



Inertial spin alignment in a circular magnetic nanotube



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ABSTRACT

In cobalt nanotubes with a curling magnetization, the orbital motion of the conduction electrons interacts with their spin. As the spin goes around the nanotube it cannot follow the magnetization, since with the Fermi velocity it moves too fast. Instead, we predict that the spin precesses about an axis that is almost parallel to the axis of the nanotube and that rotates with the angular velocity of the electron. Therefore, the (absolute) value of the magnetic energy of the spin $|\boldsymbol{\mu} \cdot \mathbf{B}|$ is strongly reduced. The physics of the ferromagnet is considerably modified.

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1. Introduction

Nanotubes and nanowires of both metals and semiconductors have been extensively studied for electric charge transport. However, the electron spin has been often ignored. How to control and manipulate the spin degree of freedom in nanostructures is of vital importance not only for fundamental science, but also for technological applications in micromagnetism and spintronics. This has stimulated much research effort in the synthesis and characterization of ferromagnetic nanowires and nanotubes. These quasi-one-dimensional magnetic nanostructures have exhibited unique and intriguing physical properties. As an example a number of magnetic nanotubes of different materials show the remarkable property that their magnetic polarization is circumferential around the axis of the tube [1–6] (see Fig. 1). For example, in our experiments [2] there was no observable magnetic field outside the cobalt nanotube, demonstrating that the magnetization is looped inside the tube.

In such a circular magnetic nanotube (CMNTB) the conduction electrons experience a change of direction of the internal exchange field \mathbf{B} during their propagation. This yields some interesting effects on the spin of the electrons.

2. Theory

For a theoretical discussion we treat the CMNTB as a tube with zero thickness and large (infinite) mean free path of the electrons.

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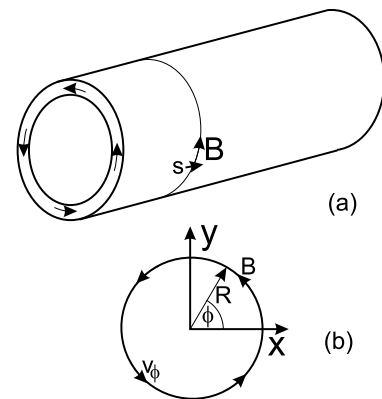


Fig. 1. a) A magnetic cobalt nanotube with the magnetization circular about the axis of the tube. b) The cross section in the limit of zero thickness.

We consider an electron with the velocity $\mathbf{v} = v_z \hat{\mathbf{z}} + v_\phi \hat{\boldsymbol{\phi}}$ where v_z is the component of the Fermi velocity v_F parallel to the axis and v_ϕ is the circular velocity. The z -component of the electron velocity v_z has no bearing on the results of the following consideration. The radius of the tube is R . Then the electron circles the CMNTB with the angular frequency $\boldsymbol{\omega}_e = (v_\phi/R) \hat{\mathbf{z}}$ where $|\boldsymbol{\omega}_e| \leq (v_F/R)$ ($v_F =$ Fermi velocity).

The electron has a spin \mathbf{s} and a magnetic moment of $\boldsymbol{\mu}$ where $\boldsymbol{\mu} = \gamma \mathbf{s}$ with $\gamma = -2\mu_B/\hbar = -e/m$ (we set the Lande factor g for the conduction electrons equal to 2). The circular magnetization acts as a magnetic field of strength B_0 on the magnetic moment of the electron. In the inertial lab frame S_0 the magnetic field causes a torque $\boldsymbol{\tau}$ on the magnetic moment of the electron

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B} = \gamma \mathbf{s} \times \mathbf{B} \quad (1)$$

Along the circular path of the electron with the angular velocity ω_e the direction of the magnetic field changes. At the position (R, ϕ, z) (in cylindrical coordinates) the magnetic field is given by

$$\mathbf{B} = B_0 (-\sin \phi, \cos \phi, 0) \quad (2)$$

As a consequence the torque constantly changes its direction for an electron whose position is given by $(R, \omega_e t, z)$ with $\phi = \omega_e t$. The fast angular velocity does not give the electron enough time to precess about the direction of the local magnetic field.

In the following we treat the motion of the electron in the frame S that rotates with the electron, i.e. with the frequency ω_e . We assign coordinate axes $(\hat{x}, \hat{y}, \hat{z})$ to the electron position which, at $t = 0$, are equal to $(\hat{e}_1, \hat{e}_2, \hat{e}_3)$ of the lab system. We attach these axes $(\hat{x}, \hat{y}, \hat{z})$ rigidly to the cylindrical surface of the tube (at the position of the electron). In the next step the cylinder, electron and local axes $(\hat{x}, \hat{y}, \hat{z})$ rotate together with frequency ω_e (so that $(\hat{x}, \hat{y}, \hat{z}) = (\hat{r}, \hat{\phi}, \hat{z})$).

In the rotating system S any vector \mathbf{Q} that is constant in the inertial system S_0 changes its value according to

$$\left(\frac{d\mathbf{Q}}{dt}\right)_S = -\omega_e \times \mathbf{Q} \quad (3)$$

(See for example [7]. An analogous consideration yields the Coriolis and centrifugal forces on the surface of the earth.)

For the spin this means that $d\mathbf{s}/dt$ in the rotating system is given by

$$\left(\frac{d\mathbf{s}}{dt}\right)_S = \left(\frac{d\mathbf{s}}{dt}\right)_{S_0} - \omega_e \times \mathbf{s} \quad (4)$$

Here $(d\mathbf{s}/dt)_{S_0}$ is the change of the spin due to the torque in the inertial system S_0 , i.e.

$$\left(\frac{d\mathbf{s}}{dt}\right)_{S_0} = \tau = \gamma \mathbf{s} \times \mathbf{B} \quad (5)$$

In the rotating system the magnetic field is constant $\mathbf{B} = (0, B_0, 0)$, and we obtain the for ds/dt

$$\left(\frac{d\mathbf{s}}{dt}\right)_S = \gamma \mathbf{s} \times \mathbf{B} + \mathbf{s} \times \omega_e = \gamma \mathbf{s} \times \left(\mathbf{B} + \frac{\omega_e}{\gamma}\right) \quad (6)$$

yielding

$$\left(\frac{d\mathbf{s}}{dt}\right)_S = \gamma \mathbf{s} \times \mathbf{B}_{\text{eff}} \quad (7)$$

with

$$\mathbf{B}_{\text{eff}} = \mathbf{B} + \frac{1}{\gamma} \omega_e = (0, B_0, \omega_e/\gamma) \quad (8)$$

The z -component of \mathbf{B}_{eff} is not an actual magnetic field but a kinematic result of the rotating coordinate system.

3. Results

The solutions to Eq. (7) (in the system S) are those for a free electron spin in a constant field \mathbf{B}_{eff} . The spin has a stable constant solution (when μ is parallel to \mathbf{B}_{eff}) and a metastable solution (when μ is anti-parallel to \mathbf{B}_{eff}). For a finite angle between μ and \mathbf{B}_{eff} the spin performs a precession about the direction of \mathbf{B}_{eff} .

In components this yields

$$\left(\frac{d}{dt}\right)_S \begin{pmatrix} s_x \\ s_y \\ s_z \end{pmatrix} = \begin{pmatrix} \omega_e s_y - \gamma B_0 s_z \\ -\omega_e s_x \\ \gamma B_0 s_x \end{pmatrix} \quad (9)$$

For the stationary solutions in the rotating system we set $(ds/dt)_S = 0$. This yields

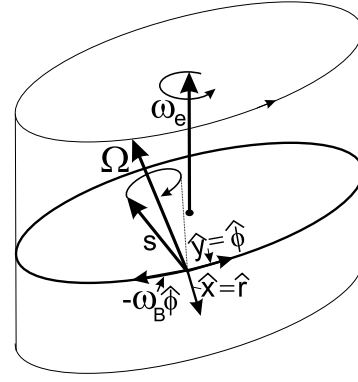


Fig. 2. The classical spin \mathbf{s} in the frame S that rotates with ω_e . The vector \mathbf{B}_{eff} is fixed in the frame S and the spin \mathbf{s} precesses about \mathbf{B}_{eff} with angular velocity $\sqrt{\omega_e^2 + \omega_B^2}$. In the inertial lab frame S_0 the precession axis \mathbf{B}_{eff} rotates itself with ω_e about the z -axis.

$$s_y = \frac{\gamma B_0}{\omega_e} s_z \quad (10)$$

$$s_x = 0 \quad (11)$$

So the unit vector of the spin is in S

$$\hat{\mathbf{s}} = \pm \frac{1}{X} (0, -\omega_B, \omega_e) \quad (12)$$

with $\hbar \omega_B = 2\mu_B B_0$ and $X = \sqrt{\omega_e^2 + \omega_B^2}$. The magnetic moment μ is parallel to \mathbf{B}_{eff} for the stable solution or anti-parallel in the metastable solution.

If the spin is not parallel or anti-parallel to \mathbf{B}_{eff} then it precesses about the effective field \mathbf{B}_{eff} with a precession frequency of

$$\omega_{\text{pcs}} = \gamma |\mathbf{B}_{\text{eff}}| = \sqrt{\omega_e^2 + (\gamma B_0)^2} = \sqrt{\omega_e^2 + \omega_B^2} \quad (13)$$

In the rotating system the effective field \mathbf{B}_{eff} is fixed with the coordinates given by Eq. (8). In the inertial system \mathbf{B}_{eff} rotates with ω_e about the z -axis. This rotation and the precession about \mathbf{B}_{eff} have opposite senses.

This is drawn in Fig. 2. Here \mathbf{B}_{eff} is the constant effective field in the rotating frame S . For \mathbf{s} parallel or anti-parallel to \mathbf{B}_{eff} the spin is stationary in a stable or metastable orientation (in S). If \mathbf{s} has an arbitrary angle with \mathbf{B}_{eff} then \mathbf{s} precesses about the axis \mathbf{B}_{eff} with the precession frequency $\sqrt{\omega_e^2 + \omega_B^2}$. (To simplify the drawing we treated \mathbf{s} and μ as parallel.)

The physical picture in the **inertial lab system** S_0 is the following. The axis of precession \mathbf{B}_{eff} rotates itself with the frequency ω_e about the z -axis. Furthermore the spin \mathbf{s} precesses about \mathbf{B}_{eff} (see Fig. 2). The rotation of the \mathbf{B}_{eff} axis with frequency ω_e and the precession about this axis with frequency $\sqrt{\omega_e^2 + \omega_B^2}$ have opposite senses.

For $\omega_e \gg \omega_B$ the actual precession in the lab system is approximately the difference

$$\omega_{\text{pcn}} \approx \sqrt{\omega_e^2 + \omega_B^2} - \omega_e \approx \frac{\omega_B^2}{2\omega_e} \quad (14)$$

This is a much smaller precession frequency than $\omega_B = |\gamma| B_0 = 2\mu_B B_0/\hbar$ which one would observe for a constant magnetic field of $B_0 \hat{z}$ in the z -direction.

In addition the lowest magnetic energy of the electron magnetic moment in the field \mathbf{B} is reduced to

$$E_{\text{mag}} = -\mu \cdot \mathbf{B} = -\mu_B B_0 \cos \theta \quad (15)$$

where θ is the angle between \mathbf{B}_{eff} and \mathbf{B} with

$$\tan \theta = \frac{\omega_e}{\omega_B}, \quad \cos \theta = \frac{\omega_B}{\sqrt{\omega_e^2 + \omega_B^2}} \quad (16)$$

Thomas precession: In our non-relativistic calculation we had two coordinate systems, $(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}})$ for the rotating system S and $(\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2, \hat{\mathbf{e}}_3)$ for the inertial system S_0 . Classically the system S is rotating with ω_e with respect to system S_0 . However, the rotating system S experiences a time-dependent acceleration \mathbf{a} . In a relativistic calculation this acceleration yields an additional precession of the axes of the orbiting system which was first calculated by Thomas [8] and is given by

$$\omega_{\text{Th}} = -\frac{1}{2c^2} \mathbf{v} \times \mathbf{a} \quad (17)$$

In our case we have

$$\mathbf{a} = \left(-R\omega_e^2 \cos(\omega_e t), -R\omega_e^2 \sin(\omega_e t), 0 \right)$$

which yields a Thomas precession about the axis of the nanotube of

$$\omega_{\text{Th}} = -\frac{R^2 \omega_e^3}{2c^2} \quad (18)$$

The Thomas precession changes the observed precession of the electron spin in the inertial lab system by ω_{Th} . Below we estimate the contribution of the different terms and conclude that the Thomas precession can be neglected.

4. Discussion

For a quantitative discussion essentially two parameters are required, the maximal angular frequency ω_e , given by the radius and the Fermi velocity and the magnetic field acting on the conduction electrons. Here one has two extreme cases:

- 1) Ferromagnets which are described by the Stoner model. Here one assumes essentially only one band which is generally the d-band of transition metal ferromagnets. The Stoner model connects the Stoner field B_0 with the Curie temperature T_C through the relation $B_0 \approx k_B T_C / \mu_B$. This yields fields in the range of a few 10^3 T. Due to the flat d-bands the Fermi velocity is generally a factor of 10 smaller than in (s,p)-metals.
- 2) A two-band ferromagnet where the magnetic properties are defined by the d-electrons and the conduction electrons are (s,p)-electrons. In this case it is more difficult to estimate the B_0 field and the Fermi velocity. In the literature values for the Fermi energy of spin-up and -down conduction electrons are given. In Ref. [9] tunnel experiments into CoFe and NiFe alloys are evaluated with values for the Fermi energy of $\varepsilon_{F\uparrow} \approx 2.2$ eV and $\varepsilon_{F\downarrow} = 0.5$ eV, yielding in a free electron model $v_{F\uparrow} \approx 0.88 \times 10^6$ m/s and $v_{F\downarrow} \approx 0.42 \times 10^6$ m/s. One of the authors [10] investigated the normal and anomalous Hall effect of amorphous Co films and obtained 0.5 conduction electrons per Co atom in the high field region where the anomalous Hall effect is saturated.

We use for the following estimate the value $v_F \approx 10^6$ m/s. This yields for a Co nanotube with the radius $R = 25$ nm the maximal value $\omega_e \approx 4.0 \times 10^{13}$ s $^{-1}$. Only when the angle between \mathbf{B}_{eff} and the z-axis is small can one use the simple relation (14) for the precession frequency in the lab frame. In Table 1 this angle $\alpha = \angle(\mathbf{B}_{\text{eff}}, \hat{\mathbf{z}})$ is given in degrees for different values of B_0 . Only for $B_0 = 10$ T is this angle small, and one obtains a smooth precession

Table 1

The ratio ω_B/ω_e and the resulting angle α between B_{eff} and the z-axis are calculated for different values of B_0 .

B_0	ω_B/ω_e	$\alpha = \tan^{-1} \frac{\omega_B}{\omega_e}$
10 T	4.4×10^{-2}	2.5°
10^2 T	4.4×10^{-1}	24°
10^3 T	4.4	77°

of $\omega_{\text{pcn}} \approx 3.9 \times 10^{10}$ s $^{-1}$. For the larger values the superposition of the rotation and precession in the rotating system S yields a complicated wobbling motion in the lab system S_0 .

For the Thomas precession frequency we obtain a value of $\omega_{\text{Th}} \approx -2.2 \times 10^8$ s $^{-1}$. This value is much smaller than precession frequency ω_{prc} and can be neglected.

5. Conclusions

The main finding of our calculation is that in a circular magnetized ferromagnetic nanotube the spin of the conduction electrons is not oriented parallel or anti-parallel to the magnetization but is tilted towards the positive or negative z-axis. The odd alignment of the electron spins in a magnetic nanotube with circular magnetization has a number of interesting effects which modify the magnetic properties. A few shall be considered here qualitatively. In the following we assume that the mean free path of the conduction electrons is sufficiently long so that the conduction electrons can circle the nanotube several times before they are scattered.

Our theory is purely classical, but we make the hypothesis that the (individual) direction of \mathbf{B}_{eff} will be the spin quantization direction in a quantum theoretical calculation. Our preliminary quantum theoretical calculation confirms this hypothesis. In the ground state the spins of the conduction electrons align parallel or antiparallel to this direction.

Our present theoretical consideration suggest a number of interesting questions:

- (1) The ground-state energy of the circular magnetic state is increased. In a regular ferromagnetic metal the conduction electrons align parallel or anti-parallel to the exchange field \mathbf{B} and lower their energy by $N_0 (\mu_B B_0)^2$ where N_0 is the (conduction) electron density of states per spin. In the nanotube with circular magnetization this energy reduction is much smaller and therefore the ground-state energy is increased by almost the same amount.
- (2) This energetic effect should be particularly important for Stoner magnets. Here the magnetic moments are band electrons (generally d-electrons) which are not localized and possess a finite (group) velocity $\mathbf{v}_d(\mathbf{k}) = (1/\hbar) \partial \varepsilon_d(\mathbf{k}) / \partial \mathbf{k}$. In a magnetic field the spin-up and -down d-electrons are shifted in opposite directions on the energy scale. The resulting magnetization acts back on the d-moments through the Coulomb exchange field, and the magnetization becomes Stoner enhanced. For a sufficiently large product of $N_d U$ (N_d = d-electron density of states, U = Coulomb exchange energy) the d-band makes a transition into a Stoner band magnet. This mechanism would be dramatically disturbed if the propagating d-electrons don't align their moments in the direction of the circular magnetization but (almost) parallel and anti-parallel to the cylinder axis. If the ϕ -component of \mathbf{v}_d is sufficiently large then half the d-electrons align their moments (roughly) parallel and the other half anti-parallel to the cylinder axis, canceling the exchange field. A conclusive answer requires, of course, a detailed band structure calculation for the Stoner system under consideration.
- (3) The interaction between spin waves and the conduction electrons will be altered. The excitation of a spin wave means

the transfer of an angular momentum \hbar from a conduction electron into the spin wave. Normally this is a simple transfer because the electron spin and the magnetization have the same quantization direction. However, in the CMNTB the two quantization directions are almost orthogonal to each other. The investigation of this interaction is to be considered in the future.

- (4) By covering the circular magnetic Co nanotube with another ferromagnet or a superconductor one can investigate a cylindrical proximity effect. We expect a considerable potential for new and interesting effects.
- (5) Another interesting system would be a circular magnetic nanoring. Our conclusions apply essentially also to these magnetic systems.

References

- [1] P. Landeros, O.J. Suarez, A. Cuchillo, P. Vargas, *Phys. Rev. B* 79 (2009) 024404.
- [2] D. Li, R.S. Thompson, G. Bergmann, J.G. Lu, *Adv. Mater.* 20 (2008) 4575.
- [3] K. Nielsch, F.J. Castaño, C.A. Ross, R. Krishnan, *J. Appl. Phys.* 98 (2005) 034318.
- [4] K.Z. Rozman, D. Pecko, L. Suhodolcan, P.J. McGuinness, S. Kobe, *J. Alloys Compd.* 509 (2011) 551.
- [5] N.A. Usov, O.N. Serebryakova, *J. Appl. Phys.* 116 (2014) 133902.
- [6] D.P. Weber, D. Ruffer, A. Buchter, F. Xue, E. Russo-Averchi, R. Huber, P. Berberich, J. Arbiol, A. Fontcuberta i Morral, D. Grundler, M. Poggio, *Nano Lett.* 12 (2012) 6139.
- [7] J.R. Taylor, *Classical Mechanics*, University Science Books, 2005, 339 pp.
- [8] L.H. Thomas, *Nature* 117 (1926) 514.
- [9] S.O. Valenzuela, D.J. Monsma, C.M. Marcus, V. Narayanamurti, M. Tinkham, *Phys. Rev. Lett.* 94 (2005) 196601.
- [10] G. Bergmann, *Phys. Lett. A* 60 (1977) 245.