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### Absolute air refractive index measurement and tracking based on variable length vacuum cell

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**Abstract.** A refractometer system using four modified Wu-type heterodyne interferometers with a variable length vacuum cell is presented. The proposed system has two working modes: (1) a moving mode for measuring the absolute air refractive index at the start of a measurement and (2) a static mode for monitoring the air refractive index fluctuation with the same bandwidth as a traditional displacement interferometer. The system requires no gas filling or pumping during the measurement and can be used for real-time refractive index compensation. Comparison experiments with empirical equations are conducted to investigate the feasibility and performance of the proposed system. The standard deviation of the measurement difference between the proposed system and empirical equation is 2.8 parts in 10<sup>7</sup>, which is close to the uncertainty of our refractive index reference based on the accuracy of the environmental sensors. The relative refractive index tracking is a few parts in 10<sup>8</sup> with a bandwidth of 10 Hz, but high bandwidths are readily achievable. © 2016 Society of Photo-Optical Instrumentation Engineers (SPIE) [DOI: 10.1117/1.OE.55.6.064112]

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#### 1 Introduction

Precise knowledge of the absolute refractive index of air is essential for an accurate measurement for most optical interferometers working under atmospheric conditions. For decades, the refractive index of air has been estimated using the empirical equations by continuous environmental monitoring in the area (temperature, pressure, humidity, carbon dioxide, and so on), which are limited based on their underlying correction algorithms to approximately several parts in  $10^8$ . However, the empirical equations are valid only for standard air with a specific gas composition and are also limited by the accuracy and bandwidth of environmental sensors. Therefore, a direct measurement is desired to achieve low measurement uncertainty and real-time refractive index compensation with bandwidths comparable to traditional displacement interferometry.

There are several approaches for directly measuring the absolute refractive index of air. For some conventional refractometers, the optical path change is continuously recorded in a gas filling or pumping process in a sealed chamber to determine the absolute refractive index, <sup>4,5</sup> which is time-consuming and must compensate for the vacuum chamber deformation. To avoid a gas filling or pumping process, permanently evacuated chambers were adopted in some refractometers. Xie et al. <sup>6</sup> employed a specific trapezoidal vacuum cavity but the accuracy is highly dependent on the precision of the fabricated cavity. Huang et al. <sup>7</sup> adopted a synthetic pseudo-wavelength method, but the measuring range relies on the designed lengths of different cavities. A similar method is adopted by Yan et al. <sup>8</sup> based on laser synthetic wavelength interferometry with Edlén equation estimation,

In this manuscript, we present a refractometer by adapting a new optical configuration to the Fujii et al. <sup>15</sup> refractometer, which combines a variable length vacuum cell and a double-pass Michelson interferometer. Our new optical configuration is simpler, free of polarization mixing problems, and immune to common mode errors. We also extend the functionality of the former refractometer by accommodating both absolute refractive index sensing and tracking into the same system. This approach can measure the absolute refractive index in several seconds before subsequently switching to a tracking mode to measure refractive index fluctuations at high bandwidth without the gas filling or pumping process, which is more suitable for real-time refractive index compensation.

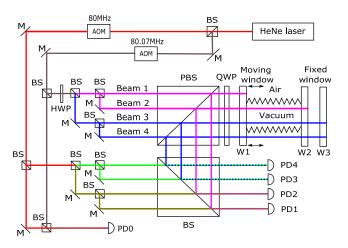
#### 2 Optical Configuration

The refractometer contains four sets of modified Wu-type interferometers  $^{16}$  as shown in Fig. 1. The vertical polarized light from a stabilized HeNe laser ( $\lambda = 633$  nm) is split equally and sent to two acoustic-optic modulators (AOMs), which creates two spatially separated beams with a 70-kHz frequency difference. The two beams pass through a 90:10 (R:T) beamsplitter. The two transmitted beams will interfere again at PD<sub>0</sub> to create a reference interference signal.

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but it also has limited measurement bandwidth. Fabry-Perot cavities<sup>9</sup> or frequency combs<sup>10,11</sup> were adopted in other systems, which can achieve high accuracy but require sophisticated and elaborate systems. Several other relative air refractive index tracking measurement methods with high measurement bandwidth and accuracy have been proposed, <sup>12–14</sup> but require another method to establish the initial absolute refractive index of air.

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**Fig. 1** Schematic of the refractometer. (BS, beamsplitter; M, mirror; AOM, acousto-optic modulator; HWP, half waveplate; QWP, quarter waveplate; PD, photodetector; and W, window.

From the initial beams, one reflected beam is horizontally polarized after passing through a half waveplate and equally split into four parallel beams using beamsplitters and mirrors. Then, the four beams transmit through a polarizing beamsplitter and a quarter waveplate toward the variable length vacuum cell. The cell consists of three windows with specific reflective coatings and a metallic bellow to form a length-changeable vacuum cavity. Beam 1 passes through the air and reflects from the first window (moving window). Beam 2 passes into the vacuum cell and reflects from the second window. Beam 3 passes into the vacuum cell and reflects from the third window. Beam 4 passes through air and reflects from the third window. After passing through the quarter wave plate for a second time, the polarization of the four beams rotates 90 deg, causing reflection at the PBS.

The second input beam to the system is vertically polarized. Then, it is split similarly into four beams that interfere with their respective beams in a 50:50 (R:T) beamsplitter. The interference signals can be detected at measurement photodetectors (PD<sub>1</sub>, PD<sub>2</sub>, PD<sub>3</sub>, and PD<sub>4</sub>) that convert optical signals to electrical signals for postprocessing. A locked-in detection or discrete Fourier transform (DFT) technique is then applied to extract the phase of the measurement signals relative to the reference signal. Finally, the optical path length (OPL) change for each beam can be determined by these accumulative phase changes  $\phi_1$ ,  $\phi_2$ ,  $\phi_3$ , and  $\phi_4$ .

#### 3 Working Principles

The refractometer has two working modes: (1) a moving mode to determine the absolute refractive index of air and (2) a static mode to track its fluctuations. The schematic of the variable length vacuum cell is shown in Fig. 2. In the moving mode, the moving window  $(W_1)$  is connected to a linear stage that has a displacement  $\Delta z_1$ . Due to the force introduced by extension or contraction of the metallic bellow, the static windows  $(W_2 \text{ and } W_3)$  will have a small displacement  $\Delta z_2$  and  $\Delta z_3$ .

Thus, the OPL change for each beam is represented by

$$\Delta OPL_1 = n\Delta z_1, \tag{1}$$

$$\Delta OPL_2 = (n-1)\Delta z_1 + \Delta z_2, \tag{2}$$

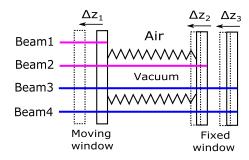


Fig. 2 Schematic of variable length vacuum cell and measurement beam paths.

$$\Delta OPL_3 = (n-1)\Delta z_1 - (n-1)\Delta z_2 + n\Delta z_3, \tag{3}$$

$$\Delta OPL_4 = n\Delta z_3,\tag{4}$$

where n is the absolute refractive index of air and  $\Delta OPL_i$  (i = 1,2,3, and 4) is converted from individual phase change  $\Delta \phi$  between measurement photodetector (PD<sub>1</sub>, PD<sub>2</sub>, PD<sub>3</sub>, and PD<sub>4</sub>) with reference photodetector PD<sub>0</sub>

$$\Delta OPL = \frac{\Delta \phi c}{4\pi f},\tag{5}$$

where c is the speed of light and f is the laser frequency. The absolute refractive index of air, n, can be determined by solving Eqs. (1)–(4)

$$n = 1 + \frac{\Delta \text{OPL}_3 - \Delta \text{OPL}_4}{\Delta \text{OPL}_1 - \Delta \text{OPL}_2}.$$
 (6)

In static mode for refractive index tracking, the OPL change between beam 1 and beam 2 is

$$\Delta OPL_{1,2} = \Delta L, \tag{7}$$

which indicates the vacuum cell length fluctuation. The OPL change between beam 3 and beam 4 is

$$\Delta OPL_{3,4} = L\Delta n + (n-1)\Delta L, \tag{8}$$

which is a combination of vacuum length change,  $\Delta L$  and refractive index fluctuation  $\Delta n$ . By solving Eqs. (7) and (8), the refractive index fluctuation can be calculated by

$$\Delta n = \frac{1}{L} [\Delta OPL_{3,4} - (n-1)\Delta OPL_{1,2}], \tag{9}$$

where L is the vacuum cell length.

#### 4 Experiments and Results

To verify the feasibility of the proposed method for absolute refractive index sensing and tracking, an experimental setup was constructed as shown in Fig. 3 (except the laser source). The laser source is a stabilized red HeNe laser (HRS015, Thorlabs, Inc.), which emits 1.2 mW of vertically polarized light with a 632.991-nm wavelength. Afterward, the equally-split beams are modulated separately by AOMs (1250C-2, Isomet Corporation) driven at 80 and 80.07 MHz. As described

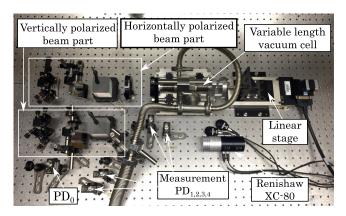


Fig. 3 Photograph of the experimental setup (except the laser source).

in Sec. 2, four horizontally polarized beams enter a customized length changeable vacuum cell that mainly consists of three windows (BK7 glass) with specific reflective coating and stainless steel bellows. 15 The cell is evacuated by a mechanical pump (2010 SD, Alcatel Adixen) and driven by a linear stage (UE41PP, Newport Corporation). The interference signals are detected using measurement photodetectors (PDA36A, Thorlabs, Inc.). The individual phase change between our measurement photodetectors (PD<sub>1</sub>, PD<sub>2</sub>, PD<sub>3</sub>, and PD<sub>4</sub>) and reference photodetector (PD<sub>0</sub>) are processed by an field-programmable gate array (FPGA)-based phasemeter (IT-PM-LI-4-1480, InSituTec, Inc.) connected to the host PC through a Universal Serial Bus (USB). This phasemeter uses single-bin DFT algorithm to extract the phase information. The Renishaw XC-80 compensator is placed next to our refractometer, which measures air temperature (range: 0°C to 40°C, accuracy:  $\pm 0.1$  °C), air pressure (range: 650 to 1150 mbar, accuracy:  $\pm 1$  mbar) and relative humidity (range: 0% to 95%, accuracy: 6% RH) every 7 s. All these environmental parameters are input into the National Institute of Standards and Technology (NIST) refractive index of air calculator to obtain an absolute value for comparison. 18

The experimental process for the refractometer is as follows:

- 1. Begin with the refractometer at rest and establish a zero point for all interferometers.
- 2. Scan the refractometer 2.5 mm.
- 3. Establish a starting absolute refractive index value.
- 4. Hold the refractometer at rest. Monitor the signals to track refractive index fluctuation relative to the initial starting value.

#### 4.1 Absolute Refractive Index—Moving Mode

To establish the absolute refractive index, the refractometer must be scanned through a preprogrammed displacement. In this case, the linear stage is scanned 2.5 mm at a speed of 0.5 mm/s. Our FPGA-based phasemeter can simultaneously process five signals (four measurement signals and one reference signal) and measure the individual phase change  $\Delta\phi_1$ ,  $\Delta\phi_2$ ,  $\Delta\phi_3$ , and  $\Delta\phi_4$  for the four beams. The OPL change ( $\Delta$ OPL) for each beam can be calculated by Eq. (5), which includes both physical path change and refractive index. Figure 4 shows the OPL change of beam 1, beam 2, beam 3 and beam 4 during moving mode, which corresponds to

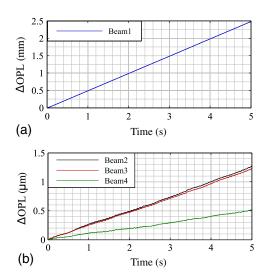


Fig. 4 The OPL change for (a) beam 1 and (b) beam 2, 3, and 4 during moving mode based on the calculation from Eq. (5).

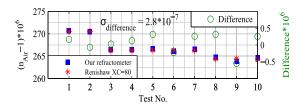


Fig. 5 The comparison of absolute refractive index between our refractometer and Renishaw XC-80.

Eqs. (1)–(4). The absolute refractive index is calculated by Eq. (6).

Figure 5 shows the absolute refractive index comparison between our refractometer and Renishaw XC-80 in ten individual tests under different environmental conditions. The standard deviation of the measurement difference between the proposed system and empirical equation is  $2.8 \times 10^{-7}$ . According to the uncertainty estimation of air refractive index based on the accuracy of environmental sensors, the final uncertainty provided by Renishaw XC-80 compensator is  $2.9 \times 10^{-7}$ . Thus, we are limited by our current environmental sensors and can validate the absolute refractive index only down to a few parts in  $10^7$ .

#### 4.2 Refractive Index Tracking—Static Mode

Once the absolute refractive index of air is established, the variable length vacuum cell can be kept in a static mode for refractive index tracking. In this case, the linear stage is held for 700 s and the phase fluctuations for the different beams are recorded. Due to the storage limit for our phasemeter, the sampling rate for this 700 s measurement is 10 Hz, but in principle this can be considerably higher with more sophisticated electronics. At the beginning of the experiment, the temperature was 21.49°C, humidity was 45%, and air pressure was 992.8 mbar. At the end, the temperature was 21.52° C, humidity was 45%, and air pressure was 992.8 mbar. The humidity and air pressure remained stable during the experiment, but the temperature increased steadily, resulting in an increase of 0.03°C. This temperature increase corresponds to approximately a  $-3 \times 10^{-8}$  change in refractive index. 19

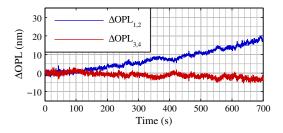


Fig. 6 The OPL change between beam 1 and 2, beam 3 and 4 during static mode based on the calculation from Eq. (5).

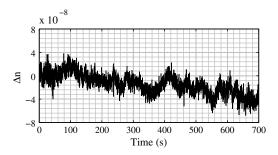


Fig. 7 The calculated refractive index fluctuation over 700 s in static mode

Figure 6 shows the OPL change between beam 1 and beam 2, beam 3 and beam 4 during static mode, which corresponds to Eqs. (7) and (8). Figure 7 shows the refractive index fluctuation calculated by Eq. (9) in static mode for 700 s, which matches the result estimated by environmental parameter change.

#### 5 Systematic Errors Discussion

The proposed schematic and equations are based on an ideal system with perfect alignment. Existing systematic errors must be eliminated or corrected in our real measurement system. In this section, three main systematic errors are discussed.

#### 5.1 Residual Gas in the Vacuum Cell

The vacuum cell is evacuated by a mechanical pump up to 0.3 Pa and may leak during the measurement, so the refractive index inside the variable length vacuum cell  $n_0$  is not exactly 1. Also, the variable length system is the system from the Fujii et al. 15 research and the condition of the seals is unknown. There is no way to measure the pressure inside the cell nor to test for leaks.

#### 5.2 Beam Alignment

Ideally, the four beams incident into the variable length vacuum cell should be parallel to each other. However, in practice there is angular misalignment among the four beams, which causes cosine errors in the OPL change. In our refractometer system, the angles between beam 1, beam 2, and beam 3 relative to beam 4 ( $\theta_{14} = 0.06^{\circ}$ ,  $\theta_{24} = 0.015^{\circ}$ , and  $\theta_{34} = 0.09^{\circ}$ ) are measured to correct for the cosine error.

#### 5.3 Extra Optical Path Length Term

Ideally, the term  $(\Delta OPL_3 - \Delta OPL_4)/(\Delta OPL_1 - \Delta OPL_2)$  in Eq. (6) should be zero if the refractometer works in moving mode with the cell open to air. However, because the moving window has pitch and yaw during translation, as well as the

unequal deformation at the incident point of beam 3 and beam 4 on the fixed window in moving mode, this term will not be zero. We measured this system error term ( $\Delta_{error} = 0.0000209$ ) with the cell open to air in moving mode.

Considering the above system errors, our final equation to calculate absolute refractive index in moving mode is

$$n = n_0 \left[ 1 + \frac{\frac{\Delta \text{OPL}_3}{\cos \theta_{34}} - \Delta \text{OPL}_4}{\frac{\Delta \text{OPL}_1}{\cos \theta_{14}} - \frac{\Delta \text{OPL}_2}{\cos \theta_{24}}} - \Delta_{\text{error}} \right]. \tag{10}$$

#### 6 Uncertainty Evaluation

#### 6.1 Absolute Refractive Index

According to Eq. (10) and the principles of the proposed method, the measurement uncertainty of absolute refractive index depends on the uncertainty of corrected optical length change ( $\Delta$ OPL corrected for cosine error) for four beams, the residual gas in vacuum cavity and an extra OPL term.

- a. Corrected  $\Delta OPL$ : According to Eq. (5), the uncertainty of  $\Delta OPL$  is dependent on laser frequency and phase measurement. The fractional uncertainty of the laser source (HRS015, Thorlabs, Inc.) is  $1\times 10^{-8}$  according to the user guide. The uncertainty of phase measurement by the FPGA-based phasemeter (IT-PM-LI-4-1480, InSituTec, Inc.) is 1 mrad based on a wire test. In this modified Wu-type heterodyne interferometer schematic, the residual level of periodic nonlinearity is less than 20 pm. The angle measurement uncertainty among the four beams is  $0.002^{\circ}$ . Thus, the contributions of each  $\Delta OPL$  after correcting for cosine error are  $3.9\times 10^{-8}$ ,  $2.2\times 10^{-8}$ ,  $2.2\times 10^{-8}$ , respectively.
- b. Residual gas in vacuum cavity: The measurement uncertainty caused by residual gas with up to 0.3 Pa would be no larger than  $8 \times 10^{-10}$ .
- c. Extra OPL term: This systematic error term is also a  $\Delta$ OPL measurement as stated above with the uncertainty contribution of  $5.4 \times 10^{-8}$ .

Overall, the combined uncertainty of the proposed absolute refractive index measurement method is  $7.6 \times 10^{-8}$ . However, it should be noted that the experimental result in Sec. 4.1 show a  $2.8 \times 10^{-7}$  standard deviation between our refractometer and the empirical equation. This discrepancy is partly caused by the calibrated uncertainty of environmental sensors and the intrinsic uncertainty of empirical equation as we mentioned in Sec. 4.1.

#### **6.2** Relative Refractive Index

According to Eq. (9), the expand relative refractive index fluctuation uncertainty is

$$u(\Delta n) = \left[ \left( \frac{n-1}{L} \right)^{2} u^{2} (\Delta OPL_{1,2}) + \left( \frac{1}{L} \right)^{2} u^{2} (\Delta OPL_{3,4}) + \left( \frac{\Delta OPL_{3,4} - (n-1)\Delta OPL_{1,2}}{L^{2}} \right)^{2} u^{2} (L) + \left( \frac{\Delta OPL_{1,2}}{L} \right)^{2} u^{2} (n) \right]^{\frac{1}{2}}.$$
(11)

Table 1 Refractometer uncertainty contributor summary.

	Component	Uncertainty contribution
Absolute refractive index	$\Delta OPL_1/\cos heta_{14}$	$3.9 \times 10^{-8}$
	$\Delta OPL_2/\cos heta_{24}$	$2.2 \times 10^{-8}$
	$\Delta {\rm OPL_3/\cos heta_{34}}$	$2.2 \times 10^{-8}$
	$\DeltaOPL_4$	$2.2 \times 10^{-8}$
	$n_0$	$8 \times 10^{-10}$
	$\Delta_{error}$	$5.4 \times 10^{-8}$
		$7.6 \times 10^{-8}$
Relative refractive index	$\Delta OPL_{1,2}$	$2.4 \times 10^{-13}$
	$\Delta OPL_{3,4}$	$8.8 \times 10^{-10}$
	L	$5 \times 10^{-11}$
	N	$7 \times 10^{-14}$
		$1.3 \times 10^{-9}$

- a. OPL difference:  $\Delta OPL_{1,2}$  and  $\Delta OPL_{3,4}$  represent the OPL change between beam 1 and beam 2, beam 3 and beam 4 with the uncertainty of  $1 \times 10^{-10}$ . Thus, the contributions of  $\triangle OPL_{1.2}$  and  $\triangle OPL_{3.4}$  are 2.4 ×  $10^{-13}$  and  $8.8 \times 10^{-10}$ .
- b. Length error of vacuum cavity: The nominal physical length of vacuum cavities is  $8 \times 10^{-2}$  m. Considering the alignment error that the axis of vacuum cavity is not parallel with the beam propagation direction (within 0.2°) and the length measurement uncertainty, the vacuum cavity length uncertainty is still less than 100  $\mu$ m, which contributes  $5 \times 10^{-11}$  to relative refractive index measurement. Additionally,  $\Delta OPL_{1,2}$ can track the vacuum cavity length change during the measurement with subnanometer resolution assuming minimal thermal expansion of material and temperature change. Thus, the material expansion error will not be a major uncertainty in our discussion.
- c. Absolute refractive index: The last term is attributed to current absolute refractive errors, either calculated by Edlén equation or measured in our moving mode. Even using the  $3 \times 10^{-7}$  uncertainty value, the final contribution is only  $7 \times 10^{-14}$ .

Thus, the combined uncertainty of relative refractive index tracking is  $1.3 \times 10^{-9}$  based on theoretical uncertainty

Table 1 summarizes the uncertainty contributor of our refractometer in absolute refractive index sensing and fluctuation tracking based on Eqs. (10) and (11).

#### 7 Conclusion

A refractometer based on variable length vacuum cell that enables both absolute refractive index sensing and tracking is presented. The system requires no gas filling or pumping process in the two working modes and can be used for real-time compensation of interferometry measurements. The comparison experiments with empirical equation for moving and static mode are conducted. Limited by the accuracy of our environmental sensors (Renishaw XC-80) and mechanical pump, the standard deviation between our absolute refractive index measurement result and empirical equation is  $2.8 \times 10^{-7}$  in moving mode. The accuracy for refractive index fluctuation tracking can be a few parts in 108.

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