Smoothed Least-laxity-first Algorithm for EV Charging

Yorie Nakahira
Computing and Mathematical Sciences
California Institute of Technology

Niangjun Chen
Computing and Mathematical Sciences
California Institute of Technology

Lijun Chen
Computer Science
University of Colorado at Boulder

Steven H. Low
Computing and Mathematical Sciences
California Institute of Technology

ABSTRACT

We formulate EV charging as a feasibility problem that meets all EVs' energy demands before departure under charging rate constraints and total power constraint. We propose an online algorithm, the smoothed least-laxity-first (sLLF) algorithm, that decides on the current charging rates based on only the information up to the current time. We characterize the performance of the sLLF algorithm analytically and numerically. Numerical experiments with real-world data show that it has significantly higher rate of generating feasible EV charging than several other common EV charging algorithms.

KEYWORDS

Online algorithm, online feasibility, resource augmentation, electric vehicle charging

1 INTRODUCTION

The electrification of transportation provides a great opportunity for energy efficiency and sustainability. There were over a million electric vehicles (EVs) worldwide as of 2015 [1], and accelerated EV proliferation is expected for many years to come. To charge a large number of EVs however presents a tremendous challenge, in terms of both its impact on power grid and management complexity. While the flexibility in charging time and rate can be exploited for coordinated EV charging to control and mitigate the impact on the grid, its efficacy often depends on accurate prediction of EV arrivals and energy demands as well as coordination across time among different EVs. However, the accurate prediction is usually either impossible or very costly (in data collection and computation), and the temporal coordination among a large number of EVs may incur prohibitively large complexity. In view of these limitations, in this paper we investigate low-complexity EV charging that does not require the prediction of EV arrivals/demands or the temporal coordination.

Specifically, we formulate EV charging as a feasibility problem that meets all EVs' energy demands before departure under individual charging rate constraints and total charging power constraint.

e-Energy'17, Hong Kong

DOI: 10.1145/3077839.3077864

We then propose an online algorithm, the smoothed least-laxity-first (sLLF) algorithm, that decides on the current charging rates based on only the information up to the current time. The laxity is defined as an EV's remaining time at the charging station minus the time needed to fully charge it at the maximum rate, and can be seen as the feasibility margin for EV charging. Without information on future EV arrivals, the sLLF algorithm makes best possible decision by maximizing the minimum resulting laxity for the next time among the EVs currently in the system.

As the sLLF algorithm does not take future EV arrivals into consideration, an (offline) feasible EV charging instance may be (online) infeasible under sLLF. We use the resource augmentation framework to study the sLLF algorithm, and characterize the minimum amount of additional resources (total power supply and charging rates) that will allow the algorithm to generate a feasible charging for any offline feasible charging instances. We further carry out numerical experiments using real-world data, and show that sLLF has significantly higher rate of generating feasible EV charging than several other common EV charging algorithms. This is expected, as the sLLF algorithm tries to leave the largest feasibility margin, so it can best accommodate arbitrary future EV arrivals.

Related Works

The existing EV charging algorithms can be categorized into either offline or online. The *offline* algorithms require complete information on all EVs to decide on the charging rates [4, 8, 16, 18, 23, 24]. However, information on future EV arrivals may not be available or very costly to obtain, which motivates *online* algorithms [2, 3, 5, 9, 13, 19, 22, 23, 23, 26, 26]. The performance of the online algorithm is generally analyzed for the worst-case [5, 13] or average-case [9, 26]. Other desirable properties of charging algorithms are low complexity in computation and memory usage, which can be achieved by sorting or bisection based methods, such as earliest-deadline-first, least-laxity-first [21], Whittle's index policy [25, 26], among others.

The *multi-processor deadline scheduling problem* [6, 7, 15] considers the scheduling of jobs on multiple processors. We can view the EV charging problem as a deadline scheduling problem by considering chargers as processors, and EVs with certain energy demand as jobs. Resource augmentation is a prominent analysis framework [10–12, 17] for analyzing the performance of online algorithms for multi-processor scheduling, we apply this framework to the EV charging problem. The main difference is that in our setting the power limit is time-varying, the maximum rates are heterogeneous, and the power limit may not necessarily be integer multiplication of the maximum rate.

Table 1: Notation

$\mathcal{I}$	EV charging problem instance
$\mathcal{V}$	set of EVs
$\mathcal{V}_t$	set of EVs remaining in the charging station at time t
$\mathcal{T}$	set of times
e_i	energy demand of EV $i \in \mathcal{V}$
$e_i(t)$	remaining energy demand of EV i at time $t \in \mathcal{T}$
$r_i(t)$	charging rate of EV i at time t
$P(t)$	power limit of the charging station at time t
a_i	arrival time of EV i
d_i	departure time of EV i

2 MODEL AND ALGORITHM

2.1 System Model

Consider a system with one charging station that serves a set of EVs, indexed by $i \in \mathcal{V} = \{1, 2, 3, \cdots\}$. We use a discrete-time model where time is divided into slots of equal sampling intervals, indexed by $t \in \mathcal{T} = \{0, 1, 2, \cdots, T\}$. EV i arrives at the charging station with an energy demand e_i at time a_i, and departs from the station at time d_i.[1] During its stay at the station, the EV is charged at a rate (or power) of $r_i(t) \geq 0$, $a_i \leq t < d_i$. For convenience, we extend this definition of $r_i(t)$ to the entire temporal domain. The notations are summarized in Table 1.

To account for limitations in the charger or battery of an EV, each EV i can only be charged up to a peak rate $\bar{r}_i$, *i.e.*

$$\begin{aligned} r_i(t) &\leq \bar{r}_i, \quad t \in [a_i, d_i), \quad i \in \mathcal{V} \\ r_i(t) &= 0, \quad t \notin [a_i, d_i), \quad i \in \mathcal{V} \end{aligned} \tag{1}$$

To account for limitations in the grid or power station, the charging station has a (possibly time-varying) power limit $P(t)$ such that[2]

$$\sum_{i \in \mathcal{V}} r_i(t) \leq P(t), \ t \in \mathcal{T}. \tag{2}$$

Furthermore, the power limit and maximum charging rates fall within the following nominal ranges:

$$\begin{aligned} P_{\min} &\leq P(t) \leq P_{\max}, \\ \bar{r}_{\min} &\leq \bar{r}_i \leq \bar{r}_{\max}, \ i \in \mathcal{V}. \end{aligned}$$

Finally, every EV's energy demands needs to be satisfied, *i.e.*,[3]

$$\sum_{t \in \mathcal{T}} r_i(t) = e_i, \ i \in \mathcal{V}. \tag{3}$$

Next, we define an EV charging problem instance as a quintuple $\mathcal{I} = \{a_i, d_i, e_i, \bar{r}_i; P(t)\}_{i \in \mathcal{V}, t \in \mathcal{T}}$. The primary goal of EV charging is to satisfy every EV's energy demands under the above power supply and peak rate constraints.

[1] Each EV leave at its departure time regardless of its charging conditions. This assumption is applicable for most slow chargers including ACN [14]. Under this assumption, we do need to explicitly model number of stations, as the speed of charging does not affect the availability of chargers for incoming EVs.

[2] All EVs at the charing station can be simultaneously as long as the constraints (1)-(2) are satisfied.

[3] The actual constraint in ACN is $\sum_{t \in \mathcal{T}} \delta r_i(t) = e_i$, $i \in \mathcal{V}$, where δ (h) is the sojourn time of sampling time intervals, e_i has unit kWh, $r_i(t)$ has unit kW [14]. Since $r_i(t)$ can always be rescaled according to δ, we set $\delta = 1$ without loss of generality.

Definition 2.1 (Feasible instance). An EV charging problem instance $\mathcal{I}$ is *offline feasible* if there exist charging rates $r = \{r_i(t) : i \in \mathcal{V}, t \in \mathcal{T}\}$ that satisfy constraints (1)-(3).

Constraints (1)-(3) are affine. Therefore, verifying the feasibility of an EV charging instance is a linear program (LP) for which many efficient algorithms exist.

2.2 Online Scheduling

In practice, the energy demand and departure time of an EV are only informed after its arrival.[4] Consequently, the charging station must use an *online algorithm* to determine an EV's current charging rate $r_i(t)$ using only information up to the current time t:

$$\mathcal{I}_t = \{a_i, d_i, e_i(\tau), \bar{r}_i; P(\tau)\}_{i \in \mathcal{V}_t, \tau \leq t}, \tag{4}$$

where $e_i(\tau) = e_i - \sum_{t=0}^{\tau-1} r_i(t)$ is the remaining energy demand of EV i at the beginning of time slot τ.

Definition 2.2 (Online algorithm). An online algorithm is a sequence of functions $\mathcal{A} = \{\mathcal{A}_t\}$ where each function $\mathcal{A}_t : \mathcal{I}_t \to r(t)$ maps the information up to the current time $\mathcal{I}_t$ to the current charging rates $r(t) = \{r_i(t)\}_{i \in \mathcal{V}_t}$.

Definition 2.3 (Feasibility of the algorithm). An (online) algorithm $\mathcal{A}$ is feasible (online feasible) on instance $\mathcal{I}$ if it gives charging rates that satisfy constraints (1)-(3).[5]

2.3 The Smoothed Least-Laxity-First Algorithm

2.3.1 The Laxity. A measure for the flexibility (or urgency) in charging of an EV is its remaining time minus the minimum remaining time needed to fully charge it (time needed to fully charge it at the maximum rate). We refer to this measure as *laxity*.

Definition 2.4 (Laxity). The laxity of an EV $i \in \mathcal{V}$ at time $t \in \mathcal{T}$ is defined as[6]

$$\ell_i(t) = \begin{cases} [d_i - t]^+ - \dfrac{e_i(t)}{\bar{r}_i}, & t \geq a_i, \\ +\infty, & t < a_i, \end{cases}$$

where "+" denotes the projection onto the set $\mathcal{R}_+$ of non-negative real numbers.

Proposition 2.5 (Feasibility condition). *The algorithm $\mathcal{A}$ is feasible on an instance $\mathcal{I}$ if and only if $\mathcal{A}$ gives charging rates that result in non-negative laxities for all EVs, i.e.,*

$$\ell_i(t) \geq 0, \quad i \in \mathcal{V}, \ \ t \in \mathcal{T}. \tag{5}$$

Proposition 2.5 suggests that the minimum laxity among all EVs can serve as a measure of the distance from infeasibility. A naive approach—referred to as the least laxity first (LLF) algorithm—is to charge EVs starting from those with the least laxity to those with the most laxity. However, the LLF algorithm may compromise the feasibility of certain offline feasible instances (see Section 4)

[4] In ACN, the energy demand and departure time of EV i is gathered from user inputs upon arrival.

[5] The feasibility is defined for an instance $\mathcal{I}$ with respect to an online algorithm $\mathcal{A}$, whereas the offline feasibility is defined for an instance $\mathcal{I}$. Offline feasibility is a necessary condition for an instance $\mathcal{I}$ to be feasible with respect to algorithm $\mathcal{A}$. An instance $\mathcal{I}$ can be online feasible with respect to algorithm $\mathcal{A}$ but infeasible with respect to another algorithm $\mathcal{A}'$.

[6] For convenience, laxity is defined on the whole temporal domain $\mathcal{T}$.

and cause excessive preemptions and oscillations in the charging rate[7], which may reduce the lifetime of certain batteries (*e.g.,* Li-ion) [13]. Alternatively, we consider maximizing the minimum laxity among all EVs in order to maximize the feasibility margin, $\max_r \min_{i\in\mathcal{V}} \ell_i(T)$. Although its solution may be non-unique, the following optimization problem produces an unique solution that is also a solution of $\max_r \min_{i\in\mathcal{V}} \ell_i(T)$.[8]

Corollary 2.6 (Equivalent problem). *Consider the optimization algorithm*

$$\max_{r} \quad \sum_{i\in\mathcal{V}} \bar{r}_i f(\ell_i(T)) \quad \text{s.t.} \quad (1),(2), \sum_{t\in\mathcal{T}} r_i(t) \le e_i,\ i\in\mathcal{V} \quad (6)$$

where f is strictly increasing. Algorithm (6) is feasible for any offline feasible instance.

However, we cannot solve (6) because of the lack of future information of incoming EVs. Instead, we replace (6) with the following online algorithm: at each time $t\in\mathcal{T}$, given $\ell_i(t), i\in\mathcal{V}$, compute[9]

$$\max_{r(t)} \sum_{i\in\mathcal{V}_t} \bar{r}_i f(\ell_i(t+1)) \quad \text{s.t. } (1),(2), r_i(t) \le e_i(t),\ i\in\mathcal{V}_t. \quad (7)$$

The optimization problem (7) also maximizes the minimum laxity $\min_{i\in\mathcal{V}_t} \ell_i(t+1)$, and thus maximizes the feasibility margin at time t.[10] Next, we show the structure of the optimal solution, which will be used to construct a scalable algorithm.

Proposition 2.7 (Valley-filling solution). *Assume that f is strictly concave, strictly increasing, and twice continuously differentiable. A solution to the optimization problem (7) is*

$$r_i^*(t) = [\bar{r}_i(L(t) - \ell_i(t) + 1)]_0^{\min(\bar{r}_i, e_i(t))}, \qquad i\in\mathcal{V}_t \quad (8)$$

where $[x]_a^b$ denotes the projection of the scalar x on interval $[a,b]$, and the value $L(t)$ satisfies

$$\begin{aligned}&\sum_{i\in\mathcal{V}_t} [\bar{r}_i(L(t) - \ell_i(t) + 1)]_0^{\min(\bar{r}_i, e_i(t))} \\ &= \sum_{i\in\mathcal{V}_t} r_i^*(t) = \min\left(P(t), \sum_{i\in\mathcal{V}_t} \min(\bar{r}_i, e_i(t))\right)\end{aligned} \quad (9)$$

Observe that for EV $i\in\mathcal{V}_t$ with $\bar{r}_i \le e_i(t)$, the charging rates (8) result in $\ell_i(t+1) = [L(t)]_{\ell_i(t)-1}^{\ell_i(t)}$. Hence, $L(t)$ can be considered as a threshold of $\ell_i(t+1)$, below which the energy is charged to EV i. Since $r_i^*(t)$ in (8) is an increasing function of $L(t)$, a binary search can be used to find the threshold $L(t)$ in (9). Given $L(t)$, the charging rates $r_i^*(t)$ is then determined using (8). We formally state this procedure in Algorithm 1, and name it as the *smoothed least-laxity-first (sLLF) algorithm.*

The computational complexity of the sLLF algorithm is $O(|\mathcal{V}_t| + \log(1/\delta))$, where δ is the level of tolerable error. Lastly, we note that the sLLF algorithm has other useful properties such as Least-laxity-first property and fairness.

[7]For example, consider a system of two EVs, where $\ell_1(0) = 1.25$, $\ell_2(0) = 0.75$ and $\bar{r}_1 = \bar{r}_2 = P(t) = 1, t\in\mathcal{T}$. EV 1 and EV 2 will be charged according to $(r_1(0), r_2(0)) = (0,1)$, $(\ell_1(1), \ell_2(1)) = (0.25, 0.75)$; $(r_1(1), r_2(1)) = (1,0)$, and so on. In this example, both EV switches in-between charging and not charging.

[8]Additionally, we can show the problem (6) also has a fairness property.

[9]For more complex form of power limits, in optimization problems (6) and (7), the power constraints (2) can be replaced by $Ar(t) \le_{e.w.} P(t)$, for element-wise inequality and positive matrix A. The Result in Corollary 2.6 also holds for $Ar(t) \le_{e.w.} P(t)$.

[10]The solution of (7) is also unique.

for $t\in\mathcal{T}$ **do**
  Update set of EVs $\mathcal{V}_t$ and laxities $\ell_i(t),\ i\in\mathcal{V}_t$
  Obtain $L(t)$ that solves (9) using bisection
  Charge according to rates $r_i(t)$ in (8)
end for

Algorithm 1: The Smoothed Least-Laxity-First (sLLF) Algorithm.

Lemma 2.8 (Least-laxity-first property). *If there exist two EVs $i,j\in\mathcal{V}$ under the sLLF algorithm such that*

$$\ell_i(t) \le \ell_j(t), \quad (10)$$

$$\ell_i(t+1) > \ell_j(t+1), \quad (11)$$

then either one of the following holds:

$$t \ge d_i \ \&\ r_i(t) = 0, \quad (12)$$

$$t < d_i \ \&\ t < d_j \ \&\ e_j(t+1) = 0 \ \&\ r_i(t) \ne 0. \quad (13)$$

Due to space constraints, the proofs are given in Appendix. The above properties will be useful in the analysis of feasibility conditions.

3 PERFORMANCE ANALYSIS

There are two extreme cases, $\bar{r}_i \to \infty,\ i\in\mathcal{V}$ and $P(t)\to\infty$, in which online algorithms can be feasible for any offline feasible instances. When $\bar{r}_i \to \infty\ i\in\mathcal{V}$, or equivalently $P(t) \le \min_{i\in\mathcal{V}_t} \bar{r}_i$ for all $t\in\mathcal{T}$, the charging problem is identical to the single processor preemptive scheduling problem where the processing capacity is time-variant. For this case, the earliest-deadline-first (EDF) algorithm is feasible for any offline feasible instances [20]. When $P(t)\to\infty$, or equivalently $P(t) \ge \sum_{i\in\mathcal{V}_t} \bar{r}_i(t)$ for all $t\in\mathcal{T}$, the sLLF algorithm is feasible for any offline feasible instances. However, beyond the above two extreme cases, no online algorithm can be feasible on all offline feasible instances [23]. The hardness of finding feasible online algorithms motivates a quantitative measure to evaluate the likelihood of an algorithm being feasible. Observe that if more resources (e.g., $P(t)$, $\bar{r}_i$) are allowed, an otherwise infeasible problem instance may become online feasible under the online algorithm. We use this (minimum) additional resource to analyze the performance of the sLLF algorithm, where either power P or both power P and peak rate $\bar{r}_i$ are augmented. The former allows more EVs to be charged simultaneously, while the latter additionally allows EVs to be charged faster. As we will demonstrate, these two ways of resource augmentation are qualitatively different and provide complementary insights into the behavior of the sLLF algorithm.

3.1 Power Augmentation

In the case of power augmentation, the online algorithm is allowed to use more power than the offline algorithm, *i.e.,* $P^{on}(t) = (1+\epsilon)P(t)$, $\bar{r}_i^{on} = \bar{r}_i$.

Definition 3.1. [ϵ-power augmented instance] Given an EV charging instance $\mathcal{I} = \{a_i, d_i, e_i, \bar{r}_i; P(t)\}_{i\in\mathcal{V}, t\in\mathcal{T}}$, we define its ϵ-power augmented instance as

$$\{a_i, d_i, e_i, \bar{r}_i; (1+\epsilon)P(t)\}_{i\in\mathcal{V}, t\in\mathcal{T}} \quad (14)$$

Definition 3.2. [ϵ-power feasibility] An online algorithm $\mathcal{A}$ is ϵ-power feasible if $\mathcal{A}$ is feasible on the ϵ-power augmented instances $\mathcal{I}_p(\epsilon)$ generated from any offline feasible instance $\mathcal{I}$.[11]

Unfortunately, there is no ϵ-power feasible online algorithm for any finite $\epsilon > 0$ [17].[12] However, under a mild assumption, ϵ-power feasibility condition can be obtained for a finite ϵ. Assume that the energy demand of each EV is bounded by X and the inter-arrival time between consecutive arrivals are greater than N, *i.e.*,

$$e_i \le X, \qquad i \in \mathcal{V}, \tag{15}$$

$$|a_i - a_j| > N, \qquad i, j \in \mathcal{V}. \tag{16}$$

We can characterize the relation between N and the sufficient amount of resource augmentation ϵ as follows.

Theorem 3.3. *Assume* (15), (16). *The sLLF algorithm is ϵ-power feasible with*

$$\epsilon = \frac{P_{\max}}{P_{\min}} \left\{ \log_{\varphi} \left(\frac{\sqrt{5}X}{NP_{\max}} + \frac{1}{2} \right) + 2 \right\} - 1,$$

where $\varphi = 1.61803$ is the golden ratio.

In particular, when $N \ge X/P_{\max}$,[13] we can further simplify the feasibility condition in Theorem 3.3.

Corollary 3.4. *If $N \ge X/P_{\max}$, then the sLLF algorithm is 3-power feasible.*

3.2 Power and Rate Augmentation

In the case of power and maximum charging rate augmentation, the online algorithm is allowed to use more power and higher maximum rate than the offline algorithm: $P^{on}(t) = (1+\epsilon)P(t)$, $\bar{r}_i^{on} = (1+\epsilon)\bar{r}_i$.

Definition 3.5. [ϵ-augmented instance] Given an EV charging instance $\mathcal{I}\{a_i, d_i, e_i, \bar{r}_i; P(t)\}_{i\in\mathcal{V}, t\in\mathcal{T}}$, we define its ϵ augmented instance as

$$\{a_i, d_i, e_i, (1+\epsilon)\bar{r}_i; (1+\epsilon)P(t)\}_{i\in\mathcal{V}, t\in\mathcal{T}} \tag{17}$$

Definition 3.6. [ϵ-feasibility] An online algorithm $\mathcal{A}$ is ϵ-feasible if $\mathcal{A}$ is feasible on the ϵ-augmented instances $\mathcal{I}_{pr}(\epsilon)$ generated from any offline feasible instance $\mathcal{I}$.

Contrary to the power augmentation, the sLLF algorithm is ϵ-feasible for a finite value of $\epsilon > 0$ without any assumptions of the arrival patterns.

Theorem 3.7. *The sLLF algorithm is ϵ-feasible with*

$$\epsilon = \max_{i\in\mathcal{V}} \left\{ \max_{\tau_1,\tau_2\in[a_i,d_i]} \frac{P(\tau_1)}{P(\tau_2)} - \max_{\tau\in[a_i,d_i]} \frac{\bar{r}_i}{P(\tau)} \right\}.$$

[11] Alternatively, the (minimum) value of ϵ can also be interpreted as the constraints on instances that are online feasible. That is, given the original resource $P(t)$, $\bar{r}_i(t)$, the algorithm is online feasible for any instances $\mathcal{I} = \{a_i, d_i, e_i, \bar{r}_i; P(t)/(1+\epsilon)\}_{i\in\mathcal{V}, t\in\mathcal{T}}$ that is offline feasible given the reduced resource $P(t)/(1+\epsilon)$, $\bar{r}_i(t)$. Large ϵ restricts possible instances, thus less likely to be online infeasible.

[12] It is shown in [17] that the LLF algorithm is not ϵ-power feasible for any $\epsilon > 0$ for uniform processors and time-invariant number of processors. Since their setting is a special case of our setting, the same results extend to our setting.

[13] If the inter-arrival time is N, and the power demand is X, the incoming energy demand per unit time is X/N. Since the total power supply is $P_{\max}$ per unit time, N should be at least X/P for offline feasiblity. Therefore, $X/P \le N$ is a mild assumption.

As we demonstrate in the next section, the actual EV instance in ACN and others requires smaller amount of resource augmentation than the worse-case upper bound in practice.

4 SIMULATION

In this section, we show the performance of the sLLF algorithm using trace-base simulation on real EV datasets and compare it to that of several heuristic online EV charging algorithms.

4.1 Experimental Setup

Our simulations use datasets from the ACN deployment (CAGarage) and Google's facilities in Mountain View (Google_mtv) and Sunnyvale (Google_svl). They include a total of 52,362 charging sessions over more than 4,000 charging days in 2016 at 104 locations. See Table 2 for a summary of the data. Each instance consists of one day of charging. We can see that there is a large degree of variation in the sojourn time and laxity of the vehicles in the instances.

For each instance, we compute the minimum power capacity in which the instance is feasible by using offline an LP, i.e., we minimize $P = P(t)$, subject to (1)-(3). This corresponds to the minimum power supply in order for the instance to be offline feasible. We use this minimum power supply to generate an offline instance, and tested if the instance is feasible under online algorithms. Besides the sLLF algorithm, we also implemented some common (online) scheduling algorithms: earliest-deadline-first (EDF), least-laxity-first (LLF), equal share (ES), remaining energy proportional (REP) [20], and an online linear program (OLP) [3]: . Due to space constraints, precise description of each algorithm is given in Appendix C.

	Instances	EV sojourn time (m)	Laxity (m)
CAGarage	92	321(11, 720)	231 (0.1, 660)
Google_mtv	3793	149 (10, 720)	35 (0.001, 694)
Google_svl	246	152 (11, 720)	38 (0.02, 676)

Table 2: Statistics of the EV charging instances. Each entry is formated as average (minimum, maximum), unit (m) denotes minutes.

4.2 Results without Augmented Resources

We first evaluate the success rate of the online algorithms without resource augmentation. We define the success rate of an algorithm as the percentage of online feasible instances under the algorithm. The sLLF algorithm achieves uniformly high success rate for all datasets compared to other online algorithms considered. The EDF, ES, and REP algorithms perform much worse in terms of finding feasible schedules (Figure 1). This is not surprising as feasibility requires online algorithms to jointly consider deadline, maximum charging rate, and remaining energy of each EV. However, none of these (the EDF, ES and REP algorithms) consider all three factors simultaneously. The low success rate of the LLF algorithm, despite its similarity to the sLLF algorithm, suggests the importance of maximizing minimum laxity (see Section 2.3).

Next, we study what characteristics of the instances affect the success rate. We find that the minimum normalized laxity and the maximum ratio between EV sojourn times have high correlations with the success rate. The maximum ratio between EV sojourn times

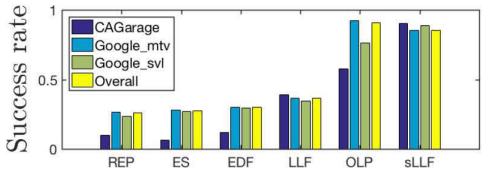
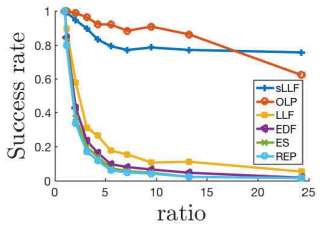
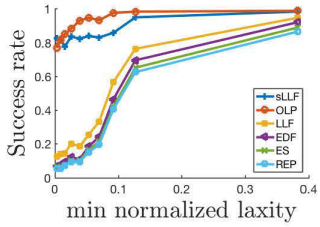


Figure 1: Success rates of finding feasible online schedule without resource augmentation.



(a)



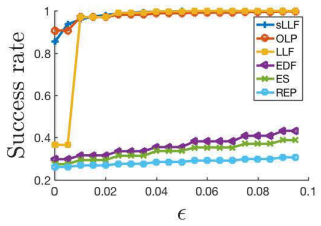
(b)

Figure 2: Success rate of finding feasible online schedule without resource augmentation.

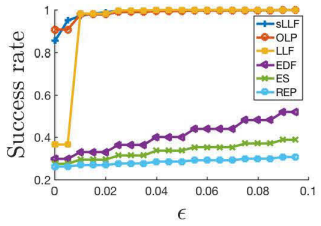
	REP	ES	EDF	LLF	OLP	sLLF
power	4.61	3.65	1.39	0.07	0.28	0.07
power and rate	4.61	3.24	0.54	0.05	0.28	0.05

Table 3: Minimum resource augmentation for online feasibility for all instances. The LLF and sLLF algorithms have the smallest ϵ among algorithms considered.

is defined as the maximum ratio between the longest and shortest EV sojourn times in the instances. The minimum normalized laxity of an EV is defined as the laxity divided by the EV sojourn times $\ell_i(a_i)/(d_i - a_i)$. Fig. 2 shows that as the minimum normalized laxity increases, all algorithms considered have improved success rates. Among these algorithms, the sLLF algorithm has one of the highest success rate for all minimum normalized laxity. Fig. 2b shows that as the maximum ratio between EV sojourn times increases, all algorithms considered have decreased success rates. Among these algorithms, the sLLF algorithm is least sensitive to the maximum ratio between EV sojourn times and maintains highest success rate across all sojourn times. Although instances with urgent schedule (small minimum normalized laxity) and large variety of EV sojourn times tend to have lower success rate, the sLLF algorithm has the best performance in almost all scenarios.



(a) ϵ-power augmentation



(b) ϵ-power and rate augmentation

Figure 3: Success rate of finding feasible online schedule under resource augmentation.

4.3 Results with Augmented Resources

While the sLLF algorithm has shown high success rate in finding feasible online EV charging schedules without resource augmentation, we further analyze the performance of online algorithms with resource augmentation in (a) power, and (b) both power and rate. Fig. 3 shows that the sLLF and OLP algorithm have the highest success rate of among other algorithms under various level of resource augmentation. We can see that to achieve 95% success rate for the sLLF algorithm, only 2% increase in resources is required. Table 3 shows that the minimum ϵ resource augmentation required for each algorithm to achieve 100% feasibility for all instances is smallest for the LLF and sLLF algorithms. Other algorithms (EDF, ES, REP and OLP) require significantly larger augmentation compared to the sLLF algorithm. While the OLP algorithm has high success rate without augmentation (Fig. 1), it requires much more resource augmentation to achieve 100% success rate (Table 3).

5 CONCLUSION

We have formulated EV charging as a feasibility problem that meets all EVs' energy demands before departure under charging rate constraints and total power constraint, and proposed an online algorithm, the sLLF algorithm, that decides on the current charging rates based on only the information up to the current time. We characterize the performance of the sLLF algorithm analytically and numerically. Numerical experiments with real-world data show that it has significantly higher rate of generating feasible EV charging than several other common EV charging algorithms. By finding feasible EV charging schedules using only a small augmentation to the absolute minimum resource needed for offline feasibility, our proposed algorithm (sLLF) can significantly reduce infrastructural cost for EV charging facilities.

REFERENCES

[1] International Energy Agency. 2016. Global EV outlook 2016. https://www.iea.org/publications/freepublications/publication/Global_EV_Outlook_2016.pdf. (2016).

[2] Rui Carvalho, Lubos Buzna, Richard Gibbens, and Frank Kelly. 2015. Congestion control in charging of electric vehicles. *arXiv preprint arXiv:1501.06957* (2015).

[3] Optimal Online Adaptive Electric Vehicle Charging. Electric vehicle public charging-time vs. energy.

[4] Niangjun Chen, Chee Wei Tan, and Tony QS Quek. 2014. Electric vehicle charging in smart grid: Optimality and valley-filling algorithms. *IEEE Journal of Selected Topics in Signal Processing* 8, 6 (2014), 1073–1083.

[5] Shiyao Chen, Yuting Ji, and Lang Tong. 2012. Large scale charging of electric vehicles. In *2012 IEEE Power and Energy Society General Meeting*. IEEE, 1–9.

[6] Robert I Davis and Alan Burns. 2011. A survey of hard real-time scheduling for multiprocessor systems. *ACM Computing Surveys (CSUR)* 43, 4 (2011), 35.

[7] Michael L. Dertouzos and Aloysius K. Mok. 1989. Multiprocessor online scheduling of hard-real-time tasks. *IEEE Transactions on software engineering* 15, 12 (1989), 1497–1506.

[8] Lingwen Gan, Ufuk Topcu, and Steven H Low. 2013. Optimal decentralized protocol for electric vehicle charging. *IEEE Transactions on Power Systems* 28, 2 (2013), 940–951.

[9] Lingwen Gan, Adam Wierman, Ufuk Topcu, Niangjun Chen, and Steven H Low. 2013. Real-time deferrable load control: handling the uncertainties of renewable generation. In *Proceedings of the fourth international conference on Future energy systems*. ACM, 113–124.

[10] Sungjin Im, Janardhan Kulkarni, and Kamesh Munagala. 2014. Competitive algorithms from competitive equilibria: non-clairvoyant scheduling under polyhedral constraints. In *Proceedings of the 46th Annual ACM Symposium on Theory of Computing*. ACM, 313–322.

[11] Sungjin Im, Janardhan Kulkarni, Kamesh Munagala, and Kirk Pruhs. 2014. Selfish-migrate: A scalable algorithm for non-clairvoyantly scheduling heterogeneous processors. In *Foundations of Computer Science (FOCS), 2014 IEEE 55th Annual Symposium on*. IEEE, 531–540.

[12] Bala Kalyanasundaram and Kirk Pruhs. 2000. Speed is as powerful as clairvoyance. *Journal of the ACM (JACM)* 47, 4 (2000), 617–643.

[13] Fanxin Kong, Qiao Xiang, Linghe Kong, and Xue Liu. 2016. On-Line Event-Driven Scheduling for Electric Vehicle Charging via Park-and-Charge. In *2016 IEEE Real-Time Systems Symposium (RTSS)2016 IEEE*. IEEE, 69–78.

[14] George Lee, Ted Lee, Zhi Low, Steven H. Low, and Christine Ortega. 2016. Adaptive Charging Network for Electric Vehicles. In *Proceedings of the IEEE Global Conference on Signal and Information Processing (GlobalSIP)*. Washington, DC.

[15] Chung Laung Liu and James W Layland. 1973. Scheduling algorithms for multiprogramming in a hard-real-time environment. *Journal of the ACM (JACM)* 20, 1 (1973), 46–61.

[16] Zhongjing Ma, Duncan Callaway, and Ian Hiskens. 2010. Decentralized charging control for large populations of plug-in electric vehicles. In *49th IEEE conference on decision and control (CDC)*. IEEE, 206–212.

[17] Phillips, Stein, Torng, and Wein. 2002. Optimal Time-Critical Scheduling via Resource Augmentation. *Algorithmica* 32, 2 (2002), 163–200. DOI :http://dx.doi.org/10.1007/s00453-001-0068-9

[18] Peter Richardson, Damian Flynn, and Andrew Keane. 2012. Local versus centralized charging strategies for electric vehicles in low voltage distribution systems. *IEEE Transactions on Smart Grid* 3, 2 (2012), 1020–1028.

[19] Lui Sha, Tarek Abdelzaher, Karl-Erik Årzén, Anton Cervin, Theodore Baker, Alan Burns, Giorgio Buttazzo, Marco Caccamo, John Lehoczky, and Aloysius K Mok. 2004. Real time scheduling theory: A historical perspective. *Real-time systems* 28, 2-3 (2004), 101–155.

[20] John A. Stankovic, Krithi Ramamritham, and Marco Spuri. 1998. *Deadline Scheduling for Real-Time Systems: EDF and Related Algorithms*. Kluwer Academic Publishers, Norwell, MA, USA.

[21] John A Stankovic, Marco Spuri, Krithi Ramamritham, and Giorgio C Buttazzo. 2012. *Deadline scheduling for real-time systems: EDF and related algorithms*. Vol. 460. Springer Science & Business Media.

[22] Sebastian Stein, Enrico Gerding, Valentin Robu, and Nicholas R. Jennings. 2012. A Model-based Online Mechanism with Pre-commitment and Its Application to Electric Vehicle Charging. In *Proceedings of the 11th International Conference on Autonomous Agents and Multiagent Systems - Volume 2 (AAMAS '12)*. International Foundation for Autonomous Agents and Multiagent Systems, Richland, SC, 669–676. http://dl.acm.org/citation.cfm?id=2343776.2343792

[23] Anand Subramanian, Manuel J Garcia, Duncan S Callaway, Kameshwar Poolla, and Pravin Varaiya. 2013. Real-time scheduling of distributed resources. *IEEE Transactions on Smart Grid* 4, 4 (2013), 2122–2130.

[24] Olle Sundström and Carl Binding. 2010. Planning electric-drive vehicle charging under constrained grid conditions. In *Power System Technology (POWERCON), 2010 International Conference on*. IEEE, 1–6.

[25] Peter Whittle. 1988. Restless bandits: Activity allocation in a changing world. *Journal of applied probability* 25, A (1988), 287–298.

[26] Zhe Yu, Yunjian Xu, and Lang Tong. 2016. Deadline Scheduling as Restless Bandits. *arXiv preprint arXiv:1610.00399* (2016).

APPENDIX

A PROOFS FOR SECTION 2

Proof (Proposition 2.5). Observe that feasibility is equivalent with the condition

$$e_i(d_i) = 0, \qquad i \in \mathcal{V}. \tag{18}$$

Condition (5) implies that for any EV $i \in \mathcal{V}$, $\ell_i(d_i) = -e_i(d_i)/\bar{r}_i \geq 0$, which yields $e_i(d_i) = 0$. Next, notice that the laxity of EV i is monotonically decreasing at $t < d_i$ and constant at $t \geq d_i$, *i.e.*,

$$\ell_i(t) = \ell_i(t+1) + 1 - r_i(t)/\bar{r}_i \geq \ell_i(t+1), \qquad t < d_i \tag{19}$$

$$\ell_i(t) = \ell_i(t+1) \qquad t \geq d_i \tag{20}$$

Therefore, condition (21) implies that $\ell_i(t) \geq 0$ at any time $t \in \mathcal{T}$. □

Proof (Corollary 2.6). From constraint $\sum_{t\in\mathcal{T}} r_i(t) \leq e_i$ and f being strictly increasing, the objective function satisfies

$$\sum_{i\in\mathcal{V}} f(\ell_i(T)) \leq \sum_{i\in\mathcal{V}} f(0).$$

Moreover, if an instance $\mathcal{I}$ is offline feasible, then there exists some charging rates that achieve $\ell_i(T) = 0, \forall i \in \mathcal{V}$. Since the laxity is monotonically decreasing at any $t \in \mathcal{T}$, such charging rates also satisfy $\ell_i(t) \geq 0, i \in \mathcal{V}, t \in \mathcal{T}$. From Proposition 2.5, $\ell_i(t) \geq 0, i \in \mathcal{V}, t \in \mathcal{T}$ implies that algorithm (6) is feasible on instance $\mathcal{I}$. Therefore, the cost $\sum_{i\in\mathcal{V}} f(\ell_i(T)) = \sum_{i\in\mathcal{V}} f(0)$ is attainable. □

Proof (Proposition 2.5). Observe that feasibility is equivalent with the condition

$$e_i(d_i) = 0, \qquad i \in \mathcal{V}. \tag{21}$$

Condition (5) implies that for any EV $i \in \mathcal{V}$, $\ell_i(d_i) = -e_i(d_i)/\bar{r}_i \geq 0$, which yields $e_i(d_i) = 0$. Next, notice that the laxity of EV i is monotonically decreasing at $t < d_i$ and constant at $t \geq d_i$, *i.e.*,

$$\ell_i(t) = \ell_i(t+1) + 1 - r_i(t)/\bar{r}_i \geq \ell_i(t+1), \qquad t < d_i \tag{22}$$

$$\ell_i(t) = \ell_i(t+1) \qquad t \geq d_i \tag{23}$$

Therefore, condition (21) implies that $\ell_i(t) \geq 0$ at any time $t \in \mathcal{T}$. □

Proof (Corollary 2.6). From constraint $\sum_{t\in\mathcal{T}} r_i(t) \leq e_i$ and f strictly increasing, the objective function satisfies

$$\sum_{i\in\mathcal{V}} f(\ell_i(T)) \leq \sum_{i\in\mathcal{V}} f(0).$$

If an instance $\mathcal{I}$ is offline feasible, then there exists certain charging rates that achieve $\ell_i(T) = 0, \forall i \in \mathcal{V}$, which yields $\sum_{i\in\mathcal{V}} f(\ell_i(T)) = \sum_{i\in\mathcal{V}} f(0)$. Since the laxity is monotonically decreasing at any $t \in \mathcal{T}$, such charging rates also satisfy condition (5). From Proposition 2.5, condition (5) implies that algorithm (6) is feasible on instance $\mathcal{I}$. □

Proof (Proposition 2.7). From the Karush-Kuhn-Tucker (KKT) conditions for the optimization problem (7),

$$r_i(t) \geq 0 \qquad i \in \mathcal{V}_t \qquad (24)$$

$$r_i(t) \leq \min(e_i(t), \bar{r}_i) \qquad i \in \mathcal{V}_t \qquad (25)$$

$$\sum_{i \in \mathcal{V}_t} r_i(t) \leq P(t) \qquad i \in \mathcal{V}_t \qquad (26)$$

$$f'(l_i(t+1)) + \bar{\lambda}_i - \underline{\lambda}_i + v = 0 \qquad i \in \mathcal{V}_t \qquad (27)$$

$$\underline{\lambda}_i \geq 0, \ \ \bar{\lambda}_i \geq 0 \qquad i \in \mathcal{V}_t \qquad (28)$$

$$\underline{\lambda}_i r_i(t) = 0, \ \ \bar{\lambda}_i \{r_i(t) - \min(e_i(t), \bar{r}_i)\} = 0 \qquad i \in \mathcal{V}_t \qquad (29)$$

where $\underline{\lambda}_i, \bar{\lambda}_i, v$ are the dual variables for constraints (24), (25), (26), respectively. We consider three mutually exclusive cases: $r_i(t) = 0$, $r_i(t) \in (0, \min(e_i(t), \bar{r}_i))$, or $r_i(t) = \min(e_i(t), \bar{r}_i)$. When $r_i(t) = 0$, $\bar{\lambda}_i = 0$ and

$$r_i(t)/\bar{r}_i = f'^{-1}(-v) - l_i(t) + 1 - \underline{\lambda}_i \leq f'^{-1}(-v) - l_i(t) + 1, \quad (30)$$

where the the inverse of f' exists since f' is strictly concave, strictly increasing, and twice continuously differentiable. When $r_i(t) \in (0, \min(e_i(t), \bar{r}_i))$, then from (29) (complementary slackness), $\bar{\lambda}_i = \underline{\lambda}_i = 0$. Substituting $\bar{\lambda}_i = \underline{\lambda}_i = 0$ into (27), we obtain

$$l_i(t) - 1 + r_i(t)/\bar{r}_i = f'^{-1}(-v) \qquad (31)$$

When $r_i(t) = \min(e_i(t), \bar{r}_i)$, $\underline{\lambda}_i = 0$ and

$$r_i(t)/\bar{r}_i = f'^{-1}(-v) - l_i(t) + 1 + \hat{\lambda}_i \geq f'^{-1}(-v) - l_i(t) + 1. \quad (32)$$

Combining (30)-(32), we obtain

$$r_i(t) = [\bar{r}_i (f'^{-1}(-v) - l_i(t) + 1)]_0^{\min(\bar{r}_i, e_i(t))}, \qquad (33)$$

Because the same value of $f'^{-1}(-v)$ is shared for all EVs at the charging station, we can define an variable $L(t) = f'^{-1}(-v)$. Since the optimal solution is attained at the boundary $\sum_{i \in \mathcal{V}_t} r_i^*(t) = \min\left(P(t), \sum_{i \in \mathcal{V}_t} \min(\bar{r}_i, e_i(t))\right)$, we obtain the optimal solution (8)-(9). □

Proof (Lemma 2.8). First notice that, by Definition 2.4, it satisfies the following relation:

$$\ell_i(t) - 1 \leq \ell_i(t+1) \leq \ell_i(t), \ i \in \mathcal{V}. \qquad (34)$$

First, consider the case $r_i(t) = 0$. The evolution of ℓ_i satisfies

$$\ell_i(t+1) = \begin{cases} \ell_i(t) - 1 & t < d_i, \\ \ell_i(t) & t \geq d_i. \end{cases} \qquad (35)$$

Suppose that $t < d_i$, combining (10) and (34) gives

$$\ell_j(t+1) \geq \ell_j(t) - 1 \geq \ell_i(t) - 1 = \ell_i(t+1),$$

which contradicts (11). Therefore, $t \geq d_i$, and (12) follows.

Next, consider the case $r_i(t) \neq 0$. Non-zero $r_i(t)$ implies $t < d_i$. If $t < d_j$, (10) and (11) jointly implies

$$\frac{r_j(t)}{\bar{r}_j(t)} < \frac{r_i(t)}{\bar{r}_i(t)}. \qquad (36)$$

Under the sLLF algorithm, (36) happens only when $e_j(t) = r_j(t)$, which leads to $e_j(t+1) = 0$. If $t \geq d_j$, then $\ell_j(t+1) = \ell_j(t) \geq \ell_i(t) \geq \ell_i(t+1)$, which contradicts (11). Therefore, (13) follows. □

B PROOFS FOR SECTION 3

Notations. Next we introduce some notation that will be used later. Denote by $A_t = \{i \in \mathcal{V} : a_i \leq t\}$ the set of EVs that have arrived by time t, $D_t = \{i \in A_t : d_i \leq t \text{ or } e_i(t) = 0\}$ the set of EVs that have either departed or finished charging by time t, $V_t = \{i \in A_t : a_i \leq t < d_i\}$ the set of EVs remaining in the charging station at time t, and $U_t = \{i \in V_t : e_i(t) > 0\}$ the set of EVs with unfulfilled energy demand at the beginning of time slot t, where we reload the notation and use $e_i(t)$ to denote the remaining energy demand of EV i at the beginning of time slot t. In addition, denote by $A_{[t_1,t_2]} = \{i \in \mathcal{V} : a_i \in [t_1, t_2]\}$ the set of EVs that arrive during time interval $[t_1, t_2]$, $t_1, t_2 \in \mathcal{T}$. See Table 4 for a summary of notation.

Denote the total energy supply to EVs in set $\mathcal{S} \subseteq \mathcal{V}$ during the interval $[t_1, t_2]$ under the (feasible) offline algorithm by

$$\Psi^*_{[t_1:t_2]}(\mathcal{S}; \mathcal{I}) := \sum_{i \in \mathcal{S}} \sum_{\tau=t_1}^{t_2} r_i(\tau),$$

and the total energy supply to EVs in set $\mathcal{S} \subseteq \mathcal{V}$ during the interval $[t_1, t_2]$ under the ϵ-power augmentation (or ϵ-augmentation) by

$$\Psi^\epsilon_{[t_1:t_2]}(\mathcal{S}; \mathcal{I}) := \sum_{i \in \mathcal{S}} \sum_{\tau=t_1}^{t_2} r_i(\tau),$$

We use superscript * to indicate variables under an (feasible) offline algorithm with original power limit $P(t)$ and maximum charging rates $\bar{r}_i$, and use superscript $^\epsilon$ to indicate variables under the augmented resources.

Table 4: Additional Notation

A_t	set of EVs arriving by time t
$A_{[t_1,t_2]}$	set of EVs arriving during interval $[t_1, t_2]$
D_t	set of EVs either departed or finished by time t
V_t	set of EVs at the charging station at time t
U_t	set of EVs unfinished charging at time t
$\Psi_{[t_1:t_2]}(\mathcal{S}; \mathcal{I})$	total energy supplied to the set of EVs $\mathcal{S}$ during the interval $[t_1, t_2]$ under instance $\mathcal{I}$
$\Psi^\epsilon_{[t_1:t_2]}(\mathcal{S}; \mathcal{I})$	total energy supplied to $\mathcal{S}$ during $[t_1, t_2]$ under instance $\mathcal{I}$ with ϵ augmented resources

B.1 Preliminaries

B.1.1 The infeasibility condition. For a charging instance $\mathcal{I} = \{a_i, d_i, e_i, \bar{r}_i; P(t)\}_{i \in \mathcal{V}, t \in \mathcal{T}}$ that is not online feasible under the sLLF algorithm, there are times when some EV has negative laxity. Denote by t_- the earliest among such times. Let $\mathcal{F} = \{i \in A_{t_-} : \ell_i(t_-) < 0\}$ denote the set of EVs arriving at the changing station by time t_- that have negative laxity, $\mathcal{S}_1 = \{i \in A_{t_-} : \ell_i(t_-) \geq 0 \ \& \ d_i \leq t_-\}$ the set of EVs with non-negative laxity that depart by time t_-, and $\mathcal{S}_2 = \{i \in A_{t_-} : \ell_i(t_-) \geq 0 \ \& \ d_i > t_-\}$ the set of EVs with non-negative laxity that remain at the charging station at time t_-. Sets $\mathcal{F}$, $\mathcal{S}_1$ and $\mathcal{S}_2$ are mutually exclusive, and $A_{t_-} = \mathcal{F} \cup \mathcal{S}_1 \cup \mathcal{S}_2$.

Lemma B.1. *When the sLLF algorithm is used on instance $\mathcal{I}$, for any $i \in \mathcal{S}_2$ and $j \in \mathcal{F}$, the laxities satisfy*

$$\ell_i(t) > \ell_j(t), \ t \in [\max(a_i, a_j), \ t_-]. \qquad (37)$$

Proof (Lemma B.1). By the construction of $\mathcal{S}_2$, relation (37) holds at $t = t_-$. By Lemma 2.8, a necessary condition for the inequality in (37) to flip at some time $t+1 \le t_-$ is for (13) to hold for EV i. We show below that this condition cannot holds for any EV in $\mathcal{F}$ or $\mathcal{S}_1$. For EVs in $\mathcal{F}$, condition $e_j(t+1) = 0$ in (13) cannot happen because negative laxity at some time implies the energy demand will not be fulfilled. For EVs in $\mathcal{S}_1$, (37) holds only after $e_j(t+1) = 0$ when they have energy demand fulfilled at time $t+1$. Consequently, condition (37) holds for all $t \in [\max(a_i, a_j), t_-]$. □

Notice that the sLLF algorithm prioritizes EVs with smaller laxity so the presence of EVs with strictly greater laxity will not impact the charging of the EVs with smaller laxity. Let $\tilde{\mathcal{V}} = \mathcal{F} \cup \mathcal{S}_1$, and use it to define another instance that does not contain the EVs in $\mathcal{S}_2$: $\tilde{\mathcal{I}} = \{a_i, d_i, e_i, \bar{r}_i; P(t)\}_{i\in\tilde{\mathcal{V}}, t\in\mathcal{T}}$. Following Corollary can be obtained as a consequence of Lemma B.1.

Corollary B.2. *Regardless of the actual instance being $\mathcal{I}$ or $\tilde{\mathcal{I}}$, the EVs in $\tilde{\mathcal{F}}$ are charged in exactly the same way under the sLLF algorithm by time t_-.*

B.1.2 The infeasibility condition of an augmented instance. Let $\mathcal{I}$ be an EV charging instances that are offline feasible. Consider using the sLLF algorithm with the ϵ augmented resources (either power augmentation $P^{on}(t) = \epsilon P(t)$, or power and rate augmentation $P^{on}(t) = \epsilon P(t), \bar{r}_i^{on}(t) = \bar{r}_i(t)$). Now, the above result from previous section, we derive a condition for the sLLF algorithm being infeasible on some online feasible instance, which holds for both power augmentation and power and rate augmentation.

Since the EVs in $\mathcal{S}_1$ are fully charged by time t_- under both the sLLF algorithm and the offline algorithm, we have

$$\Psi^{\epsilon}_{[0:t_-]}(\mathcal{S}_1; \mathcal{I}) = \Psi^{*}_{[0:t_-]}(\mathcal{S}_1; \mathcal{I}), \tag{38}$$

where $\mathcal{S}_1, \mathcal{F}$ are the sets defined above under the sLLF algorithm using augmented resources. Notice that $\ell_i(t) \ge 0$, $\forall t \in \mathcal{T}$ is a necessary condition for EV i to be feasible. Thus, for EV $i \in \mathcal{F}$, the offline algorithm must maintain $\ell_i(t_-) \ge 0$. Given that laxity $\ell_i(t)$ is strictly decreasing in the remaining energy demand $e_i(t)$, the total energy fulfilled by t_- under the offline algorithm must be strictly greater than that with the sLLF algorithm, *i.e.*,

$$\Psi^{\epsilon}_{[0:t_-]}(\{i\}; \mathcal{I}) < \Psi^{*}_{[0:t_-]}(\{i\}; \mathcal{I}),\ i \in \mathcal{F} \tag{39}$$

from which

$$\Psi^{\epsilon}_{[0:t_-]}(\mathcal{F}; \mathcal{I}) < \Psi^{*}_{[0:t_-]}(\mathcal{F}; \mathcal{I}). \tag{40}$$

Recall that $\tilde{\mathcal{V}} = \mathcal{V} \backslash \mathcal{S}_2$. Combining (38) and (40), we have

$$\Psi^{\epsilon}_{[0:t_-]}(\tilde{\mathcal{V}}; \mathcal{I}) < \Psi^{*}_{[0:t_-]}(\tilde{\mathcal{V}}; \mathcal{I}). \tag{41}$$

Corollary B.2 implies

$$\Psi^{\epsilon}_{[0:t_-]}(i; \mathcal{I}) = \Psi^{\epsilon}_{[0:t_-]}(i; \tilde{\mathcal{I}}),\ i \in \tilde{\mathcal{V}}, \tag{42}$$

$$\Psi^{\epsilon}_{[0:t_-]}(\tilde{\mathcal{V}}; \mathcal{I}) = \Psi^{\epsilon}_{[0:t_-]}(\tilde{\mathcal{V}}; \tilde{\mathcal{I}}). \tag{43}$$

Further, since the charging instance $\mathcal{I}$ is offline feasible, its sub-instance $\tilde{\mathcal{I}}$ is offline feasible too. Similar to equations (38)-(41), we can show that

$$\Psi^{\epsilon}_{[0:t_-]}(\mathcal{S}_1; \tilde{\mathcal{I}}) = \Psi^{*}_{[0:t_-]}(\mathcal{S}_1; \tilde{\mathcal{I}}), \tag{44}$$

$$\Psi^{\epsilon}_{[0:t_-]}(\{i\}; \tilde{\mathcal{I}}) < \Psi^{*}_{[0:t_-]}(\{i\}; \tilde{\mathcal{I}}),\ i \in \mathcal{F}, \tag{45}$$

$$\Psi^{\epsilon}_{[0:t_-]}(\mathcal{F}; \tilde{\mathcal{I}}) < \Psi^{*}_{[0:t_-]}(\mathcal{F}; \tilde{\mathcal{I}}), \tag{46}$$

$$\Psi^{\epsilon}_{[0:t_-]}(\tilde{\mathcal{V}}; \tilde{\mathcal{I}}) < \Psi^{*}_{[0:t_-]}(\tilde{\mathcal{V}}; \tilde{\mathcal{I}}). \tag{47}$$

B.2 Proof of Theorem 3.3

Consider the use of the sLLF algorithm on an offline feasible instance $\mathcal{I} = \{a_i, d_i, e_i, \bar{r}_i; P(t)\}_{i\in\mathcal{V}, t\in\mathcal{T}}$ under ϵ-power augmented resources. Let

$$n = (1+\epsilon)\frac{P_{\min}}{P_{\max}}. \tag{48}$$

For $m \le n$, we define the earliest time to charge at a power greater than $mP_{\max}$ for the rest of the time until t_- as

$$t_m = \min\left\{ t \in \mathcal{T} : \sum_{j\in V_t} \min(\bar{r}_j, e_j(\tau)) \ge mP_{\max}, \tau \in [t, t_-] \right\}. \tag{49}$$

Let $T_m = [t_{m-1}, t_m)$ and $\hat{T}_m = [t_m, t_-]$ and denote their lengths by $|T_m|$ and $|\hat{T}_m|$.

We first present a lemma that is used in the proof of Theorem 3.3.

Lemma B.3. *For any integer $i \le n-1$, the following two relations hold:*

$$\Psi^{*}_{[0:t_i]}(A_{T_i}; \tilde{\mathcal{I}}) - \Psi^{\epsilon}_{[0:t_i]}(A_{T_i}; \tilde{\mathcal{I}}) > P_{\max}|\hat{T}_{i+1}|, \tag{50}$$

$$|T_i| > |\hat{T}_{i+1}|. \tag{51}$$

Proof (Lemma B.3). On one hand, from definition (49),

$$\sum_{j\in V_{(t_{i-1})-1}} \min(\bar{r}_j, e_j(t_{i-1}-1)) < (i-1)P_{\max}.$$

This implies that the EVs that have arrived before t_{i-1} are charged at a total power of at most $(i-1)P_{\max}$ at t_{i-1} and after. On the other hand, from definition (49), the total power supply is at least $iP_{\max}$ during the interval $T_{i+1} = [t_i, t_{i+1}]$. Therefore, the total charing power to the EVs that arrive after t_{i-1} is at least $P_{\max}$ during T_{i+1}. Since the offline algorithm can only use a power of at most $P_{\max}$, for the EVs that arrive after t_{i-1} we obtain

$$\begin{aligned} &\Psi^{*}_{[0;t_{i+1}]}(A_{\hat{T}_{i-1}}; \tilde{\mathcal{I}}) - \Psi^{\epsilon}_{[0;t_{i+1}]}(A_{\hat{T}_{i-1}}; \tilde{\mathcal{I}}) \\ &< \Psi^{*}_{[0;t_i]}(A_{\hat{T}_{i-1}}; \tilde{\mathcal{I}}) - \Psi^{\epsilon}_{[0;t_i]}(A_{\hat{T}_{i-1}}; \tilde{\mathcal{I}}). \end{aligned} \tag{52}$$

The same argument can be applied to the interval $\hat{T}_{i+1} = [t_{i+1}, t_-]$. From definition (49), the total charging power is at least $(i+1)P_{\max}$ during $\hat{T}_{i+1}$. Therefore, during $\hat{T}_{i+1}$, the total charing power to the EVs that arrive after t_{i-1} is at least $2P_{\max}$. Since the offline algorithm can only use a power of at most $P_{\max}$, the total energy supply to EVs in $\hat{T}_{i-1}$ under the augmented resources is greater than that without augmented resources, *i.e.*,

$$\begin{aligned} 0 &< \Psi^{*}_{[0;t_-]}(A_{\hat{T}_{i-1}}; \tilde{\mathcal{I}}) - \Psi^{\epsilon}_{[0;t_-]}(A_{\hat{T}_{i-1}}; \tilde{\mathcal{I}}) \\ &< \Psi^{*}_{[0;t_{i+1}]}(A_{\hat{T}_{i-1}}; \tilde{\mathcal{I}}) - \Psi^{\epsilon}_{[0;t_{i+1}]}(A_{\hat{T}_{i-1}}; \tilde{\mathcal{I}}) - P_{\max}|\hat{T}_{i+1}|. \end{aligned} \tag{53}$$

Combining (52)-(53), we have

$$\Psi^*_{[0;t_i]}(A_{\hat{T}_{i-1}};\tilde{I}) - \Psi^{\epsilon}_{[0;t_i]}(A_{\hat{T}_{i-1}};\tilde{I}) > P_{\max}|\hat{T}_{i+1}|. \tag{54}$$

Since the set A_{T_i} is identical to the subset of $A_{\hat{T}_{i-1}}$ that contains only the EVs that have arrived by t_i,

$$\begin{aligned}&\Psi^*_{[0;t_i]}(A_{T_i};\tilde{I}) - \Psi^{\epsilon}_{[0;t_i]}(A_{T_i};\tilde{I})\\ &= \Psi^*_{[0;t_i]}(A_{\hat{T}_{i-1}};\tilde{I}) - \Psi^{\epsilon}_{[0;t_i]}(A_{\hat{T}_{i-1}};\tilde{I}).\end{aligned} \tag{55}$$

Combining (54) and (55) leads to relation (50).

Finally, as all EVs in A_{T_i} arrives after t_{i-1}, during T_i the offline algorithm can charge a total energy of at most $|T_i|P_{\max}$, we obtain

$$\Psi^*_{[0:t_i]}(A_{T_i};\tilde{I}) - \Psi^{\epsilon}_{[0:t_i]}(A_{T_i};\tilde{I}) \le |T_i|P_{\max}.$$

which together with (50) leads to (51). □

Proof (Theorem 3.3). Suppose that there exists an offline feasible instance $\mathcal{I} = \{a_i, d_i, e_i, \bar{r}_i; P(t)\}_{i\in\mathcal{V}, t\in\mathcal{T}}$ such that the sLLF algorithm is not feasible with ϵ-power augmented resources. Then, from Appendix B.1.1, there exists another offline feasible instance $\tilde{I} = \{a_i, d_i, e_i, \bar{r}_i; P(t)\}_{i\in\tilde{\mathcal{V}}, t\in\mathcal{T}}$ such that

$$\Psi^{\epsilon}_{[0:t_-]}(\{i\};\tilde{I}) < \Psi^*_{[0:t_-]}(\{i\};\tilde{I}),\ i\in\mathcal{V}. \tag{56}$$

When $m = 1$, we obtain $\sum_{j\in V_{t_1-1}} \min(\bar{r}_j, e_j(t_1-1)) < P_{\max}$. Let $\mathcal{S} = \{i \in A_{T_1} : e_i(t_1) > 0\} \subset A_{T_1}$ denote the set of EVs that arrive during T_1 and have not yet been fully charged by t_1. Because the number of EVs is upper bounded by $P_{\max}/\bar{r}_{\min}$ (from (49)), and the EVs in $A_{T_1}\setminus\mathcal{S}$ are all fully charged,

$$\begin{aligned}P_{\max}|\hat{T}_2| &\le \Psi^*_{[0:t_-]}(A_{T_1};\tilde{I}) - \Psi^{\epsilon}_{[0:t_-]}(A_{T_1};\tilde{I})\\ &= \Psi^*_{[0:t_-]}(\mathcal{S};\tilde{I}) - \Psi^{\epsilon}_{[0:t_-]}(\mathcal{S};\tilde{I})\\ &\le XP_{\max}/\bar{r}_{\min}.\end{aligned}$$

This leads to

$$|\hat{T}_2| < \frac{X}{\bar{r}_{\min}}. \tag{57}$$

At time $t < t_{m-1}$, we have

$$\sum_{j\in V_{t_{m-1}-1}} \min(\bar{r}_j, e_j(t_{m-1}-1)) < (m-1)P_{\max},$$

which implies that there are at most $(m-1)P_{\max}/\bar{r}_{\min}$ EVs with unfulfilled energy demand by time t_f. Meanwhile, at time $t \ge t_m$, we have

$$\sum_{j\in V_{t_m}} \min(\bar{r}_j, e_j(t_m)) \ge mP_{\max},$$

which implies that there are at least $mP_{\max}/\bar{r}_{\max}$ EVs with unfulfilled energy demand during T_{m-1}. Therefore, the number of EVs that arrive during $[t_{m-1}, t_m]$ is greater than the following:

$$\frac{mP_{\max}}{\bar{r}_{\max}} - \frac{(m-1)P_{\max}}{\bar{r}_{\min}} \ge \frac{P_{\max}}{\bar{r}_{\min}}. \tag{58}$$

Since the inter-arrival periods of EVs are at least N, the length of $\hat{T}_{m-1}$ satisfies

$$|\hat{T}_{m-1}| \ge \frac{P_{\max}N}{\bar{r}_{\min}}. \tag{59}$$

Now, consider the following recursion:

$$\begin{aligned}|\hat{T}_2| &= |\hat{T}_3| + |T_3|\\ &\ge |\hat{T}_3| + |\hat{T}_4| \ge 2|\hat{T}_4| + |\hat{T}_5|\\ &\ge 3|\hat{T}_5| + 2|\hat{T}_6| \ge 5|\hat{T}_6| + 3|\hat{T}_7|\\ &\ge \cdots \ge f_{k-2}|\hat{T}_{m-1}| + f_{k-3}|\hat{T}_m|,\end{aligned}$$

where f_k is the Fibonacci sequence defined by $f_1 = 1$, $f_2 = 1$ and $f_k = f_{k-1} + f_{k-2}$ for $k \ge 3$. From the above, we have

$$|\hat{T}_2| > f_{m-2}|\hat{T}_{m-1}|.$$

Combining equations (57)-(59) gives

$$\frac{X}{\bar{r}_{\min}} > |T_2| > f_{m-2}|\hat{T}_{m-1}| > f_{m-2}\frac{P_{\max}N}{\bar{r}_{\min}} \tag{60}$$

From $m \le n$ for n defined in (48), we obtain

$$\begin{aligned}\left\lfloor (1+\epsilon)\frac{P_{\min}}{P_{\max}} \right\rfloor - 2 = m-2 &= \log_\varphi\left(\sqrt{5}f_{n-2} + \frac{1}{2}\right)\\ &< \log_\varphi\left(\frac{\sqrt{5}X}{NP_{\max}} + \frac{1}{2}\right),\end{aligned}$$

which gives $(1+\epsilon)P_{\min}/P_{\max} < \log_\varphi\left(\sqrt{5}X/NP_{\max} + 1/2\right) + 2$. □

Corollary 3.4. Suppose there exists an offline feasible instance $\mathcal{I}$ that is not feasible under the sLLF algorithm with 3-power augmentation. Using the same argument of the proof for Theorem 3.3, we obtain inequality (60). However, from assumption

$$f_1\frac{P_{\max}N}{\bar{r}_{\min}} \le \frac{X}{\bar{r}_{\min}},$$

which contradicts (60). □

B.3 Proof of Theorem 3.7

Proof (Theorem 3.3). Suppose that there exists an instance $\mathcal{I} = \{a_i, d_i, e_i, \bar{r}_i; P(t)\}_{i\in\mathcal{V}, t\in\mathcal{T}}$ such that the sLLF algorithm is not feasible with ϵ-augmented resources. We then have equation (47), repeated here for convenience:

$$\Psi^{\epsilon}_{[0:t_-]}(\tilde{\mathcal{V}};\tilde{I}) < \Psi^*_{[0:t_-]}(\tilde{\mathcal{V}};\tilde{I})$$

for another instance $\tilde{I} = \{a_i, d_i, e_i, \bar{r}_i; P(t)\}_{i\in\tilde{\mathcal{V}}, t\in\mathcal{T}}$.

Let $\mathcal{S}(\tilde{\mathcal{V}})$ be the set of EVs in the instance $\tilde{I}$ that receive strictly less energy under the online algorithm than under the offline algorithm by some time t at which $\Psi^{\epsilon}_{[0:t]}(\tilde{\mathcal{V}};\tilde{I}) < \Psi^*_{[0:t]}(\tilde{\mathcal{V}};\tilde{I})$:

$$\begin{aligned}\mathcal{S}(\tilde{\mathcal{V}}) = \Big\{i\in\tilde{\mathcal{V}} : \exists t\in\mathcal{T}\ \text{s.t.}\ &\Psi^{\epsilon}_{[0:t]}(\{i\};\tilde{I}) < \Psi^*_{[0:t]}(\{i\};\tilde{I})\\ &\&\ \Psi^{\epsilon}_{[0:t]}(\tilde{\mathcal{V}};\tilde{I}) < \Psi^*_{[0:t]}(\tilde{\mathcal{V}};\tilde{I})\Big\}.\end{aligned}$$

In view of (47), $\mathcal{S}(\tilde{\mathcal{V}}) \ne \emptyset$. Consider EV $j = \operatorname{argmin}_{i\in\mathcal{S}(\tilde{\mathcal{V}})} a_i$ that arrives the earliest among those in $\mathcal{S}(\tilde{\mathcal{V}})$. There exists a time $t \in [a_j, d_j]$ such that

$$\Psi^{\epsilon}_{[0:t]}(\{j\};\tilde{I}) < \Psi^*_{[0:t]}(\{j\};\tilde{I}), \tag{61}$$

$$\Psi^{\epsilon}_{[0:t]}(\tilde{\mathcal{V}};\tilde{I}) < \Psi^*_{[0:t]}(\tilde{\mathcal{V}};\tilde{I}). \tag{62}$$

Notice that $\Psi^{\epsilon}_{[0:a_j-1]}(\tilde{\mathcal{V}};\tilde{I}) < \Psi^{*}_{[0:a_j-1]}(\tilde{\mathcal{V}};\tilde{I})$ can only happen when there is another EV in $\mathcal{S}(\tilde{\mathcal{V}})$ that arrives before EV j, which however contradicts the definitions of $\mathcal{S}(\tilde{\mathcal{V}})$ and j. So,

$$\Psi^{\epsilon}_{[0:a_j-1]}(\tilde{\mathcal{V}};\tilde{I}) \geq \Psi^{*}_{[0:a_j-1]}(\tilde{\mathcal{V}};\tilde{I}),$$

which implies

$$\Psi^{\epsilon}_{[a_j:t]}(\tilde{\mathcal{V}};\tilde{I}) < \Psi^{*}_{[a_j:t]}(\tilde{\mathcal{V}};\tilde{I}). \tag{63}$$

Now, let us take a look at the energy demand fulfilled during the interval $[a_j, t]$ under the sLLF algorithm with ϵ-augmented resources. Define the overloaded times

$$T_o = \left\{ t \in [a_j, t] : \sum_{i \in \tilde{\mathcal{V}}} r_i(t) = (1+\epsilon)P(t) \right\}$$

and underloaded times

$$T_u = \left\{ t \in [a_j, t] : \sum_{i \in \tilde{\mathcal{V}}} r_i(t) < (1+\epsilon)P(t) \right\},$$

we have $|T_o| + |T_u| = t + 1 - a_j$. The total energy demand fulfilled during the overloaded period is lower bounded by $|T_o|(1+\epsilon)\min_{\tau\in[a_j,d_j]} P(\tau)$, while that during the underloaded period is at least $|T_u|(1+\epsilon)\bar{r}_j$. Hence, the total and individual energy demands fulfilled during $[a_j, t]$ are lower bounded by

$$(1+\epsilon)\left(|T_u|\bar{r}_j + |T_o| \min_{\tau\in[a_j,d_j]} P(\tau)\right) \leq \Psi^{\epsilon}_{[a_j:t]}(\tilde{\mathcal{V}};\tilde{I}), \tag{64}$$

$$(1+\epsilon)|T_u|\bar{r}_j \leq \Psi^{\epsilon}_{[a_j:t]}(\{j\};\tilde{I}). \tag{65}$$

Next, let us take a look at the energy demand fulfilled during the interval $[a_j, t+1]$ by the offline algorithm without resource augmentation. The total energy fulfilled is upper bounded by

$$\Psi^{*}_{[a_j:t]}(\tilde{\mathcal{V}};\tilde{I}) \leq (t+1-a_j) \max_{\tau\in[a_j,d_j]} P(\tau), \tag{66}$$

and the energy fulfilled to EV j is upper bounded by

$$\Psi^{*}_{a_j:t}(j) \leq (t+1-a_j)\bar{r}_j. \tag{67}$$

By equations (61), (65) and (67), we have

$$|T_u|(1+\epsilon) < (t-a_j+1). \tag{68}$$

By equations (63) (64) and (66), we have

$$\begin{aligned}&(1+\epsilon)(|T_u|\bar{r}_j + |T_o| \min_{\tau\in[a_j,d_j]} P(\tau))\\ &< (t+1-a_j) \max_{\tau\in[a_j,d_j]} P(\tau).\end{aligned}$$

Combining equation (68) becomes

$$\begin{aligned}&(|T_u| + |T_o|)(1+\epsilon) \min_{\tau\in[a_j,d_j]} P(\tau)\\ &< (t-a_j+1)\Big(\max_{\tau\in[a_j,d_j]} P(\tau) + \min_{\tau\in[a_j,d_j]} P(\tau) - \bar{r}_j\Big)\end{aligned}$$

Notice that $|T_o| + |T_u| = t + 1 - a_j$, the above inequality leads to

$$\epsilon < \max_{\tau_1,\tau_2\in[a_j,d_j]} \frac{P(\tau_1)}{P(\tau_2)} - \min_{i\in\mathcal{V}} \max_{\tau\in[a_i,d_i]} \frac{\bar{r}_i}{P(\tau)}.$$

□

C BENCHMARK ALGORITHMS

We summarize the online algorithms evaluated in Section 4. See [20] for a review of each algorithm.

Earliest Deadline First (EDF). All EVs in $\mathcal{V}_t$ are sorted by their deadlines d_i in an increasing order. The available power $P(t)$ is assigned to EVs in this order up to $\min(\bar{r}_i, e_i(t))$.

Least laxity first algorithms (LLF). All EVs in $\mathcal{V}_t$ are sorted by their laxities $\ell_i(t)$ in an increasing order. The available power $P(t)$ is assigned to EVs in this order up to $\min(\bar{r}_i, e_i(t))$.

Equal Share (ES). The available power supply $P(t)$ is divided equally to all connected EVs able to charge more energy, each EV receives the minimum between their fair share and their maximum charging rate. Repeat until either $P(t)$ power is supplied or no more EV can be charged further.

Remaining Energy Proportional (REP). The available power $P(t)$ is divided to EVs in proportion to their remaining energy demand $e_i(t)$. Each EV receives the minimum between their proportional share and their maximum charging rate. Repeat until either $P(t)$ power is supplied or no more EV can be charged further.

Online Linear Program (OLP) [3]. At each time t, the charging rate $r_i(t)$ to EV $i \in \mathcal{V}_t$ is provided according to the solution of the following LP:

$$\begin{aligned}\min \quad & \sum_{i\in\mathcal{V}_t} \sum_{\tau=t}^{T} \tau r_i(\tau)\\ \text{subject to} \quad & \sum_{\tau=t}^{T} r_i(\tau) = e_i(t),\ \forall i \in U_t\\ & \sum_{i\in U_\tau} r_i(\tau) \leq P(\tau),\ \forall \tau = t,\dots,T\\ & 0 \leq r_i(t) \leq \bar{r}_i\end{aligned}$$

The constraints of the online LP to find a feasible schedule for all the currently active EVs assuming no EV arrivals in the future, while objective function encourages the charging station to charge EVs as early as possible.