code [9], which is known as one of the well-performing STBCs that is already incorporated in the IEEE 802.16e-2005 specifications. Although the Matrix C benefits from full-diversity and full-rate properties, its detection complexity grows as the fourth power of the signal constellation size, and this makes it impractical for low-cost wireless user terminals. In [10], the authors proposed a new high-rate STBC called *maximum transmit diversity (MTD)* code that is designed based on the linear combination of two Alamouti codes. The MTD code has an ML detection complexity of $\mathcal{O}(M^2)$. where M is the cardinality of the signal constellation, and $\mathcal{O}(\cdot)$ denotes the big omicron. Due to the non-orthogonal structure of the MTD code, its ML detection complexity increases quadratically with the constellation size. Recently, in [11], the authors proposed a 2×2 full-rate and linear-receiver (FRLR) STBC whose decoding complexity is of order $\mathcal{O}(M)$ for the ML decoder. However, the FRLR code shows a satisfactory performance for only BPSK and 4-QAM constellations and does not perform well for high-order QAM constellations which are essential to attain high spectral efficiency.

The decoding complexity is very critical for practical employment of MIMO systems. Therefore, the development of low complex decoding algorithms while providing optimal performance is always a necessity for wireless communication systems. In this letter, we propose a full-diversity full-rate STBC for a 2×2 MIMO system. Due to the orthogonal structure of the proposed code, the decoding complexity is reduced to $\mathcal{O}(M)$, which has significant impact on the energy consumption of the receiver especially for higher order modulation schemes. As our numerical evaluation indicate, the proposed code not only achieve significantly lower decoding complexity but also provides a similar or better bit-error-rate performance compared to that of existing rate-2 STBCs.

Notation: Throughout this letter, we use capital, $\mathbf{X}$, and lower, $\mathbf{x}$, boldface letters, for matrices and vectors, respectively. $(\cdot)^T$, $(\cdot)^*$, $||\cdot||_F$, $\det(\cdot)$, and $\mathrm{Re}\{\cdot\}$ denote the transpose, conjugate, Frobenius norm, determinant, and real operators, respectively.

II. Proposed Space-Time Block Code

We consider a single-user 2×2 MIMO system. We construct every 2×2 codeword matrix from four information symbols $\{s_1, s_2, s_3, s_4\}$ that will be sent during $T = 2$ time slots from two antennas. A 2×2 block code, which consists of four symbols, is transmitted by two transmit antennas during $T = 2$ time slots, i.e.,

$$\mathbf{X} = \sqrt{P}\begin{bmatrix} c_1(1) & c_2(1) \\ c_1(2) & c_2(2) \end{bmatrix} = \sqrt{P}\begin{bmatrix} s_1\gamma_1 - s_2^*\eta_1 & s_3\gamma_2 - s_4^*\eta_2 \\ -s_3^*\gamma_2 + s_4\eta_2 & s_1^*\gamma_1 - s_2\eta_1 \end{bmatrix}, \quad (1)$$

where P is the transmit power per antenna, $\gamma_i = \sin(\theta_i)$, $\eta_i = \cos(\theta_i)$, for $i \in \{1, 2\}$. These choices for γ_i and η_i ensure that there is no transmit energy increase, i.e., $\gamma_i^2 + \eta_i^2 = 1$. To simplify the analysis and implementation issues with regard to the proposed code, we consider only a real-valued γ_i and η_i. However, our design procedure can be generalized to a complex-valued γ_i and η_i as well.

Here, we present the methodology for selecting the parameters θ_1 and θ_2 to maximize the diversity and coding gains of the proposed MIMO system. Let us denote two distinct sets of symbols by $\{s_1, s_2, s_3, s_4\}$ and $\{u_1, u_2, u_3, u_4\}$ and construct two distinct STBC codewords $\mathbf{X}$ and $\mathbf{U}$ using (1). Applying the approach in [12], we find the optimal value for θ_i by forming the following optimization problem that maximizes the coding gain and guarantees full diversity

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$$\{\theta_1^o, \theta_2^o\} = \underset{\theta_1,\theta_2 \in [0,\pi/2]}{\operatorname{argmax}} \ \min_{\mathbf{X}\neq\mathbf{U}} |\det[(\mathbf{X}-\mathbf{U})]|$$
$$= \underset{\theta_1,\theta_2 \in [0,\pi/2]}{\operatorname{argmax}} \ \min_{\mathbf{X}\neq\mathbf{U}} \Big| |d_1|^2\gamma_1^2 + |d_2|^2\eta_1^2 + |d_3|^2\gamma_2^2$$
$$+ |d_4|^2\eta_2^2 - 2\mathrm{Re}\{d_1 d_2 \gamma_1 \eta_1\} - 2\mathrm{Re}\{d_3 d_4 \gamma_2 \eta_2\}\Big|, \quad (2)$$

where $d_m = s_m - u_m$, for $m \in \{1, 2, 3, 4\}$. Similar to [10], we introduce the constraint that $\theta_1 + \theta_2 = \pi/2$. We then define

$$f = \max_{\theta_1} \min_{\mathbf{X}\neq\mathbf{U}} |\det[(\mathbf{X}-\mathbf{U})]|$$
$$= \min_{\mathbf{X}\neq\mathbf{U}} \max_{\theta_1} \Big| (|d_1|^2 + |d_4|^2)\sin^2(\theta_1)$$
$$+ (|d_2|^2 + |d_3|^2)\cos^2(\theta_1)$$
$$- (\mathrm{Re}\{d_1 d_2\} + \mathrm{Re}\{d_3 d_4\})\sin(2\theta_1)\Big|. \quad (3)$$

By differentiating (3) with respect to θ_1 and setting the result to zero, the solution for (2) is given by

$$\theta_1^o = \frac{1}{2}\arctan\frac{2(\mathrm{Re}\{d_1 d_2\} + \mathrm{Re}\{d_3 d_4\})}{(|d_1|^2 + |d_4|^2) - (|d_2|^2 + |d_3|^2)}. \quad (4)$$

By substituting (4) in (3), we get

$$f = \min_{d_1,d_2,d_3,d_4} \Big| (|d_1|^2 + |d_4|^2)\sin^2(\theta_1^o)$$
$$+ (|d_2|^2 + |d_3|^2)\cos^2(\theta_1^o)$$
$$- (\mathrm{Re}\{d_1 d_2\} + \mathrm{Re}\{d_3 d_4\})\sin(2\theta_1^o)\Big|. \quad (5)$$

Remark 1: Minimizing (5) with respect to d_m, for $m \in \{1, 2, 3, 4\}$, one can find the optimum rotation angle, θ_1^o, that maximizes the coding gain. Note that this optimization problem requires an exhaustive search over M^8 possible constellation points and can be carried out once offline and not in real-time. The optimal value for θ_1^o is 63.4° for the 4-QAM constellation and 76° for the 16-QAM constellation.

III. Reduced ML Decoding Method

In this section, we formulate the ML decoding problem for the proposed code. The received signal at the i-th antenna, $\mathbf{y}_i \triangleq [y_i(1),\ y_i(2)]^T$, for $i = 1, 2$, can be written as

$$\mathbf{y}_i = \sqrt{P}\begin{bmatrix} s_1\gamma_1 - s_2^*\eta_1 & s_3\gamma_2 - s_4^*\eta_2 \\ -s_3^*\gamma_2 + s_4\eta_2 & s_1^*\gamma_1 - s_2\eta_1 \end{bmatrix}\mathbf{h}_i + \mathbf{z}_i, \quad (6)$$

where $\mathbf{h}_i \triangleq [h_{i,1},\ h_{i,2}]^T$ is the channel vector, and $\mathbf{z}_i \triangleq [z_i(1),\ z_i(2)]^T$ represents the noise vector with independent identically distributed (i.i.d) elements from $\mathcal{CN}(0, N_0)$. The decoder in Fig. 1, receives the signals $\mathbf{y}_1$ and $\mathbf{y}_2$ during $T = 2$ time slots as shown in (6). Assuming perfect CSI at the receiver, the joint ML decoder is given by

$$(\hat{s}_1, \hat{s}_2, \hat{s}_3, \hat{s}_4) = \underset{s_1,s_2,s_3,s_4}{\operatorname{argmin}} \sum_{i=1}^{2} ||\mathbf{y}_i - \mathbf{X}\mathbf{h}_i||_F^2. \quad (7)$$

The ML decoder in (7) requires an exhaustive search over s_1, s_2, s_3, s_4 and consequently has a computational complexity of order $\mathcal{O}(M^4)$. Next, we show that the structure of the proposed code allows us to reduce the ML detection complexity to $\mathcal{O}(M^2)$.

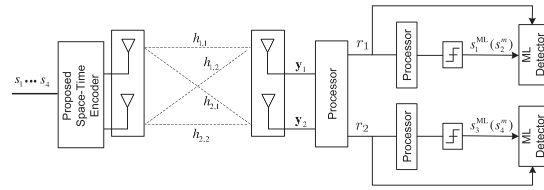

Fig. 1. Block diagram of 2×2 MIMO using the proposed low-complexity detection technique.

The received signal in (6) can be rewritten as follows:

$$\begin{bmatrix} y_i(1) \\ y_i^*(2) \end{bmatrix} = \sqrt{P}\begin{bmatrix} h_{i,1} & h_{i,2} \\ h_{i,2}^* & -h_{i,1}^* \end{bmatrix}\begin{bmatrix} s_1\gamma_1 - s_2^*\eta_1 \\ s_3\gamma_2 - s_4^*\eta_2 \end{bmatrix} + \begin{bmatrix} z_i(1) \\ z_i^*(2) \end{bmatrix}. \quad (8)$$

Note that the two columns of the channel matrix in (8) are orthogonal. By using this orthogonality property, we are able to decouple the symbols transmitted from different antennas. To do so, let us define

$$\begin{bmatrix} a_i(1) \\ a_i(2) \end{bmatrix} \triangleq \begin{bmatrix} h_{i,1}^* & h_{i,2} \\ h_{i,2}^* & -h_{i,1} \end{bmatrix}\begin{bmatrix} y_i(1) \\ y_i^*(2) \end{bmatrix},$$
$$\triangleq \sqrt{P}||\mathbf{h}_i||^2\begin{bmatrix} s_1\gamma_1 - s_2^*\eta_1 \\ s_3\gamma_2 - s_4^*\eta_2 \end{bmatrix} + \begin{bmatrix} w_i(1) \\ w_i(2) \end{bmatrix}, \quad (9)$$

where $w_i(1) \triangleq h_{i,1}^* z_1(1) + h_{i,2} z_1^*(2)$ and $w_i(2) \triangleq h_{i,2}^* z_1(1) - h_{i,1} z_1^*(2)$. Then, the sufficient statistics of $(s_1\gamma_1 - s_2^*\eta_1)$ is given by

$$r_1 = \frac{1}{\sqrt{4}}\sum_{i=1}^{2} a_i(1) = \sqrt{\frac{P}{4}}||\mathbf{H}||_F^2(s_1\gamma_1 - s_2^*\eta_1) + \tilde{z}_1 \quad (10)$$

where $\mathbf{H} \triangleq [\mathbf{h}_1, \mathbf{h}_2]$ is the channel matrix and $\tilde{z}_1 \triangleq \Big(h_{1,1}^* z_1(1) + h_{1,2} z_1^*(2) + h_{2,1}^* z_2(1) + h_{2,2} z_2^*(2)\Big)/\sqrt{4}$. Similarly, the sufficient statistics of $(s_3\gamma_2 - s_4^*\eta_2)$ is

$$r_2 = \frac{1}{\sqrt{4}}\sum_{i=1}^{2} a_i(2) = \sqrt{\frac{P}{4}}||\mathbf{H}||_F^2(s_3\gamma_2 - s_4^*\eta_2) + \tilde{z}_2 \quad (11)$$

where $\tilde{z}_2 \triangleq \Big(h_{1,2}^* z_1(1) - h_{1,1} z_1^*(2) + h_{2,2}^* z_2(1) - h_{2,1} z_2^*(2)\Big)/\sqrt{4}$. It can be seen from (10) and (11) that the signals r_1 and r_2 are either dependent on (s_1, s_2) or (s_3, s_4), which reduces the ML decoding complexity to $\mathcal{O}(M^2)$. Moreover, as it can be seen from (10) and (11), the transmit symbols are multiplied by a positive coefficient $||\mathbf{H}||_F^2$, which enables us to use a low complexity threshold comparator in combination with conditional ML decoding [5] to further reduce the decoding complexity. To apply the conditional ML decoding, let us compute the following intermediate signals for a given value of the symbol s_{2i}, e.g. s_{2i}^m, as

$$\tilde{r}_i^m = r_i - \sqrt{\frac{P}{4}}||\mathbf{H}||_F^2(-(s_{2i}^m)^*\eta_i), \qquad \text{for} \quad i = 1, 2 \quad (12)$$

where m is one of the M constellation point. Signals $\tilde{r}_1$ and $\tilde{r}_2$ can be used as inputs to a low complexity threshold comparator to get the ML estimate of the symbol s_1 conditional on s_2 and the symbol s_3 conditional on s_4, respectively. As a result,

Algorithm 1. Conditional ML Decoding

for $i = 1$ to 2 **do**
  for $m = 1$ to M-cardinality of signal constellation-**do**
    Step 1: Select s_{2i}^{m} from the signal constellation set.
    Step 2: Compute $\tilde{r}_i^m$ via (12)
    Step 3: Supply $\tilde{r}_i^m$ to a monotone detector to obtain the estimate of s_{2i-1} conditional on s_{2i}^m, denoted by $s_{2i-1}^{ML}|_{s_{2i}^m}$.
    Step 4: Compute the cost function in (14) for $s_{2i-1}^{ML}|_{s_{2i}^m}$ and s_{2i}^m.
  end for
  Step 5: $s_{2i-1}^{ML}|_{s_{2i}^m}$ and s_{2i}^m, $m \in [1 \cdots M]$, that correspond to the minimum cost function value are estimates of s_{2i-1} and s_{2i}.
end for

instead of minimizing the cost function in (7) over all possible pairs (s_{2i-1}, s_{2i}), for $i = 1, 2$, we first obtain the estimate of s_{2i-1} using a threshold comparator or look up table, denoted by $s_{2i-1}^{ML}|_{s_2^m}$, and then compute the cost function for $(s_{2i-1}^{ML}|_{s_{2i}^m}, s_{2i}^m)$, for $m = 1, 2, \cdots, M$. The optimal solution can be obtained as

$$\hat{s}_{2i} = \underset{m}{\operatorname{argmin}}\, f\left(s_{2i-1}^{ML}|_{s_{2i}^m}, s_{2i}^m\right), \qquad \text{for} \quad i = 1, 2 \quad (13)$$

where

$$f\left(s_{2i-1}^{ML}|_{s_{2i}^m}, s_{2i}^m\right) = \left| r_i - \sqrt{\frac{P}{4}} ||\mathbf{H}||_F^2 \left(s_{2i-1}^{ML}|_{s_{2i}^m}\gamma_i - \left(s_{2i}^m\right)^* \eta_i\right)\right|^2. \quad (14)$$

Algorithm 1 summarizes the conditional ML approach.

To summarize, we reduced the detection complexity to $\mathcal{O}(M^2)$ since the search needs to be carried out over two symbols simultaneously. Moreover, the application of the conditional ML further reduces the complexity of the detector for the proposed code[1]. More specifically, if we determine the ML detection complexity for the proposed code, which uses conditional ML, we note that $39M$ or $\mathcal{O}(M)$ arithmetic operations are needed ($26M$ multiplications and $13M$ additions). In comparison, the MTD approach in [10] already uses the conditional ML decoding to reach a decoding complexity of $\mathcal{O}(M^2)$. Moreover, the Matrix C approach in [8] requires searching simultaneously for four symbols and cannot take advantage of the conditional ML, since it requires the application of a monotone receiver in both the in-phase and quadrature paths. As such, the Matrix C approach has an overall complexity of $\mathcal{O}(M^4)$. Table I summarizes the rate, diversity, and complexity comparison.

IV. Simulation Results

Here, we present simulations to demonstrate the performance of the proposed STBC and compare it to the Matrix C [8], MTD [10], Ren *et al.* [13], FRLR [11], and Alamouti codes [1]. In order to simulate a wireless channel with different degrees of scattering richness, the channel response is modeled as a sum of line-of-sight (LoS) and non-line-of-sight (NLoS) components. For example, in a purely scattering environment, the LoS component vanishes and only NLoS component constitutes the entire channel response. The channel coefficient between transmit antenna i and receive antenna j is given [15]

[1]Note that application of conditional ML does not impact the system performance.

TABLE I
Rate and Decoding Complexity Comparison

Scheme	Rate	Decoding complexity
Proposed Code	2	$\mathcal{O}(M)$
MTD [10]	2	$\mathcal{O}(M^2)$
FRLR [11]	2	$\mathcal{O}(M)$
Ren *et al.* [13]	2	$\mathcal{O}(M^2)$
Matrix C [8]	2	$\mathcal{O}(M^4)$
Golden (ML) [4]	2	$\mathcal{O}(M^4)$
Golden (near-ML) [14]	2	$\mathcal{O}(M^2)$

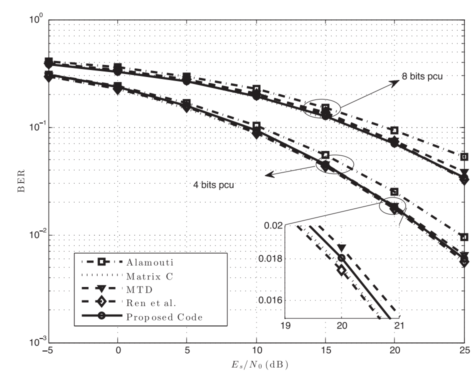

Fig. 2. BER performance of the proposed code with spectral efficiency of 4 and 8 bpcu.

$$h_{i,j} = \sqrt{\frac{K(f_c)}{s}\left(\frac{d_0}{d}\right)^{\gamma}} \left(\sqrt{\frac{K_R}{K_R+1}} h_{i,j}^L + \sqrt{\frac{1}{K_R+1}} h_{i,j}^N\right),$$

where

- $K(f_c) \triangleq (\frac{\lambda}{4\pi d_0})^2$, $\lambda = \frac{c}{f_c}$ is the wavelength, c is the speed of light, f_c is the carrier frequency, d_0 is the reference distance, d is the distance between transmitter and receiver, and γ is the path loss exponent.
- s is a log-normally distributed random variable with mean μ_s and standard deviation σ_s which models the shadowing effect.
  K_R is the Rician factor expressing the ratio of powers of the free-space signal and the scattered waves.
- $h_{i,j}^N$ and $h_{i,j}^L$ denote random and the deterministic components, respectively. The former accounts for the scattered signals with its entries being modeled as an independent and identically distributed (i.i.d) complex Gaussian random variable with zero mean and unit variance. The latter, $h_{i,j}^L$, models the LoS component.

In all experiments, we consider a 2×2 MIMO structure. The distance between transmitter and receiver is set to 25 meters, while the carrier frequency is set to $f_c = 60$ GHz, path loss exponent is set to $\gamma = 4$, and the shadowing effect parameter is assumed to have mean $\mu_s = 0$ and variance $\sigma_s = 9$ dB [15]. The results are given in terms of bit error rate (BER) versus E_s/N_0, where E_s is the symbol energy and N_0 is the noise power spectral density.

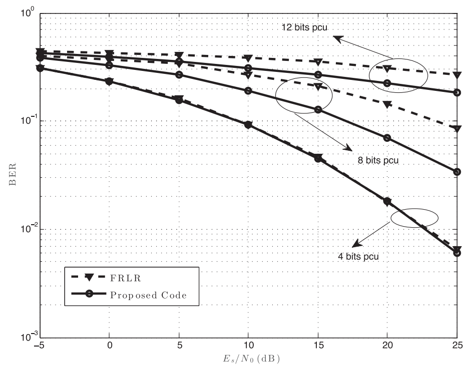

Fig. 3. BER performance of the proposed code with spectral efficiency of 4, 8 and 12 bpcu.

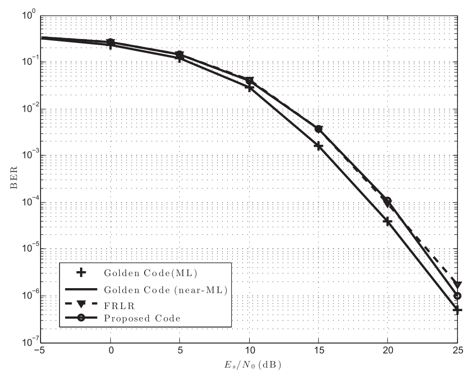

Fig. 4. BER performance of the proposed code with spectral efficiency of 4 bpcu in Rayleigh fading channels.

Fig. 2 illustrates the BER performance of the proposed code in comparison with the performance of the Alamouti, Matrix C, Ren *et al.* and MTD schemes with K-factor equal to 5 dB for spectral efficiency of 4 and 8 bits per channel use (bpcu)[2]. As it can be seen from this figure, the performance of the proposed code is very close to that of Ren *et al.* and Matrix C while outperforming the MTD approach. This performance is achieved with a significantly lower ML detection complexity compared to MTD, Ren *et al.* and Matrix C.

Fig. 3 compares the BER performance of the proposed code with the FRLR code [12]. As shown in this figure, the proposed code achieves the same performance as the FRLR code for QPSK modulation. However, for higher order modulation like 16-QAM or 256-QAM, the proposed coding performs significantly better than the FRLR. In particular, at a bit error rate of 10^{-1}, the proposed code outperforms the FRLR [12] by nearly 5 dB when the bandwidth efficiency is 8 bpcu. The FRLR code suffers from the lack of the non-vanishing determinant property that is a key parameter in designing a full-rate STBC across QAM constellation. This performance improvement demonstrates the superiority of our proposed scheme for high-order QAM constellations which are essential to achieve high spectral efficiency.

Fig. 4 compares the BER performance of the proposed code with the FRLR [11], MTD [10], and the Golden code using the ML [4] and near-ML [14] decoding schemes. As it can be seen from this figure, the proposed coding scheme has marginal performance loss compared to the Golden code. Since the decoding complexity order of Golden code with ML decoder is $\mathcal{O}(M^4)$ and with near-ML decoder is $\mathcal{O}(M^2)$ and our code is $\mathcal{O}(M)$, this performance loss can be viewed as a small penalty to be paid for the complexity reduction.

[2]For 4 bpcu case, we use 16-QAM for Alamouti and QPSK for the rest of the codes. For 8 bpcu case, we use 256-QAM for Alamouti and 16-QAM for the rest of the codes.

V. Conclusion

We proposed a full-rate full-diversity STBC for 2×2 MIMO systems. Due to the structure of the proposed code, we reduce the ML decoding complexity to $\mathcal{O}(M)$, which has significant impact on the the delay and energy consumption of the receiver especially for higher order modulations. Our simulations indicate that the proposed scheme outperforms FRLR [11], and maintain almost a similar performance as that of Ren *et al.* [13], Matrix C [8] and MTD [10], which have a decoding complexity of $\mathcal{O}(M^2)$, $\mathcal{O}(M^4)$ and $\mathcal{O}(M^2)$, respectively. Future research directions can be defined as the extension of the proposed code for more than two antennas and its analytical evaluation.

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