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Minimizing Age-of-Information in Multi-Hop Wireless Networks

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***Abstract*—Timely exchange of information over multi-hop wireless networks is gaining increasing relevance with growing interests in applications such as internet of things (IoT) and autonomous vehicular networks. Age-of-information (AoI) is a recently proposed performance metric that measures information freshness at the destination node. AoI at a destination node is the time since last update was received. We study AoI for multi-hop networks with general interference constraints with $\mathcal{R}$ source-destination pairs, and derive simple stationary policies in which links are activated according to a stationary probability distribution. We first consider a line network with a single source-destination pair, and characterize AoI as a convex function of link activation rates. We then use this result to obtain the optimal policy, in the class of stationary policies, for multi-hop network, with several source-destination pairs. We prove an important *separation principle*, which says that the optimal scheduling policy for the multi-hop problem can be obtained by solving an equivalent problem in which all source-destination pairs are single-hop away.**

I. Introduction

Exchanging fresh information updates over multi-hop wireless networks is gaining increasing relevance with advent of ad-hoc networked wireless systems such as internet of things (IoT), vehicular networks, and network of unmanned aerial vehicles. In unmanned aerial vehicular networks, for example, exchanging position, velocity, and other control information in a timely fashion can help in collision avoidance and efficient path planning [1], [2]. In IoT, and other cyber physical systems, timely feedback of sensor data is vital to the overall system performance.

These systems differ from the traditional communication systems in two ways. In traditional communication systems, data or packet arrival is assumed to be an exogenous process that cannot be controlled. However, in these networks, the update packets, such as sensor data, can be generated at will. Generating update packets at the right rate may be more efficient [3], as high rate of generation results in network clogging and low rate results in updates being sent too infrequently.

Secondly, traditional communication systems use packet centric performance measures such as throughput or delay to characterize performance. These performance measures do not fully capture the information freshness paradigm. For example, delay of a stale packet, that got caught in the network due to network clogging, doesn't need to be accounted for as long as the intended ground station gets fresh information regularly via other, promptly received, update packets.

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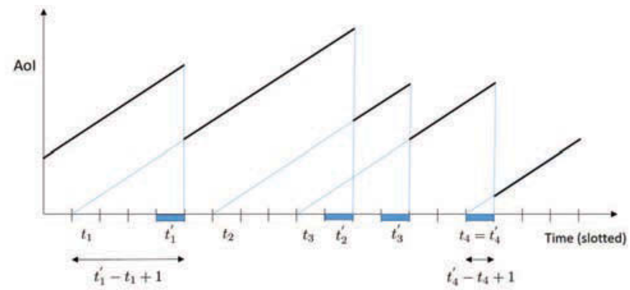

Fig. 1. Age of Information (AoI) as a function of time. Here, t_i is the time of generation of the ith packet at the source, and t_i' is the time of its reception at the destination node.

A new performance measure, called Age of information (AoI), was proposed in [3], [4] to measure information freshness at the destination node. AoI at the destination node at time t, is the time elapsed since the last received update packet was generated. Figure 1, plots AoI evolving in time. Whenever the destination node receives a fresh update packet, the AoI drops to the time elapsed since the received packet's generation time, while it grows linearly otherwise.

AoI was first studied in [4] for a vehicular network using simulation. Vehicles periodically generated update packets to be transmitted to other nodes in the network. An optimal rate of packet generation was observed. To better understand this phenomena, [3] modeled the network between the source and destination as a single first-in-first-out (FIFO) queue, and proved that there is indeed an optimal rate at which AoI is minimized.

Since then, most of the work on AoI has focused on single queue models. Age for FIFO M/M/1, M/D/1, and D/M/1 queues was analyzed in [3], multiclass FIFO M/G/1 and G/G/1 queues were studied in [5], while last-in-first-out (LIFO) queues under various arrival and service time distributions were studied in [6]–[8]. AoI for M/M/2 and M/M/∞ queues was analyzed in [9], [10], which primarily studied the impact of out-of-order delivery of packets on age. Effects of packet error or packet drop on age for the M/M/1 queue, with FIFO service, was studied in [11].

AoI over multi-hop networks with general interference constraints, however, has received very little attention. In [12], a switch type network was considered under phys-

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ical interference constraints, and the problem of scheduling finitely many update packets was shown to be NP-hard for this network. Multiple access type network with nodes and a single base station, where only a single link can be activated at any given time, was studied in [13], [14]. Slotted ALOHA-like random access for AoI minimization was studied in [15]. To the best of our knowledge, AoI over multi-hop networks has been considered only in [16] where LIFO queue service was shown to reduce age.

We consider the age minimization problem over a multi-hop network with general interference constraint. We consider a set of $\mathcal{R}$ source-destination pairs or flows, with each flow associated with a source-destination path. Our objective is to device simple scheduling policy that minimizes weighted age over flows. We therefore limit ourselves to simple, stationary policies in which links are activated according to a stationary probability distribution.

We first consider a L-hop line network with a single flow with general interference constraints. We completely characterize the peak and average age, two popular measures of AoI, as a convex function of link activation rates, and show that the two measures are equal. We then apply these results to general, $\mathcal{R}$-flow multi-hop network, with general interference constraints, and formulate a weighted age minimization problem. We prove an important *separation principle*, that says that the optimal stationary scheduling policy for multi-hop age can be obtained by solving a single-hop problem, where each link is considered to be a flow. To the best of our knowledge, this is the first work to consider scheduling policies for age minimization problem for multi-hop networks, with general interference constraints.

We describe the system model in Section II. The specific case of line network with general interference constraint is considered in Section III. In Section IV, we extend the results obtained for the line network to general networks with general interference constraint. We conclude in Section V.

II. System Model

Consider a network $G = (V, E)$, where V denotes the set of communicating nodes and E denotes the set of directed communication links. Not all links in E can be activated simultaneously in the network due to interference constraints. We call $m \subset E$ a *feasible activation set* if all links in m can be activated simultaneously without interference. We let $\mathcal{A}$ to denote the collection of all feasible activation sets. We assume time to be slotted with slot durations normalized to unity.

The network G contain a set $\mathcal{R} = \{1, 2, \dots R\}$ source-destination pairs. For every $r \in \mathcal{R}$, let $s(r)$ and $d(r)$ denote the source and the destination nodes, respectively. The source $s(r)$ has to send update packets to the destination $d(r)$. We refer to $\mathcal{R}$ as set of flows and a $r \in \mathcal{R}$ as flow. The destination $d(r)$ may be multiple hops away from the source $s(r)$, and as a result, the updates need to be routed to the destination node with the aid of other nodes in the network. The source

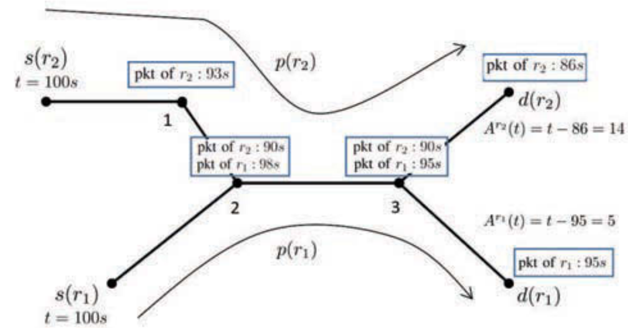

Fig. 2. Illustration of age propagation in a network with $\mathcal{R} = \{r_1, r_2\}$. Snapshot of the network taken at time $t = 100$. Also shown is the time of generation of the last received packet at each node, for each flow.

$s(r)$ chooses a path $p(r)$ in G from $s(r)$ to $d(r)$ to send update packets.

The source $s(r)$ generates and transmits update packet. Update packets are time stamped at the time of their generation to ensure that every receiving node, especially the destination node, knows the age of the information contained in the packet. We assume all the nodes in the network to be *active*, i.e., they retain the latest generated packet for every flow $r \in \mathcal{R}$. In particulate, the source nodes always have a fresh update packet for transmission. In Figure 2, for example, nodes 2 has retained the latest update packet for both flows, namely, r_1 and r_2, that pass through it. The last received update packets at node 2 were generated at $t = 98$ and $t = 90$ for flows r_1 and r_2, respectively.

For every flow r, the age of a node $i \in V$ at time t is the time since the last received update packet of that flow was generated; we denote it by $A_i^r(t)$. In Figure 2, we see that nodes 2 and 3 have update packets of flow r_1 that were generated at times $t = 98$ and $t = 95$, respectively. The age now (at $t = 100$) is the time elapsed since then. Therefore, $A_2^{r_1}(t) = t - 98 = 100 - 98 = 2$ and $A_3^{r_1}(t) = t - 95 = 100 - 95 = 5$. The age evolution equation can be written as

$$A_i^r(t+1) = \begin{cases} t - G_i^r(t) + 1 & \text{if } i \text{ receives a pkt. at } t \\ A_i^r(t) + 1 & \text{otherwise} \end{cases}, \quad (1)$$

where $G_i^r(t)$ is the time of generation of the update packet delivered to node i in slot t. The evolution of age $A_i^r(t)$ is illustrated in Figure 3 for a line network. Age $A_i^r(t)$ drops abruptly when the destination node receives a fresh packet and increases linearly at rate 1 when it doesn't. Notice that not every transmission results in reduction of age as the transmitting node may not have received a new update since the last transmission.

We define three measures of age. Since the goal is to send fresh update packets to the destination node $d(r)$ for each flow, all the age measures are a function of age at the destination $A_{d(r)}^r(t)$, for all $r \in \mathcal{R}$. The weighted average age is given by

$$A^{\text{ave}} = \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} w^r A_{d(r)}^r(t), \quad (2)$$

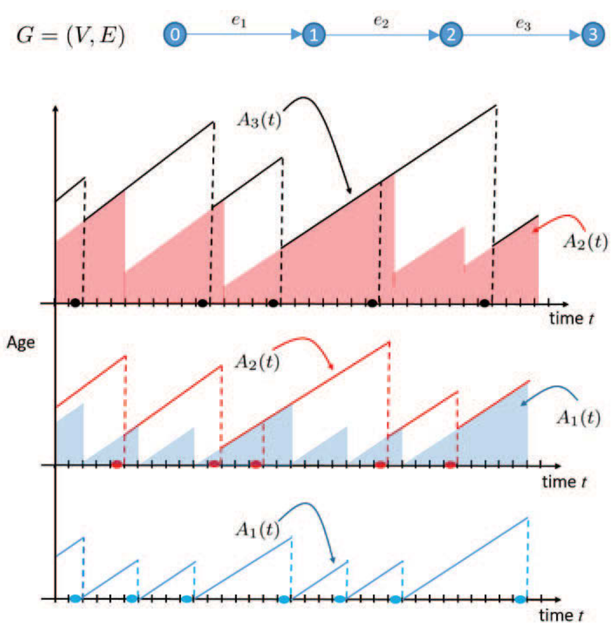

Fig. 3. Age evolution at nodes in a three link line graph $G = (V, E)$, where $V = \{0, 1, 2, 3\}$ and $E = \{e_1, e_2, e_3\}$, and a single flow $\mathcal{R} = \{r\}$ with $s(r) = 0$, $d(r) = 3$, and path $p(r) = \{e_1, e_2, e_3\}$. Figure plots age at node 1, 2, and 3 as a function of time; denoted as $A_1(t)$, $A_2(t)$, and $A_3(t)$, respectively.

where w^r is a positive weight assigned to each $r \in \mathcal{R}$, the weighted peak age is given by

$$A^{\text{p}} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} w^r A^r_{d(r)}(S^r(i)), \tag{3}$$

where $S^r(i)$ denote the time slots in which an update packet was received by $d(r)$,[1] and the average instantaneous age given by

$$A^{\text{inst}} = \lim_{t \to \infty} \mathbb{E}\left[\sum_{r \in \mathcal{R}} w^r A^r_{d(r)}(t)\right]. \tag{4}$$

It is important to note that all the age measures depend on the policy π of activating links. This dependence can be made explicit by using the notation $A^{\text{ave}}(\pi)$, $A^{\text{p}}(\pi)$, and $A^{\text{inst}}(\pi)$ for average, peak, and instantaneous age, respectively. In general, the age measures as stated in (2), (3), and (4) may not be well defined for any policy π. However, for the stationary policies considered in this paper, these limits are indeed well defined.

A. Stationary Policies

We shall restrict our attention to simple stationary policies. This is primarily because they are easy to implement using random access. Let

$$B_e(t) = \{\text{link } e \text{ is successfully activate at time } t\}, \tag{5}$$

[1]This is not the average of peaks in the age evolution curve, but average of age sampled at times when an update is delivered to the destination node. This definition coincides with the classical peak age definition when every flow is a single-hop flow.

be the event that link $e \in E$ is activated successfully at time t. We define stationary policies as follows.

Definition A policy π is stationary if

1) For each $e \in E$, events $B_e(t)$ are independent across t
2) $\mathbb{P}[B_e(t)] = \mathbb{P}[B_e(t')]$ for all slots t, t', and links e

Note that every stationary policy π is associated with

$$f_e = \mathbb{P}[B_e(t)], \tag{6}$$

for all $e \in E$. We call f_e as the *link activation frequency* of link e, and use $\mathbf{f} = (f_e)_{e \in E}$ to denote the vector of link activation frequencies. The following are two examples of stationary policies.

Stationary Centralized Policies: Let $\mathbf{x} \in \mathbb{R}^{|\mathcal{A}|}$ be a probability distribution over the set of all feasible activation sets $\mathcal{A}$. In each slot t, activate a feasible activation set $m \in \mathcal{A}$ with probability x_m, independent across t. This is a stationary policy, and the probability $\mathbb{P}[B_e(t)]$ can written as

$$\mathbb{P}[B_e(t)] = (M\mathbf{x})_e, \tag{7}$$

where M is a $|E| \times |\mathcal{A}|$ matrix such that

$$M_{e,m} = \begin{cases} 1 & \text{if } e \in m \\ 0 & \text{otherwise} \end{cases}. \tag{8}$$

In order to minimize age over the space of all centralized stationary policies Π_C, we need to characterize the set of all achievable link activation frequencies by such policies; namely, the set

$$\mathcal{F}_C = \left\{\mathbf{f} \in \mathbb{R}^{|E|} \mid f_e = f_e(\pi) \text{ for some } \pi \in \Pi_C\right\}. \tag{9}$$

From (7), we can conclude that the set $\mathcal{F}_C$ is given by

$$\mathcal{F}_C = \left\{\mathbf{f} \in \mathbb{R}^{|E|} \mid \mathbf{f} = M\mathbf{x}, \mathbf{1}^T\mathbf{x} \leq 1, \text{ and } \mathbf{x} \geq 0\right\}. \tag{10}$$

It is not always possible to implement stationary centralized policy in wireless networks without a centralized controlling node. We, thus, consider distributed policies.

Stationary Distributed Policies: For each $e \in E$ let $a_e \in (0, 1)$ be the probability that link e attempts transmission in a slot t with probability a_e, independent across slot t. Under the pairwise interference model, in which for each $e \in E$ there is a subset of links $N_e \subset E$ that interfere with it, the link activation frequencies can be written as

$$f_e = \mathbb{P}[B_e(t)] = a_e \prod_{e' \in N_e} (1 - a_{e'}), \tag{11}$$

for all $e \in E$ and slots t. Therefore, the space of all link activation frequencies achievable by stationary distributed policies is given by

$$\mathcal{F}_D = \Big\{\mathbf{f} \in \mathbb{R}^{|E|} \mid f_e = a_e \prod_{e' \in N_e} (1 - a_{e'}) \text{ and } 0 \leq a_e \leq 1 \;\forall\, e \in E\Big\}. \tag{12}$$

In the next section, we derive average, peak, and instantaneous age for a line network under a stationary policy. We show all the age measures to be simple convex functions of link activation frequencies. The general network case is then considered in Section IV.

III. Age Minimization for a Line Network

Consider line network $G = (V, E)$, where $V = \{0, 1, \dots L\}$ and $E = \{(0,1), (1,2), \dots (L-1, L)\}$ denote the $L+1$ nodes and L links, respectively. For convenience, we shall use e_l to denote link $(l-1, l)$ for all $l = 1, 2, \dots L$. The network contains a single flow with source node $s = 0$ and destination node $d = L$. The source s generates fresh update packets that are transmitted over the line network to reach the destination node d.

The age evolution given in (1) can be simplified as follows.

Lemma *1:* The age evolution of (1) can be written as

$$A_l(t+1) = \begin{cases} A_{l-1}(t) + 1 & \text{if } e_l \text{ activated at } t \\ A_l(t) + 1 & \text{otherwise} \end{cases},$$

for all $l \in \{1, 2, \dots L\}$.

Proof: If link e_l is not activated then from (1) it follows that $A_l(t+1) = A_l(t) + 1$. In the case when link e_l is activated at time t two possibilities arise. Either node $(l-1)$ has received a fresh update since the last activation of e_l or it hasn't. If it has then this fresh update packet is transmitted to node l at time t, and we get $A_l(t+1) = A_{l-1}(t)+1$. However, if node $(l-1)$ hasn't received a fresh update since the last activation of link e_l, then we have $A_l(t) = A_{l-1}(t)$. This is because both nodes have the same update packet that was communicated in the last activation of e_l. The age evolution, in the absence of a fresh update therefore becomes $A_l(t+1) = A_l(t) + 1 = A_{l-1}(t) + 1$. ■

The age evolution equation of Lemma 1 is true irrespective of the scheduling policy. We now focus on stationary policies described in Section II-A. We saw in Section II-A that every stationary policy π is associated with a link activation frequency f_e, for each link e. Using this, we now characterize the average instantaneous age at the destination node d.

Theorem *1:* If $f_l > 0$ is the link activation frequency of link $e_l = (l-1, l)$ under a stationary policy π then the average instantaneous age at node d is given by

$$A_d^{\text{inst}} = \mathbb{E}\left[A_d(t)\right] = \sum_{l=1}^{L} \frac{1}{f_l}. \tag{13}$$

Proof: Let π be a stationary policy, and f_l be the link activation frequency of link $e_l = (l-1, 1)$ under policy π, for all $l \in \{1, 2, \dots L\}$. Further, let $\tau_l(t)$ denote the last instance, when link e_l was activated. For example, if link activations occurred at time slots $2, 10, 14$, and 21 then $\tau_l(t) = 10$ for all $t = 11, 12, 13$, and 14.

Since π is a stationary policy the inter-activation times must be geometrically distributed with mean $\frac{1}{f_l}$. The memoryless property, therefore, implies that

$$\mathbb{P}\left[\tau_l(t) = t - k\right] = f_l \left(1 - f_l\right)^{k-1}, \tag{14}$$

for all $k = 1, 2, \dots$. Thus, $\tau_l(t)$ has the same distribution as $t - X_{e_l}$, where X_{e_l} is a geometrically distributed random variable given by

$$\mathbb{P}\left[X_{e_l} = k\right] = f_l \left(1 - f_l\right)^{k-1}, \tag{15}$$

for all $k \in \{1, 2, \dots\}$, with mean

$$\mathbb{E}\left[X_{e_l}\right] = \frac{1}{f_l}, \tag{16}$$

for all $l \in \{1, 2, \dots L\}$.

Consider age at node 1. Since the source node always transmits fresh information, the age of node 1 is the just the time elapsed since the last activation of link $e_1 = (0, 1)$. This is given by

$$A_1(t) = t - \tau_1(t), \tag{17}$$

as $\tau_1(t)$ was the last time a fresh update packet was sent to node 1.

Next, consider age at node 2. By Lemma 1, whenever link e_2 is activated, node 2 resets its age to node 1's age $A_1(\cdot)$. Thus, $A_2(t)$ is given by

$$A_2(t) = t - \tau_2(t) + A_1\left(\tau_2(t)\right), \tag{18}$$

where $A_1\left(\tau_2(t)\right)$ is the age of node 1 at the time of the last activation of link $e_2 = (1, 2)$, namely $\tau_2(t)$, and $t - \tau_2(t)$ is the time elapsed since then. Substituting (17) in (18), we obtain

$$A_2(t) = t - \tau_2(t) + \left[\tau_2(t) - \tau_1\left(\tau_2(t)\right)\right], \tag{19}$$

$$= t - \tau_1\left(\tau_2(t)\right). \tag{20}$$

Iterating this over l links, we get

$$A_l(t) = t - \tau_1\left(\tau_2\left(\cdots \tau_l\left(t\right) \cdots\right)\right), \tag{21}$$

for all t and $l \in \{1, 2, \dots L\}$. Taking expectation, we get

$$\mathbb{E}\left[A_l(t)\right] = t - \mathbb{E}\left[\tau_1\left(\tau_2\left(\cdots \tau_l\left(t\right) \cdots\right)\right)\right], \tag{22}$$

$$= \sum_{k=1}^{l} \frac{1}{f_k}, \tag{23}$$

where the last equality follows from the following Lemma 2, and substituting $l = L$ we obtain the result.

Lemma *2:* $\tau_l(t)$ for $l \in \{1, 2, \dots L\}$ satisfy

$$\mathbb{E}\left[\tau_1\left(\tau_2\left(\cdots \tau_l\left(t\right) \cdots\right)\right)\right] = t - \sum_{k=1}^{l} \frac{1}{f_l}. \tag{24}$$

Proof of Lemma 2 is given in Appendix A. ■

We now show that for a stationary policy, all the three age measures are equal.

Theorem *2:* If $f_l > 0$ is the link activation frequency of link $e_l = (l-1, l)$ under a stationary policy π then the peak and average age at node d are equal, and is given by

$$A_d^{\text{ave}} = A_d^{\text{p}} = A_d^{\text{inst}} = \sum_{l=1}^{L} \frac{1}{f_l}. \tag{25}$$

Proof: Consider age at node l. Peak age is the average of $A_l(t)$ sampled at activation times of link e_l. Since e_l is activated at epoches of a memoryless process with geometric inter-activation times, the poisson-arrivals-see-time-average (PASTA) property implies that peak age and average age are equal:

$$A_l^{\text{ave}} = A_l^{\text{p}}. \tag{26}$$

Furthermore, the process $\mathbf{A}(t) = (A_1(t), A_2(t), \dots A_L(t))$ is ergodic and stationary, as $\mathbf{A}(t)$ is an aperiodic, positive recurrent Markov chain with a single communicating class. Therefore, average age equals instantaneous age:

$$A_l^{\text{ave}} = A_l^{\text{inst}}. \tag{27}$$

Substituting $l = d$ in (26) and (27) yields the result. In Appendix B, we give an alternative derivation of the average age, which may be of interest. ■

A. Age Minimization

Theorem 2 imply that all age measures are equivalent under a stationary policy π. Thus, all three age measures can be minimized simultaneously if we restrict to stationary policies for scheduling. This minimization problem can be formulated as:

Line-Network Age Problem

$$\begin{aligned} &\underset{\mathbf{f}}{\text{Minimize}} && \sum_{l=1}^{L} \frac{1}{f_l}, \\ &\text{subject to} && \mathbf{f} \in \mathcal{F} \end{aligned} \tag{28}$$

where $\mathcal{F}$ is $\mathcal{F}_C$ for stationary centralized policies and is $\mathcal{F}_D$ for stationary distributed policies.

Notice that we have ignored the flow weight w^1 as the network contains only one flow, i.e., $\mathcal{R} = \{1\}$. The set $\mathcal{F}$ is the space of feasible link activation frequencies which depend on the interference constraints, and the type of policy. As we saw in Section II-A, that the space $\mathcal{F}$ can be significantly different for centralized and distributed policies, respectively. We discuss the solvability of (28) in the next section, where we consider age minimization for a general network.

IV. Age Minimization for General Network

In this section, we consider the problem of age minimization for a general network, where we have a network $G = (V, E)$ with a collection of feasible activation sets $\mathcal{A}$, and $\mathcal{R} = \{1, 2, \dots R\}$ flows. Given that every flow is assigned a path we obtain the optimal stationary scheduling policy.

Before we proceed, we briefly review the single-hop age problem from [17], in which all flows are single-hop, i.e., for every flow $r \in \mathcal{R}$ the source $s(r)$ and destination $d(r)$ share a link, and the path is that very link. We discuss the multi-hop case in Section IV-B.

A. Single-Hop Age Problem

Assume that all the flows in $\mathcal{R}$ are just single-hop flows, i.e., for each flow $r \in \mathcal{R}$ the source $s(r)$ and destination $d(r)$ share a link $e(r) \triangleq (s(r), d(r))$, and the assigned path for the flow is $p(r) = \{(s(r), d(r))\}$.

Without loss of generality we assume that each link in G corresponds to a source destination pair. If a link $e \in E$ does not correspond to a source destination pair then it also doesn't contribute to age for any flow, as every flow is one hop. In this case, the problem can be reduced by eliminating the link activation frequency variable f_e associated with link e. Therefore, each link $e \in E$ corresponds to a source destination pair $r \in \mathcal{R}$; which implies $|E| = |\mathcal{R}|$.

A single-hop flow is a special case of a line network. Thus, Theorem 2 imply that for a stationary policy π, all three age measures are equal, and are given by

$$A_r^{\text{inst}} = A_r^{\text{p}} = A_r^{\text{ave}} = \frac{1}{f_{e(r)}}, \tag{29}$$

for all $r \in \mathcal{R}$, where $f_{e(r)}$ is the link activation frequency of link $e(r)$ under policy π. The weighted average age (2), weighted peak age (3), and the weighted instantaneous age (4) are all given by

$$\sum_{r \in \mathcal{R}} \frac{w^r}{f_{e(r)}}. \tag{30}$$

This can also be written as

$$\sum_{e \in E} \frac{w_e}{f_e}, \tag{31}$$

where w_e is the weight of link e, and is given by $w_{e(r)} = w^r$. Therefore, the age minimization problem can be written as:

Single-Hop Age Problem

$$\begin{aligned} &\underset{\mathbf{f}}{\text{Minimize}} && \sum_{e \in E} \frac{w_e}{f_e}, \\ &\text{subject to} && \mathbf{f} \in \mathcal{F} \end{aligned} \tag{32}$$

where $\mathcal{F}$ is $\mathcal{F}_C$ for stationary centralized policies and is $\mathcal{F}_D$ for stationary distributed policies.

Notice that the single-hop age problem and the line-network age problem are equivalent if all link weights w_e are equal. This implies that the optimal stationary policy that minimizes age for the line network can be obtained by considering each link in the line network to be a separate flow. As we shall see in Section IV-B, this is in fact true in for the general network with $\mathcal{R}$ flows.

We now consider the two cases of stationary centralized and stationary distributed policies, and discuss solvability of the single-hop age problem.

1) Stationary Centralized Policy under Matching Constraints: The single-hop age problem when $\mathcal{F} = \mathcal{F}_C$ can be written as

$$\begin{aligned} \underset{\mathbf{f}\in\mathbb{R}^{|E|}}{\text{Minimize}} \quad & \sum_{e\in E} \frac{w_e}{f_e}, \\ \text{subject to} \quad & \mathbf{f} = M\mathbf{x}, \\ & \mathbf{1}^T\mathbf{x} \leq 1 \text{ and } \mathbf{x} \geq 0 \end{aligned} \tag{33}$$

where M is a $|E| \times |\mathcal{A}|$ matrix give in (8). Here, $\mathbf{x}$ can be interpreted as a probability distribution over the activation sets $\mathcal{A}$.

We let the interference in the network to be such that two links interfere if they share a node. This implies that a feasible activation must necessarily be a matching in the network graph. Therefore, $\mathcal{A}$, in this case, is a collection of all matchings in G. Under this structure on $\mathcal{A}$, the set of all link activation frequencies $\mathcal{F}_C$ can be shown to be the matching polytope [18]. Therefore, the problem (33) reduces to minimizing a convex objective, namely,

$$\sum_{e\in E} \frac{w_e}{f_e}, \tag{34}$$

over a matching polytope. This can be efficiently solved by using the Frank-Wolfe algorithm [19], and the separation oracle for matching polytope developed in [20].

2) Stationary Distributed Policy under Pairwise Interference Constraints: We can obtain an optimal stationary distributed policy, under pairwise interference constraint, by solving the single-hop age problem for $\mathcal{F} = \mathcal{F}_D$. This can be written as:

$$\begin{aligned} \underset{\mathbf{a}\in[0,1]^{|E|},\mathbf{f}\in\mathbb{R}^{|E|}}{\text{Minimize}} \quad & \sum_{e\in E} \frac{w_e}{f_e} \\ \text{subject to} \quad & f_e = a_e \prod_{e'\in N_e} (1 - a_{e'}) \ \forall\ e \in E \\ & f_e \geq 0 \text{ for all } e \in E \end{aligned} \tag{35}$$

This is a non-convex program, as the constraint set is non-convex. However, substituting $b_e = 1 - a_e$ the optimization problem (35) reduces to

$$\begin{aligned} \underset{\mathbf{a}\geq 0,\mathbf{b}\geq 0}{\text{Minimize}} \quad & \sum_{e\in E} \frac{w_e}{a_e \prod_{e'\in N_e} b_{e'}} \\ \text{subject to} \quad & a_e + b_e \leq 1\ \forall e \in E \end{aligned} \tag{36}$$

This is a convex program and can be solved using standard techniques [21].

B. Multi-Hop Age Problem

We now consider the general multi-hop age minimization problem. Where the paths $p(r)$ may be more than a single-hop away. Let π be a stationary policy with link activation frequencies $\mathbf{f}$. Let f_e^r denote the fraction of times link e activates successfully to transmit update of source-destination pair r. If $e \notin p(r)$ then $f_e^r = 0$. Since f_e is the net link activation frequency, we have

$$f_e = \sum_{r\in\mathcal{R}, e\in p(r)} f_e^r, \tag{37}$$

for all $e \in E$. Now, using Theorem 1, the instantaneous age can be written as

$$A^{\text{inst}} = \mathbb{E}\left[\sum_{r\in\mathcal{R}} w^r A^r(t)\right] = \sum_{r\in\mathcal{R}} w^r \sum_{e\in p(r)} \frac{1}{f_e^r}, \tag{38}$$

which is equivalent to

$$A^{\text{inst}} = \sum_{e\in E}\sum_{r\in\mathcal{R}} \frac{w^r}{f_e^r} \mathbb{I}_{\{e\in p(r)\}}, \tag{39}$$

where we interpret $\frac{0}{0}$ as 0. Theorem 2 tells us that the average and peak are also given by (39). Therefore, we define the age minimization problem as follows:

General-Network Age Problem:

$$\begin{aligned} \underset{\mathbf{f}}{\text{Minimize}} \quad & \sum_{e\in E}\sum_{r\in\mathcal{R}} \frac{w^r}{f_e^r} \mathbb{I}_{\{e\in p(r)\}}, \\ \text{subject to} \quad & \sum_{r\in\mathcal{R}} f_e^r \mathbb{I}_{\{e\in p(r)\}} \leq f_e\ \forall e \in E \\ & \mathbf{f} \in \mathcal{F} \end{aligned} \tag{40}$$

where $\mathcal{F}$ is $\mathcal{F}_C$ for stationary centralized policies and is $\mathcal{F}_D$ for stationary distributed policies.

The following theorem proves that the general-network age problem is equivalent to the single-hop age problem.

Theorem *3:* The general-network age problem is equivalent to the single-hop age problem with

$$w_e = \left(\sum_{r\in\mathcal{R}} \sqrt{w^r}\mathbb{I}_{e\in p(r)}\right)^2, \tag{41}$$

for all $e \in E$. Further, if $\mathbf{f}^*$ solves the single-hop age problem then the optimal $(f_e^{r*} | e \in E, r \in \mathcal{R})$ for the general-network age problem are given by

$$f_e^{r*} = \left[\frac{\sqrt{w^r}}{\sum_{u\in\mathcal{R}} \sqrt{w^u}\mathbb{I}_{\{e\in p(u)\}}}\right] f_e^*, \tag{42}$$

for all $e \in p(r)$ and $r \in \mathcal{R}$.

Proof: See Appendix C. ■

Theorem 3 proves an important *separation principle* in designing stationary policies for age minimization over general network. It states that the optimal stationary policy can be obtained by converting all the flows to single-hop, with suitable edge weights. Another implication of Theorem 3 is that given the link activation frequency f_e^*, the per-flow frequency f_e^{r*} can be determined locally.

V. Conclusions

We considered the problem of minimizing age-of-information (AoI) for a general multi-hop network with $\mathcal{R}$ source-destination pairs, with general interference constraints. We derived AoI optimal stationary scheduling policies, in which links are activated according to a stationary probability distribution.

We first considered a simple line network, with a single source-destination pair, and showed that the age is a simple separable convex function of link activation frequencies. We then used this result to prove an important *separation principle* for the general multi-hop network with $\mathcal{R}$ source-destination pairs. The separation principle states that the optimal stationary policy for the multi-hop network can be obtained by solving an equivalent problem in which all source-destination pairs are just a single-hop away.

REFERENCES

[1] R. Talak, S. Karaman, and E. Modiano, "Speed limits in autonomous vehicular networks due to communication constraints," in *Proc. CDC*, pp. 4998–5003, Dec. 2016.

[2] I. Bekmezci, O. K. Sahingoz, and S. Temel, "Flying ad-hoc networks (FANETs): A survey," *Ad Hoc Networks*, vol. 11, pp. 1254–1270, May 2013.

[3] S. Kaul, R. Yates, and M. Gruteser, "Real-time status: How often should one update?," in *Proc. INFOCOM*, pp. 2731–2735, Mar. 2012.

[4] S. Kaul, M. Gruteser, V. Rai, and J. Kenney, "Minimizing age of information in vehicular networks," in *Proc. SECON*, pp. 350–358, Jun. 2011.

[5] L. Huang and E. Modiano, "Optimizing age-of-information in a multi-class queueing system," in *Proc. ISIT*, pp. 1681–1685, Jun. 2015.

[6] S. K. Kaul, R. D. Yates, and M. Gruteser, "Status updates through queues," in *Proc. CISS*, pp. 1–6, Mar. 2012.

[7] E. Najm and R. Nasser, "Age of information: The gamma awakening," *ArXiv e-prints*, Apr. 2016.

[8] M. Costa, M. Codreanu, and A. Ephremides, "Age of information with packet management," in *Proc. ISIT*, pp. 1583–1587, Jun. 2014.

[9] C. Kam, S. Kompella, and A. Ephremides, "Age of information under random updates," in *Proc. ISIT*, pp. 66–70, Jul. 2013.

[10] C. Kam, S. Kompella, and A. Ephremides, "Effect of message transmission diversity on status age," in *Proc. ISIT*, pp. 2411–2415, Jun. 2014.

[11] K. Chen and L. Huang, "Age-of-information in the presence of error," *ArXiv e-prints*, May 2016.

[12] Q. He, D. Yuan, and A. Ephremides, "Optimizing freshness of information: On minimum age link scheduling in wireless systems," in *Proc. WiOpt*, pp. 1–8, May 2016.

[13] I. Kadota, E. Uysal-Biyikoglu, R. Singh, and E. Modiano, "Minimizing the age of information in broadcast wireless networks," in *Proc. Allerton*, pp. 844–851, Sep. 2016.

[14] Y.-P. Hsu, E. Modiano, and L. Duan, "Age of information: Design and analysis of optimal scheduling algorithms," pp. 1–5, Jun. 2017.

[15] S. K. Kaul and R. D. Yates, "Status updates over unreliable multiaccess channels," *ArXiv e-prints*, May 2017.

[16] A. M. Bedewy, Y. Sun, and N. B. Shroff, "Age-optimal information updates in multihop networks," in *Proc. ISIT*, pp. 576–580, Jun. 2017.

[17] R. Talak, S. Karaman, and E. Modiano, "Optimizing information freshness in wireless networks under general interference constraints," *Under Review*, 2017.

[18] B. Korte and J. Vygen, *Combinatorial Optimization: Theory and Algorithms*. Springer Publishing Company, Incorporated, 4th ed., 2007.

[19] D. Garber and E. Hazan, "A linearly convergent variant of the conditional gradient algorithm under strong convexity, with applications to online and stochastic optimization," *SIAM J. on Opt.*, vol. 26, no. 3, pp. 1493–1528, 2016.

[20] B. Hajek and G. Sasaki, "Link scheduling in polynomial time," *IEEE Trans. Inf. Theory*, vol. 34, pp. 910–917, Sep. 1988.

[21] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

APPENDIX

A. Proof of Lemma 2

From (14), we know that

$$\tau_l(t) \stackrel{d}{=} t - X_{e_l}, \tag{43}$$

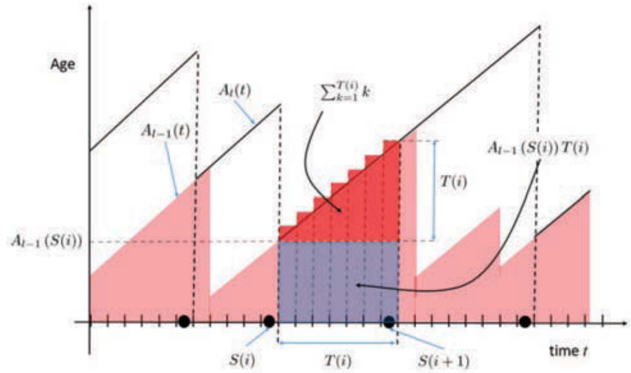

Fig. 4. Computation of average age. We compute the area under the age curve $A_l(t)$ between two successive activations of link e_l, namely, between time $S(i)$ and $S(i+1)$, and then add them up.

where X_{e_l} is a geometrically distributed random variable given by (15), and therefore,

$$\mathbb{E}\left[\tau_l(t)\right] = \mathbb{E}\left[t - X_{e_l}\right] = t - \frac{1}{f_l}, \tag{44}$$

for all $l \in \{1, 2, \ldots L\}$. Now from (43), we can obtain

$$\tau_l\left(\tau_{l-1}(t)\right) \stackrel{d}{=} \tau_{l-1}(t) - X_{e_l}, \tag{45}$$

and therefore,

$$\mathbb{E}\left[\tau_l\left(\tau_{l-1}(t)\right)\right] = \mathbb{E}\left[\tau_{l-1}(t)\right] - \mathbb{E}\left[X_{e_l}\right], \tag{46}$$

$$= t - \frac{1}{f_{l-1}} - \frac{1}{f_l}, \tag{47}$$

where the last equality follows from (44) and (16). Iterating this l times we obtain the result.

B. Proof of Theorem 2

Figure 4 plots age $A_{l-1}(t)$ and $A_l(t)$ as a function of time. Also shown are the activation times of link e_l as black dots on time axis. In order to compute the average age A_l^{ave}, we need to compute the area under the age curve $A_l(t)$. Let $S(i)$ denote the ith activation time of link e_l under the stationary policy π. Area under the age curve $A_l(t)$ can be computed by adding up area under $A_l(t)$ between times $S(i-1)$ and $S(i)$, and dividing by $T(i)$ to get the average age per slot; see Figure 4.

Consider a node $l \neq 0$ and link e_l. Let $M(t)$ be a counting process that counts the number of times link e_l was activated until, and including, time t. Let $S(i)$ denote the epoches of $M(t)$ and $T(i)$ be the time between ith and $(i+1)$th activation of link e_l, i.e.,

$$T(i) = S(i+1) - S(i). \tag{48}$$

Then the average age is given by

$$A_l^{\text{ave}} = \lim_{N\to\infty} \frac{\sum_{i=1}^{N}\left[A_{l-1}\left(S(i)\right)T(i) + \sum_{k=1}^{T(i)} k\right]}{\sum_{i=1}^{N} T(i)}, \quad (49)$$

$$= \lim_{N\to\infty} \frac{\sum_{i=1}^{N} A_{l-1}\left(S(i)\right)T(i) + \frac{1}{2}T(i)\left(T(i)-1\right)}{\sum_{i=1}^{N} T(i)}, \quad (50)$$

as this is the area under the age curve $A_l(t)$, where $A_l\left(S(i)\right)$ is the age evaluated at the ith epoch of $M(t)$. We illustrate this in Figure 4. This can be shown to be equal to[2]

$$A_l^{\text{ave}} = \frac{\mathbb{E}\left[A_{l-1}\left(S(i)\right)T(i)\right] + \frac{1}{2}\mathbb{E}\left[T(i)\left(T(i)-1\right)\right]}{\mathbb{E}\left[T(i)\right]}. \quad (51)$$

Note that $A_{l-1}\left(S(i)\right)$ and $T(i)$ are independent random variables as $A_{l-1}\left(S(i)\right)$ is dependent only on events prior to the epoch $S(i)$ while $T(i)$ is the time between ith and $i+1$th epoch of M_{e_l}. Therefore, (51) reduces to

$$A_l^{\text{ave}} = \frac{\mathbb{E}\left[A_{l-1}\left(S(i)\right)\right]\mathbb{E}\left[T(i)\right] + \frac{1}{2}\mathbb{E}\left[T(i)\left(T(i)-1\right)\right]}{\mathbb{E}\left[T(i)\right]},$$

$$= \mathbb{E}\left[A_{l-1}\left(S(i)\right)\right] + \frac{1}{2}\frac{\mathbb{E}\left[T(i)\left(T(i)-1\right)\right]}{\mathbb{E}\left[T(i)\right]}. \quad (52)$$

Using Theorem 1, it can be deduced that

$$\mathbb{E}\left[A_{l-1}\left(S(i)\right)\right] = \sum_{k=1}^{l-1}\frac{1}{f_k}. \quad (53)$$

Further, $\{T(i)\}_{i\geq 1}$ are i.i.d. and geometrically distributed as

$$\mathbb{P}\left[T(i) = k\right] = f_l\left(1-f_l\right)^{k-1}, \quad (54)$$

for all $k \in \{1, 2, \ldots\}$, where f_l is the link activation frequency of link e_l. From this we can compute

$$\frac{1}{2}\frac{\mathbb{E}\left[T(i)\left(T(i)-1\right)\right]}{\mathbb{E}\left[T(i)\right]} = \frac{1}{f_l}. \quad (55)$$

Using (53) and (55) we obtain

$$A_l^{\text{ave}} = \sum_{k=1}^{l}\frac{1}{f_k}. \quad (56)$$

Substituting $l = L = d$ yields the result.

C. Proof of Theorem 3

The problem (40) can be written as

$$\underset{\mathbf{f}\in\mathcal{F}}{\text{Minimize}} \quad \sum_{e\in E} G_e(f_e), \quad (57)$$

where $G_e(f_e)$ are given by

$$G_e(f_e) = \begin{array}{ll} \underset{f_e^r\ \forall r\in\mathcal{R}}{\text{Minimize}} & \sum_{r\in\mathcal{R}} \frac{w^r}{f_e^r}\mathbb{I}_{\{e\in p(r)\}} \\ \text{subject to} & \sum_{r\in\mathcal{R}} f_e^r \mathbb{I}_{\{e\in p(r)\}} \leq f_e. \end{array} \quad (58)$$

[2]This follows because $\{(A_l\left(S(i)\right), T(i))\}_{i\geq 0}$ is a stationary ergodic process.

The minimum for the optimization problem (58) is attained at

$$f_e^r = \left[\frac{\sqrt{w^r}}{\sum_{u\in\mathcal{R}}\sqrt{w^u}\mathbb{I}_{\{e\in p(u)\}}}\right] f_e, \quad (59)$$

for all $r \in \mathcal{R}$. Substituting this in (58), we obtain

$$G_e(f_e) = \frac{\left[\sum_{r\in\mathcal{R}}\sqrt{w^r}\mathbb{I}_{\{e\in p(r)\}}\right]^2}{f_e}. \quad (60)$$

Therefore, the optimization problem (57), and therefore (40), can be written as

$$\underset{\mathbf{f}\in\mathcal{F}}{\text{Minimize}} \quad \sum_{e\in E}\frac{w_e}{f_e}, \quad (61)$$

where $w_e = \left[\sum_{r\in\mathcal{R}}\sqrt{w^r}\mathbb{I}_{\{e\in p(r)\}}\right]^2$ for all $e \in E$.