

Multi-Leader Multi-Follower Game-Based ADMM for Big Data Processing

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Abstract—Alternating direction method of multipliers (ADMM) is a promising approach to solve “big data” problems due to its efficient variable decomposition and fast convergence. However, it is subject to the following two fundamental assumptions: no contradiction among multiple controllers’ objectives and ideal feedback from the agents to the controllers. In this paper, a multiple-leader multiple-follower (MLMF) game-based ADMM is developed to balance the conflicting objectives among the controllers as well as those between the controllers and the agents. Both analytical and simulation results verify that the proposed method reaches a hierarchical social optimum and converges at a linear speed. More importantly, the convergence rate is independent of the network size, which indicates that the MLMF game-based ADMM can be used in a very large network for big data processing.

Index Terms—Big data, large-scale network, game theory, ADMM

I. INTRODUCTION

In recent years, the alternating direction method of multipliers (ADMM) [1]–[3] has been recognized as a useful algorithm to deal with “big data” problems, such as data classification, due to its effectiveness in both fast convergence and variable decomposition. Through ADMM, a controller, with an objective function to optimize, can divide the needed computation into smaller problems and implement them at different agents in a distributed manner [1]. Even though the variable updates of ADMM can be performed separately on a number of agents, the method of updating is specified by the controller. Hence, ADMM cannot work properly when multiple controllers want to optimize multiple conflicting objectives simultaneously or the agents have their own objectives to achieve and do not feed variables back as the controllers expect.

Fig. 1 shows an example of a mobile crowd sourcing network [4], in which the controllers (as servers) collect sensing data from the agents (as mobile devices) to achieve their systematic objectives. We denote by $x_{i,j}$ the data collected from agent j to controller i . $(-g_{i,j}(x_{i,j}))$ is defined as the gain when controller i obtains $x_{i,j}$ from agent j . Hence,

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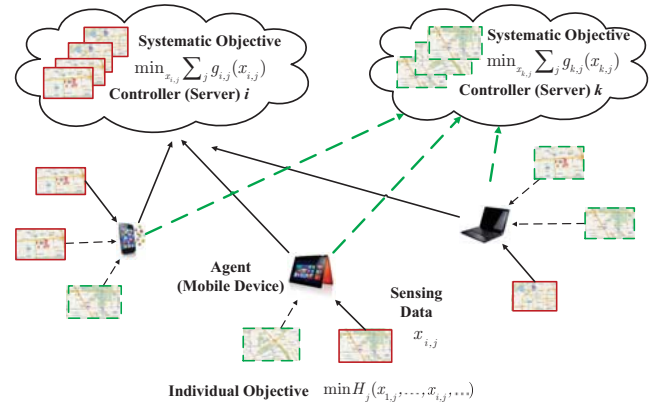


Fig. 1. Problem illustration in a mobile crowd sourcing network with multiple controllers and multiple agents.

the systematic objective for controller i can be expressed as $\min_{x_{i,j}} \sum_j g_{i,j}(x_{i,j})$. However, each agent can only sense limited data (expressed as $\sum_i x_{i,j} = x_j$ for each agent j) due to the sensing capacity constraint. Therefore, the controllers have to compete for the sensing services of the agents, where controller i 's minimization on $g_{i,j}(x_{i,j})$ may increase $g_{k,j}(x_{k,j})$ for another controller k . Thus, those servers cannot run the ADMM independently for data collection due to the *first limitation* above. Besides, when there are multiple controllers in the system, each agent has the opportunity to select desirable controllers to collect data. This incentivizes each agent to work for the controllers with the highest payments/benefits. Therefore, without incentives, each agent j is usually willing to update variables $x_{i,j}$ to minimize its own objective, denoted by $H_j(x_{1,j}, \dots, x_{i,j}, \dots)$, rather than $g_{i,j}(x_{i,j})$ for each controller i due to the *second limitation* above.

To balance the objectives among different controllers, and overcome the influence of the individual objectives of the agents, incentive mechanisms, such as Stackelberg games [5], [6], can be applied for the controllers to motivate the agents to update the variables as the controllers expect. The controllers can be regarded as buyers that pay the agents for optimizing the objectives for the controllers. Thus, the controllers'

objectives can be driven towards some rational goal, such as the Pareto optimum or the social optimum [6], [7]. Although the existing game-theoretic methods have found some ways to balance objectives among the multiple controllers and helped design incentive mechanisms, they require a prohibitively large number of iterations to converge, especially when the network size is very large. This can hardly meet the needs of big data networks [8], [9].

From the above discussion, it is natural to combine the ADMM with game theory to address the tradeoff and convergence issues. To this end, we develop a multi-leader multi-follower (MLMF) game-based ADMM as an incentive mechanism, which exploits the advantages of both ADMM and game theory. In summary, we have the following major contributions in this paper:

- We develop an MLMF game-based ADMM to achieve a hierarchical social optimum, which theoretically connect optimization methods (such as the ADMM) and game theory for big data networks.
- We demonstrate its linear convergence regardless of the network size as long as the objective functions are separable and strongly convex.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

We consider a big data network with multiple controllers and multiple agents, as shown in Fig. 1. The number of controllers and agents can be very large. Each agent holds resources and leases them to the controllers. We denote by $x_{i,j}$ the resources leased from agent j to controller i . We assume that the system consists of K controllers and N agents. Obviously, with large K and N , there exist millions or billions of variables $\{x_{i,j} | i = 1, 2, \dots, K; j = 1, 2, \dots, N\}$ to be processed, which makes centralized optimization impractical. For convenience, we use a $K \times N$ matrix $\mathbf{X} \triangleq (x_{i,j})_{i,j=1}^{K,N}$ to represent all resources.

We define the objective for each controller in the leasing process as a general minimization problem. Mathematically, the objective of controller i can be modeled as a typical ADMM-form optimization problem [1] expressed as

$$\min_{\mathbf{x}_{i,*}} G_i(\mathbf{x}_{i,*}) \triangleq \sum_{j=1}^N g_{i,j}(x_{i,j}), \quad (1)$$

$$\text{s.t. } \sum_{j=1}^N A_{i,j}x_{i,j} - B_i = 0, \quad (1a)$$

where $\mathbf{x}_{i,*} \triangleq (x_{i,1}, x_{i,2}, \dots, x_{i,N})$ is a row vector that represents the set of resources from N agents, $g_{i,j}(x_{i,j})$ can be regarded as the cost (or $-g_{i,j}(x_{i,j})$ as the gain) of controller i when it obtains resource $x_{i,j}$ from agent j , and $G_i(\mathbf{x}_{i,*})$ is defined as the cost function for controller i . The linear constraint for controller i as in (1a) can be regarded as the largest requirement, B_i , where all $\{A_{i,j} | j = 1, 2, \dots, N\}$ and B_i are real and scalar constants.

The objective for each agent j can be also written as a general minimization problem,

$$\min_{\mathbf{x}_{*,j}} H_j(\mathbf{x}_{*,j}) \triangleq \sum_{i=1}^K h_{i,j}(x_{i,j}), \quad (2)$$

$$\text{s.t. } \sum_{i=1}^K C_{i,j}x_{i,j} - D_j = 0, \quad (2a)$$

where $h_{i,j}(x_{i,j})$ is the cost for agent j when agent j leases resource $x_{i,j}$ to controller i , $H_j(\mathbf{x}_{*,j})$ denotes the cost function of agent j , and $\mathbf{x}_{*,j} \triangleq (x_{1,j}, x_{2,j}, \dots, x_{K,j})^T$. The linear constraint in (2a) for agent j indicates the limited total resources held by agent j , where all $\{C_{i,j} | i = 1, 2, \dots, K\}$ and D_j are real and scalar constants.

It is worth pointing out that the ADMM cannot be implemented directly to reach the controllers' objectives in (1) since resources leased by K controllers from the same agent j , are coupled with the constraint, $\sum_{i=1}^K C_{i,j}x_{i,j} = D_j$, in (2a). Note that the controllers' objectives in (1) and the agents' objectives in (2) may conflict as well. We denote by $\mathbf{X}^* \triangleq (x_{i,j}^*)_{i,j=1}^{K,N}$ the optimal matrix of resources for the controllers' objectives in (1), and by $\hat{\mathbf{X}} \triangleq (\hat{x}_{i,j})_{i,j=1}^{K,N}$ the optimal point for agents' objectives in (2). Since the resources are held by agents, the value of $\mathbf{X}$ is determined by agents. Thus, the result probably reaches $\hat{\mathbf{X}}$ instead of the optimal matrix, $\mathbf{X}^*$. To motivate agents to change their actions in reaching $\mathbf{X}^*$ but not $\hat{\mathbf{X}}$, the controllers need to provide incentives for the agents [10].

We formulate the incentive mechanism design problem as a multiple-leader multiple-follower game so that the optimal result of the game corresponds to a hierarchical social optimum, which balances the controllers' objectives in (1) and simultaneously considers the agents' objectives in (2).

Mathematically, the incentive mechanism is determined by a set of incentive functions. Denote by $\Phi_j(H_j(\mathbf{x}_{*,j}), \theta_{*,j})$ an incentive function to describe the interaction between all controllers and agent j , where $\theta_{i,j}$ denotes the incentive factor adjusted by controller i to influence $H_j(\mathbf{x}_{*,j})$, and $\theta_{*,j} \triangleq (\theta_{1,j}, \theta_{2,j}, \dots, \theta_{K,j})^T$ represents the incentive factor corresponding to agent j . For the financial incentive in [10], $\theta_{*,j}$ is the actual payment or payment per unit (prices) for agent j ¹.

B. Problem Formulation

With the incentive mechanism in Fig. 1, our problem can be formulated as an MLMF game as

$$\text{Leaders' Game: } \underset{\theta_{i,*}}{\operatorname{argmin}} G_i(\mathbf{x}_{i,*}), \quad (3a)$$

$$\text{Followers' Game: } \underset{\mathbf{x}_{*,j}}{\operatorname{argmin}} \Phi_j(H_j(\mathbf{x}_{*,j}), \theta_{*,j}), \quad (3b)$$

$$\text{Leaders' Constraints: } \text{s.t. } \sum_{j=1}^N A_{i,j}x_{i,j} - B_i = 0, \quad (3c)$$

$$\text{Followers' Constraints: } \text{s.t. } \sum_{i=1}^K C_{i,j}x_{i,j} - D_j = 0, \quad (3d)$$

¹In this paper, we adopt the financial incentive for the incentive mechanism design [10].

for all $i = 1, 2, \dots, K$ and $j = 1, 2, \dots, N$, where the controllers and the agents are regarded as the leaders and the followers, respectively².

There typically exists no feasible solution that simultaneously minimizes all objective functions for the leaders and incentive functions for the followers. Instead, we will find the *hierarchical social optimum* [6], [7] as a reasonable result, as given below.

Definition 1: Hierarchical Social Optimum. Denote by $\mathcal{X}$ and Θ the feasible set of the resource matrices, $\mathbf{X}$, and the feasible set of the matrices of incentive factors, $\Theta \triangleq (\theta_{i,j})_{i,j=1}^{K,N}$, respectively. A point $(\mathbf{X}^*, \Theta^*) \in (\mathcal{X}, \Theta)$ is a *hierarchical social optimum* if the following two conditions are simultaneously satisfied:

- 1) There is no other point, $(\mathbf{X}, \Theta) \in (\mathcal{X}, \Theta)$, such that $\sum_{i=1}^K G_i(\mathbf{x}_{i,*}) < \sum_{i=1}^K G_i(\mathbf{x}_{i,*}^*)$; and
- 2) Given an optimal point of incentive factors, $\Theta^* \triangleq (\theta_{i,j}^*)_{i,j=1}^{K,N}$, there exists no other point $(\mathbf{X}, \Theta^*) \in (\mathcal{X}, \Theta)$, such that for at least one follower j , $\Phi_j(H_j(\mathbf{x}_{*,j}), \theta_{*,j}^*) < \Phi_j(H_j(\mathbf{x}_{*,j}^*), \theta_{*,j}^*)$, and for other followers $\Phi_n(H_n(\mathbf{x}_{*,n}), \theta_{*,n}^*) \leq \Phi_n(H_n(\mathbf{x}_{*,n}^*), \theta_{*,n}^*)$, $n \in \{1, 2, j-1, j+1, \dots, N\}$.

The first condition in the above assures that the sum of the costs for the leaders is minimized and the second one indicates that all followers reach a Nash equilibrium given the optimal incentive factors from the controllers [11]. Even though the result of the hierarchical social optimum is defined for a centralized goal of all leaders, it can also balance the objectives among different leaders, since the social optimum can guarantee a Pareto optimum among leaders [7].

C. Assumptions

To ensure the convergence of the MLMF game-based ADMM, we need the following assumptions:

- 1) Each term in each leader's objective function in (1), $g_{i,j}(x_{i,j})$, is strongly convex in variable $x_{i,j}$. That is to say, there exists a $\gamma > 0$, for each $g_{i,j}(x_{i,j})$, such that given any $x_{i,j}$ and $x'_{i,j}$,

$$g_{i,j}(x'_{i,j}) \geq g_{i,j}(x_{i,j}) + \nabla_{x_{i,j}} g_{i,j}(x_{i,j})(x'_{i,j} - x_{i,j}) + \frac{\gamma}{2} \|x'_{i,j} - x_{i,j}\|^2. \quad (4)$$

- 2) Each term in each follower's objective function in (2), $h_{i,j}(x_{i,j})$, is a convex function with a feasible minimum point $h_{i,j}^* > -\infty$. Mathematically, given any $x_{i,j}$ and $x'_{i,j}$, the following inequality is satisfied:

$$h_{i,j}(x'_{i,j}) \geq h_{i,j}(x_{i,j}) + \nabla_{x_{i,j}} h_{i,j}(x_{i,j})(x'_{i,j} - x_{i,j}). \quad (5)$$

- 3) The objective function of each follower satisfies a uniform Lipschitz gradient condition [3], [12]. There exists $\kappa > 0$, for each $H_j(\mathbf{x}_{*,j})$, given any $\mathbf{x}_{*,j} \preceq \mathbf{x}'_{*,j}$,

$$\nabla_{\mathbf{x}_{*,j}} H_j(\mathbf{x}'_{*,j}) - \nabla_{\mathbf{x}_{*,j}} H_j(\mathbf{x}_{*,j}) \preceq \kappa (\mathbf{x}'_{*,j} - \mathbf{x}_{*,j}), \quad (6)$$

²For convenience, we call the controllers the leaders and the agents the followers in the rest of this paper.

where $\mathbf{y} \preceq \mathbf{z}$ denotes that each element of vector $\mathbf{y}$ is less than or equal to the corresponding one of vector $\mathbf{z}$.

The assumptions above clarify the generality and feasibility of our framework. We emphasize that all the assumptions are common [2], [3], [12] and many general functions satisfy these assumptions³.

III. MULTIPLE-LEADER MULTIPLE-FOLLOWER GAME-BASED ADMM

In this section, we first present the incentive function design. Then, we describe the iteration process of our MLMF game-based ADMM. Finally, we discuss the linear convergence of our algorithm.

A. Incentive Function Design

Leader i wants follower j to optimize $g_{i,j}(x_{i,j})$ through a payment, which is proportional to the resources provided from follower j , as $\theta_{i,j} x_{i,j}$, where $\theta_{i,j}$ is the payment per unit, i.e., the *price* [10]. Denote $\theta_{i,j}^{(p)}$ the price at the iteration step p . With the payment, follower j helps to minimize $g_{i,j}(x_{i,j})$ and simultaneously considers its pure individual part $h_{i,j}(x_{i,j})$. As a result, the incentive function for each follower j can be constructed at step p as

$$\Phi_j \left(H_j(\mathbf{x}_{*,j}), \theta_{*,j}^{(p)} \right) = \sum_{i=1}^K \left(g_{i,j}(x_{i,j}) + h_{i,j}(x_{i,j}) - \theta_{i,j}^{(p)} x_{i,j} \right). \quad (7)$$

B. MLMF Game-based ADMM

The operation process consists of a two-tier iteration. The process to achieve $\mathbf{X}^{(p)}$ at each step p is called the *inner loop*, where t is the iteration index for the inner loop. The one to update Θ from step p to step $p+1$ is called the *outer loop*. The two-tier iteration can be described as follows.

- 1) *Inner loop*: The leaders and the followers find optimum resource allocation to minimize the incentive functions in (7) with an ADMM iterative process at each step p .
 - a) When $t = 0$: each follower j sets $\mathbf{x}_{*,j}^{(p)}(t) = \mathbf{x}_{*,j}^{(p-1)}$ and each leader i sets $\lambda_i^{(p)}(t) = \lambda_i^{(p-1)}$, where $\mathbf{x}_{*,j}^{(p-1)}$ and $\lambda_i^{(p-1)}$ are the optimal point for the resource allocation and dual variable, respectively, at step $p-1$.
 - b) When $t > 0$: The followers update the primal variables, $\mathbf{X}$, sequentially,

$$\begin{aligned} & \mathbf{x}_{*,j}^{(p)}(t+1) \\ &= \underset{\mathbf{x}_{*,j}}{\operatorname{argmin}} \Phi_j \left(H_j(\mathbf{x}_{*,j}), \theta_{*,j}^{(p)} \right) - \sum_{i=1}^K \lambda_i^{(p)}(t) A_{i,j} x_{i,j} \\ &+ \frac{\rho}{2} \sum_{i=1}^K \left\| \sum_{n=1, n \neq j}^N A_{i,n} x_{i,n}^{(p)}(t) + A_{i,j} x_{i,j} - B_i \right\|_2^2, \\ & \text{s.t.} \quad \sum_{i=1}^K C_{i,j} x_{i,j} - D_j = 0, \end{aligned} \quad (8)$$

³Many widely used functions in communication networks and big data areas satisfy these assumptions, such as the segmental log-function $-\log(1+x)$ with $x_L \leq x \leq x_H$ and the second order norm $\|x\|_2^2$ [12].

Algorithm 1: MLMF game-based ADMM**Input:** $\theta_{i,j} = 0, i = 1, 2, \dots, K, j = 1, 2, \dots, N, p = 0$ **Output:** $\theta_{i,j}^*, x_{i,j}^*, i = 1, 2, \dots, K, j = 1, 2, \dots, N$ **while** $\left\| \sum_{i=1}^K G_i(\mathbf{x}_{i,*}^{(p)}) - \sum_{i=1}^K G_i(\mathbf{x}_{i,*}^{(p-1)}) \right\| \geq \varepsilon$ **do**

(a) Optimization with ADMM (inner loop):

To achieve $\mathbf{X}^{(p)}$ and $\{\lambda_i^{(p)} | i = 1, 2, \dots, K\}$;

(b) Followers' feedback and leaders' incentive factors design (outer loop):

 $\theta_{i,j}^{(p+1)} = \nabla_{x_{i,j}} h_{i,j}(x_{i,j}^{(p)});$ (c) $p = p + 1$;**end****Result:**Optimal values of variables: $\mathbf{X}^* = \mathbf{X}^{(p)}$;Optimal value of incentive factors: $\Theta^* = \Theta^{(p)}$.

for $j = 1, 2, \dots, N, \rho > 0$ is a damping factor, and $\hat{t} = t + 1$ when $n < j$ and $\hat{t} = t$ when $n > j$.

The leaders update the dual variables, λ , concurrently,

$$\lambda_i^{(p)}(t+1) = \lambda_i^{(p)}(t) - \rho \left(\sum_{j=1}^N A_{i,j} x_{i,j}^{(p)}(t+1) - B_i \right) \quad (9)$$

for $i = 1, 2, \dots, K$.

Repeat steps a) and b) until each $\mathbf{x}_{*,j}^{(p)}$ and each $\lambda_i^{(p)}$ have no significant change.

- 2) *Outer loop:* After the leaders and the followers achieve a current optimal point of resources, $\mathbf{X}^{(p)}$, each follower j feeds back its marginal cost of resources to each leader i , which is used to update the incentive factor as

$$\theta_{i,j}^{(p+1)} = \nabla_{x_{i,j}} h_{i,j}(x_{i,j}^{(p)}), \quad (10)$$

for $i = 1, 2, \dots, K$, and $j = 1, 2, \dots, N$.

The outer loop stops at step p when the sum of the leaders' objective functions cannot change much, which mathematically satisfies a primal stopping criterion given as follows:

$$\left\| \sum_{i=1}^K G_i(\mathbf{x}_{i,*}^{(p)}) - \sum_{i=1}^K G_i(\mathbf{x}_{i,*}^{(p-1)}) \right\| < \varepsilon, \quad (11)$$

where ε is a pre-determined threshold. Further details on the MLMF game-based ADMM are provided in Algorithm 1.

C. Convergence

In this section, we demonstrate the linear convergence of the MLMF game-based ADMM regardless of the network size, which is critical for big data processing in a very large network.

We at first define an augmented Lagrangian function $L(\mathbf{X}, \lambda, \mu)$ as a potential function for the whole network as

in [12] as the following:

$$\begin{aligned} L(\mathbf{X}, \lambda, \mu) = & \sum_{i=1}^K G_i(\mathbf{x}_{i,*}) - \sum_{i=1}^K \lambda_i \left(\sum_{j=1}^N A_{i,j} x_{i,j} - B_i \right) \\ & - \sum_{j=1}^N \mu_j \left(\sum_{i=1}^K C_{i,j} x_{i,j} - D_j \right) \\ & + \sum_{i=1}^K \frac{\rho}{2} \left\| \sum_{j=1}^N A_{i,j} x_{i,j} - B_i \right\|_2^2, \end{aligned} \quad (12)$$

where $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_K)$ and $\mu = (\mu_1, \mu_2, \dots, \mu_N)$ are dual variables to guarantee the constraints in (1a) and constraints in (2a), respectively, are satisfied.

Then, we show that the augmented Lagrangian function decreases at a linear speed before it reaches its minimum point.

Theorem 1: Given any positive small scalar ε_0 , when the primal residue of the augmented Lagrangian function in (12) is less than ε_0 , that is,

$$\|L(\mathbf{X}^{(p)}, \lambda^{(p)}, \mu^{(p)}) - L(\mathbf{X}^*, \lambda^*, \mu^*)\| < \varepsilon_0, \quad (13)$$

the iteration step,

$$p < \frac{\log((L(\mathbf{X}^0, \lambda^0, \mu^0) - L(\mathbf{X}^*, \lambda^*, \mu^*)) / \varepsilon_0)}{\log \frac{1}{\tau}}. \quad (14)$$

where $\tau = \frac{1}{1+2\gamma}$ is the step size, and $\mathbf{X}^0, \lambda^0$, and μ^0 are the initial values of the primal variables and dual variables.

It can be derived directly that the relation between ε_0 and ε in the stopping criterion in (11), is $\varepsilon_0 = \frac{\varepsilon}{1-\tau}$.

We omit the proof here due to space limitations.

IV. SIMULATION RESULTS

In this section, we illustrate the performance of the MLMF game-based ADMM. Without loss of generality, we use the mobile crowd network in Fig. 1 as an example in our simulation. The leaders (servers) collect the transmit power information, $\mathbf{X}$, from the followers (users) to perform statistical power control. The followers want to maximize their capacity through optimizing their transmission power.

The objective functions of the leaders in (1) are selected as commonly used functions in big data statistics and learning problems, such as in the Tikhonov regularization for the model fitting problem [1],

$$G_i(\mathbf{x}_{i,*}) = \|\mathbf{x}_{i,*}\|_2^2 - \phi_i^T \mathbf{x}_{i,*}, \quad (15)$$

where ϕ_i is a random column constant, with an independent uniform distribution between 0 and 1 in our simulation.

Shannon capacity for an additive Gaussian channel is used as the objective function of each follower in (2), that is,

$$H_j(\mathbf{x}_{*,j}) = - \sum_{i=1}^K \log(1 + \varphi_{i,j} x_{i,j}), \quad (16)$$

where $\varphi_{i,j}$ represents a channel-to-noise ratio and is uniformly distributed between 0 and 10 for different indices i or j in our simulation.

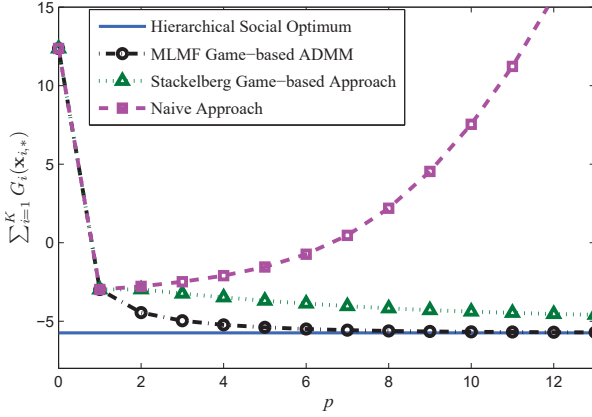


Fig. 2. Comparison of the convergence speeds of different algorithms, when $K = 10$, $N = 30$, and $\varepsilon = 10^{-2}$.

$A_{i,j}$, B_i , $C_{i,j}$, and D_j in constraints (1a) and (2a) are random parameters, with independent uniform distributions between 0 and 1 for different indices i or j .

Fig. 2 compares the convergence speeds of the MLMF game-based ADMM, the Stackelberg game-based ADMM, and the naive approach. The naive approach is generated from the intuitive logic in the game theory [11], where each follower j increases $x_{i,j}$ with a small value $\delta = 0.1$ for leader i when the incentive factor $\theta_{i,j}$ is positive, and it decreases $x_{i,j}$ otherwise. Correspondingly, each leader i increases the incentive factor $\theta_{i,j}$ for follower j with a small positive value $\delta = 0.1$ when its marginal cost is positive, that is, $\nabla_{x_{i,j}} h_{i,j}(x_{i,j}) > 0$, and decreases $\theta_{i,j}$ otherwise. The Stackelberg game-based approach [5] guarantees that the inner loop in Algorithm 1 reaches a current optimal point for the followers at each step. However, the adjustment of the incentive factors, Θ , is the same as in the naive approach. The hierarchical social optimal is represented by the optimal value of the sum of leaders' objectives. From the figure, the naive approach may not converge. In addition, our MLMF game-based approach converges much faster than the Stackelberg game-based approach since we carefully select the incentive factors in (10) in our proposed method.

Fig. 3 demonstrates the required number of iterations versus the accuracy, ε , for stopping iterations when $K = 10, 30$ and $N = 10, 30, 100$, respectively. The results in the figure are averaged over 100 trials. From the figure, the relation between the required number of iterations and $\log_{10}(\frac{1}{\varepsilon})$ is linear, which confirms the linear convergence as indicated in Theorem 14. Given the prescribed accuracy, for example, $\varepsilon = 10^{-3}$, the required number of iterations is always about 26, which is independent of the numbers of leaders or followers. This indicates that the MLMF game-based ADMM can deal with the network with a large size.

V. CONCLUSION

In this paper, we have proposed an MLMF game-based ADMM for networks with multiple leaders and multiple

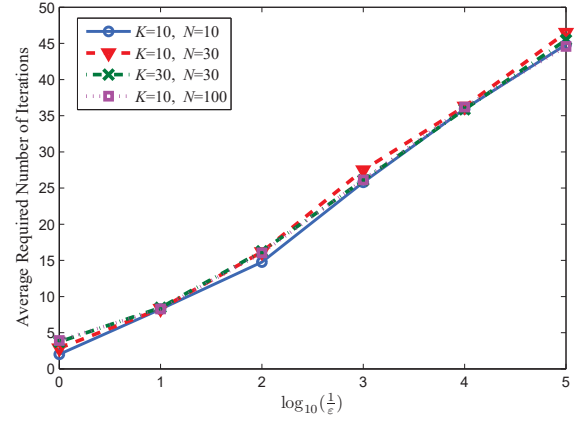


Fig. 3. The relation between the accuracy ε in stopping criterion in (11) and the required number of iteration, when $K = 10, 30$ and $N = 10, 30, 100$.

followers. The proposed method can reach the hierarchical social optimum in a distributed manner. It converges linearly and regardless of the network size. Therefore, the proposed MLMF game-based ADMM can efficiently cope with big data networks with large numbers of leaders (controllers) and followers (agents). In addition to the mobile crowd network scenario, many other applications such as small cell caching for 5G virtualized networks can be implemented with our algorithm for service providers to induce the network operators to share their storage resources.

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