Channel Estimation and Rate Analysis for Multipair Massive MIMO Relaying with One-Bit Quantization

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Abstract— **We study the impact of using one-bit analog-to-digital and digital-to-analog converters in a multipair amplify-and-forward MIMO relaying system. The relay estimates the channel state information using training data, and then uses the channel estimate to perform maximum ratio combining and maximum ratio transmission. An exact achievable rate is derived for the system under general assumptions on the quantization noise, and then a closed-form asymptotic approximation is derived, which enables efficient evaluation of the impact of key parameters on system performance. Contrary to the conventional unquantized systems, the performance is seen to depend on the specific pilot sequences that are employed. In addition, the sum rate gap between the double quantized relay system and an ideal unquantized system is shown to be a factor of $4/\pi^2$ in the low source power regime.**

I. Introduction

Multipair multiple-input multiple-output (MIMO) relaying systems have recently attracted considerable attention since they can provide a cost-effective way of achieving performance gains and maintaining a uniform quality of service. For instance, [1] derived the ergodic rate of the system when maximum ratio combining/maximum ratio transmission (MRC/MRT) beamforming was employed and showed that the energy efficiency gain scales with the number of relay antennas in Rayleigh fading channels. Then, [2] extended the analysis to the Ricean fading case and obtained similar power scaling behavior. For full-duplex systems, [3], [4] analytically compared the performance of MRC/MRT and zero-forcing reception/transmission and characterized the impact of the number of user pairs on the spectral efficiency.

All the aforementioned works are based on the assumption of infinite-resolution analog-to-digital converters (ADCs) and digital-to-analog converters (DACs). Since the fabrication cost, chip area and power consumption of the ADCs and DACs grow roughly exponentially with the number of quantization bits, the cumulative cost and power required to implement a relay with a very large array can be prohibitive. Therefore, it is desirable to investigate the use of cheaper and more energy-efficient one-bit ADCs and DACs.

For the case of one-bit ADCs, [5] reformulated the nonlinear quantization using a second-order statistically equivalent linear operator, and derived a linear minimum mean-squared error (LMMSE) channel estimator. The work [6] examined the impact of one-bit ADCs on wideband channels with frequency-selective fading. For one-bit DACs, [7], [8] showed that even simple MRT precoding can achieve reasonable results. In [9], an LMMSE precoder was proposed by taking the quantization non-linearities into account, and different precoding schemes were compared in terms of uncoded bit error rate.

All these prior works deal with single hop systems, and hence the impact of one-bit ADCs and DACs in relaying systems remains unknown. To fill this gap, we consider a multipair amplify-and-forward (AF) relaying system where the relay uses both one-bit ADCs and one-bit DACs. Our main contributions are summarized as follows: 1) We show that the channel estimation accuracy of the quantized system depends on the specific orthogonal pilot matrix that is used. By considering the specific case of identity and Hadamard pilot matrices, we show that the identity training scheme provides better channel estimation performance for users with weaker than average channels, and vice versa; 2) We present an exact achievable rate by using the arcsine law. Then, we use asymptotic arguments to provide an approximate closed-form expression for the achievable rate; 3) We show that the sum rate of the relay system with one-bit ADCs and DACs is $4/\pi^2$ times that achievable with perfect ADCs and DACs in the low source power regime.

Notation: We use bold upper case letters to denote matrices, bold lower case letters to denote vectors and lower case letters to denote scalars. The notation $(\cdot)^H$, $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^{-1}$ respectively represent the conjugate transpose operator, the conjugate operator, the transpose operator, and the matrix inverse. The Euclidian norm is denoted by $\|\cdot\|$, and the absolute value by $|\cdot|$. Also, $\mathbf{x} \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Sigma})$ denote a circularly symmetric complex Gaussian random vector with zero mean and covariance matrix $\mathbf{\Sigma}$. The terms $\Im(\mathbf{C})$ and $\mathcal{C}(\mathbf{C})$ stand for the real and imaginary part of $\mathbf{C}$, respectively. Finally, the statistical expectation operator is represented by $\mathtt{E}\{\cdot\}$, and the variance operator is $\mathrm{Var}(\cdot)$.

II. System Model

Consider a multipair massive MIMO relaying system serving K single-antenna user pairs, denoted as S_k and D_k, $k = 1, \ldots, K$. The relay is equipped with M receive antennas with one-bit ADCs and M transmit antennas with one-bit DACs. The one-bit ADCs cause errors in the channel estimation stage and subsequently in the reception of the uplink data; then, after a linear transformation, the one-bit DACs produce distortion when the downlink signal is coarsely quantized. Thus, the system we study is double quantized. We assume that direct links between S_k and D_k do not exist due to large obstacles or severe shadowing. In addition, we further assume that the relay operates in half-duplex mode, and hence it cannot receive and transmit signals simultaneously [10]. Accordingly, information transmission from S_k to D_k is completed in two separate phases. In the first phase, the K sources transmit independent data symbols to the relay, and in the second phase the relay broadcasts the double-quantized signals $\tilde{\mathbf{x}}_{\mathrm{R}}$ to the destinations. The signals at the relay's receive antennas and at the destinations before quantization are respectively given by

$$\mathbf{y}_{\mathrm{R}} = \sqrt{p_{\mathrm{S}}}\mathbf{G}_{\mathrm{SR}}\mathbf{x}_{\mathrm{S}} + \mathbf{n}_{\mathrm{R}}, \tag{1}$$

$$\mathbf{y}_{\mathrm{D}} = \gamma \mathbf{G}_{\mathrm{RD}}^{T}\tilde{\mathbf{x}}_{\mathrm{R}} + \mathbf{n}_{\mathrm{D}}, \tag{2}$$

where γ is chosen to satisfy the total power constraint p_{R} at the relay, i.e., $\mathsf{E}\left\{\|\gamma\tilde{\mathbf{x}}_{\mathrm{R}}\|^2\right\} = p_{\mathrm{R}}$, which will be specified shortly. The source symbols are represented by $\mathbf{x}_{\mathrm{S}} = [x_{\mathrm{S},1}, \ldots, x_{\mathrm{S},K}]^T$, whose elements are assumed to be Gaussian distributed with zero mean and unit power, and p_{S} denotes the transmit power of the sources. The vectors $\mathbf{n}_{\mathrm{R}}$ and $\mathbf{n}_{\mathrm{D}}$ represent the additive white Gaussian noise (AWGN) at the relay and destinations, whose elements are identically and independently distributed (i.i.d.) $\mathcal{CN}(0,1)$. The matrices $\mathbf{G}_{\mathrm{SR}} = [\mathbf{g}_{\mathrm{SR},1}, \ldots, \mathbf{g}_{\mathrm{SR},K}]$ and $\mathbf{G}_{\mathrm{RD}} = [\mathbf{g}_{\mathrm{RD},1}, \ldots, \mathbf{g}_{\mathrm{RD},K}]$ respectively represent the uncorrelated Rayleigh fading channels from the K sources to the relay with $\mathbf{g}_{\mathrm{SR},k} \in \mathcal{CN}(0, \beta_{\mathrm{SR},k}\mathbf{I}_M)$ and the channels from the relay to the K destinations with $\mathbf{g}_{\mathrm{RD},k} \in \mathcal{CN}(0, \beta_{\mathrm{RD},k}\mathbf{I}_M)$. The terms $\beta_{\mathrm{SR},k}$ and $\beta_{\mathrm{RD},k}$ model the large-scale path-loss, which is assumed to be constant over many coherence intervals and known a priori.

A. Channel Estimation

During each coherence interval of length τ_{c} (in symbols), all sources simultaneously transmit their mutually orthogonal pilot sequences $\mathbf{\Phi}_{\mathrm{S}} \in \mathbb{C}^{\tau_{\mathrm{p}} \times K}$ satisfying $\mathbf{\Phi}_{\mathrm{S}}^H\mathbf{\Phi}_{\mathrm{S}} = \tau_{\mathrm{p}}\mathbf{I}_K$ to the relay while the destinations remain silent ($\tau_{\mathrm{p}} \approx K$). Afterwards, all destinations simultaneously transmit their mutually orthogonal pilot sequences $\mathbf{\Phi}_{\mathrm{D}} \in \mathbb{C}^{\tau_{\mathrm{p}} \times K}$ satisfying $\mathbf{\Phi}_{\mathrm{D}}^H\mathbf{\Phi}_{\mathrm{D}} = \tau_{\mathrm{p}}\mathbf{I}_K$ to the relay while the sources remain silent.

Since the channels $\mathbf{G}_{\mathrm{SR}}$ and $\mathbf{G}_{\mathrm{RD}}$ are estimated in the same fashion, we focus only on the first link $\mathbf{G}_{\mathrm{SR}}$. The received training signal at the relay is given by

$$\mathbf{Y}_{\mathrm{p}} = \sqrt{p_{\mathrm{p}}}\mathbf{G}_{\mathrm{SR}}\mathbf{\Phi}_{\mathrm{S}}^T + \mathbf{N}_{\mathrm{p}}, \tag{3}$$

where p_{p} represents the transmit power of each pilot symbol, and $\mathbf{N}_{\mathrm{p}}$ denotes the noise at the relay, which has i.i.d. $\mathcal{CN}(0,1)$ elements. After vectorizing the matrix $\mathbf{Y}_{\mathrm{p}}$, we obtain

$$\mathbf{y}_{\mathrm{p}} = \mathrm{vec}\left(\mathbf{Y}_{\mathrm{p}}\right) = \bar{\mathbf{\Phi}}_{\mathrm{S}}\bar{\mathbf{g}}_{\mathrm{SR}} + \bar{\mathbf{n}}_{\mathrm{p}}, \tag{4}$$

where $\bar{\mathbf{\Phi}}_{\mathrm{S}} = \mathbf{\Phi}_{\mathrm{S}} \otimes \sqrt{p_{\mathrm{p}}}\mathbf{I}_M$, $\bar{\mathbf{g}}_{\mathrm{SR}} = \mathrm{vec}\left(\mathbf{G}_{\mathrm{SR}}\right)$, and $\bar{\mathbf{n}}_{\mathrm{p}} = \mathrm{vec}\left(\mathbf{N}_{\mathrm{p}}\right)$.

1) One-bit ADCs: After the one-bit ADCs, the quantized signal can be expressed as

$$\mathbf{r}_{\mathrm{p}} = \mathcal{Q}\left(\mathbf{y}_{\mathrm{p}}\right), \tag{5}$$

where $\mathcal{Q}(\cdot)$ denotes the one-bit quantization operation, which separately processes the real and imaginary parts of the signal. Therefore, the output set of the one-bit ADCs is $\frac{1}{\sqrt{2}}\{\pm 1 \pm 1j\}$. Using the Bussgang decomposition [11], [12], $\mathbf{r}_{\mathrm{p}}$ can be represented by a linear signal component and an uncorrelated quantization noise $\mathbf{q}_{\mathrm{p}}$:

$$\mathbf{r}_{\mathrm{p}} = \mathbf{A}_{\mathrm{p}}\mathbf{y}_{\mathrm{p}} + \mathbf{q}_{\mathrm{p}}, \tag{6}$$

where $\mathbf{A}_{\mathrm{p}}$ is the linear operator obtained by minimizing the power of the quantization error $\mathsf{E}\left\{\|\mathbf{q}_{\mathrm{p}}\|^2\right\}$:

$$\mathbf{A}_{\mathrm{p}} = \mathbf{R}_{\mathbf{y}_{\mathrm{p}}\mathbf{r}_{\mathrm{p}}}^H\mathbf{R}_{\mathbf{y}_{\mathrm{p}}\mathbf{y}_{\mathrm{p}}}^{-1}, \tag{7}$$

with $\mathbf{R}_{\mathbf{y}_{\mathrm{p}}\mathbf{r}_{\mathrm{p}}}$ denoting the cross-correlation matrix between the received signal $\mathbf{y}_{\mathrm{p}}$ and the quantized signal $\mathbf{r}_{\mathrm{p}}$, and $\mathbf{R}_{\mathbf{y}_{\mathrm{p}}\mathbf{y}_{\mathrm{p}}}$ representing the covariance matrix of $\mathbf{y}_{\mathrm{p}}$. It is easy to show that $\mathbf{R}_{\mathbf{y}_{\mathrm{p}}\mathbf{y}_{\mathrm{p}}}$ can be computed by

$$\mathbf{R}_{\mathbf{y}_{\mathrm{p}}\mathbf{y}_{\mathrm{p}}} = \bar{\mathbf{\Phi}}_{\mathrm{S}}\tilde{\mathbf{D}}_{\mathrm{SR}}\bar{\mathbf{\Phi}}_{\mathrm{S}}^H + \mathbf{I}_{M\tau_{\mathrm{p}}}, \tag{8}$$

where $\tilde{\mathbf{D}}_{\mathrm{SR}} = \left(\mathbf{D}_{\mathrm{SR}} \otimes \mathbf{I}_M\right)$ and $\mathbf{D}_{\mathrm{SR}}$ is a diagonal matrix whose elements are $[\mathbf{D}_{\mathrm{SR}}]_{kk} = \beta_{\mathrm{SR},k}$ for $k = 1, \ldots, K$.

For one-bit quantization, by invoking the results in [13, Chapter 10] and applying the arcsine law [14], we have

$$\mathbf{R}_{\mathbf{y}_{\mathrm{p}}\mathbf{r}_{\mathrm{p}}} = \frac{2}{\pi}\mathbf{R}_{\mathbf{y}_{\mathrm{p}}\mathbf{y}_{\mathrm{p}}}\mathrm{diag}\left(\mathbf{R}_{\mathbf{y}_{\mathrm{p}}\mathbf{y}_{\mathrm{p}}}\right)^{-1/2}, \tag{9}$$

$$\mathbf{R}_{\mathbf{r}_{\mathrm{p}}\mathbf{r}_{\mathrm{p}}} = \frac{2}{\pi}\left(\arcsin\left(\mathbf{J}\right) + j\arcsin\left(\mathbf{K}\right)\right), \tag{10}$$

where

$$\mathbf{J} = \mathrm{diag}\left(\mathbf{R}_{\mathbf{y}_{\mathrm{p}}\mathbf{y}_{\mathrm{p}}}\right)^{-1/2}\Re\left\{\mathbf{R}_{\mathbf{y}_{\mathrm{p}}\mathbf{y}_{\mathrm{p}}}\right\}\mathrm{diag}\left(\mathbf{R}_{\mathbf{y}_{\mathrm{p}}\mathbf{y}_{\mathrm{p}}}\right)^{-1/2}, \tag{11}$$

$$\mathbf{K} = \mathrm{diag}\left(\mathbf{R}_{\mathbf{y}_{\mathrm{p}}\mathbf{y}_{\mathrm{p}}}\right)^{-1/2}\Im\left\{\mathbf{R}_{\mathbf{y}_{\mathrm{p}}\mathbf{y}_{\mathrm{p}}}\right\}\mathrm{diag}\left(\mathbf{R}_{\mathbf{y}_{\mathrm{p}}\mathbf{y}_{\mathrm{p}}}\right)^{-1/2}. \tag{12}$$

Substituting (9) into (7), and after some simple mathematical manipulations, we have

$$\mathbf{A}_{\mathrm{p}} = \sqrt{\frac{2}{\pi}}\mathrm{diag}\left(\mathbf{R}_{\mathbf{y}_{\mathrm{p}}\mathbf{y}_{\mathrm{p}}}\right)^{-1/2}. \tag{13}$$

Since $\mathbf{q}_{\mathrm{p}}$ is uncorrelated with $\mathbf{y}_{\mathrm{p}}$, we have

$$\mathbf{R}_{\mathbf{q}_{\mathrm{p}}\mathbf{q}_{\mathrm{p}}} = \mathbf{R}_{\mathbf{r}_{\mathrm{p}}\mathbf{r}_{\mathrm{p}}} - \mathbf{A}_{\mathrm{p}}\mathbf{R}_{\mathbf{y}_{\mathrm{p}}\mathbf{y}_{\mathrm{p}}}\mathbf{A}_{\mathrm{p}}^H. \tag{14}$$

Substituting (10) into (14) yields

$$\mathbf{R}_{\mathbf{q}_{\mathrm{p}}\mathbf{q}_{\mathrm{p}}} = \frac{2}{\pi}\left(\arcsin\left(\mathbf{J}\right) + j\arcsin\left(\mathbf{K}\right)\right) - \frac{2}{\pi}\left(\mathbf{J} + j\mathbf{K}\right). \tag{15}$$

2) LMMSE estimator: Based on the observation $\mathbf{r}_{\mathrm{p}}$ and the training pilots $\mathbf{\Phi}_{\mathrm{S}}$, we use the LMMSE technique to estimate $\mathbf{G}_{\mathrm{SR}}$. Hence, the estimated channel $\hat{\mathbf{g}}_{\mathrm{SR}}$ is given by

$$\hat{\mathbf{g}}_{\mathrm{SR}} = \mathbf{R}_{\bar{\mathbf{g}}_{\mathrm{SR}}\mathbf{r}_{\mathrm{p}}}\mathbf{R}_{\mathbf{r}_{\mathrm{p}}\mathbf{r}_{\mathrm{p}}}^{-1}\mathbf{r}_{\mathrm{p}}. \tag{16}$$

As a result, the covariance matrix of the estimated channel $\hat{\mathbf{g}}_{\mathrm{SR}}$ is expressed as

$$\mathbf{R}_{\hat{\mathbf{g}}_{\mathrm{SR}}\hat{\mathbf{g}}_{\mathrm{SR}}} = \tilde{\mathbf{D}}_{\mathrm{SR}}\tilde{\mathbf{\Phi}}_{\mathrm{S}}^H\left(\tilde{\mathbf{\Phi}}_{\mathrm{S}}\tilde{\mathbf{D}}_{\mathrm{SR}}\tilde{\mathbf{\Phi}}_{\mathrm{S}}^H + \mathbf{A}_{\mathrm{p}}\mathbf{A}_{\mathrm{p}}^H + \mathbf{R}_{\mathbf{q}_{\mathrm{p}}\mathbf{q}_{\mathrm{p}}}\right)^{-1}\tilde{\mathbf{\Phi}}_{\mathrm{S}}\tilde{\mathbf{D}}_{\mathrm{SR}}, \tag{17}$$

where $\tilde{\mathbf{\Phi}}_{\mathrm{S}} = \mathbf{A}_{\mathrm{p}}\bar{\mathbf{\Phi}}_{\mathrm{S}}$.

From (17), we can see that $\mathbf{R}_{\hat{\mathbf{g}}_{\mathrm{SR}}\hat{\mathbf{g}}_{\mathrm{SR}}}$ is a non-trivial function of $\tilde{\mathbf{\Phi}}_{\mathrm{S}}$, which indicates that the quality of the channel estimates depends on the specific choice of the pilot sequence. This phenomenon is in sharp contrast to the unquantized systems where any set of orthogonal pilot sequences gives the same result.

In the following, we study the performance of two specific pilot sequences to show how the pilot matrix affects the channel estimation. Here, we choose $\tau_{\mathrm{p}} = K$, which is the minimum possible length of the pilot sequence.

a) Identity Matrix. In this case, $\mathbf{\Phi}_{\mathrm{S}} = \sqrt{K}\mathbf{I}_K$, and hence we have

$$\mathbf{R}_{\mathbf{y}_{\rm p}\mathbf{y}_{\rm p}} = Kp_{\rm p}\tilde{\mathbf{D}}_{\rm SR} + \mathbf{I}_{MK}. \tag{18}$$

Consequently,

$$\mathbf{A}_{\rm p} = \sqrt{\frac{2}{\pi}}\left(Kp_{\rm p}\tilde{\mathbf{D}}_{\rm SR} + \mathbf{I}_{MK}\right)^{-1/2} = \bar{\mathbf{A}}_{\rm p} \otimes \mathbf{I}_M, \tag{19}$$

$$\mathbf{R}_{\mathbf{q}_{\rm p}\mathbf{q}_{\rm p}} = \left(1 - \frac{2}{\pi}\right)\mathbf{I}_{MK}, \tag{20}$$

where $\bar{\mathbf{A}}_{\rm p}$ is a diagonal matrix with $\left[\bar{\mathbf{A}}_{\rm p}\right]_{kk} = \alpha_{{\rm p},k} = \sqrt{\frac{2}{\pi}\frac{1}{Kp_{\rm p}\beta_{{\rm SR},k}+1}}$. Substituting (19) and (20) into (17), we obtain

$$\mathbf{R}_{\hat{\mathbf{g}}_{\rm SR}\hat{\mathbf{g}}_{\rm SR}} = \mathbf{Q}_{\rm SR}^{(1)} \otimes \mathbf{I}_M, \tag{21}$$

where $\mathbf{Q}_{\rm SR}^{(1)}$ is a diagonal matrix with elements

$$\left[\mathbf{Q}_{\rm SR}^{(1)}\right]_{kk} = \sigma_{{\rm SR},k}^2 = \frac{2}{\pi}\frac{Kp_{\rm p}\beta_{{\rm SR},k}^2}{Kp_{\rm p}\beta_{{\rm SR},k}+1}. \tag{22}$$

b) <u>*Hadamard Matrix*</u>. In this case, every element of $\mathbf{\Phi}_{\rm S}$ is $+1$ or -1, and hence we have

$$\mathbf{A}_{\rm p} = \sqrt{\frac{2}{\pi}\frac{1}{p_{\rm p}\sum_{n=1}^{K}\beta_{{\rm SR},k}+1}}\mathbf{I}_{MK}, \tag{23}$$

$$\mathbf{R}_{\mathbf{q}_{\rm p}\mathbf{q}_{\rm p}} \approx \left(1 - \frac{2}{\pi}\right)\mathbf{I}_{MK}, \tag{24}$$

where the approximation in (24) holds for low $p_{\rm p}$. Substituting (23) and (24) into (17), we obtain

$$\mathbf{R}_{\hat{\mathbf{g}}_{\rm SR}\hat{\mathbf{g}}_{\rm SR}} = \mathbf{Q}_{\rm SR}^{(2)} \otimes \mathbf{I}_M, \tag{25}$$

where $\mathbf{Q}_{\rm SR}^{(2)}$ is a diagonal matrix with entries

$$\left[\mathbf{Q}_{\rm SR}^{(2)}\right]_{kk} = \kappa_{{\rm SR},k}^2 = \frac{K\bar{\alpha}_{\rm p}^2\beta_{{\rm SR},k}^2p_{\rm p}}{K\bar{\alpha}_{\rm p}^2\beta_{{\rm SR},k}p_{\rm p} + \bar{\alpha}_{\rm p}^2 + 1 - \frac{2}{\pi}}, \tag{26}$$

where

$$\bar{\alpha}_{\rm p} = \sqrt{\frac{2}{\pi}\frac{1}{p_{\rm p}\sum_{k=1}^{K}\beta_{{\rm SR},k}+1}}. \tag{27}$$

For both cases, the channels from the sources to the relay $\mathbf{g}_{{\rm SR},k}$ can be decomposed as

$$\mathbf{g}_{{\rm SR},k} = \hat{\mathbf{g}}_{{\rm SR},k} + \mathbf{e}_{{\rm SR},k}, \tag{28}$$

where $\mathbf{e}_{{\rm SR},k}$ is the estimation error vector. The elements of $\hat{\mathbf{g}}_{{\rm SR},k}$ and $\mathbf{e}_{{\rm SR},k}$ are respectively distributed as $\mathcal{CN}(0, \sigma_{{\rm SR},k}^2)$ and $\mathcal{CN}(0, \tilde{\sigma}_{{\rm SR},k}^2)$ when $\mathbf{\Phi}_{\rm SR}$ is an identity matrix, while they are distributed as $\mathcal{CN}(0, \kappa_{{\rm SR},k}^2)$ and $\mathcal{CN}(0, \tilde{\kappa}_{{\rm SR},k}^2)$ when $\mathbf{\Phi}_{\rm SR}$ is a Hadamard matrix, where $\tilde{\sigma}_{{\rm SR},k}^2 = \beta_{{\rm SR},k} - \sigma_{{\rm SR},k}^2$ and $\tilde{\kappa}_{{\rm SR},k}^2 = \beta_{{\rm SR},k} - \kappa_{{\rm SR},k}^2$. In what follows, we define $\hat{\mathbf{G}}_{\rm SR} = [\hat{\mathbf{g}}_{{\rm SR},1}, \ldots, \hat{\mathbf{g}}_{{\rm SR},K}]$ and $\mathbf{E}_{\rm SR} = [\mathbf{e}_{{\rm SR},1}, \ldots, \mathbf{e}_{{\rm SR},K}]$.

For the channel from the k-th source to the relay, the mean-square error (MSE) is given by

$$\text{MSE}_{{\rm SR},k} = \mathbb{E}\left\{\left\|\hat{\mathbf{g}}_{{\rm SR},k} - \mathbf{g}_{{\rm SR},k}\right\|^2\right\}. \tag{29}$$

Based on the above results, we have $\text{MSE}_{{\rm SR},k} = \tilde{\sigma}_{{\rm SR},k}^2$ for the identity matrix and $\text{MSE}_{{\rm SR},k} = \tilde{\kappa}_{{\rm SR},k}^2$ for the Hadamard matrix. The following proposition compares the MSE of the two approaches.

Proposition 1: For estimating the channel $\mathbf{g}_{{\rm SR},k}$, the identity matrix is preferable to the Hadamard matrix for user k if $\beta_{{\rm SR},k} < \frac{1}{K}\sum_{i=1}^{K}\beta_{{\rm SR},i}$, and vice versa.

Proof: The proof is straightforward since $\tilde{\sigma}_{{\rm SR},k}^2 < \tilde{\kappa}_{{\rm SR},k}^2$ if $\beta_{{\rm SR},k} < \frac{1}{K}\sum_{i=1}^{K}\beta_{{\rm SR},i}$. □

Proposition 1 reveals that the scaled identity matrix is beneficial for any user with higher path loss than the average. This is because a weak user benefits from being the only one transmitting at a given time, without the presence of stronger users that dominate the behavior of the ADC. In the case of the Hadamard matrix, all users are transmitting simultaneously, resulting in an average quantization noise level for all users jointly, which is advantageous for users with stronger channels.

The question of optimizing the pilot sequence for a given performance metric is very interesting, but is beyond the scope of the current paper. For simplicity, use identity matrix as the training sequence. Therefore, the channels from the relay to the destinations $\mathbf{g}_{{\rm RD},k}$ can be decomposed as

$$\mathbf{g}_{{\rm RD},k} = \hat{\mathbf{g}}_{{\rm RD},k} + \mathbf{e}_{{\rm RD},k}, \tag{30}$$

where $\hat{\mathbf{g}}_{{\rm RD},k}$ and $\mathbf{e}_{{\rm RD},k}$ are the estimated channel and estimation error vectors. The elements of $\hat{\mathbf{g}}_{{\rm RD},k}$ and $\mathbf{e}_{{\rm RD},k}$ are distributed as $\mathcal{CN}(0, \sigma_{{\rm RD},k}^2)$ and $\mathcal{CN}(0, \tilde{\sigma}_{{\rm RD},k}^2)$, where $\sigma_{{\rm RD},k}^2 = \frac{2}{\pi}\frac{Kp_{\rm p}\beta_{{\rm RD},k}^2}{Kp_{\rm p}\beta_{{\rm RD},k}+1}$ and $\tilde{\sigma}_{{\rm RD},k}^2 = \beta_{{\rm RD},k} - \sigma_{{\rm RD},k}^2$. We also define $\hat{\mathbf{G}}_{\rm RD} = [\hat{\mathbf{g}}_{{\rm RD},1}, \ldots, \hat{\mathbf{g}}_{{\rm RD},K}]$ and $\mathbf{E}_{\rm RD} = [\mathbf{e}_{{\rm RD},1}, \ldots, \mathbf{e}_{{\rm RD},K}]$.

B. Data Transmission

1) Quantization with One-bit ADCs: With one-bit ADCs at the receiver, the resulting quantized signals can be expressed as

$$\tilde{\mathbf{y}}_{\rm R} = \mathcal{Q}(\mathbf{y}_{\rm R}) = \mathbf{A}_{\rm a}\mathbf{y}_{\rm R} + \mathbf{q}_{\rm a}, \tag{31}$$

where $\mathbf{A}_{\rm a}$ is the linear operator, which is uncorrelated with $\mathbf{y}_{\rm R}$. By adopting the same technique as in the previous subsection, we have

$$\mathbf{A}_{\rm a} = \sqrt{\frac{2}{\pi}}\text{diag}\left(\mathbf{R}_{\mathbf{y}_{\rm R}\mathbf{y}_{\rm R}}\right)^{-1/2}, \tag{32}$$

$$\mathbf{R}_{\mathbf{q}_{\rm a}\mathbf{q}_{\rm a}} = \frac{2}{\pi}\left(\arcsin(\mathbf{X}) + j\arcsin(\mathbf{Y})\right) - \frac{2}{\pi}\left(\mathbf{X} + j\mathbf{Y}\right), \tag{33}$$

where

$$\mathbf{X} = \text{diag}\left(\mathbf{R}_{\mathbf{y}_{\rm R}\mathbf{y}_{\rm R}}\right)^{-1/2}\Re\left(\mathbf{R}_{\mathbf{y}_{\rm R}\mathbf{y}_{\rm R}}\right)\text{diag}\left(\mathbf{R}_{\mathbf{y}_{\rm R}\mathbf{y}_{\rm R}}\right)^{-1/2},$$
$$\mathbf{Y} = \text{diag}\left(\mathbf{R}_{\mathbf{y}_{\rm R}\mathbf{y}_{\rm R}}\right)^{-1/2}\Im\left(\mathbf{R}_{\mathbf{y}_{\rm R}\mathbf{y}_{\rm R}}\right)\text{diag}\left(\mathbf{R}_{\mathbf{y}_{\rm R}\mathbf{y}_{\rm R}}\right)^{-1/2},$$
$$\mathbf{R}_{\mathbf{y}_{\rm R}\mathbf{y}_{\rm R}} = p_{\rm S}\mathbf{G}_{\rm SR}\mathbf{G}_{\rm SR}^H + \mathbf{I}_M.$$

2) Digital Linear Processing: We assume that the relay adopts an AF protocol to process the quantized signals $\tilde{\mathbf{y}}_{\rm R}$ by one-bit ADCs, yielding

$$\mathbf{x}_{\rm R} = \mathbf{W}\tilde{\mathbf{y}}_{\rm R}, \tag{34}$$

where $\mathbf{W} = \hat{\mathbf{G}}_{\rm RD}^*\hat{\mathbf{G}}_{\rm SR}^H$ for MRC/MRT beamforming.

3) Quantization with One-bit DACs: Assuming one-bit DACs at the transmitter, the resulting quantized signals to be transmitted by the relay can be expressed as

$$\tilde{\mathbf{x}}_{\rm R} = \mathcal{Q}\left(\mathbf{x}_{\rm R}\right) = \mathbf{A}_{\rm d}\mathbf{x}_{\rm R} + \mathbf{q}_{\rm d}, \tag{35}$$

where $\mathbf{A}_{\rm d}$ is the linear operator, and $\mathbf{q}_{\rm d}$ is the quantization noise at the relay's transmit antennas, which is uncorrelated with $\mathbf{x}_{\rm R}$. Due to the one-bit DACs, we have $\mathtt{E}\left\{\|\tilde{\mathbf{x}}_{\rm R}\|^2\right\} = M$. Therefore, the normalization factor γ (c.f. (2)) can be expressed as

$$\gamma = \sqrt{\frac{p_{\rm R}}{M}}. \tag{36}$$

Following the same fashion as that of the ADCs derivations, we obtain

$$\mathbf{A}_{\rm d} = \sqrt{\frac{2}{\pi}}\mathrm{diag}\left(\mathbf{R}_{\mathbf{x}_{\rm R}\mathbf{x}_{\rm R}}\right)^{-1/2}, \tag{37}$$

$$\mathbf{R}_{\mathbf{q}_{\rm d}\mathbf{q}_{\rm d}} = \frac{2}{\pi}\left(\mathrm{arcsin}\left(\mathbf{U}\right) + j\mathrm{arcsin}\left(\mathbf{V}\right)\right) - \frac{2}{\pi}\left(\mathbf{U} + j\mathbf{V}\right), \tag{38}$$

where

$$\mathbf{U} = \mathrm{diag}\left(\mathbf{R}_{\mathbf{x}_{\rm R}\mathbf{x}_{\rm R}}\right)^{-1/2}\Re\left(\mathbf{R}_{\mathbf{x}_{\rm R}\mathbf{x}_{\rm R}}\right)\mathrm{diag}\left(\mathbf{R}_{\mathbf{x}_{\rm R}\mathbf{x}_{\rm R}}\right)^{-1/2},$$
$$\mathbf{V} = \mathrm{diag}\left(\mathbf{R}_{\mathbf{x}_{\rm R}\mathbf{x}_{\rm R}}\right)^{-1/2}\Im\left(\mathbf{R}_{\mathbf{x}_{\rm R}\mathbf{x}_{\rm R}}\right)\mathrm{diag}\left(\mathbf{R}_{\mathbf{x}_{\rm R}\mathbf{x}_{\rm R}}\right)^{-1/2},$$
$$\mathbf{R}_{\mathbf{x}_{\rm R}\mathbf{x}_{\rm R}} = \mathbf{W}\mathbf{R}_{\tilde{\mathbf{y}}_{\rm R}\tilde{\mathbf{y}}_{\rm R}}\mathbf{W}^H,$$
$$\mathbf{R}_{\tilde{\mathbf{y}}_{\rm R}\tilde{\mathbf{y}}_{\rm R}} = \mathbf{A}_{\rm a}\mathbf{R}_{\mathbf{y}_{\rm R}\mathbf{y}_{\rm R}}\mathbf{A}_{\rm a}^H + \mathbf{R}_{\mathbf{q}_{\rm a}\mathbf{q}_{\rm a}}.$$

III. ACHIEVABLE RATE ANALYSIS

In this section, we investigate the achievable rate of the considered system. In particular, we first provide an expression for the exact achievable rate, which is applicable to arbitrary system configurations. Then we use asymptotic arguments to derive an approximate rate and provide some key insights.

A. Exact Achievable Rate Analysis

We consider the realistic case where the K destinations do not have access to the instantaneous CSI, which is a typical assumption in the massive MIMO literature since the dissemination of instantaneous CSI leads to excessively high computational and signaling costs for very large antenna arrays. Hence, D_k uses only statistical CSI to decode the desired signal. Combining (1), (2), (31), (34), (35), and (36) yields the received signal at the k-th destination

$$y_{{\rm D},k} = \underbrace{\gamma\sqrt{p_{\rm S}}\mathtt{E}\left\{\mathbf{g}_{{\rm RD},k}^T\mathbf{A}_{\rm d}\mathbf{W}\mathbf{A}_{\rm a}\mathbf{g}_{{\rm SR},k}\right\}x_{{\rm S},k}}_{\text{desired signal}} + \underbrace{\tilde{n}_{{\rm D},k}}_{\text{effective noise}}, \tag{39}$$

where where $\tilde{n}_{{\rm D},k} =$

$$\underbrace{\gamma\sqrt{p_{\rm S}}\left(\mathbf{g}_{{\rm RD},k}^T\mathbf{A}_{\rm d}\mathbf{W}\mathbf{A}_{\rm a}\mathbf{g}_{{\rm SR},k} - \mathtt{E}\left\{\mathbf{g}_{{\rm RD},k}^T\mathbf{A}_{\rm d}\mathbf{W}\mathbf{A}_{\rm a}\mathbf{g}_{{\rm SR},k}\right\}\right)x_{{\rm S},k}}_{\text{estimation error}}$$
$$+ \underbrace{\gamma\sqrt{p_{\rm S}}\sum_{i\neq k}\mathbf{g}_{{\rm RD},k}^T\mathbf{A}_{\rm d}\mathbf{W}\mathbf{A}_{\rm a}\mathbf{g}_{{\rm SR},i}x_{{\rm S},i}}_{\text{interpair interference}} + \underbrace{\gamma\mathbf{g}_{{\rm RD},k}^T\mathbf{A}_{\rm d}\mathbf{W}\mathbf{A}_{\rm a}\mathbf{n}_{\rm R}}_{\text{noise at the relay}}$$
$$+ \underbrace{\gamma\mathbf{g}_{{\rm RD},k}^T\mathbf{A}_{\rm d}\mathbf{W}\mathbf{q}_{\rm a}}_{\text{quantization noise of ADCs}} + \underbrace{\gamma\mathbf{g}_{{\rm RD},k}^T\mathbf{q}_{\rm d}}_{\text{quantization noise of DACs}} + \underbrace{n_{{\rm D},k}}_{\text{noise at }k\text{-th destination}},$$

where $n_{{\rm D},k}$ is the k-th element of the noise vector $\mathbf{n}_{\rm D}$. Noticing that the "desired signal" and the "effective noise" in (39) are uncorrelated, and capitalizing on the fact that the worst-case uncorrelated additive noise is independent Gaussian, we obtain the following achievable rate for the k-th destination:

$$R_k = \frac{\tau_{\rm c} - 2\tau_{\rm p}}{2\tau_{\rm c}}\log_2\left(1 + \frac{A_k}{B_k + C_k + D_k + E_k + F_k + \frac{1}{\gamma^2}}\right), \tag{40}$$

where

$$A_k = p_{\rm S}\left|\mathtt{E}\left\{\mathbf{g}_{{\rm RD},k}^T\mathbf{A}_{\rm d}\mathbf{W}\mathbf{A}_{\rm a}\mathbf{g}_{{\rm SR},k}\right\}\right|^2, \tag{41}$$
$$B_k = p_{\rm S}\mathrm{Var}\left[\mathbf{g}_{{\rm RD},k}^T\mathbf{A}_{\rm d}\mathbf{W}\mathbf{A}_{\rm a}\mathbf{g}_{{\rm SR},k}\right], \tag{42}$$
$$C_k = p_{\rm S}\sum_{i\neq k}\mathtt{E}\left\{\left|\mathbf{g}_{{\rm RD},k}^T\mathbf{A}_{\rm d}\mathbf{W}\mathbf{A}_{\rm a}\mathbf{g}_{{\rm SR},i}\right|^2\right\}, \tag{43}$$
$$D_k = \mathtt{E}\left\{\left\|\mathbf{g}_{{\rm RD},k}^T\mathbf{A}_{\rm d}\mathbf{W}\mathbf{A}_{\rm a}\right\|^2\right\}, \tag{44}$$
$$E_k = \mathtt{E}\left\{\mathbf{g}_{{\rm RD},k}^T\mathbf{A}_{\rm d}\mathbf{W}\mathbf{R}_{\mathbf{q}_{\rm a}\mathbf{q}_{\rm a}}\mathbf{W}^H\mathbf{A}_{\rm d}^H\mathbf{g}_{{\rm RD},k}^*\right\}, \tag{45}$$
$$F_k = \mathtt{E}\left\{\mathbf{g}_{{\rm RD},k}^T\mathbf{R}_{\mathbf{q}_{\rm d}\mathbf{q}_{\rm d}}\mathbf{g}_{{\rm RD},k}^*\right\}. \tag{46}$$

B. Asymptotic Simplifications

As can be observed, all the matrices $\mathbf{R}_{\mathbf{q}_{\rm a}\mathbf{q}_{\rm a}}$, $\mathbf{A}_{\rm d}$, and $\mathbf{R}_{\mathbf{q}_{\rm d}\mathbf{q}_{\rm d}}$ involve arcsine functions, which do not give much insight into how the rate changes with various parameters. Hence, to facilitate the analysis, we focus on the asymptotic regime for a large number of users, in which (8) can be approximated by

$$\mathbf{R}_{\mathbf{y}_{\rm R}\mathbf{y}_{\rm R}} \to \mathrm{diag}\left(\mathbf{R}_{\mathbf{y}_{\rm R}\mathbf{y}_{\rm R}}\right) \to \left(1 + p_{\rm S}\sum_{k=1}^{K}\beta_{{\rm SR},k}\right)\mathbf{I}_M. \tag{47}$$

Substituting (47) into (32) and (33), we have

$$\mathbf{A}_{\rm a} \to \sqrt{\frac{2}{\pi}}\sqrt{\frac{1}{1 + p_{\rm S}\sum_{k=1}^{K}\beta_{{\rm SR},k}}}\mathbf{I}_M = \alpha_{\rm a}\mathbf{I}_M, \tag{48}$$

$$\mathbf{R}_{\mathbf{q}_{\rm a}\mathbf{q}_{\rm a}} \to \left(1 - \frac{2}{\pi}\right)\mathbf{I}_M. \tag{49}$$

Similarly, asymptotically we have

$$\mathbf{R}_{\mathbf{x}_{\rm R}\mathbf{x}_{\rm R}} \to \mathrm{diag}\left(\mathbf{R}_{\mathbf{x}_{\rm R}\mathbf{x}_{\rm R}}\right) \to \hat{\alpha}_{\rm d}\mathbf{I}_M, \tag{50}$$

where

$$\hat{\alpha}_{\rm d} = M\left(\alpha_{\rm a}^2 + 1 - \frac{2}{\pi}\right)\sum_{k=1}^{K}\sigma_{{\rm SR},k}^2\sigma_{{\rm RD},k}^2 + M\alpha_{\rm a}^2p_{\rm S}\sum_{k=1}^{K}\sigma_{{\rm SR},k}^2\sigma_{{\rm RD},k}^2\left(M\sigma_{{\rm SR},k}^2 + \sum_{i=1}^{K}\beta_{{\rm SR},i}\right). \tag{51}$$

As a result, the matrices $\mathbf{A}_{\rm d}$ and $\mathbf{R}_{\mathbf{q}_{\rm d}\mathbf{q}_{\rm d}}$ can be respectively approximated by

$$\mathbf{A}_{\rm d} \to \sqrt{\frac{2}{\pi\hat{\alpha}_{\rm d}}}\mathbf{I}_M = \alpha_{\rm d}\mathbf{I}_M, \tag{52}$$

$$\mathbf{R}_{\mathbf{q}_{\rm d}\mathbf{q}_{\rm d}} \to \left(1 - \frac{2}{\pi}\right)\mathbf{I}_M. \tag{53}$$

C. Approximate Rate Analysis

In this section, we derive a simpler closed-form approximation for the achievable rate. Substituting (48), (49), (52), and (53) into (40), the exact achievable rate R_k can be approximated by

$$\tilde{R}_k = \frac{\tau_c - 2\tau_p}{2\tau_c}\log_2\left(1+\frac{\tilde{A}_k}{\tilde{B}_k+\tilde{C}_k+\tilde{D}_k+\tilde{E}_k+\tilde{F}_k+\tilde{G}_k}\right), \tag{54}$$

where

$$\tilde{A}_k = p_S \left|\mathbb{E}\left\{\mathbf{g}_{RD,k}^T\mathbf{W}\mathbf{g}_{SR,k}\right\}\right|^2, \tag{55}$$

$$\tilde{B}_k = p_S \mathrm{Var}\left(\mathbf{g}_{RD,k}^T\mathbf{W}\mathbf{g}_{SR,k}\right), \tag{56}$$

$$\tilde{C}_k = p_S \sum_{i\neq k}\mathbb{E}\left\{\left|\mathbf{g}_{RD,k}^T\mathbf{W}\mathbf{g}_{SR,i}\right|^2\right\}, \tag{57}$$

$$\tilde{D}_k = \mathbb{E}\left\{\left\|\mathbf{g}_{RD,k}^T\mathbf{W}\right\|^2\right\}, \tag{58}$$

$$\tilde{E}_k = \left(1-\frac{2}{\pi}\right)\frac{1}{\alpha_a^2}\mathbb{E}\left\{\left\|\mathbf{g}_{RD,k}^T\mathbf{W}\right\|^2\right\}, \tag{59}$$

$$\tilde{F}_k = \left(1-\frac{2}{\pi}\right)\frac{1}{\alpha_a^2\alpha_d^2}\mathbb{E}\left\{\left\|\mathbf{g}_{RD,k}\right\|^2\right\}, \tag{60}$$

$$\tilde{G}_k = \frac{1}{\gamma^2\alpha_a^2\alpha_d^2}. \tag{61}$$

To this end, by invoking tools from random matrix theory, we present a closed-form approximate rate for the *k*-th destination, as formalized in the following theorem.

Theorem 1: With one-bit ADCs and DACs at the relay, the approximate achievable rate of the *k*-th destination is given by (54), where

$$\tilde{A}_k = p_S M^4\sigma_{SR,k}^4\sigma_{RD,k}^4, \tag{62}$$

$$\tilde{B}_k = p_S M^2\left(M\sigma_{SR,k}^4\sigma_{RD,k}^2\beta_{RD,k}+\beta_{SR,k}t_k\right), \tag{63}$$

$$\tilde{C}_k = M^2 p_S\sum_{i\neq k}\left(M\sigma_{SR,i}^4\sigma_{RD,i}^2\beta_{RD,k}+\beta_{SR,i}t_k\right), \tag{64}$$

$$\tilde{D}_k = M^2 t_k, \tag{65}$$

$$\tilde{E}_k = \left(\frac{\pi}{2}-1\right)\left(1+p_S\sum_{k=1}^K\beta_{SR,k}\right)M^2 t_k, \tag{66}$$

$$\tilde{F}_k = \beta_{RD,k}\left(\frac{\pi}{2}-1\right)M^3 p_S\sum_{k=1}^K\sigma_{SR,k}^4\sigma_{RD,k}^2 + \beta_{RD,k}\frac{M^2\pi}{2}\left(\frac{\pi}{2}-1\right)\left(1+p_S\sum_{k=1}^K\beta_{SR,k}\right)\sum_{k=1}^K\sigma_{SR,k}^2\sigma_{RD,k}^2, \tag{67}$$

$$\tilde{G}_k = \frac{M^3\pi p_S}{2p_R}\sum_{k=1}^K\sigma_{SR,k}^4\sigma_{RD,k}^2 + \frac{M^2\pi^2}{4p_R}\left(1+p_S\sum_{k=1}^K\beta_{SR,k}\right)\sum_{k=1}^K\sigma_{SR,k}^2\sigma_{RD,k}^2, \tag{68}$$

with $t_k = M\sigma_{RD,k}^4\sigma_{SR,k}^2+\beta_{RD,k}\sum_{n=1}^K\sigma_{SR,n}^2\sigma_{RD,n}^2$.

From Theorem 1, we can readily observe the impact of key parameters on the achievable rate. For instance, $\tilde{R}_k$ decreases with the number of user pairs K. This is expected since a higher number of users increases the amount of inter-user interference. In addition, $\tilde{R}_k$ is an increasing function of M, which reveals that increasing the number of relay's antennas always boosts up the system performance. As p_S approaches infinity, $\tilde{R}_k$ converges to a constant. In this case, the system becomes interference-limited.

To quantify the impact of the double quantization on system performance, we first present the achievable rate with perfect ADCs and DACs in the following corollary:

Corollary 1: With perfect ADCs and DACs, the achievable rate of the *k*-th destination can be expressed as

$$R_k^p = \frac{\tau_c-2\tau_p}{2\tau_c}\log_2\left(1+\frac{\hat{A}_k}{\hat{B}_k+\hat{C}_k+\hat{D}_k+\frac{\tilde{\alpha}_d}{\gamma^2}}\right), \tag{69}$$

where

$$\tilde{\alpha}_d = M\sum_{k=1}^K\hat{\sigma}_{SR,k}^2\hat{\sigma}_{RD,k}^2 + M\sum_{k=1}^K\hat{\sigma}_{SR,k}^2\hat{\sigma}_{RD,k}^2 p_S\left(M\hat{\sigma}_{SR,k}^2+\sum_{i=1}^K\beta_{SR,i}\right), \tag{70}$$

with $\hat{\sigma}_{SR,k}^2 = \frac{K\beta_{SR,k}^2 p_p}{K\beta_{SR,k}p_p+1}$ and $\hat{\sigma}_{RD,k}^2 = \frac{K\beta_{RD,k}^2 p_p}{K\beta_{RD,k}p_p+1}$; $\hat{A}_k$, $\hat{B}_k$, $\hat{C}_k$, $\hat{D}_k$ can be obtained by replacing $\sigma_{SR,k}^2$ and $\sigma_{RD,k}^2$ with $\hat{\sigma}_{SR,k}^2$ and $\hat{\sigma}_{RD,k}^2$ in $\tilde{A}_k$, $\tilde{B}_k$, $\tilde{C}_k$, $\tilde{D}_k$, respectively.

Armed with Corollary 1 and Theorem 1, we now compute the rate ratio between the one-bit and perfect ADC/DAC cases for the low signal-to-noise ratio (SNR) situation where massive MIMO systems are likely to operate.

Proposition 2: With $p_S \to 0$ and $M \to \infty$, we have

$$\frac{\tilde{R}_k}{R_k^p} = 4/\pi^2. \tag{71}$$

Proposition 2 indicates that the rate of the double-quantized system is $4/\pi^2$ times the rate of the system with perfect ADCs/DACs, in the low SNR regime. Interestingly, this result coincides with that reported for the single quantized system [15], suggesting that in the low SNR regime, the performance degradation due to double quantization is insignificant.

IV. Numerical Results

In this section, we present numerical results to validate the derived analytical results and study the impact of coarse quantization on the system performance.

Fig. 1 illustrates the MSE of each channel from the sources to the relay versus the transmit power of each pilot symbol. We set $K = 4$, and $\beta_{SR} = [0.6, 0.3, 0.1, 0.9]$. For $\beta_{SR,k} = \{0.1, 0.3\}$ which are less than the average large scale fading value of 0.475, the identity matrix pilot outperforms the Hadamard matrix, which is in agreement with Proposition 1. In addition, observing the curves associated with the Hadamard matrix, we can see that the approximate results nearly overlap with the exact results in the low p_p regime, indicating the validity of our theoretical analysis. However, if p_p increases, the gap between the approximate and exact results grows.

Fig. 2 shows the sum rate versus the number of user pairs K. For simplicity, unless otherwise specified, we set the large-scale fading coefficients as $\beta_{SR,k} = \beta_{RD,k} = 1$. The curves associated with "Exact numerical results" and "Approximate numerical results" are respectively generated by Monte-Carlo simulations according to (40) and (54) by averaging over 10^3 independent channel realizations, and the "Theoretical results" curves are obtained based on Theorem 1. As can be seen, there exists a gap between "Exact numerical results" (where the matrices $\mathbf{R}_{\mathbf{q}_a\mathbf{q}_a}$ and $\mathbf{R}_{\mathbf{q}_d\mathbf{q}_d}$ are not diagonal, which means that the quantization noise is correlated) and "Approximate

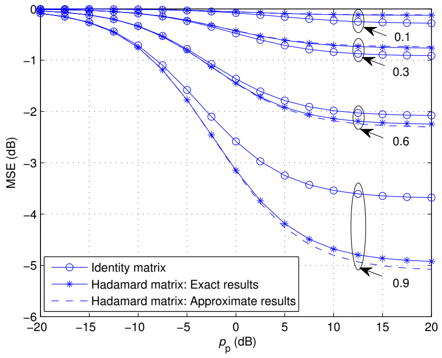

Fig. 1. MSE versus p_{p} for $K = 4$ and $M = 128$.

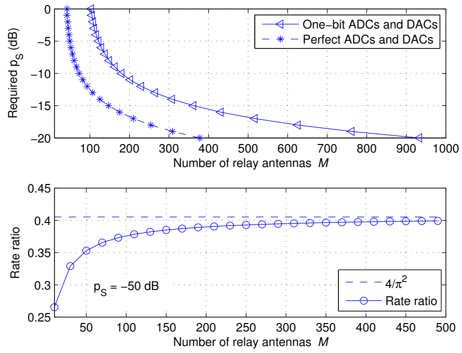

Fig. 3. Required p_{S} and rate ratio versus the number of relay antennas M for $K = 5$, $p_p = 10$ dB, and $p_{\mathrm{R}} = 10$ dB.

numerical results" (where the matrices $\mathbf{R}_{\mathbf{q}_a\mathbf{q}_a}$ and $\mathbf{R}_{\mathbf{q}_d\mathbf{q}_d}$ are approximated by identity matrices) when the number of user pairs is small, while the gap narrows and finally disappears as K becomes large. The reason is that the correlation effect is stronger with smaller K and weaker with larger K. In this example, our approximate model is very accurate when the number of user pairs is greater than 15, which is a reasonable number for this size of array. In addition, we observe that the "Approximate numerical results" curve overlaps with that for the "Theoretical results", which verifies our analytical derivations in Theorem 1.

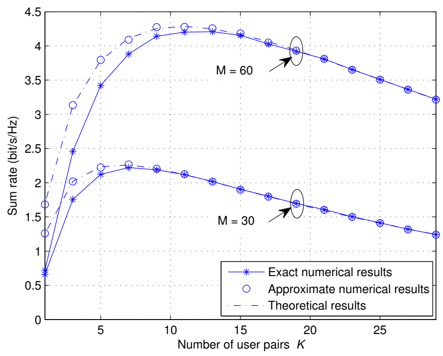

Fig. 2. Sum rate versus the the number of user pairs K for $p_{\mathrm{S}} = 10$ dB, $p_{\mathrm{R}} = 10$ dB, and $p_p = 10$ dB.

Fig. 3 shows the transmit power p_{S} of each source required to maintain a given sum rate of 5 bit/s/Hz, and the rate ratio in the low power regime. As can be seen, when the number of relay antennas increases, the required p_{S} is significantly reduced. In addition, we observe that the rate ratio converges to a nonzero limit $4/\pi^2$, which is consistent with Proposition 2. This property provides an efficient way to predict the sum rate with one-bit quantization according to the known sum rate of perfect ADC/DAC systems in low source transmit power regimes.

V. Conclusions

We have analyzed the achievable rate of a multipair half-duplex massive MIMO relaying system assuming that one-bit ADCs and DACs are deployed at the relay. An approximate closed-form expression for the achievable rate was derived, based on which the impact of key system parameters was characterized. It was shown that the identity training scheme provides better channel estimation performance for users with weaker than average channels. Furthermore, we revealed that the sum rate with one-bit ADCs and DACs is $4/\pi^2$ times that achieved by an unquantized system in the low source power regime.

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