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Distributed MPC for Coordinated Energy Efficiency Utilization in Microgrid Systems

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***Abstract*—To improve the renewable energy utilization of distributed microgrid systems, this paper presents an optimal Distributed Model Predictive Control (DMPC) strategy to coordinate energy management among microgrid systems. In particular, through information exchange among systems, each microgrid in the network, which includes renewable generation, storage systems and some controllable loads, can maintain its own system-wide supply and demand balance. With our mechanism, the closed-loop stability of the distributed microgrid systems can be guaranteed. In addition, we provide evaluation criteria of renewable energy utilization to validate our proposed method. Simulations show that the supply-demand balance in each microgrid is achieved while, at the same time, the system operation cost is reduced, which demonstrates the effectiveness and efficiency of our proposed policy.**

***Index Terms*—Microgrids, Energy Management Strategy, Distributed Model Predictive Control, Renewable Energy Sources, Coordinated Scheduling.**

I. INTRODUCTION

THE proliferation of Distributed Energy Resources (DERs) and the advent of controllable loads have brought about the concept of microgrids [1], [2]. A microgrid is defined as a cluster of distributed generation (DG), distributed storage (DS) and loads, serviced by a distribution system, and can be operated in either islanded or grid-connected mode [3]. DG within microgrids include photovoltaics (PVs) and wind turbines (WTs). DS units are mainly batteries. Sound operation of a microgrid requires an Energy Management Strategy (EMS). The essential problem of EMS is energy balance by coordinating power among the DER units in order to supply the loads with required energy in real time.

The development and evolution of microgrids will result in the plug-and-play integration of intelligent structures called multi-microgrid systems, which will be linked with each other through particular channels for power, information and control signal exchange [4]. Energy management in microgrids is needed not only to optimize of each microgrid, but also to achieve global optimization by coordinating power flow among microgrids. Therefore, many researchers have focused their interests on energy management for multi-microgrid systems.

In the literature, there are two approaches for the development of microgrid energy management. The first one is the centralized approach, which ensures economic operation and maintains the balance between the power production and consumption [5]–[13]. To that end, Model Predictive Control (M-PC) is an effective control policy for multi-microgrids which can handle the uncertainties between supply and demand as well as the system constraints, such as generator capacity and ramp rate. [8] introduces a look-ahead optimal control algorithm for dispatching the available generation resources with the objective of minimizing generation and environmental costs. In [9], a novel mixed integer linear approach embedded within an MPC framework is proposed to optimize microgrid operations efficiently while satisfying time-varing request and operation constraints. [10], [11] present an MPC-based EMS for multiple microgrids to optimally manage and coordinate energy supply and demand, which aims at minimizing the overall costs of each microgrid. Especially, a power flow management method for a network of cooperating microgrids is proposed by formulating the problem in a centralized MPC framework and the supply and demand is balanced with maximization of Renewable Energy Source (RES) [12]. However, a centralized EMS requires advanced capabilities at the MicroGrid Central Controller (MGCC) [14]. As the number of devices in microgrids increase and microgrids are combined to form smart grids, the centralized solutions might be less efficient [15].

The second approach for microgrid energy management is the distributed control architecture, which is efficient, scalable and privacy preserving, especially for multi-microgrids with large size [16]–[19]. [16] proposes a distributed power scheduling approach so that the aggregate demand equals the supply, but ignores the power distribution network and system operational constraints. A privacy-preserving energy scheduling problem is formulated with privacy constraints in [17]. In [18], a distributed peer-to-peer multi-agent framework is proposed for managing the power sharing in microgrids with inverter-interfaced distributed energy resources. A flexible and modular control scheme based on distributed model predictive control is presented to allow virtual power plant (VPP) operation in [20]. In [21], DMPC is herein extended to calculate market-based on-line energy pricing while minimizing the generation cost and emissions. [22] presents a control scheme based on distributed MPC for coordinating flexible heterogeneous DER in the Smart Grid with minimum system

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integration effort.

Above all, many centralized approaches are applied in keeping the supply-demand balance with maximization of renewable energy efficiency through appropriate information exchange. However, as the number of microgrids increases, the calculation burden is heavy and the demand for communication bandwidth is high. So, it is impractical to apply centralized methods in microgrid systems of such a large scale. Considering the practical requirements and distribution, distributed MPC appears to be an appropriate framework for the optimization problem. Nevertheless, existing DMPC schemes mainly focus on minimizing the economic cost and ignore the utilization of surplus RESs from other subsystems. Few works have attempted to improve the RES utilization to meet the supply-demand balance by using DMPC methods.

This paper not only presents a collective energy dispatch solution embedded within an DMPC framework for coordinating renewable energy among microgrids [23], [24], but also verifies the performance of the proposed algorithm. To summarize, the novel contributions of this paper are as follows: Taking the coordinated information exchange among microgrids into account, we develop a DMPC algorithm for microgrids to optimize utilization of renewable energy sources. The proposed methodology can handle multiple constraints in microgrids, reduce the operation cost of energy management, as well as guarantee the overall optimal performance of the system. Deep charging of the battery in the DSM is also explicitly defined. Furthermore, we not only derive the condition to guarantee closed-loop stability of the overall system, but also provide an evaluation function of renewable energy utilization. Corresponding distributed model predictive controllers are also presented.

The remainder of this paper is organized as follows. Section II presents the system description and modelling. Section III introduces the distributed MPC. In Section IV, the numerical results are provided. Section V concludes the paper summarizing the major findings.

II. System Description and Modelling

In this section, the system topology followed by the DG, DS and load is given, then the dynamics and behavior of microgrid components are modeled, and finally the objective of the EMS is proposed.

A. System Overview

A network of microgrids including m nodes is illustrated in Fig. 1. It should be noted that this methodology can be extended to a larger number of microgrids as well. It is assumed that microgrids supply loads both in islanded and in grid connected modes. Each microgrid contains batteries as the DS units, controllable loads and DG units. For microgrid i, the generated power at each time instant depends on renewable generation power (PV, WT, fuel cells, etc.) and surplus power from neighbours. All microgrids are connected to the distribution network, which interfaces with the main grid at the point of common coupling.

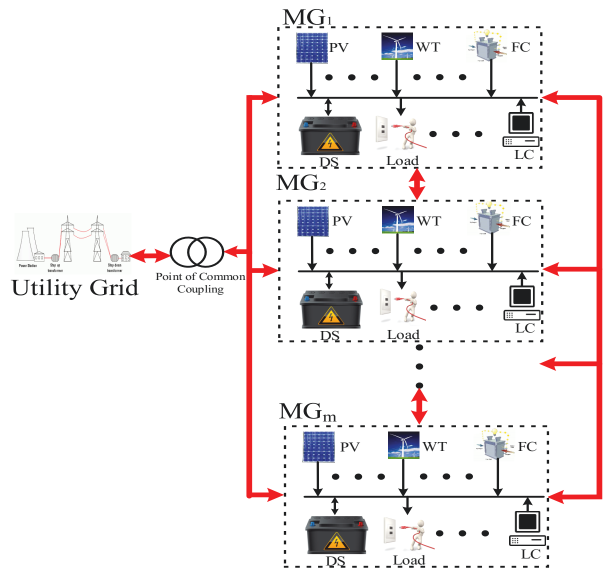

Fig. 1. A typical microgrid system.

B. System Dynamics and Constraints

1) DS Model: The storage system is a key ingredient for all microgrids since it allows to smooth intermittent RES power flow and provide peak power load shaving. As its basic core benefit, it will make microgrids successful in matching supply and demand over a 24 hour period. In this case of microgrids, storage system acts as the backup while all the power generated cannot meet the load demand. Hence, let $E_i(k)$ denote the energy stored in the ith battery at time k and $P_i^b(k)$ the electric power exchanged with the storage. Then, the dynamics of the storage capacities are modelled as follows:

$$E_i(k+1) = E_i(k) + \eta_i P_i^b(k), \tag{1}$$

$$\underline{E} \leq E_i(k) \leq \overline{E}, \tag{2}$$

where

$$\eta_i = \begin{cases} \eta^{ch}, & P_i^b(k) > 0, \\ 1/\eta^{dch}, & otherwise. \end{cases}$$

Typically η_i is the efficiency of the energy charging/discharging process, that is, η^{ch} is the efficiency of the charging process and η^{dch} is the efficiency of the discharging process, with $0 < \eta^{ch}, \eta^{dch} < 1$. $\underline{E}$ and $\overline{E}$ are the minimum and maximum allowed energy stored in the battery, respectively.

A cost function to capture the damages to the battery is considered, where deep discharging is the main concern. The battery cost is modelled as

$$C_b(P_i^b) \triangleq \sum_{k=1}^{\infty} \Big(\min\Big(E_i(k) - \alpha_b \overline{E}, 0\Big)\Big)^2, \tag{3}$$

where α_b is a positive constant and represents a deep discharging penalty when the energy stored in the battery $E_i(k)$ is less than α_b of the battery capacity $\overline{E}$.

2) Load Model: We consider interruptible loads as one type of controllable loads. For an interruptible load, the consumed

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energy is bounded by

$$\underline{P}_i^L \leq P_i^L(k) \leq \overline{P}_i^L, \tag{4}$$

where $\underline{P}_i^L$ and $\overline{P}_i^L$ are the minimum and maximum active power, respectively.

For each load, a cost function measures the dissatisfaction of the customer using the demand schedule $P_i^L(k)$. The cost function of an interruptible load can be defined as

$$C_l(P_i^L) \triangleq \sum_{k=1}^{\infty} \alpha_l \Bigg(\min \Big(P_i^{Lf}(k) - P_i^L(k), 0 \Big) \Bigg)^2, \tag{5}$$

where P_i^{Lf} is the satisfactory level and α_l is a positive constant. The daily load profile can be obtained in real time using short-term electricity demand forecasting techniques [12]. The above cost function is nonzero only when there is load shedding, i.e., $P_i^L(k) > P_i^{Lf}(k)$.

3) DG Model: Renewable DGs are considered in microgrids. Controllable DGs such as fuel cells are dispatchable. It is assumed that the output power $P_i^C(k)$ over the scheduling horizon is given.

PV generators or wind turbines are not controllable and their output power is dependent on the utilization of the natural sources (i.e., sun irradiance or wind). Hence, their future profiles over a certain finite horizon time interval can be obtained in real time using several forecasting models [25], where the precision of forecast is significant in the energy management. The renewable DG can be modelled using the method of exponential smoothing, which is designed to use the forecast error in the previous period to correct and improve the forecast of the current period. In equation form:

$$P_i^{RES}(k+1) = P_i^{RES}(k) + \alpha_r(P_i^r(k) - P_i^{RES}(k)), \tag{6}$$

$$\underline{P}_i^{RES} \leq P_i^{RES}(k) \leq \overline{P}_i^{RES}, \tag{7}$$

where $P_i^{RES}(k)$ is the predictive power produced by the ith renewable source, α_r is a smoothing constant $(0 < \alpha_r < 1)$, $P_i^r(k)$ is the actual active power during period k, and $\underline{P}_i^{RES}$ and $\overline{P}_i^{RES}$ are the minimum and maximum output power produced by the renewable source, respectively.

4) Power Balance: It is assumed that all the microgrids supply loads are in island modes. Based on the cooperative microgrid systems, the supply and demand balance is achieved not only in the local microgrid whose power is generated from DGs and storage, but also in the neighbor systems which have surplus renewable energy. Hence, the following equality constraints hold, respectively for the electric components:

$$P_i^b(k) = P_i^S(k) + P_{ij}(k) + P_i^C(k), \tag{8}$$

$$P_i^S(k) = P_i^{RES}(k) - P_i^L(k), \tag{9}$$

$$P_{ij}(k) = \sum_{j \neq i} a_{ij} P_j^S(k), \tag{10}$$

$$P_{ij}(k) \geq 0. \tag{11}$$

Above, we denote the difference between renewable generation and local demand as $P_i^S(k)$. Note that the exchanged power with the storage $P_i^b(k)$ at each time instant depends on the energy mismatch $P_i^S(k)$, the controllable generation $P_i^C(k)$ and the surplus power from other microgrids $P_{ij}(k)$, with coordinated factor a_{ij}. Due to the priority use of renewable power in order to satisfy the local demand, $P_{ij}(k)$ is greater than or equal to zero.

C. System Modelling

According to the above system dynamics in (1) and (6), power balance constraints $(8) - (10)$, we have the following subsystem model:

$$\begin{aligned} x_i(k+1) =& M_i x_i(k) + Q_i u_i(k) + \sum_{j=1, j\neq i}^{m} A_{ij} x_j(k) \\ &+ \sum_{j=1, j\neq i}^{m} B_{ij} u_j(k) + L_i w_i(k), \quad i = 1, \ldots, m, \end{aligned} \tag{12}$$

where

$$\begin{aligned} x_i(k) &= [(1/\eta_i)E_i(k) \quad P_i^{RES}(k) \quad P_i^C(k)]^T, \\ u_i(k) &= [P_i^L(k)], \\ w_i(k) &= [P_i^r(k)], \\ M_i &= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1-\alpha_{ri} & 0 \\ 0 & 0 & 1 \end{bmatrix}, Q_i = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}, \\ A_{ij} &= \begin{bmatrix} 0 & a_{ij} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B_{ij} = \begin{bmatrix} -a_{ij} \\ 0 \\ 0 \end{bmatrix}, \\ L_i &= \begin{bmatrix} 0 \\ \alpha_{ri} \\ 0 \end{bmatrix}, \quad i, j = 1, \ldots, m. \end{aligned}$$

Here $x_i(k)$ are the states of subsystem i, $u_i(k)$ is the ith subsystem input, and $w_i(k)$ are external inputs from other subsystems that influence subsystem i at sample step k. The matrices M_i, Q_i, L_i are the relevant state-space matrices.

III. DISTRIBUTED MODEL PREDICTIVE CONTROL

The optimal schedule will be affected by uncertainties due to RESs, storage, and load forecasting. Thus, an DMPC strategy with operation constraints is proposed to coordinate the surplus energy among the microgrid systems, where local controllers not only have to take into account local information, but also exchange the state information among the microgrid systems.

In this section, the implementation of a DMPC controller is first outlined, and then in the following section the corresponding control policy is discussed.

A. Prediction Model

By augmenting the states in each microgrid, we obtain the prediction model for all the microgrids, which is equal to

$$\overline{x}(k+1|k) = A\overline{x}(k|k) + B\overline{u}(k|k), \tag{13}$$

where,

$$\overline{x}(k|k) = [x_1(k|k) \quad \cdots \quad x_m(k|k)]^T,$$

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$$\overline{u}(k|k) = [u_1(k|k) \quad \cdots \quad u_m(k|k)]^T,$$

and the system matrices A and B are as follows:

$$A = \begin{bmatrix} M_1 & A_{12} & \cdots & A_{1m} \\ A_{21} & M_2 & \cdots & A_{2m} \\ \vdots & \cdots & \ddots & \vdots \\ A_{m1} & A_{m2} & \cdots & M_m \end{bmatrix},$$

$$B = \begin{bmatrix} Q_1 & B_{12} & \cdots & B_{1m} \\ B_{21} & Q_2 & \cdots & B_{2m} \\ \vdots & \cdots & \ddots & \vdots \\ B_{m1} & B_{m2} & \cdots & Q_m \end{bmatrix}.$$

Note that in the equation above that is used for prediction, $L_i w_i(k)$ is omitted since it describes the uncertainties from RES which are neglected here. Based on the model in (13), the predicted states, $\overline{x}(k+n|k)$ can be calculated by:

$$A^n \overline{x}(k|k) + A^{n-1} B \overline{u}(k|k) + \cdots + B \overline{u}(k+n-1|k), \quad (14)$$

where $\overline{u}(k+n|k) \equiv \overline{u}(k+N_u-1|k)$ if $N_u < n \leq N_p$. Both N_p and N_u are integers, representing the prediction horizon and control horizon, respectively. By defining the augmented vectors as

$$X(k) = [\overline{x}(k+1|k) \quad \cdots \quad \overline{x}(k+N_p|k)]^T,$$
$$U(k) = [\overline{u}(k|k) \quad \cdots \quad \overline{u}(k+N_u-1|k)]^T,$$

the predicted state sequency $X(k)$ can be also expressed in the form of augmented matrices as follows:

$$X(k) = F_x \overline{x}(k|k) + G_x U(k), \quad (15)$$

where

$$F_x = [A^T \quad \cdots \quad (A^{N_u})^T \quad \cdots \quad (A^{N_p})^T]^T,$$

$$G_x = \begin{bmatrix} B & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ A^{N_u-1}B & A^{N_u-2}B & \cdots & B \\ \vdots & \vdots & \vdots & \vdots \\ A^{N_p-1}B & A^{N_p-2}B & \cdots & A^{N_p-N_u}B+ \\ & & & \cdots + B \end{bmatrix}.$$

Note that the future predicted states $X(k)$ depend on the information at the current time $\overline{x}(k|k)$, namely, an n-step ahead prediction based on the measurements at time k, which simplifies the calculation of the prediction model.

B. Control Policy

The control policy for each microgrid selects the control variables based on information available at the current time. Known information includes measured states of the DERs and loads from the local and neighbor microgrids. This information is then used to calculate and minimize the total cost. Through this optimization process, the control policy determines the power schedules of each controller onsite.

Due to the low cost of power generated from renewable sources, each microgrid will supply the requested power by the renewable sources generated. When at some point the amount of produced renewable power within the microgrid i is less than the demand, we first shed the loads if the demand is above some maximum level. Otherwise, we find a neighbor, say microgrid j, whose renewable generation is higher than its demand; in this case, microgrid i will obtain the surplus power from microgrid j. Moreover, if the total RESs supplied from the local system and neighbors is still less than the power demand of system loads, fuel cells and storage inside the local microgrid are regarded as generators which supply power to loads.

Four power sources are mainly considered in the microgrid: PV, WT, fuel cells and battery. Because of the renewable energy benefits (less gas emission and low operation lost), PV and WT are considered as the prior source, and fuel cells and battery are considered as backup sources.

According to daily predictions of the available power, energy from the RESs, and energy of the loads, the electric power $P_i^b(k)$ exchanged with the storage must be determined. Algorithm 1 shows flow of the power coordination scheduling. The task of the primary control is to prioritize the use of renewable energy. The primary control is included inside each local energy management of electric components.

Algorithm 1 Power Coordination Scheduling

1: **Initialization** $E_i(0)$, $P_i^{RES}(0)$, $P_i^C(0)$, $P_i^L(0)$, $P_i^S(0)$, $\forall i = 1, \ldots, m$
2: **for** $k = 1$ **do**
3: Measure $P_i^{RES}(k)$, $P_i^L(k)$ and compute $P_i^S(k)$.
4: **if** $P_i^{RES}(k) > P_i^L(k)$ **then**
5: $P_i^b(k) = P_i^S(k)$.
6: **else if** $P_i^L(k) > P_i^{Lf}(k)$ **then**
7: Shed the loads.
8: **else if** $\left|P_i^S(k)\right| < P_{ij}(k)$ **then**
9: $P_i^b(k) = P_i^S(k) + P_{ij}(k)$.
10: **else if** $\left|P_i^S(k)\right| < P_{ij}(k) + P_i^C(k)$ **then**
11: $P_i^b(k) = P_i^S(k) + P_{ij}(k) + P_i^C(k)$.
12: **else**
13: Discharge local battery.
14: **end if**
15: $k \leftarrow k+1$ and go to step 3.
16: **end for**

Case 1: If the amount of produced renewable power within the microgrid i is higher than the demand, priority is given to local renewable energy and at such time fuel cells and the battery are backup. The electric power exchanged with the storage satisfies

$$P_i^b(k) = P_i^S(k) > 0. \quad (16)$$

Case 2: When the amount of produced renewable power within the microgrid i is less than the demand, first we will shed the loads if the demand is greater than a corresponding maximum level. If or not, priority is given to the neighbor microgrids, whose renewable generation is higher than their own demand and the surplus RES $P_j^S(k)$ can provide enough power to microgrid i. The power exchanged with the storage:

$$P_i^b(k) = P_i^S(k) + P_{ij}(k). \tag{17}$$

Case 3: If the surplus power from neighbor microgrids is not enough to meet the local demand, it uses its fuel cells to satisfy the demand, and the power exchanged with the storage satisfies

$$P_i^b(k) = P_i^S(k) + P_{ij}(k) + P_i^C(k). \tag{18}$$

Case 4: Otherwise, the local system will use the electricity discharged from the battery.

C. Optimization

The purpose of the research is to design an on-line method for active-power dispatching to maintain the renewable generation power as close as possible to load demand for all the microgrids. Hence the objective of the EMS is to: 1) improve the availability of renewable energy sources; 2) minimize the dissatisfactions of the customers in the DSM; 3) minimize deep discharging of the battery. Then the optimization problem for each subsystem can be written as:

$$\begin{aligned} \min \quad & \xi_r \sum_{n=1}^{\infty} (P_i^L(k+n|k) - P_i^{RES}(k+n|k))^2 \\ & \qquad + \xi_b C_b(P_i^b(k+n|k)) + \xi_L C_L(P_i^L(k+n|k)) \\ s.t. \quad & (2), (4), (7), (11), (13). \end{aligned} \tag{19}$$

where ξ_r, ξ_b and ξ_L are the parameters to trade off among the electric components.

A complete description of the proposed distributed EMS can be found in Algorithm 2, where the private information of the DERs and the loads is stored at the local controller. The MGCC solves the problem using the system information, such as the topology, the measured state information, etc. The information exchanged between the MGCC and the LCs include only the control signals and the schedules.

Note that by Algorithm 2, two objectives are derived: Firstly, the renewable power generation and the state of the storage can be predicted; secondly, difference between the demand and the renewable generation of each microgrid can be compensated through proper coordination.

D. Stability of Multi-Microgrid Systems

Definition 1. *The system in* (13) *is stable if for all finite $x(k)$, there exists a positive definite Lyapunov quadratic form $V(k)$ such that*

$$\Delta V(k) := V(k+1) - V(k) < 0. \tag{20}$$

Theorem 1. *The augmented system in* (13) *is stable if there exists a state feedback control law*

$$\begin{aligned} \overline{u}(k+n|k) = & -(B^T PB + S_1)^{-1}((B^T PA - S_2)\overline{x}(k+n|k) \\ & - S_5^T) \end{aligned} \tag{21}$$

and the following optimization problem subject to the LMI constraints is solvable:

$$\min_{\gamma, Q} \gamma \tag{22}$$

Algorithm 2 Proposed Distributed MPC

1: **Initialization** $E_i(0)$, $P_i^{RES}(0)$, $P_i^C(0)$, $P_i^S(0)$, $\forall i = 1, \ldots, m$
2: **for** $k = 1$ **do**
3: Measure $P_i^{RES}(k)$ for the previous iteration. Set $x_i(k|k) = x_i(k)$.
4: Make a measurement of the state $E_i(k)$ to make energy charging or discharging decisions.
5: Solve the corresponding optimization problem (19).
6: Send $\widehat{P}_i^S(k)$ to neighboring agents j, collect $\widehat{P}_j^S(k)$ from them, and go to step 5.
7: **until** convergence
8: Implement the optimal control action $u_i^*(k)$.
9: $k \leftarrow k+1$ and go to step 3.
10: **end for**

subject to:

$$\begin{bmatrix} -\gamma & \overline{x}^T(k) \\ * & -Q \end{bmatrix} < 0, \tag{23}$$

$$\begin{bmatrix} -Q & \Delta_{12} & \Delta_{13} & \Delta_{14} \\ * & -S_5 S_1^{-1} S_5^T + S_6 & (BS_1^{-1}S_5^T)^T & 0 \\ * & * & -Q - BS_1^{-1}B^T & 0 \\ * & * & * & -I \end{bmatrix} < 0, \tag{24}$$

$$\begin{bmatrix} -S_3 & S_2^T \\ S_2 & -S_1 \end{bmatrix} < 0, \tag{25}$$

where the symbol $*$ depicts a symmetric structure, I is an appropriately dimensioned identity matrix, $Q \succ 0, S_1 \succ 0, S_2, S_3 \succ 0, S_4, S_5$ are the matrix variables with appropriate dimensions, S_6 is positive scalar variable, $\gamma > 0$ and:

$$\begin{aligned} \Delta_{12} &= -QS_2^T S_1^{-1} S_5^T - QS_4^T, \\ \Delta_{13} &= Q(A + BS_1^{-1}S_2)^T, \\ \Delta_{14} &= Q(S_3 - S_2^T S_1^{-1} S_2)^{1/2}, \\ Q &= P^{-1}, \quad e = \alpha_b \overline{E}, \quad f_i(k) = P_i^{Lf}(k), \\ l_r &= [0 \quad 1 \quad 0], \quad l_b = [\eta_i \quad 0 \quad 0], \\ S_1 &= \begin{bmatrix} \xi_r + \xi_L & & \\ & \ddots & \\ & & \xi_r + \xi_L \end{bmatrix}, \\ S_2 &= \begin{bmatrix} \xi_r l_r & & \\ & \ddots & \\ & & \xi_r l_r \end{bmatrix}, \\ S_3 &= \begin{bmatrix} l_r^T \xi_r l_r + l_b^T \xi_b l_b & & \\ & \ddots & \\ & & l_r^T \xi_r l_r + l_b^T \xi_b l_b \end{bmatrix}, \\ S_4 &= \begin{bmatrix} e\xi_b l_b & \cdots & e\xi_b l_b \end{bmatrix}, \\ S_5 &= \begin{bmatrix} f_1(k)\xi_L & \cdots & f_m(k)\xi_L \end{bmatrix}, \\ S_6 &= me^2\xi_b + \sum_{i=1}^{m} (f_i^2(k)\xi_L). \end{aligned}$$

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Proof. The optimization problem can be divided into four cases. We choose the worst case among of them, i.e., there exists deep discharging in the battery and load shedding. Rewriting the objective function in (19) into the augmented matrix based on the distributed discrete-time system model (13), we have

$$\begin{aligned}
J(k) =& \sum_{n=0}^{\infty}\sum_{i=1}^{m}(\parallel u_i(k+n|k) - l_r x_i(k+n|k) \parallel_{\xi_r}^2 \\
&+ \parallel l_b x_i(k+n|k) - e \parallel_{\xi_b}^2 + \parallel u_i(k+n|k) - f_i(k) \parallel_{\xi_L}^2) \\
=& \sum_{n=0}^{\infty}(\sum_{i=1}^{m}(u_i^T(k+n|k)(\xi_r+\xi_L)u_i(k+n|k) \\
&- 2u_i^T(k+n|k)\xi_r l_r x_i(k+n|k) + e^2\xi_b + f_i(k)^2\xi_L \\
&+ x_i^T(k+n|k)(l_r^T\xi_r l_r + l_b^T\xi_b l_b)x_i(k+n|k) \\
&- 2e\xi_b l_b x_i(k+n|k) - 2f_i(k)\xi_L u_i(k+n|k))) \\
=& \sum_{n=0}^{\infty}[\overline{u}^T(k+n|k)S_1\overline{u}(k+n|k) \\
&- 2\overline{u}^T(k+n|k)S_2\overline{x}(k+n|k) \\
&+ \overline{x}^T(k+n|k)S_3\overline{x}(k+n|k) \\
&- 2S_4\overline{x}(k+n|k) - 2S_5\overline{u}(k+n|k) + S_6] \geq 0.
\end{aligned} \tag{26}$$

Let a positive definite quadratic function $V(k) = \overline{x}^T(k)P\overline{x}(k)$. Then we have the following equation:

$$\begin{aligned}
&V(k+n+1|k) - V(k+n|k) + J(k+n|k) \\
=&\overline{u}^T(k+n|k)(B^TPB+S_1)\overline{u}(k+n|k) \\
&+ 2\overline{u}^T(k+n|k)(B^TPA-S_2)\overline{x}(k+n|k) \\
&- 2S_5\overline{u}(k+n|k) + \overline{x}^T(k+n|k)(A^TPA \\
&- P + S_3)\overline{x}(k+n|k) - 2S_4\overline{x}(k+n|k) + S_6 \\
=&(\overline{u}(k+n|k) + (B^TPB+S_1)^{-1}((B^TPA-S_2)\overline{x}(k+n|k) \\
&- S_5^T))^T(B^TPB+S_1)(\overline{u}(k+n|k) \\
&+ (B^TPB+S_1)^{-1}((B^TPA-S_2)\overline{x}(k+n|k) - S_5^T)) \\
&+ \begin{bmatrix} \overline{x}(k+n|k) \\ I \end{bmatrix}^T \Xi \begin{bmatrix} \overline{x}(k+n|k) \\ I \end{bmatrix},
\end{aligned} \tag{27}$$

where

$$\begin{aligned}
&\Xi = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ * & \Omega_{22} \end{bmatrix}, \\
&\Omega_{11} = S_3 - P - S_2^TS_1^{-1}S_2 \\
&\quad + (A+BS_1^{-1}S_2)^T(P^{-1}+BS_1^{-1}B^T)^{-1}(A+BS_1^{-1} \\
&\quad S_2), \\
&\Omega_{12} = -S_2^TS_1^{-1}S_5^T - S_4^T \\
&\quad + (A+BS_1^{-1}S_2)^T(P^{-1}+BS_1^{-1}B^T)^{-1}BS_1^{-1}S_5^T, \\
&\Omega_{22} = S_6 - S_5S_1^{-1}S_5^T + S_5S_1^{-1}B^T(P^{-1}+BS_1^{-1}B^T)^{-1} \\
&\quad BS_1^{-1}S_5^T.
\end{aligned}$$

If the matrix inequality (24) holds, pre-multiplying and post-multiplying (24) by $diag(P, I, I, I)$, we obtain:

$$\begin{bmatrix}
-P & -S_2^TS_1^{-1}S_5^T - S_4^T & (A+BS_1^{-1}S_2)^T & (S_3 - S_2^TS_1^{-1}S_2)^{1/2} \\
* & -S_5S_1^{-1}S_5^T + S_6 & (BS_1^{-1}S_5^T)^T & 0 \\
* & * & -P^{-1} - BS_1^{-1}B^T & 0 \\
* & * & * & -I
\end{bmatrix} < 0. \tag{28}$$

By Schur complements and combining (25) with (28), it is easy to derive the following inequality:

$$\Xi < 0. \tag{29}$$

If the controller is taken as

$$\begin{aligned}
\overline{u}(k+n|k) =& -(B^TPB+S_1)^{-1}((B^TPA-S_2)\overline{x}(k+n|k) \\
&- S_5^T),
\end{aligned}$$

then we have

$$\begin{aligned}
&V(k+n+1|k) - V(k+n|k) + J(k+n|k) \\
&= \begin{bmatrix} \overline{x}(k+n|k) \\ I \end{bmatrix}^T \Xi \begin{bmatrix} \overline{x}(k+n|k) \\ I \end{bmatrix} \\
&< 0,
\end{aligned}$$

which implies

$$V(k+n+1|k) - V(k+n|k) < -J(k+n|k) \leq 0 \tag{30}$$

since $J(k+n|k) \geq 0$. The system stability is guaranteed. By Schur Complements, we can rewrite the LMI constraint in (23) as $V(k) < \gamma$. Summarizing (30) from $n = 0$ to $n = \infty$, we have

$$V(\infty|k) - V(k|k) < -J(k),$$

which yields

$$J(k) < V(k) < \gamma. \tag{31}$$

If the inequality constraint (31) holds for a given γ, the optimization problem (22) is solvable. Thus, the proof is completed. □

IV. VALIDATION RESULTS AND DISCUSSION

A. Simulation Setup

We use the Matlab optimization toolbox to verify the effectiveness of our proposed control algorithm. The simulation was run using the continuous solver ode23tb. The system composed of three microgrids with PV, WT, fuel cells, battery and local loads is illustrated in Fig. 2, which implies that there is collaboration within the network. For these simulations, we used an MPC time horizon of $N = 24$, a length of the optimization window $N_p = 4$, and control horizon $N_u = 1$. This corresponds to one hour intervals over a 24 hour period and is a typical horizon and time step for schedule updates. It should be noted that all the values reported in this section are converted to power unit (p.u.).

The capacity of the battery $\underline{E}$ is 5 p.u., $\overline{E}$ is chosen to be 60 p.u., and $\eta^{ch} = 0.7, \eta^{dch} = 0.65$. The initial value of the battery is set to $E_i(0) = 40$ p.u., $\forall i = 1, 2, 3$. The capacity of

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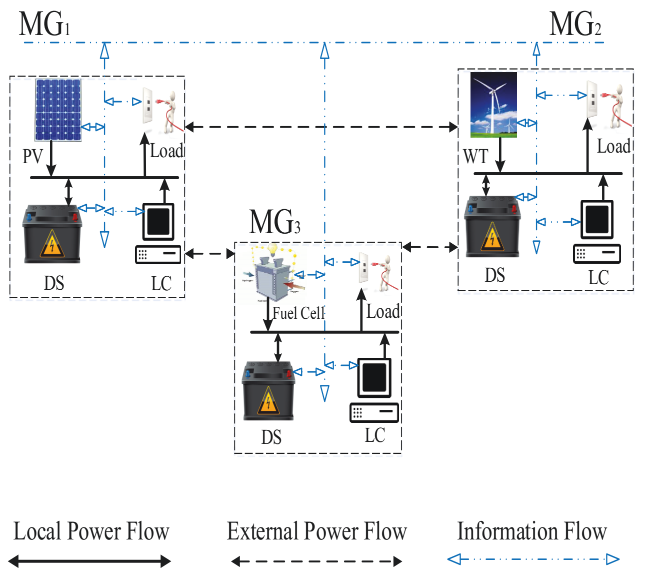

Fig. 2. An example illustrating the cooperation between microgrids in a network.

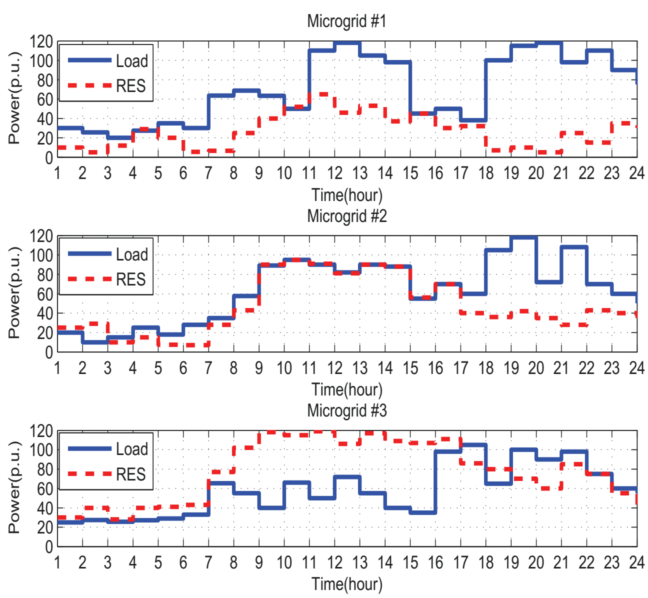

Fig. 3. Demand profile and daily power generated by the renewable energy sources.

the fuel cell is 10 p.u.. Let the positive constant of interruptible loads be $\alpha_l = 10$, the depth of the discharge $\alpha_b = 0.2$ and the smoothing constant is $\alpha_r = 0.3$.

B. Results

We compute the DMPC using the setup described above. The parameters in the algorithm are chosen as $\xi_r = 0.5, \xi_b = 0.5, \xi_l = 1$. The difference between the demand and the renewable generation is used to generate the reference trajectory for each microgrid shown in Fig. 3, where solid lines represent demand and dashed lines represent RESs. It is easy to see that:

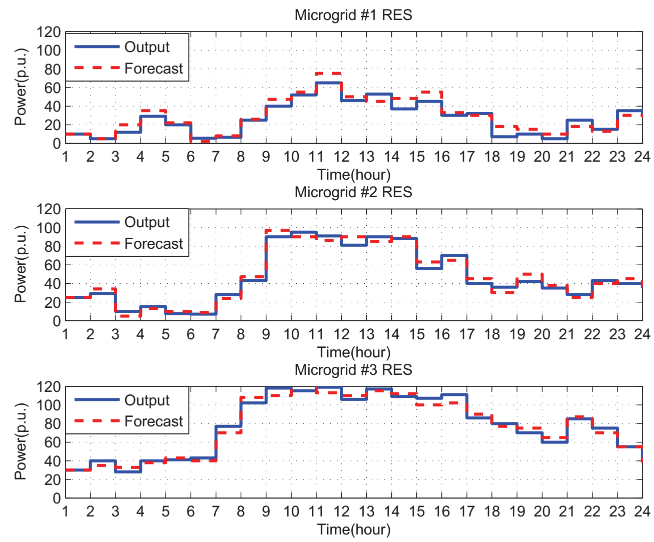

Fig. 4. Predicted and actual renewable sources.

the produced renewable power within microgrid $\sharp 1$ is less than the demand from 7 am to 7 pm, while the renewable energy nearly equals the demand in microgrid $\sharp 2$, while renewable generation in microgrid $\sharp 3$ is higher than the demand.

In order to compensate for the difference between the demand and RESs, the microgrid systems need to coordinate by exchanging energy. We evaluate the predictive values of system states from each microgrid by taking advantage of our proposed DMPC algorithm. Fig. 4 shows a comparison of the daily forecasted and actual power generation for the renewable source of each microgird. The forecasted curves are shown by red dashed lines while the actual profiles are shown in blue solid lines. It can be seen that the predicted renewable generation at each time step can track the actual value very well. In addition, the optimization of battery is shown in Fig. 5. When renewable sources from local and other microgrids are able to meet users' demand, the battery is considered as a backup device; otherwise the battery is discharging. According to the simulation result, it is clear that deep dischaging is penalized when the energy stored in the battery is less than 12 p.u. (20% of the total capacity of the battery).

Fig. 6 illustrated how the loads are shedded, where the load is reduced in microgrid $\sharp 1$ from 11 am to 3 pm and from 6 pm to midnight, microgrid $\sharp 2$ from 9 am to 3 pm and from 6 pm to 10 pm, and microgrid $\sharp 3$ from 4 pm to 10 pm. As can be expected, the loads are shedded in response to $P_i^L(k) > P_i^{Lf}(k)$, which is mainly used to balance the local supply.

Then the coordinated energy distribution based on the proposed control policy is shown in Fig. 7, where PV, WT, external grid exchanges P_{ij}, fuel cells and battery are shown with blue, blue-green, black, green and yellow respectively. It can be seen that when renewable supply is enough to meet the demand (red curve), e.g., in microgrid $\sharp 2$ from 9 am to 2 pm, the renewable sources are the only ones being utilized. If the produced renewable power is unable to meet users' demand, e.g., in microgrid $\sharp 1$ from 8 am to 4 pm, it receives surplus

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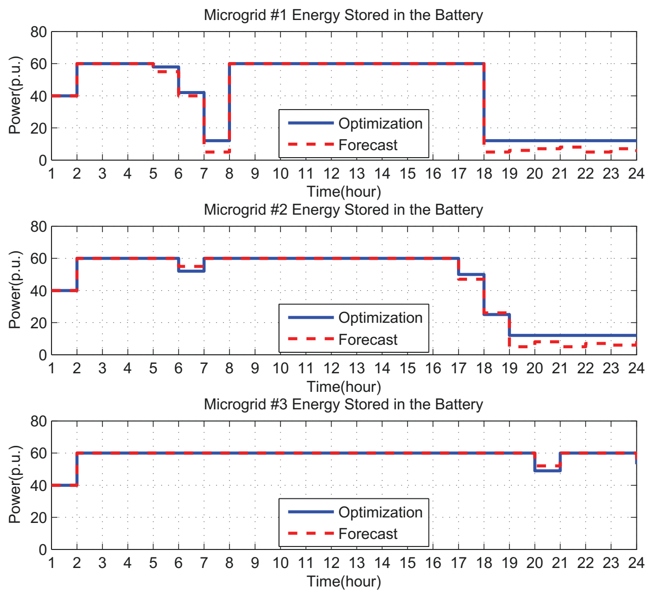

Fig. 5. Profiles of state of storage device.

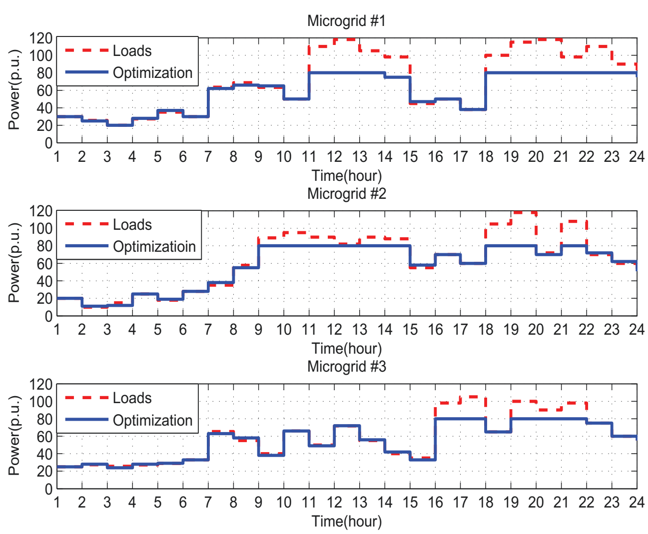

Fig. 6. Demand reduction of the loads.

power from others. Note that the surplus power from neighbor microgrids is not enough to meet the local demand at 7 pm, at that time it will use local fuel cells first and then the battery.

In order to assess the level of RESs effficiency utilization, we define $REUR$ (Renewable Energy Utilization Ratio) as

$$REUR = \frac{\sum_{i=1}^{m} P_i^{REU}(k)}{\sum_{i=1}^{m} P_i^{L}(k)}, \tag{32}$$

where $P_i^{REU}(k)$ denotes total consumption of RESs at time instant k. In Fig. 8, we compare the RESs utilization under our coordination strategy with noncooperative plannning. It can be seen that our coordinated policy significantly improve the

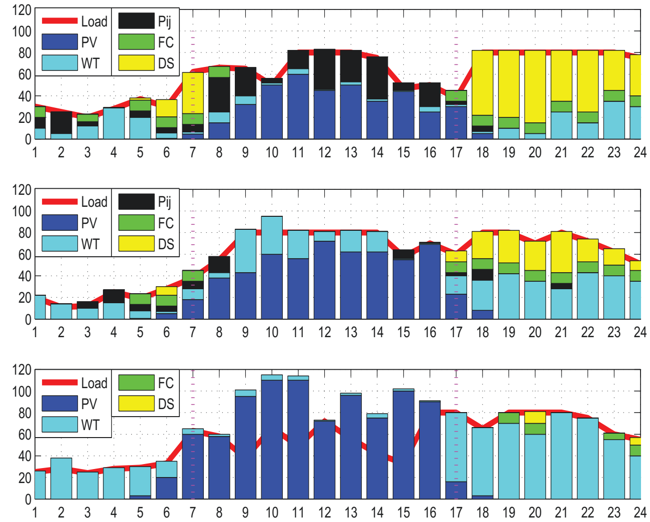

Fig. 7. Profiles of energy dispatch coordinated scheduling.

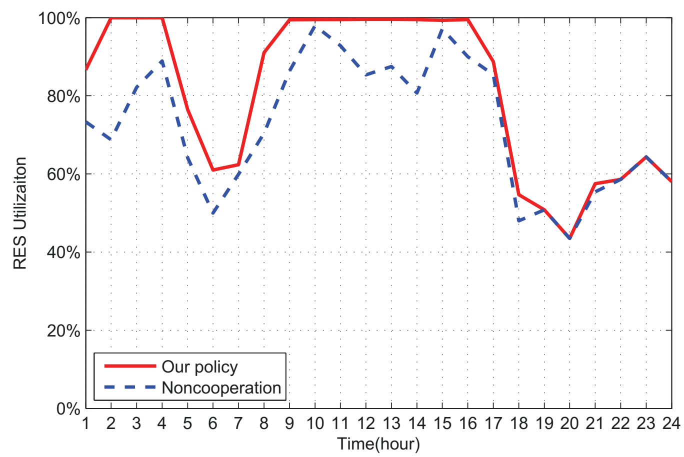

Fig. 8. Comparison of RES utilization.

utilization of renewable energy in all microgrids, especially during that time when there are sufficient solar and wind sources.

Stability analysis results are shown in Fig. 9-10. The value of the γ upper bound based on Theorem 1 is 9.3. The reduced operation cost is 2.7. Fig. 9 gives the state responses of RESs in each subsystem. We can see that the trends of the state responses are in line with that shown in Fig. 4. It is clear that the performance of the system has been changed with the variance of RESs. Finally, the states of all three subsystems converged together to their equilibrium points. In addition, optimal control inputs of the three subsystems are also shown in Fig. 10.

V. CONCLUSION

In this paper, a distributed model predictive control strategy is proposed to optimize energy management in microgrids, where the surplus renewable energy exchange has been taken into account among a cooperative network of heterogeneous

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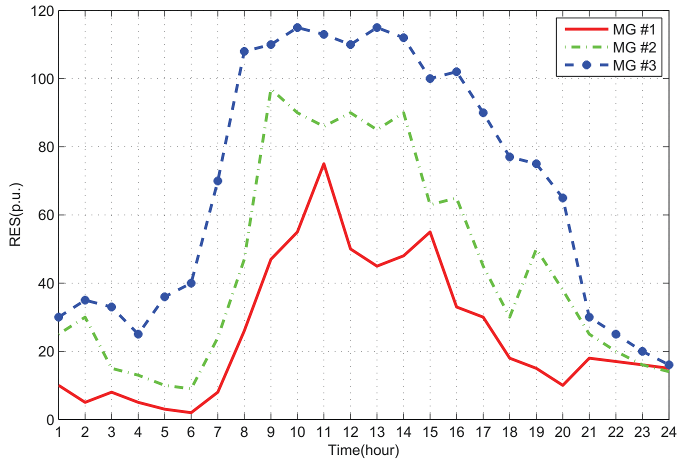

Fig. 9. State response of the subsystems.

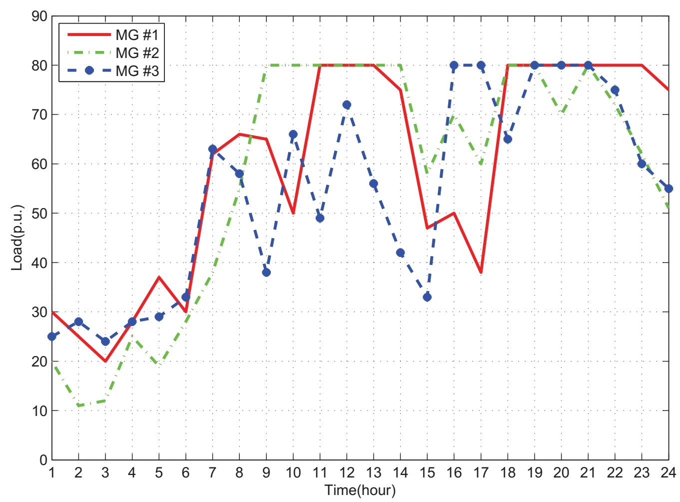

Fig. 10. Control inputs of the subsystems.

microgrids. According to the dynamics and behaviour of microgrid components, a discrete time formulation is presented. Moreover, a multi-objective function is introduced to improve the availability of renewable energy sources. It is shown that the proposed distributed MPC strategy guarantees closed-loop stability of the overall system. The simulation results demonstrate the effectiveness of our proposed method.

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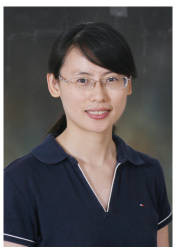

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