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Application of magnetoelastic materials in spatiotemporally modulated phononic crystals for nonreciprocal wave propagation

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Abstract

In this paper, a physical platform is proposed to change the properties of phononic crystals in space and time in order to achieve nonreciprocal wave transmission. The utilization of magnetoelastic materials in elastic phononic systems is studied. Material properties of magnetoelastic materials change significantly with an external magnetic field. This property is used to design systems with a desired wave propagation pattern. The properties of the magnetoelastic medium are changed in a traveling wave pattern, which changes in both space and time. A phononic crystal with such a modulation exhibits one-way wave propagation behavior. An extended transfer matrix method (TMM) is developed to model a system with time varying properties. The stop band and the pass band of a reciprocal and a nonreciprocal bar are found using this method. The TMM is used to find the transfer function of a magnetoelastic bar. The obtained results match those obtained via the theoretical Floquet–Bloch approach and numerical simulations. It is shown that the stop band in the transfer function of a system with temporal varying property for the forward wave propagation is different from the same in the backward wave propagation. The proposed configuration enables the physical realization of a class of smart structures that incorporates nonreciprocal wave propagation.

Keywords: phononic crystals, nonreciprocal wave propagation, magnetoelastic materials, transfer matrix, stop bands, pass bands, spatiotemporal modulation

(Some figures may appear in colour only in the online journal)

1. Introduction

By nature, acoustic and elastic waves tend to propagate symmetrically in all spatial directions, a phenomenon that is attributed to elastodynamic reciprocity. In other words, wave transmission between two points is not a function of direction. If sound can be transmitted from point A to point B in a given medium, we expect it to be transmittable from point B to point A as well under the same conditions. While we frequently experience this phenomenon in our daily life, reciprocity might not be always desirable. As a result, there has been a recent spurt of activity addressing the design of non-reciprocal material systems where waves are engineered to propagate in only one direction. For example, Liang [1] utilized a superlattice with a nonlinear medium to synthesize a one-dimensional acoustic diode. Boechler [2] studied a mechanism for tunable rectification that uses bifurcations and chaos. Fleury [3] showed acoustic isolation and nonreciprocal sound transmission using a resonant ring cavity biased by a circulating fluid. Nonreciprocity has also been demonstrated in the context of static loading where Coulais [4] utilized large nonlinearities and geometrical asymmetries to achieve static nonreciprocal mechanical metamaterials.

Achieving one-way wave propagation motivated the design of acoustic diodes, analogous to their electric counterparts. It has been shown, however, that the transmission-reflection symmetry is not broken in such linear systems [5]. Systems comprising time-varying material fields, however, have been shown to break the elastic reciprocity of the propagating waves in linear systems because of the induced parametric pumping [6, 7]. Combined with stop bands resulting from material variations in space, such spatiotemporally modulated systems have been recently introduced as a way to achieve one-way phonon isolation [8]. Among such efforts, Nassar [9] investigated wave propagation in periodic laminates where the modulus of elasticity and mass density are modulated in space and time. Chaunsali [7] showed that nonreciprocal behavior of 1D space-time dependent systems can be linked to topological nature of stop band. Croenne *et al* [10] studied the application of spatiotemporal modulation of electrical boundaries in 1D phononic crystals to achieve nonreciprocal behavior and showed its effects are analogous to the classical acousto-optic Brillouin scattering. Although wave propagations in both reciprocal and nonreciprocal phononic crystals have been investigated by several groups [1–3, 7, 11, 12], to the best of our knowledge, a realizable physical configuration for imposing the spatiotemporal property variations in the modulated phononic systems has not yet been proposed.

In this paper, we envision creating a traveling wave like magnetic field through a mechanism similar to linear induction motors (LIM) that will yield any desired property variation in a magnetoelastic medium. Mechanical properties of magnetoelastic materials have been shown to change significantly when subjected to an external magnetic field [13–15]. Terfenol-D ($Tb_{.3}Dy_{.7}Fe_2$) is a good candidate of magnetoelastic materials that can be utilized to build a bar with a varying modulus of elasticity. The modulus of elasticity of Terfenol-D can be changed to a desired value by changing the magnetic field surrounding the material [13–15], which in turn provides a physical mechanism to change the modulus of elasticity of a bar simultaneously in space and time. The key thrust of this approach is in the real-time configurability of the magnetoelastic medium. Magnetoelastic materials have been of interest as building blocks for periodic structures due to the degree of flexibility they can introduce in stop band control. Among others, Wang *et al* [16] studied wave propagation in magnetoelectroelastic phononic crystals by assuming quasi-static approximations of magnetoelectric field. In the current approach, however, since material property modulations in both space and time are induced by an externally imposed traveling wave-like magnetic field, they are no longer restrained. This paradigm change opens up the possibility of near-instantaneous control of material properties leading up to intriguing wave dispersion characteristics such as, in this case, one-way propagation. Such capabilities can find potential applications in targeted signal conditioning, back-scattering elimination, and achieving unprecedented noise rejection standards in elastic periodic structures, also known as phononic crystals.

The magnetoelastic system studied here is a periodic structure. Periodic structures comprise a number of identical sub-structure components, also known as cells, connected in a periodic fashion to form an artificial discrete or continuous structure. These structures exhibit a unique wave propagation behavior [17]. At specific frequencies, referred to 'pass band', waves are able to freely propagate along these periodic structures while they will be blocked and rapidly attenuated at other frequency ranges, referred to 'stop band'. Extensive studies on wave propagation in periodic structures were first conducted by Brillouin [17], and a plethora of studies have since then been extended to different aspects of these materials. For instance, Mead [18] studied the free wave propagation in periodically supported infinite beams for isolating the vibration transmitted from sea waves to offshore platform. Roy [19] studied the attenuation behavior of periodic structures specifically the attenuation of bending waves in a dissipation-free beam with flexible ribs attached to it. Ponge *et al* [20] investigated controlling of elastic wave propagation in piezomagnetic phononic crystals. Their results showed that the stop band of the 1D phononic crystals could be tuned and controlled by connected external impedances. Bou Matar *et al* [21] studied the tuning of stop band in phononic crystals made of magnetoelastic materials by applying an external magnetic field. They showed that some elastic constants of Terfenol-D could vary more than fifty percent by changing the applied magnetic field. Wright [22] developed a closed-form solution to the acoustic wave transmission through a 1D time-varying phononic crystal.

The transfer matrix method (TMM) is a mathematical tool that is widely adopted to analyze the vibrations of periodic bar and beam-type systems [23–25]. Nouh [26] and Yu [27] used the TMM to study the vibration characteristics of Euler–Bernoulli beams with periodic resonators. The approach has been also used to study several other configurations of beams [28, 29] and plates [30, 31] with periodic elements, and locally resonant metamaterials [32, 33]. Recently, Trainiti [11] employed a Floquet–Bloch form solution [34] accompanied by a Fourier series expansion to conduct the dispersion analysis of nonreciprocal wave propagation in elastic structures. However, the directional transfer function of the structure was not addressed. Furthermore, while suitable for certain modulation waveforms, the truncated Fourier series is an optimistic approximation for square-wave modulations especially when higher frequency stop bands are of interest. Here, we propose an approximate analytical procedure based on an extended adaptation of the TMM approach to (1) extract dispersion curves and (2) find the directional transfer functions for a nonreciprocal structure. The method proves to be highly computationally efficient and is shown to stay reasonably accurate in terms of dispersion analysis for sufficiently slow temporal modulation, regardless of the dispersion band of interest. Through this approach, we investigate creating a traveling-wave form variation of material properties through use of Terfenol-D bar and an external magnetic field that breaks the symmetry of wave dispersion spectrum and onsets one-way propagation behavior in the phononic crystal. The goal is to have, within a certain mathematically predictable frequency range, waves that can travel in a given direction but not in the opposing one. We show that the dispersion relations parity in such systems has been broken and that the transfer functions for forward and backward traveling waves in a finite realization of the modulated system have become inherently different.

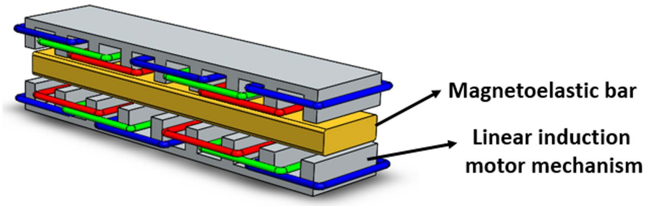

Figure 1. Physical platform for generating traveling waves for a magnetoelastic bar.

The paper is organized as follows: first, the device configuration and governing equation of the wave propagation in a magnetoelastic bar are discussed. Then, an approximate solution for the governing equation is provided, followed by the development of extended TMM for dispersion analysis and finding the transfer function in the proposed medium where material properties oscillate in both space and time. Two case studies are solved using the extended TMM and the results are validated with the Fourier expansion method and real-time simulations by a commercial finite element code, COMSOL multi-physics. A future experimental work is discussed and the transfer function for an under construction prototype is calculated. At the end, the accuracy of results are examined in comparison with the Fourier-expansion method at various temporal modulation velocities.

2. Device configuration and governing equations

Figure 1 shows a configuration similar to three phase linear induction motors to generate traveling magnetic wave along a bar made of magnetoelastic materials. The linear induction mechanism provides the required magnetic field for variation in the modulus of elasticity of the bar. LIM consists of Primary (equivalent to stator) and Secondary (equivalent to rotor). For our application, motion of the secondary (the moving plate) is not required and only the primary part of the motor is used to generate a traveling-wave magnetic field. The primary consists of a three-phase coil assembly. These coil windings are shown by the colors blue, green, and red in figure 1. The three-phase coil windings are inserted into a steel lamination stack. When three-phase AC power is applied to the LIM primary, a traveling-wave magnetic flux is induced along the motor. The wave length of the traveling-wave is the same as the pole pitch. Pole pitch of the motor is the distance between the centers of two adjacent poles. The speed of the traveling-wave is adjusted by the frequency of the drive and the amplitude of the wave, which is controlled by the drive [35]. The proposed configuration generates a traveling-wave transverse magnetic field along the length of the bar which results in the desired variation of modulus of elasticity of the bar along its length. A closed loop controller is utilized to control the generated magnetic field to ensure that the magnitude of the field is not affected by the magneto-electric couplings in the system. The Lagrangian of the magnetoelastic bar is, hence, defined as $L = T - U$, where T is the total kinetic energy and U is the total strain energy in the bar:

$$\begin{cases} T = \frac{A}{2}\int_0^{L_{\mathrm{Tot}}} \rho(x, t)\left(\frac{\partial u(x,t)}{\partial t}\right)^2 \mathrm{d}x, \\ U = \frac{A}{2}\int_0^{L_{\mathrm{Tot}}} \sigma(x, t)\frac{\partial u(x,t)}{\partial x}\mathrm{d}x, \end{cases} \tag{1}$$

where L_{Tot} is the length of the bar, A is the cross section of the bar, ρ is the density, u is the axial deformation, σ is the normal stress, x is the spatial coordinate, and t is time. Under the assumption of linear elasticity, small displacements, and Kelvin–Voigt damping, the stress at the cross section x and time t in the bar is given by [36]

$$\sigma(x, t) = E(x, t)\frac{\partial u(x, t)}{\partial x} + C_D\frac{\partial^2 u(x, t)}{\partial x \partial t} - E(x, t)\,\lambda(x, t), \tag{2}$$

where E is the modulus of elasticity, C_D is Kelvin–Voigt damping coefficient, λ is the magnetostriction due to the applied magnetic field. Therefore, Lagrangian of the system is an integral of function $f\left(x, t, u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial t}\right)$ over the length of the bar.

$$L = \int_0^{L_{\mathrm{Tot}}} f\left(x, \mathrm{t}, u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial \mathrm{t}}\right)\mathrm{d}x, \tag{3}$$

The governing equation of longitudinal vibration for an unforced bar is, thus, calculated using Hamilton's principle for a continuous system [37]:

$$\frac{\partial f}{\partial u} - \frac{\partial}{\partial t}\left(\frac{\partial f}{\partial_t u}\right) - \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial_x u}\right) = 0. \tag{4}$$

Using equations (1) and (2) to substitute for f in equation (4) yields the governing equation of motion for the magnetoelastic bar.

$$\frac{\partial}{\partial x}\left(E(x, t)\frac{\partial u(x, t)}{\partial x} + C_D\frac{\partial^2 u(x, t)}{\partial x \partial t} - E(x, t) \times \lambda(x, t)\right) = \frac{\partial}{\partial t}\left(\rho(x, t)\frac{\partial u(x, t)}{\partial t}\right). \tag{5}$$

Magnetostriction of the bar (λ) is proportional to the applied magnetic field [36]. Figure 2(a) shows the variation of λ in Terfenol-D with applied magnetic field and therefore, magnetostriction is a nonlinear function of the magnetic field. In order to simplify the governing equation, the applied magnetic field is assumed to stay within the range corresponding to region II in figure 2(a), where a linear correlation between the magnetostriction of the bar and the applied magnetic field can be expected [38, 39]. Figure 2(b) shows the effect of applied magnetic field on the elastic modulus of Terfenol-D. This effect is called ΔE effect. For a wide range (less than 100 Gpa), the relation is a linear and in this paper, the non-reciprocal magnetoelastic phononic crystal is designed and studied under this assumption.

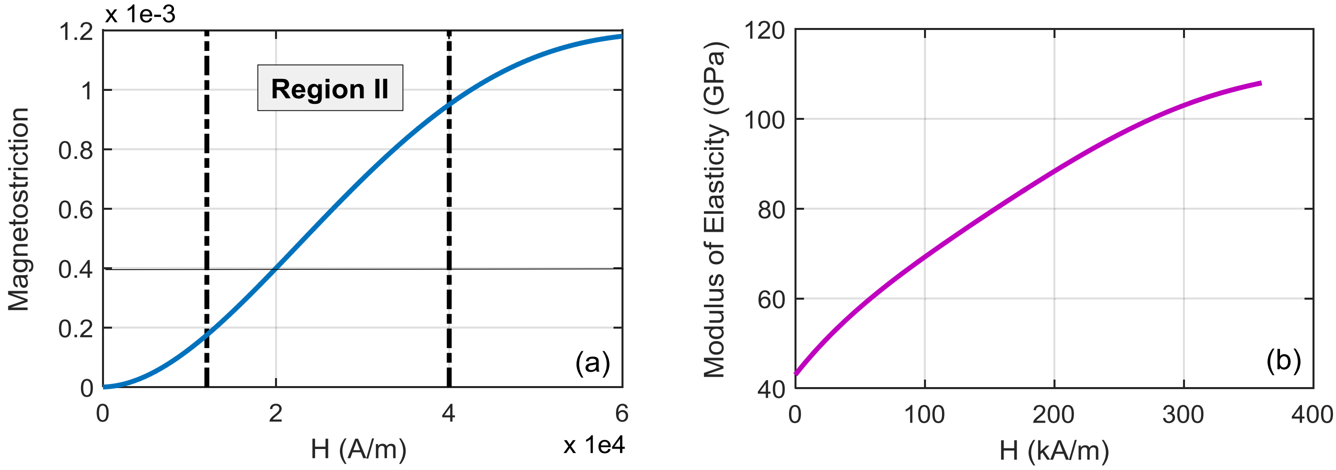

Figure 2. (a) Magnetostriction of Terfenol-D with respect to the applied magnetic field [36]. (b) Effect of applied magnetic field on the modulus of elasticity of Terfenol-D, ΔE effect [40].

3. Approximate solution to the governing equation

In this section, an approximate solution is provided to study the wave propagation in a bar with periodic modulus of elasticity in space and time. Trainiti [11] used a free wave Floquet–Bloch solution with appropriately modulated amplitude for wave propagation given by,

$$u(x,\,t) = \mathrm{e}^{\mathrm{i}(\omega t - kx)} \sum_{n=-\infty}^{+\infty} \hat{u}_n \mathrm{e}^{\mathrm{i}n(\omega_m t - k_m x)}, \tag{6}$$

where, ω_m, k_m, $\hat{u}_n$ are the temporal modulation frequency, spatial modulation frequency and nth Fourier expansion coefficient for the modulated amplitude respectively. Also, ω and k are the temporal and spatial frequencies of assumed free wave. This solution can be rewritten as $u = \sum_{n=1}^{\infty} U_n(x)\mathrm{e}^{\mathrm{i}\omega_n t}$, where $U_n(x)$ is the spatial and $\mathrm{e}^{\mathrm{i}\omega_n t}$ is the temporal part of the solution. $\omega_n = \omega + n\omega_m$ is the corresponding nth frequency of the temporal part that consists of both frequency of the external excitation and multiples of the temporal modulation frequency. If the material properties only change in space, the temporal part consists of only one frequency, which is the frequency of the input excitation ($\mathrm{e}^{\mathrm{i}\omega t}$). Since this way, we exclude the effect of temporal modulation from the assumed solution, it can provide an approximate solution when the modulus of elasticity of the bar also changes in time. This approximation, however, allows us to extend TMM and provides the possibility to approximately calculate directional transfer functions for wave propagation in a nonreciprocal bar. The transfer functions can be effectively used for designing nonreciprocal elastic structures. Therefore, if only the first dominant temporal term ($\mathrm{e}^{\mathrm{i}\omega t}$) is considered, then we can write equation (5) as:

$$(E + \mathrm{i}\omega C_D)\frac{\mathrm{d}^2 U}{\mathrm{d}x^2} + \frac{\partial E}{\partial x}\frac{\mathrm{d}U}{\mathrm{d}x} + \left(\omega^2 \rho - \mathrm{i}\omega\frac{\partial \rho}{\partial t}\right)U - \left(\frac{\partial E}{\partial x}\lambda + E\frac{\partial \lambda}{\partial x}\right)\mathrm{e}^{-\mathrm{i}\omega t} = 0. \tag{7}$$

4. Extended TMM

The TMM can effectively be used to find the ratio between the deformation of one end of the bar to the other end [25], or the transfer function (TF). In this paper, the conventional TMM is extended to solve for cases in which the modulus of elasticity varies in both space and time. Two cases are studied in this section. In the first case, the modulus of elasticity of the bar is only a function of space. In the second case, the modulus of elasticity varies both in space and time. In order to extend the TMM, a new variable is introduced as the traveling time, $\tau(x)$. The traveling time is the time taken by the wave to get to point x in the bar. We can write $\frac{x}{\tau(x)} = c_{\mathrm{avg}}$, where c_{avg} is the average of the wave speed in the bar from $x = 0$ to x and $t = 0$ to $t = \tau$. For an infinitesimal section, the variation of the traveling time with respect to the spatial coordinate is written as

$$\frac{\mathrm{d}\tau}{\mathrm{d}x} = \frac{1}{c}, \tag{8}$$

where, $c = \sqrt{E(x,\,\tau)/\rho}$. Appending this differential equation to the equation of motion in (7), allows us to use the TMM for structures with spatiotemporal modulation. Therefore, by adding a new state to the conventional state-space form of equation (7), we have:

$$\frac{\mathrm{d}}{\mathrm{d}x}\begin{bmatrix} U \\ \frac{\mathrm{d}U}{\mathrm{d}x} \\ \tau \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -\omega^2\rho/(E + \mathrm{i}\omega C_D) & -\frac{\partial E}{\partial x}\frac{1}{(E + \mathrm{i}\omega C_D)} & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} U \\ \frac{\mathrm{d}U}{\mathrm{d}x} \\ \tau \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{(E + \mathrm{i}\omega C_D)}\left(\frac{\partial E}{\partial x}\lambda + E\frac{\partial \lambda}{\partial x}\right)\mathrm{e}^{-\mathrm{i}\omega\tau} \\ \frac{1}{c} \end{bmatrix}. \tag{9}$$

By solving the above equation, a matrix is obtained that relates the value of the states, i.e. $[U,\,\mathrm{d}U/\mathrm{d}x,\,\tau]^T$ at any point on the bar to the corresponding values at a reference point. As finding the exact solution for (9) is rather complicated, we numerically calculated entries of matrix A [41], hence,

$$A(L_{\mathrm{Tot}}, 0) = \left[a^1(L_{\mathrm{Tot}}) \;\; a^2(L_{\mathrm{Tot}})\right] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}, \tag{10}$$

where $a^1(L_{\mathrm{Tot}})$ and $a^2(L_{\mathrm{Tot}})$ are the solution of the system (equation (9)) at the end of the bar with the initial conditions of $a^1(0) = [1,\ 0,\ 0]^T$ and $a^2(0) = [0,\ 1,\ 0]^T$ respectively. In a 1D elastic structure, the stress–strain relationship can be simplified to $U/\mathrm{d}x = F/EA$. Thus, after finding matrix A, we can write equation (11) for displacement and force in the bar:

$$\begin{bmatrix} U \\ F \end{bmatrix}_{L_{\mathrm{Tot}}} = \begin{bmatrix} a_{11} & a_{12}\frac{1}{AE(0)} \\ a_{21}AE(L_{\mathrm{Tot}}) & a_{22}\frac{E(L_{\mathrm{Tot}})}{E(0)} \end{bmatrix} \begin{bmatrix} U \\ F \end{bmatrix}_0. \tag{11}$$

The matrix that relates the displacement and force of one end of a bar to the other end is called the transfer matrix and denoted by T:

$$T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}, \tag{12}$$

where, $T_{11} = a_{11}$, $T_{12} = a_{12}\frac{1}{AE(0)}$, $T_{21} = a_{21}AE(L_{\mathrm{Tot}})$, and $T_{22} = a_{22}\frac{E(L_{\mathrm{Tot}})}{E(0)}$. The matrix T provides useful information about wave propagation in the bar. By looking at the eigenvalues of transfer matrix, λ_1 and λ_2, it is found that they satisfy $\lambda_1\lambda_2 = 1$. Therefore, a pass band is defined as the range of frequency, at which the magnitude of both eigenvalues is one. Otherwise, the waves gets attenuated along the bar which defines a stop band condition [25]. The physical meaning of the eigenvalues of the matrix T is better revealed by rewriting them in a polar form of $\lambda = \mathrm{e}^{\mu} = \mathrm{e}^{\alpha+\mathrm{i}\beta}$. Where μ is named as the wave propagation constant. The real part, α reflects the logarithmic decay of the states corresponding to a stop band, while a purely imaginary μ indicates a pass band.

Here, we considered a bar with base excitation at one end and free on the other end, which means $F_{L_{\mathrm{Tot}}} = 0$. Thus, the transfer function of the displacement of one end of the bar to the displacement of the other end is written as:

$$\frac{U_{L_{\mathrm{Tot}}}}{U_0} = \mathrm{TF} = T_{11} - T_{12}\, T_{21}\, T_{22}{}^{-1}. \tag{13}$$

The transfer function can provide the natural frequencies, as well as the stop and pass bands.

4.1. Spatial modulation

First, we assume a bar in which the modulus of elasticity only varies in space. Modulus of elasticity is defined as a periodic function $E = E_0[\sin(k_m x) + C_c]$, where E_0 is the amplitude of the harmonic variation, k_m is the wavenumber, C_c is the coefficient representing constant part of the modulus. To provide the required magnetic field for varying modulus of elasticity in the magnetoelastic medium, a magnetic field in the form of $H = H_c + H_v \sin(\psi(x))$ is considered to be applied to the magnetoelastic bar. H_c and H_v are the amplitude of the constant and variable parts of the magnetic field, and $\psi(x)$ denotes the phase difference between the coils. The required magnetic field can be found from figure 2(b) to vary

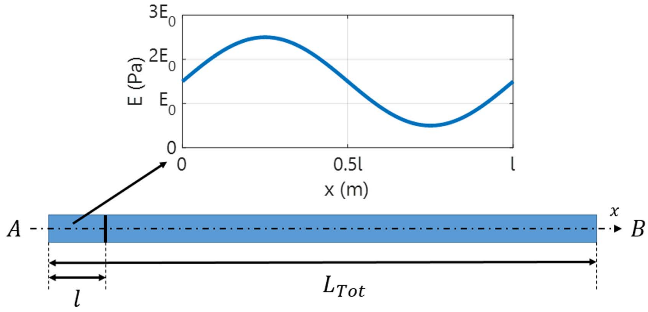

Figure 3. The elastic bar made from magnetoelastic materials with varying modulus of elasticity.

modulus of elasticity based on the required characteristics of the design.

For spatial modulation case, equation (9) is solved just for a length of the bar that covers a single full cycle of the harmonic property variation. We call this length an *'element'*. Figure 3 shows that the bar with the length of L_{Tot} consists of several *elements* with the length of l. The T matrix for the whole bar is calculated by $T = T_e^n$ [25], where T_e is the transfer matrix for the *element*, and n is the number of *elements* in the bar. Matrix T_e is found by solving equation (9) for only one *element* using solutions of the homogeneous system for independent initial conditions [42].

To study wave transmission and attenuation inside a bar with continuously varying properties in space, a finite element approach can alternatively be used to find the matrix T_e of an *element* and therefore, transfer function TF of the bar. In this method, instead of solving equation (9) for an e*lement* to find the T_e matrix, we discretize each *element* into finite sections where the properties can be assumed constant along each section. The transfer matrix for a bar with constant properties is already known [18, 43]:

$$[T_j] = \begin{bmatrix} \cos(k_j x_j) & \frac{1}{z_j\omega}\sin(k_j x_j) \\ -z_j\omega\sin(k_i x_i) & \cos(k_j x_j) \end{bmatrix}, \tag{14}$$

where, $k_j = \sqrt{\frac{\rho_j}{E_j}}\omega$ is the wave number, $z_j = A_j\sqrt{E_j\rho_j}$ is the mechanical impedance and A_j is the cross section of the jth section of the *element*. The total transfer matrix of the *element* is found by multiplying the transfer function of each section as,

$$T_e = T_n T_{n-1} \ldots . T_2 T_1, \tag{15}$$

where T_j ($j = 1,\ 2, \ldots, n$) is the transfer function of the jth section shown in figure 4 and n is the total number of discretized sections per element.

4.2. Spatiotemporal modulation

In the second case, the modulus of elasticity of the magnetoelastic material is considered to be a function of space and time. A modulus of elasticity is assumed in the form of $E = E_0[\sin(k_m x + \omega_m t) + C_c]$, where ω_m is the temporal modulation frequency. A magnetic field in the form of $H = H_c + H_v \sin(\omega_m t + \psi(x))$ is induced to the

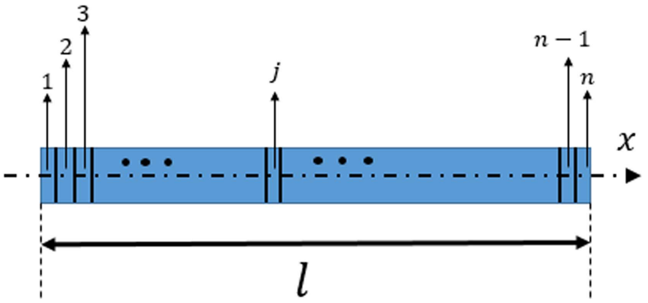

Figure 4. Dividing an *element* to finite small sections.

magnetoelastic bar to create such variation in elasticity. Since the modulus of elasticity is a function of space and time, finding a periodic *element* is not as convenient as the spatial case. In other words, we need to find an *element* in which the modulus of elasticity has at least one full harmonic period in both space and time. Such an *element* might also be longer than the length of the bar. Consequently, looking at the transfer function of the bar (equation (13)) provides a more convenient way of finding the pass and stop bands.

5. Case studies

Two different cases are studied here. The first one is a bar with the length of L_{Tot} made from magnetoelastic materials with a constant density of $\rho = 9200$ kg m^{-3}. The modulus of elasticity is spatially modulated with $E = E_0[\sin(k_m x) + C_c]$, $k_m = 20\frac{2\pi}{L_{\mathrm{Tot}}}$ and $C_c = 2.5$. The value of k_m implies that the modulus of elasticity has 20 full harmonic cycles along the bar, therefore, rendering a bar with 20 elements. Equation (9) is solved to find the T matrix for one element $\left(l = \frac{L_{\mathrm{Tot}}}{20}\right)$. Figure 5 shows the variation of eigenvalues with the dimensionless frequency $\Omega = \frac{\omega\,\lambda_m}{c_0}$, where ω is the excitation frequency, $\lambda_m = \frac{2\pi}{k_m}$, and $c_0 = \sqrt{\frac{C_c E_0}{\rho}}$ [11]. In the band structure shown in figure 5, frequency range that corresponds to $\lambda = 1$ is the pass band and the stop band exists where $\lambda \neq 1$. The Frequency range with a non-zero real part for μ indicates wave attenuation in the bar. Figure 6(a) shows the absolute value of TF calculated from equation (13). Each peak indicates a natural frequency for the bar. The low amplitude transfer function for Ω between 0.43 and 0.53 shows that the bar absorbs the waves within that frequency range and the waves do not propagate to the other end of the bar. For the case in which the modulus only varies spatially, there is no difference between the TF of forward and backward wave propagations and the bar shows a reciprocal behavior. Figure 6(b) shows the dispersion diagram for this case obtained both theoretically by Floquet–Bloch theorem and numerically by COMSOL-Multiphysics. In the time-domain numerical simulation, the spatially modulated bar is excited by a Gaussian wave packet that comprises the same frequency range included in the TF analysis and the dispersion contours are obtained via a two-dimensional Fourier transforms [44]. The x-axis is the dimensionless wavenumber, $\overline{\beta} = \kappa\lambda_m$ where κ is the wavenumber. No solution is found for Ω between 0.43 and 0.53, which reflects a stop band.

Similarly, the second case is a bar with the length of L_{Tot} made from magnetoelastic materials. In this case, the modulus of elasticity is considered to be a function of both space and time with $E = E_0[\sin(k_m x + \omega_m t) + C_c]$, $k_m = 20\frac{2\pi}{L_{\mathrm{Tot}}}$, $\omega_m = \nu_m k_m$, and $C_c = 2.5$. The transfer functions are found for $\nu_m = 0$ and $\nu_m = 0.2\,c_0$. Figure 7(a) shows three different transfer functions. The blue solid line is the previously discussed spatially modulated structure ($\nu_m = 0$). The peaks show the natural frequencies of the bar and the region with the lower amplitude indicates the stop band frequency range. The dashed green line and the dotted red line are the forward and backward transfer function for $\nu_m = 0.2\,c_0$. The difference between the stop Bands of the forward and the backward transfer functions signify the nonreciprocal behavior. In other words, some waves propagate from one side (side A in figure 3) of the bar to the other side (side B in figure 3). However, the same waves are attenuated when the source and receiver are swapped and the bar acts as a one-way stop band for those frequencies. Figure 7(b) shows the dispersion diagrams using an exact solution for a bar with time-varying properties and is confirmed by the transient COMSOL simulations [11]. The forward and backward stop bands match very well with those in figure 7(a).

6. Future experimental work

An experimental prototype of the modulated magnetoelastic device is currently under construction to physically realize and validate the nonreciprocal wave propagation in practice. The configuration of the experimental prototype resembles the system displayed in figure 1. A three-phase linear motor is designed with 19.05 cm pole pitches and total of eight poles. The pole size defines the wave length of the traveling wave (λ_m). The total length of the motor is 194.31 cm which is longer than 152.4 cm (8 poles times 19.05) to account for the effect of the non-uniform magnetic field around the edges of the motor. An AC drive is used for this prototype. The drive is a 480 V three phase AC voltage class and has a maximum output current of 22 amps. The drive is used to provide a sinusoidal current to the motor. The frequency of the drive defines the frequency of the traveling wave (ω_m). The maximum frequency of the drive is 590 Hz. A rod of Terfenol-D with the total length of 194.31 cm and the diameter of 1 cm is used as the magnetoelastic material. The density of Terfenol-D is 9200 kg m^{-3} and the average modulus of elasticity is 75 GPa. To give a quick insight into the working frequencies and the expected operating range of the experimental device, the forward and backward transfer functions are recalculated for the stated parameters, as opposed to the dimensionless analysis presented earlier. Figure 8 shows the simulated results for the transfer function of the experimental prototype for the case where ω_m is equal to 500 Hz.

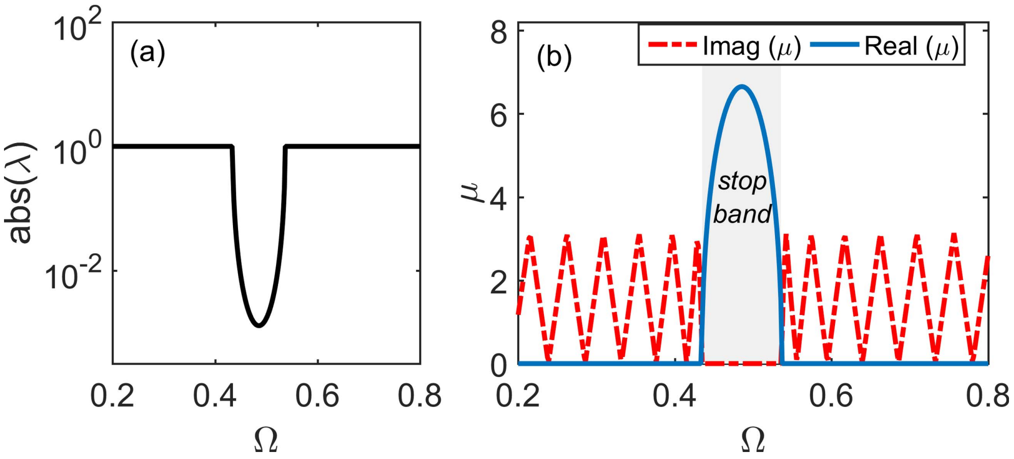

Figure 5. (a) The absolute value of λ (b) real part (solid blue line) and imaginary part (dashed red line) of the propagation constant versus dimensionless frequency.

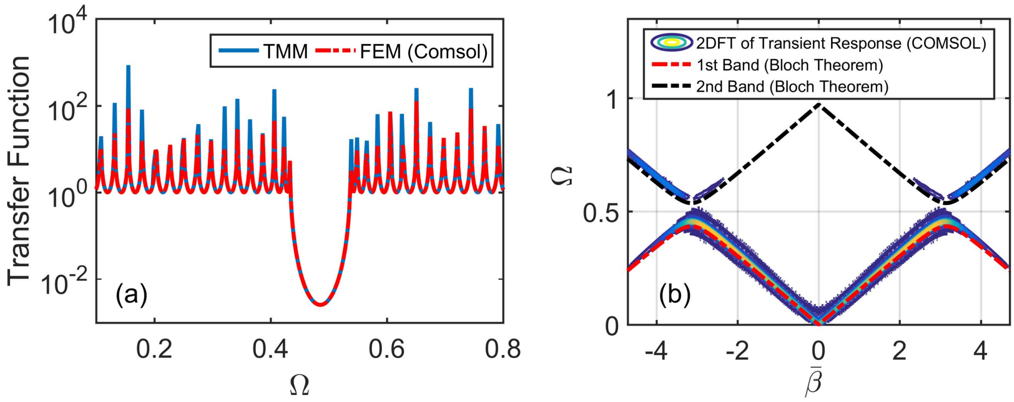

Figure 6. (a) TMM results (solid blue line) and numerical results from COMSOL (dashed red line) for the transfer function of a bar with varying modulus of elasticity in space. (b) Dispersion diagram for the bar with $E = E_0[\sin(k_m x) + C_c]$.

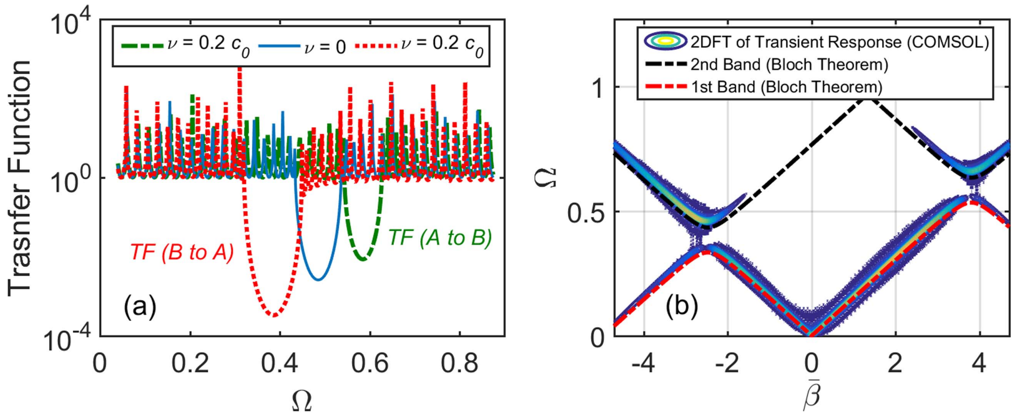

Figure 7. (a) The transfer function of a bar with varying modulus of elasticity in space and time. (b) Dispersion diagram for the bar with $E = E_0[\sin(k_m x + \omega_m t) + C_c]$.

7. Approximation error

When the material properties of the bar are only a function of space ($\omega_m = 0$), the temporal term of u consists of only one single harmonic. In this case, the results from TMM match the theoretical Flouet–Bloch approach. However, when the property of the bar is a function of both space and time, the temporal part of the solution ($e^{i\omega_n t}$) consists of infinite number of harmonics. In order to use the TMM method, only the dominant frequency (ω) is assumed to exist in the solution. The comparison of the approximate results and the theoretical Floquet–Bloch approach are shown in figure 7. In this section, we study the error between current approach and the theoretical Floquet–Bloch approach [11] as a function of ν_m. By

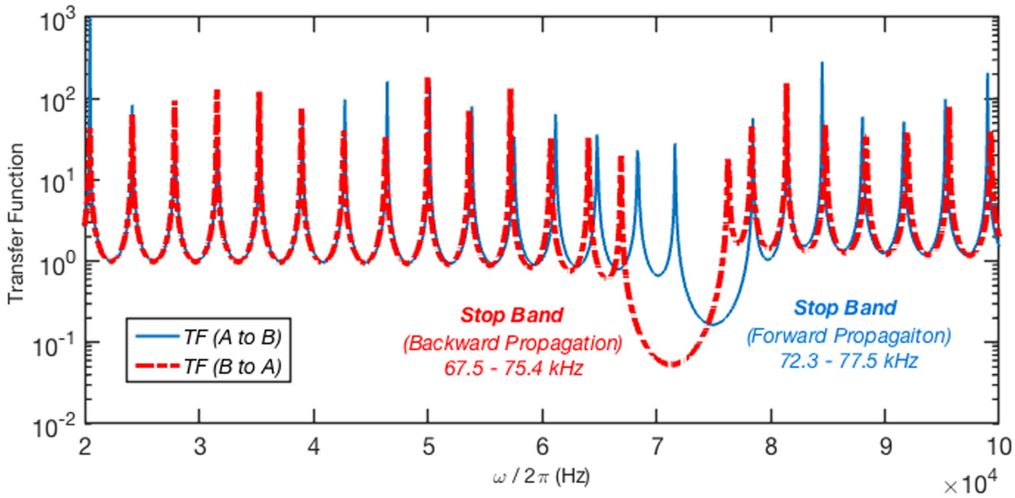

Figure 8. Simulation results for forward and backward TF of a Terfenol-D rod with spatiotemporal varying modulus of elasticity.

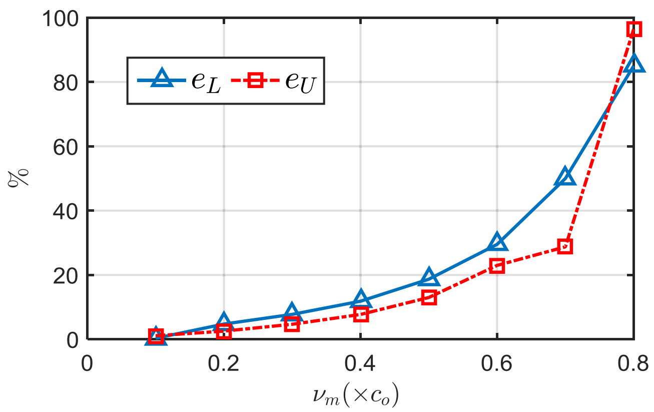

Figure 9. Effect of increase in the frequency of variation of the material property in time on lower bound and upper bound error.

increasing ν_m, the temporal modulation frequency (ω_m) is increased, which in turn increases the effect of ω_m in the frequency components of the temporal part, $e^{i\omega_n t}$. Hence, we define two errors corresponding to the upper and lower bounds of the stop bands as:

$$e_L = \frac{|\Omega^L_{\text{Theo}} - \Omega^L_{\text{TMM}}|}{\Omega^L_{\text{Theo}}} \times 100,$$

$$e_U = \frac{|\Omega^U_{\text{Theo}} - \Omega^U_{\text{TMM}}|}{\Omega^L_{\text{Theo}}} \times 100, \tag{16}$$

where e_L is the error for the lower bound and e_U is the error for the upper bound of the stop band. Ω^L_{ext} and Ω^L_{TMM} denote the lower bound dimensionless frequencies of the stop band from the theoretical solution and the TMM, respectively. Ω^U_{Theo} and Ω^U_{TMM} are the upper bound dimensionless frequencies of the stop band from the theoretical solution and the TMM. Figure 9 shows these two errors for backward wave propagation for the second case study. As ω_m increases, the error between TTM and theoretical results [11] increases. Thus, a threshold of $\nu_m = 0.4\ c_0$ is considered reasonable for using extended TMM when spatiotemporal modulation is present.

8. Conclusion

In this paper, the nonreciprocal behavior of a magnetoelastic bar was studied by using TMM. A physical platform inspired by LIM was proposed to change the modulus of elasticity of a magnetoelastic bar in space and time. The governing equation of wave propagation in a 1D magnetoelastic bar in the presence of a magnetic field were studied and an approximate solution was discussed. Two different case studies were considered. In the first case, the modulus of elasticity was assumed to be varying in space only, while in the second case, the modulus of elasticity was assumed to be varying in both space and time. An extended TMM was introduced and employed to find the directional transfer functions of the bar. For the first case, an alternative solution utilizing finite element method was proposed to find the transfer function. The second case exhibited the expected nonreciprocal wave propagation. The results of both cases were validated against numerical simulations and the theoretical Floquet–Bloch method. The transfer function for a prototype under construction was calculated. At the end, the error from single harmonic approximation with respect to the speed of variation of modulus of elasticity in time was discussed and justified.

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