

Assessing Liquefaction Susceptibility Using the CPT Soil Behavior Type Index



Brett W Maurer

Department of Civil and Environmental Engineering, University of Washington, Seattle, WA, USA

Russell A Green

Department of Civil and Environmental Engineering, Virginia Tech, Blacksburg, VA, USA

Sjoerd van Ballegooy

Tonkin & Taylor Ltd, Auckland, New Zealand

Liam Wotherspoon

Department of Civil and Environmental Engineering, University of Auckland, Auckland, New Zealand

ABSTRACT

Cone Penetration Test (CPT) based simplified liquefaction evaluation procedures offer significant advantages over variants based on alternative in-situ test indices. However, the main drawback of CPT-based variants is that soil samples are typically not recovered during CPT sounding, and as a result, soils are often not characterized directly or tested further in the laboratory. The need thus arises to infer soil properties from CPT indices without physical confirmation. Utilizing an unprecedented database of field and laboratory test data from Christchurch, New Zealand, this study proposes deterministic and probabilistic correlations relating the soil behavior type index (I_c) to liquefaction susceptibility as defined by published criteria. The findings show that I_c can be used with reasonable accuracy as a proxy for laboratory index-test based criteria for assessing liquefaction susceptibility. Additionally, to assess liquefaction hazards in a fully probabilistic manner, the uncertainty of I_c for defining liquefaction susceptibility can be accounted for using an approach similar to that used to create fragility functions for performance-based earthquake engineering. The approaches used herein to develop deterministic and probabilistic I_c relationships for liquefaction susceptibility are not limited to soils from New Zealand, but rather, can be applied worldwide.

1 INTRODUCTION

Cone Penetration Test (CPT) based simplified liquefaction evaluation procedures (e.g., Stark and Olson 1995; Robertson and Wride 1998; Moss et al. 2006; Idriss and Boulanger 2008; and Boulanger and Idriss 2014) offer significant advantages over variants based on alternative in-situ test indices. This position is supported by NRC (2016): “CPT soundings offer advantages over other methods of estimating liquefaction resistance in both the detection of thin layers that may affect liquefaction triggering and subsequent pore pressure redistribution and in the reproducibility of results. CPT results are less dependent on the equipment operator or setup than most other in situ test methods, and CPT can be performed with relative speed and economy.”

Antecedent to using any CPT-based liquefaction evaluation procedure, liquefaction susceptible soils must first be identified. CPT-based liquefaction procedures are intended to evaluate the potential for liquefaction triggering in soils susceptible to liquefaction. They should not be applied to high plasticity, fine grained, “non-liquefiable” soils, which could result in less accurate liquefaction hazard assessments (Maurer et al. 2015a), and for which other more appropriate methods exist to predict cyclic behavior (e.g., Boulanger and Idriss 2007).

However, CPT-based variants of the simplified procedure have disadvantages too. Namely, because soil samples are typically not recovered during CPT sounding, soils are often not characterized directly or tested further in the laboratory. While disturbed and/or undisturbed soil specimens can be obtained using special cone tooling or

by performing borings adjacent to the CPT sounding, this can be prohibitively expensive for many projects, including hazard assessments for low risk project and/or those involving many sites. As a result, the standard-of-practice for many projects is to assess liquefaction susceptibility using CPT indices (e.g., Soil Behavior Type Index, I_c).

The focus of the study presented herein is to assess:

- (a) How well the I_c index (Robertson, 1990; Robertson and Wride, 1998) segregates soils susceptible to liquefaction from soils not susceptible to liquefaction, as defined by the Polito (2001) (P01), Seed et al. (2003) (Sea03), Bray and Sancio (2006) (BS06), and Boulanger and Idriss (2006)/Idriss and Boulanger (2008) (BI06/IB08) criteria (i.e., the degree to which I_c is correlated to these criteria); and
- (b) The I_c thresholds, or cutoffs, at which the efficiencies of these segregations are optimized.

Towards this end, the authors use the wealth of data from the 2010 - 2011 Canterbury, New Zealand, earthquake sequence (CES) (e.g., Green et al. 2011; Wotherspoon et al. 2011; Bray et al. 2014; Cubrinovski et al. 2014) to develop deterministic and probabilistic I_c correlations for predicting liquefaction susceptibility per P01, Sea03, BS06, and BI06/IB08.

In the following sections of this paper, background information on I_c and the CES data is first presented. Next, commonly used liquefaction susceptibility criteria are summarized. Deterministic and probabilistic assessments of using I_c as a proxy for the commonly used liquefaction criteria are then made.

2 BACKGROUND

2.1 Soil Behavior Type Index, I_c

First proposed by Jeffries and Davies (1993) to define soil type boundaries, I_c was subsequently modified by Robertson and Wride (1998) to better fit the Robertson (1990) $Q - F$ classification scheme (Figure 1), with the latter version becoming widely used in practice. Per Robertson and Wride (1998) I_c is defined as:

$$I_c = \sqrt{(3.47 - \log_{10} Q)^2 + (1.22 + \log_{10} F)^2} \quad [1]$$

where Q and F are the normalized CPT penetration resistance and normalized CPT friction ratio, respectively. As shown in Figure 1, I_c represents the radial distance between any point on this chart and the point defined by $Q = 10^{3.47}$ and $F = 10^{-1.22}$ (i.e., $Q = 2951$ and $F = 0.06026\%$). Circular arcs defined by constant I_c values approximate the boundaries between different soil behavior types, with $I_c = 2.60$ shown in Figure 1 as the approximate boundary between soil behavior types 4 and 5 (4: Silt Mixtures; 5: Sand Mixtures) (Robertson 2009).

I_c is often used to assess liquefaction susceptibility, with soils with $I_c < 2.6$ being inferred to be liquefiable and can be evaluated using the simplified liquefaction evaluation procedures (Robertson and Wride 1998). However, because I_c boundaries between soil types are approximate and may need regional refinement (e.g., Yi 2014), the $I_c < 2.6$ criterion may in some cases be inappropriate (e.g., Zhang et al. 2002; Li et al. 2007; Pease 2010). For this reason, Youd et al. (2001) recommended that soils with $I_c > 2.4$ be sampled and tested to evaluate their susceptibility.

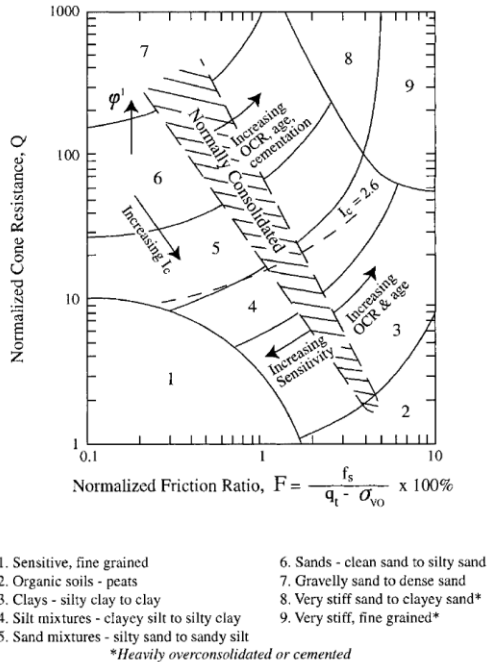


Figure 1. CPT-based Soil Behavior Type chart (Robertson, 1990).

2.2 Canterbury Earthquake Sequence Data

The 2010-2011 CES began with the 4 September 2010, $M_w 7.1$ Darfield earthquake and included up to ten events that triggered liquefaction (Quigley et al. 2013). However, most notably, widespread liquefaction was triggered by the $M_w 7.1$, 4 September 2010 Darfield and the $M_w 6.2$, 22 February 2011 Christchurch earthquakes. Due to the severity and spatial extent of liquefaction resulting from the 2010 Darfield earthquake, the New Zealand Earthquake Commission (EQC) initiated an extensive geotechnical characterization program to assess regional liquefaction hazards and inform land-use planning. The resulting data are compiled in the New Zealand Geotechnical Database (NZGD 2016).

A subset of the subsurface characterization data from the New Zealand Geotechnical Database is utilized herein. The data used includes (a) Borings with standard penetration testing (SPT) performed at 825 sites throughout Christchurch, from which 2,620 samples were obtained in the split-spoon sampler; (b) laboratory measurements performed on each sample, to include fines content (FC), Liquid Limit (LL), Plastic Limit (PL), and natural moisture content (w_n); and (c) CPT soundings performed adjacent to each of the 825 borings. With respect to (b), Atterberg Limits were determined for 2,097 of the 2,620 samples collected.

Several maximum offset distances between the borings and CPT locations were considered in assembling the database used in this study. The advantage using a larger offset distance is that more borehole-CPT pairs could be included in the database, but the disadvantage is the increase in uncertainty due to the lateral spatial variability of the deposits that the I_c value corresponded to the soil sampled and tested in the laboratory. Conversely, the disadvantage of using a smaller maximum offset distance is that less borehole-CPT pairs could be included in the database, but the advantage is a decrease potential for the influence of heterogeneity of soil conditions (spatial variability) on the uncertainty that the I_c value corresponded to the soil sampled and tested in the laboratory. Ultimately, a maximum horizontal offset of 5 m was used to assemble the database used herein as a compromise of these advantages and disadvantages. The reader is referred to Lees et al. (2015) for more discussion about the maximum offset distance used.

The near-continuous nature of CPT sounding data (i.e., 1 or 2 cm measurement intervals) can result in significant changes in the measured I_c over short depth intervals. Accordingly, to study relationships between I_c and laboratory test indices, the measured I_c was averaged over a 300 mm depth interval (e.g., Boulanger et al., 1997), centered on the midpoint of the split-spoon sample. In addition, samples with large variation in I_c were identified by computing the standard deviation (σ) of I_c within the sample interval and removed from the database before the analyses were undertaken.

3 LIQUEFACTION SUSCEPTIBILITY CRITERIA

Various studies have proposed criteria for assessing liquefaction susceptibility based on Plasticity Index (PI) values derived from Atterberg Limit laboratory tests performed on recovered soil samples from borings. The criteria proposed by P01, Sea03, BS06, and BI06/IB08 are summarized as follows:

- (a) Polito (2001): P01 proposed that soils with (i) $PI < 7$ and (ii) $LL < 25$ are “liquefiable;” ; soils with (i) $7 < PI < 10$ and (ii) $25 < LL < 35$ are “potentially liquefiable” and require further testing; and soils whose properties lie outside these bounds are not susceptible to liquefaction.
- (b) Seed et al. (2003): Sea03 proposed that soils with (i) $PI \leq 12$ and (ii) $LL \leq 37$ are “potentially liquefiable” if $w_n/LL > 0.8$; soils with (i) $PI \leq 20$ and (ii) $LL \leq 47$ are transitional and require further testing if $w_n/LL > 0.85$, and soils whose properties lie outside these bounds are not susceptible to liquefaction but may be vulnerable to strength loss.
- (c) Bray and Sancio (2006): BS06 proposed that soils with (i) $PI < 12$ and (ii) $w_n/LL > 0.85$ are “susceptible” to liquefaction; soils with (i) $12 < PI < 18$ and (ii) $w_n/LL > 0.8$ may be “moderately susceptible” to liquefaction and require further testing; and soils whose properties fall outside these bounds are not susceptible to liquefaction but may undergo deformation.
- (d) Boulanger and Idriss (2006) and Idriss and Boulanger (2008): BI06 proposed that soils with $PI < 3$ exhibit “sand-like” behavior; soils with $PI \geq 7$ exhibit “clay-like” behavior, but if a soil classifies as CL-ML according to the Unified Soil Classification System (USCS) (ASTM D-2487-11; ASTM, 2011), this criterion may be reduced to $PI \geq 5$; and soils with $3 < PI < 6$ may exhibit intermediate behavior and should be tested further. Based on this Idriss and Boulanger (2008) proposed that in the absence of cyclic laboratory testing on undisturbed samples, soils with $PI < 7$ can be conservatively assumed to exhibit “sand-like” behavior and be evaluated using the simplified procedure.

Although all the criteria summarized above are “liquefaction susceptibility” criteria, there are subtle differences in their intended use. All the criteria focus on the cyclic response of the soil, where Boulanger and Idriss (2006) use the terms “sand-like” and “clay-like” to distinguish cyclic responses. Soils that exhibit “sand-like” behavior experience a significant reduction in stiffness at elevated excess pore pressures at low to moderate strains. In contrast, soils that exhibit “clay-like” behavior retain moderate stiffness for all strain ranges even at high excess pore pressures. However, beyond just specifying criteria to characterize the cyclic response of soils with high fines contents, Boulanger and Idriss (2006) and Idriss and Boulanger (2008) explicitly state their criterion defines when the simplified procedure is valid to assess liquefaction potential; the other studies only imply that their criteria does this. More detailed overviews of commonly used susceptibility criteria are given in Armstrong and Malvick (2014, 2016), Green and Ziotopoulou (2015), and NRC (2016), among others.

4 USING I_c AS PROXY FOR LIQUEFACTION SUSCEPTIBILITY CRITERIA

4.1 Deterministic Assessment

As stated previously, this study seeks to determine (a) how well the I_c index segregates soils susceptible to liquefaction from soils not susceptible to liquefaction, as defined by the P01, Sea03, BS06, and BI06/IB08 criteria (i.e., the degree to which I_c is correlated to these criteria); and (b) the I_c thresholds, or cutoffs, at which the efficiencies of these segregations are optimized. To make these determinations, a standard analysis is needed to assess the performance of diagnostic tests. Receiver operating characteristic (ROC) analyses are herein adopted for this purpose. ROC analyses have been widely used to study the performance of classifier systems, including extensive use in medical diagnostics (e.g., Zou 2007), but by comparison, their use in geotechnical engineering is limited (Chen et al. 2007; Oommen et al. 2010; Mens et al. 2012; Maurer et al. 2015a; Zhu et al. 2015).

In any ROC curve application, the distributions of “positives” (e.g., soil is susceptible to liquefaction per one of the criteria summarized above) and “negatives” (e.g., soil is not susceptible to liquefaction per one of the criteria summarized above) overlap when the frequency of the distributions are expressed as a function of index test results (e.g., I_c values). In such cases, optimal decision thresholds for the index test are selected considering the rates of true positives (R_{TP}) (e.g., soil is predicted to be susceptible to liquefaction based on I_c and is predicted to be susceptible to liquefaction per one of the criteria summarized above) and false positives (R_{FP}) (e.g., soil is predicted to be susceptible to liquefaction based on I_c but is predicted to not be susceptible to liquefaction per one of the criteria summarized above). Setting the I_c threshold too high will result in a higher R_{FP} , the cost of which could be excessive spending on site remediation. Conversely, setting the threshold too low results in a higher rate of false negatives (e.g., soil is predicted to be not susceptible to liquefaction based on I_c but is predicted to be susceptible to liquefaction per one of the criteria summarized above), the cost of which is liquefaction-induced damage during a future event. Thresholds should thus be selected to minimize these costs.

ROC curves plot R_{TP} versus R_{FP} for varying threshold values. Figure 2 illustrates the relationship among the positive and negative distributions, the threshold value, and the ROC curve. Figure 2b also illustrates how a ROC curve is used to assess the efficiency of a diagnostic test and to select an optimum threshold. In ROC space, random guessing is indicated by a 1:1 line through the origin (i.e., equivalent number of correct and incorrect predictions), while a perfect model plots as a bi-linear curve that pass through the points (0,0; 0,1; 1,1), indicating the existence of a threshold value which perfectly segregates the dataset (e.g., all soils susceptible to liquefaction have I_c below the threshold; all soils not susceptible to liquefaction have I_c above the threshold).

While no single parameter can fully characterize model performance, the area under a ROC curve (AUC)

is commonly used for this purpose, where AUC is statistically equivalent to the probability that “positives” have lower index test values than “negatives” (e.g., Fawcett 2005). As such, increasing AUC indicates better model performance. The optimum decision threshold is defined herein as the threshold which minimizes the rate of misprediction [i.e., $R_{FP} + (1 - R_{TP})$]. As such, contours of the quantity $[R_{FP} + (1 - R_{TP})]$ map points of equivalent performance in ROC space, as shown in Figure 2b. Notably, this definition implicitly treats the costs of false positives and false negatives to be approximately equal. For further overview of ROC analyses, and for demonstration of how project-specific misprediction consequences can be incorporated into ROC analyses, the reader is referred to Fawcett (2005) and Maurer et al. (2015b).

ROC analyses were performed to determine how well the I_c index correlates to liquefaction susceptibility per the P01, Sea03, BS06, and BI06/IB08 criteria. Shown in Figure 3 are frequency distributions of the 2,097 classified samples, plotted as a function of measured I_c . ROC analysis of the “susceptible” and “not susceptible” distributions are plotted in Figure 4. The performance of I_c in assessing liquefaction susceptibility is indicated by $AUC = 0.89$ (Sea03) to 0.92 (BI06/IB08) (i.e., there is about a 90% probability that the measured I_c of an “not susceptible” soil is greater than that of a “susceptible” soil). In other words, I_c is well-correlated to the Atterberg Limit criteria, with BI06/IB08 exhibiting the strongest correlation.

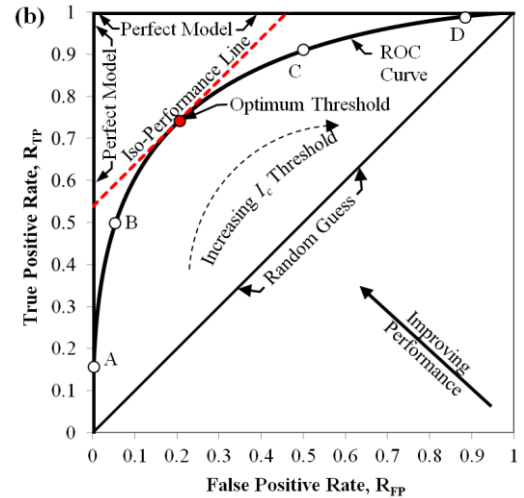
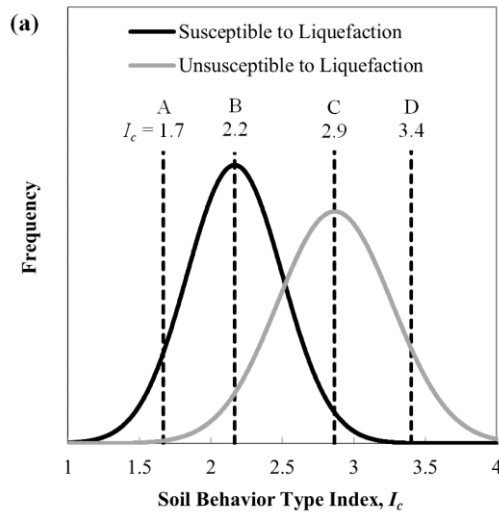
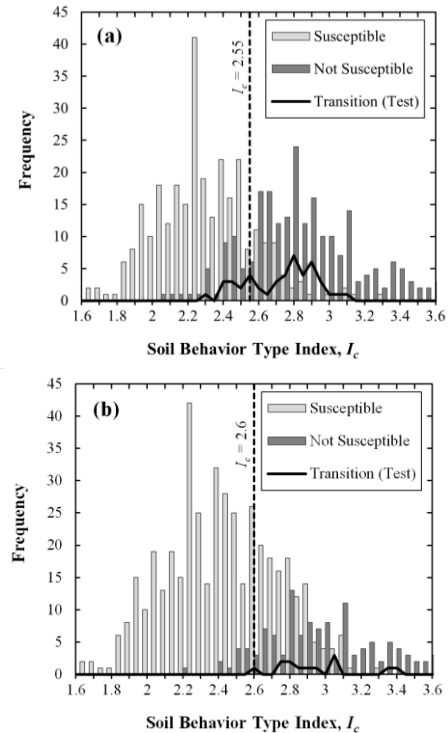


Figure 2. ROC analyses: (a) frequency distributions of soils susceptible and not susceptible to liquefaction per BI06 as a function of measured I_c , with four different threshold I_c values shown; (b) corresponding ROC curve, and illustration of how a ROC curve is used to assess the efficiency of a diagnostic test. The optimum decision threshold is that for which the rate of misprediction is minimized.



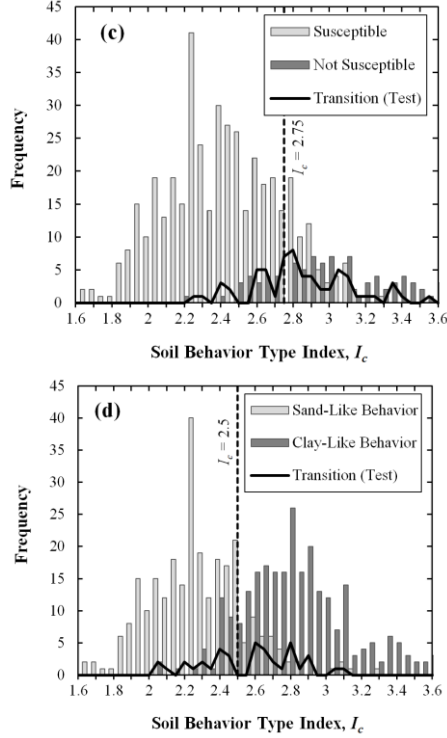


Figure 3. Frequency distributions of samples classified by liquefaction-susceptibility criteria based on Atterberg Limits, plotted as a function of measured I_c : (a) P01; (b) Sea03; (c) BS06; and (d) BI06/IB08. Classifications in (a) – (d) are as defined in the text. Optimal I_c thresholds for identifying liquefaction-susceptible soils are identified in (a) – (d).

As highlighted in Figure 3, the optimal I_c thresholds corresponding to the P01, Sea03, BS06, and BI06/IB08 criteria are 2.55, 2.60, 2.75, and 2.50, respectively, which are generally consistent with common I_c criteria used in practice and discussed previously. However, it should be recognized that these thresholds are those minimizing the misprediction rate and are only optimal if false positives and false negatives have equal cost. That is to say, the proposed thresholds are not inherently conservative in nature. For example, if the costs of false negatives were instead greater than the costs of false positives, the optimum I_c threshold would increase in response. Operating at their respective optimal thresholds, the I_c classifiers have overall accuracy (OA) ranging from 0.76 (Sea03) to 0.86 (BI06/IB08), where OA indicates the percentage of samples correctly classified (Figure 4). The superior performance of BI06/IB08 should not be interpreted to mean that it is more appropriate for assessing liquefaction susceptibility, but rather, that it better correlates to I_c . It should be emphasized that while Atterberg Limit criteria offer a more direct characterization of soil behavior, they are not a definitive test of susceptibility. Ideally, cyclic laboratory tests on undisturbed samples would also be performed to corroborate or re-calibrate susceptibility thresholds. Nonetheless, these analyses suggest that I_c can be an

efficient and cost-effective index of liquefaction-susceptibility.

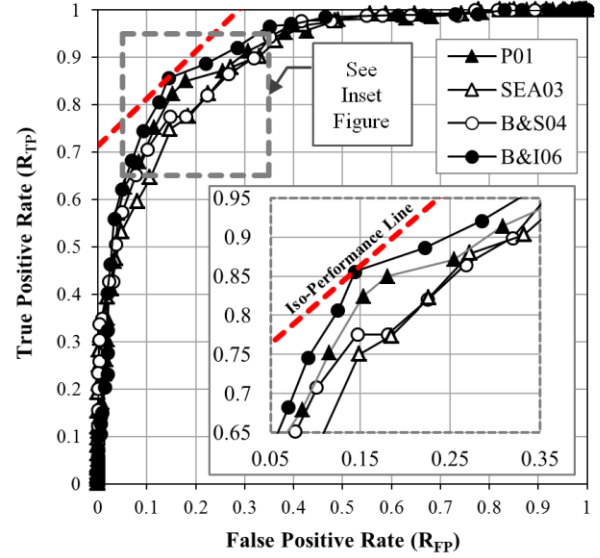


Figure 4. ROC analysis of I_c index performance in segregating soils susceptible to liquefaction from soils unsusceptible to liquefaction per P01, Sea03, BS06, and BI06/IB08 criteria.

4.2 Probabilistic Assessment

To assess liquefaction hazards in a fully probabilistic manner, the uncertainty of liquefaction susceptibility should be adequately accounted for. A probabilistic correlation is developed herein using an approach similar to that described by Porter (2016) to create fragility functions for performance-based earthquake engineering. While the adopted approach is outlined below, the reader is referred to Porter (2016) for complete details.

The probability that a soil is “not susceptible” to liquefaction, given a measured I_c value, is denoted by $F_{not_susceptible}(I_c)$ and idealized by a log-normal distribution, as is typical for fragility functions (e.g., Bradley, 2010):

$$F_{not_susceptible}(I_c) = \Phi \left[\frac{\ln(\frac{I_c}{x_m})}{\beta} \right] \quad [2]$$

where Φ denotes the Gaussian cumulative distribution function; x_m is the median value of the distribution; and β is the logarithmic standard deviation. While several approaches exist for fitting functions to data, this study utilizes the maximum likelihood method described in Porter (2016), which identifies the model parameters with the highest likelihood of producing the observed data. Specifically, the soil samples are grouped into m bins of similar I_c , where bins have index i , average value I_{ci} , and contain n_i samples, of which f_i are samples “not susceptible” to liquefaction. Assuming quantity f_i can be estimated from a binomially-distributed random variable, F_i , Eq. 3 gives the probability of observing quantity f_i among n_i samples, if the probability of an individual sample being “not susceptible” is given by Eq. 2.

$$P[F_i = f_i] = \frac{n_i!}{f_i!(n_i - f_i)!} \cdot p_i^{f_i} \cdot (1 - p_i)^{n_i - f_i} \quad [3]$$

In Eq. 3, p_i is defined by Eq. 2, evaluated at I_{ci} . Lastly, the values of parameters x_m and β that maximize the likelihood of producing the observed data are determined. This likelihood is given by the product of the probabilities in Eq. 3, multiplied over all bins:

$$L(X_m, \beta) = \prod_{i=1}^m P[F_i = f_i] \quad [4]$$

The probability that a soil is “susceptible” to liquefaction, denoted as $F_{\text{susceptible}}(I_c)$, is then given by:

$$F_{\text{susceptible}}(I_c) = 1 - F_{\text{not susceptible}}(I_c) \quad [5]$$

Using the approach outlined above, probabilistic correlations were developed from the “susceptible” and “not susceptible” distributions shown in Figure 3. The resulting functions are plotted in Figure 5 and defined by Eqs. 2 and 5 in conjunction with the criteria-specific coefficients in Table 1. The proposed functions allow for the uncertainty of whether a soil is susceptible to liquefaction to be incorporated into fully probabilistic hazard assessments. It can be seen from Figure 5 and Table 1 that the I_c values corresponding to a 50% probability of susceptibility (as indicated by the P01, Sea03, BS06; and BI06/IB08 criteria) are similar to the deterministic thresholds developed from ROC analysis. For I_c thresholds traditionally used in practice (i.e., $2.4 \leq I_c \leq 2.6$), the probability of susceptibility ranges from 0.60 (BI06/IB08) to 0.78 (BS06) at $I_c = 2.4$, and from 0.40 (BI06/IB08) to 0.61 (BS06) at $I_c = 2.6$. It can therefore be expected that the rates of false positives and false negatives will be roughly equal using thresholds in this range. As with the deterministic approach, it should be emphasized that Atterberg Limit criteria are not a definitive test of susceptibility. Ideally, cyclic laboratory tests on undisturbed samples would also be used to classify susceptibility and develop I_c functions.

5 CONCLUSIONS

Utilizing an unprecedented database of field and laboratory test data from Christchurch, New Zealand, this study assessed the use of the CPT soil behavior type index as a proxy for laboratory index test based criteria for assessing liquefaction susceptibility. Towards this end, deterministic and probabilistic I_c relationships were developed for predicting liquefaction susceptibility for four commonly used laboratory index test based criteria: P01, Sea03, BS06, and BI06/IB08. To predict liquefaction susceptibility, deterministic I_c thresholds of 2.55, 2.60, 2.75, and 2.50 were proposed for the P01, Sea03, BS06, and BI06/IB08 susceptibility criteria, respectively. Probabilistic correlations were proposed in Eqs. 2 and 3, and are plotted in Figure 5. The proposed deterministic thresholds approximately correspond to a 50% probability that the tested soil is liquefaction-susceptible. The correlations developed in this study are particularly pertinent to liquefaction-related research derived from the CES, which will play a transformative role in geotechnical

research for many years. However, the methods demonstrated herein to develop deterministic and probabilistic correlations are not limited to parts of New Zealand, but rather, can be applied worldwide.

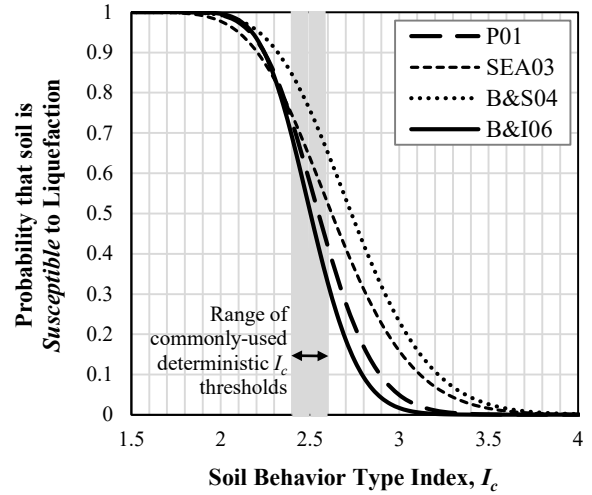


Figure 5. The probability of liquefaction susceptibility as a function of measured I_c . Susceptibility is as defined by the P01, Sea03, BS06, and BI06/IB08 criteria. The range of deterministic I_c thresholds commonly used in practice is also highlighted.

Table 1. Criteria-specific coefficients for use in Eq 2.

Criteria	β	x_m
P01	0.0988	2.5474
Sea03	0.1348	2.6214
BS06	0.1275	2.7315
BI06/IB08	0.0851	2.5031

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