Nonlinear Precoding for Multipair Relay Networks with One-bit ADCs and DACs

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***Abstract*—We consider a multipair half-duplex relay communication network, where the relay is deployed with one-bit ADCs and one-bit DACs. To suppress the interpair interference and quantization artifacts, we propose nonlinear precoding schemes to forward the quantized signals at the relay. We first present a technique based on gradient projection, and then show how to refine the solution using successive decoding and perturbation methods. For the single-user case with BPSK symbols, we obtain a closed-form solution for the optimal transmit vector. Numerical results verify that the proposed precoding design significantly outperforms quantized linear precoding strategies.**

I. Introduction

Multipair massive multiple-input multiple-output (MIMO) relays can greatly improve communication system performance and expand the coverage of wireless networks [1], [2], since massive antennas at the relay are capable of reaping large array and spatial multiplexing gains. Such enhancement comes at the cost of increased power consumption and hardware complexity. One promising approach to address this issue is to deploy one-bit ADCs and one-bit DACs at the relay, since this substantially reduces the required complexity and power usage. Furthermore, one-bit quantization eliminates the need for highly linear and energy inefficient power amplifiers, automatic gain controls, and can simplify the baseband digital signal processing and reduce on-chip data transfers. These savings come of course at the cost of significant quantization noise that must be accounted for.

Most previous work has adopted linear beamforming schemes for multipair massive MIMO relaying systems [3], [4], such as the maximum ratio conbiming/maximum ratio transmission (MRC/MRT), and zero-forcing reception/zero-forcing transmission (ZFR/ZFT), since such schemes can be implemented with low computational complexity and still provide good performance. However, when the system employs low-resolution ADCs/DACs, simply quantizing the output of the linear approaches is likely insufficient to mitigate the inter-user interference [5], [6]. Thus, it is desirable to investigate the use of nonlinear precoding techniques in multipair relaying networks.

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Some recent contributions have studied the nonlinear precoding problem for massive MIMO systems with one-bit quantization. For example, [7] formulates a problem that maximizes the product of the distances to the decision thresholds via the gradient projection algorithm, while [8] focuses on the minimum distance to the decision threshold and then optimizes it using the branch-and-bound method. The work in [9] presents two nonlinear one-bit precoding algorithms based on biconvex relaxation to minimize the mean-square error at the receiver. Other techniques based on coding theory and vector perturbation are provided in [10] and [11]. However, with the exception of [6], all of the work cited above focuses only on the massive MIMO downlink, and not on the one-bit multipair relay problem, where the data to be relayed has been quantized. In this paper we present several nonlinear precoding methods for amplify-and-forward (AF) multipair massive MIMO relays, based on the objective of minimizing the bit-error rate (BER) at the single antenna receivers. For the special case of a single user, we analytically solve the optimization problem and provide a closed-form optimal solution.

The remainder of the paper is organized as follows. Section II introduces the considered multipair AF relaying system. Section III formulates the optimization problem to design the transmit vector at the relay, and presents several algorithms for solving the problem. Numerical results illustrating the algorithms' performance are provided in Section IV.

Notation: We use bold upper case letters to denote matrices, bold lower case letters to denote vectors and lower case letters to denote scalars. The notation $(\cdot)^H$, $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^{-1}$ respectively represents the conjugate transpose, the conjugate, the transpose, and the matrix inverse. We let $\mathbf{x} \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Sigma})$ denote a circularly symmetric complex Gaussian random vector with zero mean and covariance matrix $\mathbf{\Sigma}$, while $\mathbf{I}_k$ is the identity matrix of size k. Finally, we define $[\cdot]^{\Re} = \Re(\cdot)$ and $[\cdot]^{\Im} = \Im(\cdot)$ as the real and imaginary parts.

II. System Model

Consider a multipair relaying system, where K single-antenna sources simultaneously transmit information to K single-antenna destinations via a shared relay, as shown in Fig. 1. The relay is equipped with M antennas with one-bit ADCs on receive and one-bit DACs for transmit. We assume that direct links between the sources and destinations do not exist. In addition, we assume that the relay operates in half-duplex mode; hence it cannot receive and transmit signals simultaneously. Accordingly, the whole information transmission is completed in two phases. In the first phase, the K sources transmit signals to the relay, and the $M \times 1$ received vector at the relay after one-bit quantization is

$$\mathbf{y}_{\mathrm{R}} = \mathcal{Q}\left(\sqrt{p_{\mathrm{S}}}\mathbf{G}_{\mathrm{SR}}\mathbf{s} + \mathbf{n}_{\mathrm{R}}\right), \tag{1}$$

where $\mathcal{Q}(\cdot)$ denotes the one-bit quantization operation, which separately processes the real and imaginary parts of the signal. Therefore, the elements of $\mathbf{y}_{\mathrm{R}}$ lie in the set $\frac{1}{\sqrt{2}}\{\pm 1 \pm 1j\}$. The matrix $\mathbf{G}_{\mathrm{SR}} = [\mathbf{g}_{\mathrm{SR},1}, \ldots, \mathbf{g}_{\mathrm{SR},K}] \in \mathbb{C}^{M \times K}$ represents the channel coefficients from the K sources to the relay with $\mathbf{g}_{\mathrm{SR},k} \in \mathcal{CN}(\mathbf{0}, \beta_{\mathrm{SR},k}\mathbf{I}_M)$, where $\beta_{\mathrm{SR},k}$ models the path-loss and shadow fading which is assumed to be constant over many coherence intervals and known a priori, $\mathbf{s}$ denotes the transmitted signal vector from the K sources, p_{S} is the transmit power of the sources, and $\mathbf{n}_{\mathrm{R}}$ is additive white Gaussian noise (AWGN) at the relay, whose elements are identically and independently distributed (i.i.d.) $\mathcal{CN}(0, 1)$.

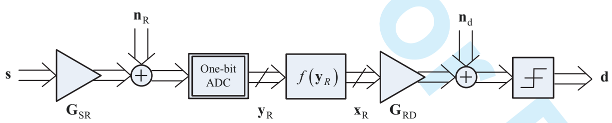


Fig. 1: Illustration of the multipair relaying system with one-bit ADCs and DACs

The relay adopts a general nonlinear precoding technique to process the signals, where the function $f(\mathbf{y}_{\mathrm{R}})$ maps $\mathbf{y}_{\mathrm{R}}$ to $\mathbf{x}_{\mathrm{R}}$. Due to the use of one-bit DACs, the elements of $\mathbf{x}_{\mathrm{R}}$ also lie in the set $\frac{1}{\sqrt{2}}\{\pm 1 \pm 1j\}$. In the second phase, the relay forwards the signal $\mathbf{x}_{\mathrm{R}}$ to the K destinations. As a result, the $K \times 1$ received vector at the destinations is given by

$$\mathbf{y}_{\mathrm{d}} = \sqrt{\frac{p_{\mathrm{R}}}{M}}\mathbf{G}_{\mathrm{RD}}^T\mathbf{x}_{\mathrm{R}} + \mathbf{n}_{\mathrm{d}}, \tag{2}$$

where the matrix $\mathbf{G}_{\mathrm{RD}} = [\mathbf{g}_{\mathrm{RD},1}, \ldots, \mathbf{g}_{\mathrm{RD},K}] \in \mathbb{C}^{M \times K}$ represents the channel coefficients from the relay to the K destinations with $\mathbf{g}_{\mathrm{RD},k} \in \mathcal{CN}(0, \beta_{\mathrm{RD},k}\mathbf{I}_M)$, $\mathbf{x}_{\mathrm{R}} \in \mathbb{C}^{M \times 1}$ denotes the transmitted signal from the relay, where $\beta_{\mathrm{RD},k}$ models the path-loss and shadow fading which, like $\beta_{\mathrm{SR},k}$, is assumed to be constant over many coherence intervals and known a priori, p_{R} is the transmit power of the relay, and $\mathbf{n}_{\mathrm{d}}$ is AWGN at the destination, whose elements are i.i.d. $\mathcal{CN}(0, 1)$. Afterwards, the signal $\mathbf{y}_{\mathrm{d}}$ is decoded as $\mathbf{d}$ by applying the hard-decision rule $\mathbf{d} = \sqrt{1/2}\,\mathrm{sign}(\mathbf{y}_{\mathrm{d}})$ applied elementwise and separately for the real and imaginary parts.

III. Nonlinear Precoding Design

Given $\mathbf{y}_{\mathrm{R}}$ and $\mathbf{x}_{\mathrm{R}}$, the probability of correct decoding at the K receivers can be expressed as

$$P(\mathbf{d} = \mathbf{s} | \mathbf{y}_{\mathrm{R}}, \mathbf{x}_{\mathrm{R}}) \overset{(a)}{=} \prod_{\mathbf{v} \in \mathcal{V}} P(\mathbf{s} = \mathbf{v} | \mathbf{y}_{\mathrm{R}}) \cdot P(\mathbf{d} = \mathbf{v} | \mathbf{x}_{\mathrm{R}}) \tag{3}$$

$$= \prod_{\mathbf{v} \in \mathcal{V}} \frac{P(\mathbf{s} = \mathbf{v})}{P(\mathbf{y}_{\mathrm{R}})} P(\mathbf{y}_{\mathrm{R}} | \mathbf{s} = \mathbf{v}) \cdot P(\mathbf{d} = \mathbf{v} | \mathbf{x}_{\mathrm{R}}) \tag{4}$$

where in (a) we used the fact that $\mathbf{s}$ is independent of $\mathbf{x}_{\mathrm{R}}$ and $\mathbf{d}$ is independent of $\mathbf{y}_{\mathrm{R}}$, and the vector $\mathbf{v} \in \mathcal{V}$ is determined by the adopted modulation technique. For instance, the set $\mathcal{V}$ consists of 4^K possible vectors for QPSK signaling, while it contains 2^K vectors for BPSK. The probabilities $P(\mathbf{y}_{\mathrm{R}} | \mathbf{s} = \mathbf{v})$ and $P(\mathbf{d} = \mathbf{v} | \mathbf{x}_{\mathrm{R}})$ are respectively given by

$$P(\mathbf{y}_{\mathrm{R}} | \mathbf{s} = \mathbf{v}) = \prod_{c \in \{\Re, \Im\}} \prod_{i=1}^{M} \Phi\left(\sqrt{2p_{\mathrm{S}}}\, y_{\mathrm{R},i}^c \left[\mathbf{g}_{\mathrm{SR},i}\mathbf{v}\right]^c\right)$$

$$P(\mathbf{d} = \mathbf{v} | \mathbf{x}_{\mathrm{R}}) = \prod_{c \in \{\Re, \Im\}} \prod_{j=1}^{K} \Phi\left(\sqrt{\frac{2p_{\mathrm{R}}}{M}}\, v_j^c \left[\mathbf{g}_{\mathrm{RD},j}^T\mathbf{x}_{\mathrm{R}}\right]^c\right),$$

where $\Phi(z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z} e^{-u^2/2} du$, $y_{\mathrm{R},i}$ denotes the i-th element of $\mathbf{y}_{\mathrm{R}}$, and v_j is the j-th element of $\mathbf{v}$.

Our goal is to design the transmit signals $\mathbf{x}_{\mathrm{R}}$ to maximize the probability $P(\mathbf{d} = \mathbf{s} | \mathbf{y}_{\mathrm{R}}, \mathbf{x}_{\mathrm{R}})$. Since $P(\mathbf{s} = \mathbf{v})$ and $P(\mathbf{y}_{\mathrm{R}})$ are fixed, the optimization problem can be formulated as

$$\mathcal{P}_1 : \underset{\mathbf{x}_{\mathrm{R}}}{\text{maximize}} \prod_{\mathbf{v} \in \mathcal{V}} \prod_{c \in \{\Re, \Im\}} \prod_{i=1}^{M} \Phi\left(\sqrt{2p_{\mathrm{S}}}\, y_{\mathrm{R},i}^c \left[\mathbf{g}_{\mathrm{SR},i}\mathbf{v}\right]^c\right) \tag{5}$$

$$\cdot \prod_{c \in \{\Re, \Im\}} \prod_{j=1}^{K} \Phi\left(\sqrt{\frac{2p_{\mathrm{R}}}{M}}\, v_j^c \left[\mathbf{g}_{\mathrm{RD},j}^T\mathbf{x}_{\mathrm{R}}\right]^c\right)$$

$$\text{subject to} \quad \mathbf{x}_{\mathrm{R}} \in \frac{1}{\sqrt{2}}\{\pm 1 \pm 1j\}. \tag{6}$$

Problem $\mathcal{P}_1$ is not convex since the constraint $\mathbf{x}_{\mathrm{R}} \in \frac{1}{\sqrt{2}}\{\pm 1 \pm 1j\}$ is non-convex.

A. Gradient projection algorithm

Solving $\mathcal{P}_1$ is prohibitively expensive, since: 1) the objective function is a sum of numerous terms (for example, the number of terms is 4^K for QPSK signals), and 2) a direct solution would require a search over 4^M possible vectors $\mathbf{x}_{\mathrm{R}}$. To address these issues, we first narrow down the set $\mathcal{V}$ to $\mathbf{v} = \hat{\mathbf{s}}$, where $\hat{\mathbf{s}}$ is the estimated signal at the relay, obtained in this case using coherent detection $\hat{\mathbf{s}} = \mathcal{Q}(\mathbf{F}\mathbf{y}_{\mathrm{R}})$, where $\mathbf{F}$ is the standard MRC receiver $\mathbf{F} = \mathbf{G}_{\mathrm{SR}}^H$. Then, we relax the QPSK constraint on $\mathbf{x}_{\mathrm{R}}$ to $|x_{\mathrm{R},i}^{\Re}| \le \frac{1}{\sqrt{2}}$ and $|x_{\mathrm{R},i}^{\Im}| \le \frac{1}{\sqrt{2}}$ to further reduce the complexity, where $x_{\mathrm{R},i}$ is the i-the element of $\mathbf{x}_{\mathrm{R}}$. The relaxed optimization problem reads as

$$\mathcal{P}_2 : \underset{\mathbf{x}_{\mathrm{R}}}{\text{maximize}} \prod_{c \in \{\Re, \Im\}} \prod_{j=1}^{K} \Phi\left(\sqrt{\frac{2p_{\mathrm{R}}}{M}}\, \hat{s}_j^c \left[\mathbf{g}_{\mathrm{RD},j}^T\mathbf{x}_{\mathrm{R}}\right]^c\right) \tag{7}$$

$$\text{subject to} \quad |x_{\mathrm{R},i}^{\Re}| \le \frac{1}{\sqrt{2}}, |x_{\mathrm{R},i}^{\Im}| \le \frac{1}{\sqrt{2}}, \tag{8}$$

where $\hat{s}_j$ is the j-th element of $\hat{\mathbf{s}}$.

We further reformulate the problem by taking the logarithm of (7) and decomposing it into real and imaginary parts:

$$\mathcal{P}_3 : \underset{\tilde{\mathbf{x}}_{\mathrm{R}}}{\text{maximize}} \prod_{j=1}^{K} \log\left(\Phi\left(\sqrt{\frac{2p_{\mathrm{R}}}{M}}\, \hat{s}_j^{\Re}\mathbf{a}_j^T\tilde{\mathbf{x}}_{\mathrm{R}}\right)\right) \tag{9}$$

$$+ \prod_{j=1}^{K} \log\left(\Phi\left(\sqrt{\frac{2p_{\mathrm{R}}}{M}}\, \hat{s}_j^{\Im}\mathbf{b}_j^T\tilde{\mathbf{x}}_{\mathrm{R}}\right)\right)$$

$$\text{subject to} \quad |\tilde{x}_{\mathrm{R},i}| \le \frac{1}{\sqrt{2}}, \tag{10}$$

where $\tilde{x}_{\mathrm{R},i}$ is the i-th element of $\tilde{\mathbf{x}}_{\mathrm{R}}$, and

$$\mathbf{a}_j = \left[\left(\mathbf{g}_{\mathrm{RD},j}^{\Re}\right)^T, -\left(\mathbf{g}_{\mathrm{RD},j}^{\Im}\right)^T \right]^T \quad (11)$$

$$\mathbf{b}_j = \left[\left(\mathbf{g}_{\mathrm{RD},j}^{\Im}\right)^T, \left(\mathbf{g}_{\mathrm{RD},j}^{\Re}\right)^T \right]^T \quad (12)$$

$$\tilde{\mathbf{x}}_{\mathrm{R}} = \left[\left(\mathbf{x}_{\mathrm{R}}^{\Re}\right)^T, \left(\mathbf{x}_{\mathrm{R}}^{\Im}\right)^T \right]^T. \quad (13)$$

Problem $\mathcal{P}_3$ is convex (detailed proof is omitted here). Thus, we resort to the gradient projection algorithm to perform the maximization. Before proceeding, we first need to compute the first derivative of (9) with respect to $\tilde{\mathbf{x}}_{\mathrm{R}}$, which is

$$\mathbf{d} = \prod_{j=1}^{K} \left(\mathbf{d}_{1,j} + \mathbf{d}_{2,j}\right), \quad (14)$$

where

$$\mathbf{d}_{n,j} = \frac{\sqrt{\frac{2p_{\mathrm{R}}}{M}}\mathbf{q}_{n,j} \exp\left(-\left(\sqrt{\frac{2p_{\mathrm{R}}}{M}}\mathbf{q}_{n,j}^T\tilde{\mathbf{x}}_{\mathrm{R}}\right)^2/2\right)}{\sqrt{2\pi}\Phi\left(\sqrt{\frac{2p_{\mathrm{R}}}{M}}\mathbf{q}_{n,j}^T\tilde{\mathbf{x}}_{\mathrm{R}}\right)}, \quad (15)$$

with $n \in \{1,2\}$, $\mathbf{q}_{1,j} = \hat{s}_j^{\Re}\mathbf{a}_j$, and $\mathbf{q}_{2,j} = \hat{s}_j^{\Im}\mathbf{b}_j$. When the number of relay antennas is very large or p_{R} is very high, the values of $\Phi\left(\sqrt{\frac{2p_{\mathrm{R}}}{M}}\mathbf{q}_{n,j}^T\tilde{\mathbf{x}}_{\mathrm{R}}\right)$ approach 1, which leads to a zero-valued cost-function. This causes $\mathcal{P}_3$ to be unsolvable since zero is the function's maximum value. To overcome this difficulty, we make the following approximations:

$$\log\left(\Phi\left(\sqrt{\frac{2p_{\mathrm{R}}}{M}}\mathbf{q}_{n,j}^T\tilde{\mathbf{x}}_{\mathrm{R}}\right)\right) \approx -\Phi\left(-\sqrt{\frac{2p_{\mathrm{R}}}{M}}\mathbf{q}_{n,j}^T\tilde{\mathbf{x}}_{\mathrm{R}}\right),$$

by invoking the properties $\log(1+x) \approx x$ as $x \to 0$ and $\Phi(x) = 1 - \Phi(-x)$. Based on the above discussions, we summarize our approach in Algorithm 1.

Algorithm 1 Gradient projection algorithm for $\mathcal{P}_3$

1: **Initialization**.

1.1: Define a tolerance ϵ and iteration step μ.

2.2: Set $\tilde{\mathbf{x}}_{\mathrm{R}}^{(1)} = \left[\mathcal{Q}\left(\Re\{\mathbf{W}\mathbf{y}_{\mathrm{R}}\}\right)^T, \mathcal{Q}\left(\Im\{\mathbf{W}\mathbf{y}_{\mathrm{R}}\}\right)^T\right]^T$, where $\mathbf{W} = \mathbf{G}_{\mathrm{RD}}^*\mathbf{G}_{\mathrm{SR}}^H$. Set $i = 1$.

2: **Calculation**. Substitute $\tilde{\mathbf{x}}_{\mathrm{R}}^{(1)}$ into (9) and (14), and then compute the objective function $c^{(1)}$, and the first derivative of the cost function $\mathbf{d}^{(1)}$.

3: **Iteration** i.

3.1: $\tilde{\mathbf{x}}_{\mathrm{R}}^{(i+1)} = \tilde{\mathbf{x}}_{\mathrm{R}}^{(i)} + \mu\mathbf{d}^{(i)}$.

3.2: Let $\tilde{\mathbf{x}}_{\mathrm{R}}^{(i+1)} = \sqrt{1/2}\mathrm{sign}\left(\tilde{\mathbf{x}}_{\mathrm{R}}^{(i+1)}\right)$.

3.3: Calculate the cost function (9).

3.4: If $c^{(i+1)} < c^{(i)}$ then $\mu = \mu/2$.

4: **Stopping criterion**. If $\left|c^{(i+1)} - c^{(i)}\right|/c^{(i)} < \epsilon$, set $\tilde{\mathbf{x}}_{\mathrm{R}}^{\mathrm{opt}} = \tilde{\mathbf{x}}_{\mathrm{R}}^{(i+1)}$ and stop; otherwise, go to step 5.

5: **Update initial values**. Set $i = i + 1$. Go to step 3.

Algorithm 1 provides the optimal solution of problem $\mathcal{P}_3$, which can be expressed as $\tilde{\mathbf{x}}_{\mathrm{R}}^{\mathrm{opt}} = \left[\tilde{x}_{\mathrm{R},1}^{\mathrm{opt}}, \tilde{x}_{\mathrm{R},2}^{\mathrm{opt}}, \ldots, \tilde{x}_{\mathrm{R},2M}^{\mathrm{opt}}\right]^T$, where $-\sqrt{1/2} \le \tilde{x}_{\mathrm{R},m}^{\mathrm{opt}} \le \sqrt{1/2}$. Due to the one-bit quantization, we adjust the elements of the vector to be either $\pm\sqrt{1/2}$ using a hard limiter:

$$\mathbf{x}_{\mathrm{R}}^{*,\mathrm{g}} = \left[x_{\mathrm{R},1}^{*,\mathrm{g}} + jx_{\mathrm{R},M+1}^{*,\mathrm{g}}, \ldots, x_{\mathrm{R},M}^{*,\mathrm{g}} + jx_{\mathrm{R},2M}^{*,\mathrm{g}}\right]^T, \quad (16)$$

where $x_{\mathrm{R},i}^{*,\mathrm{g}} = \sqrt{1/2}\mathrm{sign}\left(\tilde{x}_{\mathrm{R},i}^{\mathrm{opt}}\right)$.

The computational complexity of each iteration of Algorithm 1 is $O(MK)$, which is of the same order as calculating the non-iterative quantized linear precoders. While the solution to Algorithm 1 is optimal for problem $\mathcal{P}_3$, it does not satisfy the discrete QPSK constraint imposed by the quantization. Due to this and the discrete nature of the solution space (a vector with QPSK entries), there are many local minima in the vicinity of the solution provided by Algorithm 1. While Algorithm 1 finds a reasonable operating point, perturbing the solution around this operating point will often lead to a higher value for the cost function. The following are two methods for implementing this perturbation.

Successive decoding – Instead of crudely applying the $\mathrm{sign}(\cdot)$ function when shaping the vector $\tilde{\mathbf{x}}_{\mathrm{R}}^{\mathrm{opt}}$, here the criterion for judging each element of $\tilde{\mathbf{x}}_{\mathrm{R}}^{\mathrm{opt}}$ as positive or negative is based on which value makes the cost function larger. Since elements of $\tilde{\mathbf{x}}_{\mathrm{R}}^{\mathrm{opt}}$ with larger absolute values have a greater impact on the cost function, we successively determine each element of $\tilde{\mathbf{x}}_{\mathrm{R}}^{\mathrm{opt}}$ in descending order.

Vector perturbation – We examine small perturbations to the transmit vector near $\sqrt{1/2}\mathrm{sign}\left(\tilde{\mathbf{x}}_{\mathrm{R}}^{\mathrm{opt}}\right)$ to improve performance. In this case, we first obtain $2M$ vectors by changing the sign of only one element of $\sqrt{1/2}\mathrm{sign}\left(\tilde{\mathbf{x}}_{\mathrm{R}}^{\mathrm{opt}}\right)$ while keeping the others fixed. Then, among these $2M+1$ vectors ($2M$ perturbed versions of $\sqrt{1/2}\mathrm{sign}\left(\tilde{\mathbf{x}}_{\mathrm{R}}^{\mathrm{opt}}\right)$ and itself), we select the candidate vector that maximizes the cost function (9).

The two above approaches can enhance the system performance with relatively minimal computational cost since they operate one element of the vector at a time. When changing only one element of the vector, one need not compute the entire cost function again; all that is required is a simple addition and subtraction. As a result, the additional computational cost for these methods is only $O(M)$, less than one iteration of the gradient search.

B. Single-user case

For the single-user case, the input signal at the source is denoted by s, the received signal at the destination is represented by d, and the channels from the source to the relay and from the relay to the destination are respectively expressed as $\mathbf{g}_{\mathrm{SR}} \in \mathcal{CN}(\mathbf{0}, \beta_{\mathrm{SR}}\mathbf{I}_M)$ and $\mathbf{g}_{\mathrm{RD}} \in \mathcal{CN}(\mathbf{0}, \beta_{\mathrm{RD}}\mathbf{I}_M)$, where β_{SR} and β_{RD} model the path-loss and shadow fading that follow the same distributions as $\beta_{\mathrm{SR},k}$ and $\beta_{\mathrm{RD},k}$. Therefore, for BPSK input signals, the probability of correct decoding is expressed as

$$P(d = s|\mathbf{y}_{\mathrm{R}}, \mathbf{x}_{\mathrm{R}}) = \prod_{v\in\{+1,-1\}} \frac{P(\mathbf{y}_{\mathrm{R}}|s = v)P(s = v)}{P(\mathbf{y}_{\mathrm{R}})} P(d = v|\mathbf{x}_{\mathrm{R}}), \quad (17)$$

where v denotes the possible transmit symbol from the source, and lies in the set $v \in \{+1, -1\}$,

$$P(\mathbf{y}_{\mathrm{R}}|s=v) = \prod_{c\in\{\Re,\Im\}} \prod_{i=1}^{M} \Phi\left(v\sqrt{2p_{\mathrm{S}}}\, y^{c}_{\mathrm{R},i} g^{c}_{\mathrm{SR},i}\right) \tag{18}$$

$$P(d=v|\mathbf{x}_{\mathrm{R}}) = \Phi\left(v\sqrt{2p_{\mathrm{R}}/M}\left[\mathbf{g}_{\mathrm{RD}}^{T}\mathbf{x}_{\mathrm{R}}\right]^{\Re}\right) \tag{19}$$

$$P(s=v) = \frac{1}{2} \tag{20}$$

$$P(\mathbf{y}_{\mathrm{R}}) = \sum_{v\in\{+1,-1\}} P(\mathbf{y}_{\mathrm{R}}|s=v)\, P(s=v), \tag{21}$$

with $g_{\mathrm{SR},i}$ denoting the i-th element of $\mathbf{g}_{\mathrm{SR}}$. By letting $\frac{P(\mathbf{y}_{\mathrm{R}}|s=+1)P(s=+1)}{P(\mathbf{y}_{\mathrm{R}})} = \alpha$, we have

$$\begin{aligned} &P(d=s|\mathbf{y}_{\mathrm{R}},\mathbf{x}_{\mathrm{R}}) \\ &= \alpha P(d=+1|\mathbf{x}_{\mathrm{R}}) + (1-\alpha)\, P(d=-1|\mathbf{x}_{\mathrm{R}}). \end{aligned} \tag{22}$$

Thus, problem $\mathcal{P}_1$ reduces to

$$\begin{aligned} \mathcal{P}_4 : \underset{\mathbf{x}_{\mathrm{R}}}{\text{maximize}} \quad & \alpha\Phi\left(\sqrt{2p_{\mathrm{R}}/M}\left[\mathbf{g}_{\mathrm{RD}}^{T}\mathbf{x}_{\mathrm{R}}\right]^{\Re}\right) \\ & + (1-\alpha)\,\Phi\left(-\sqrt{2p_{\mathrm{R}}/M}\left[\mathbf{g}_{\mathrm{RD}}^{T}\mathbf{x}_{\mathrm{R}}\right]^{\Re}\right) \end{aligned} \tag{23}$$

$$\text{subject to} \quad \mathbf{x}_{\mathrm{R}} \in \frac{1}{\sqrt{2}}\{\pm 1 \pm 1j\}. \tag{24}$$

The following theorem provides the solution to $\mathcal{P}_4$.

Theorem 1: The solution of problem $\mathcal{P}_4$ is given by

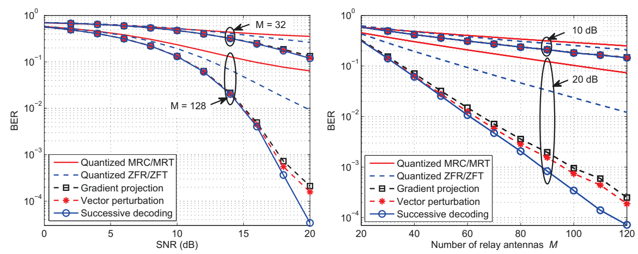
$$\mathbf{x}_{\mathrm{R}}^{*} = \begin{cases} \mathcal{Q}(\mathbf{g}_{\mathrm{RD}}^{*}), & \alpha > 0.5, \\ -\mathcal{Q}(\mathbf{g}_{\mathrm{RD}}^{*}), & \alpha < 0.5. \end{cases} \tag{25}$$

Proof: By observing that $g(x) = \alpha\Phi(x) + (1-\alpha)\,\Phi(-x)$ is an increasing function for $\alpha > 0.5$ and a decreasing function for $\alpha < 0.5$, problem $\mathcal{P}_4$ can be reduced to maximizing $\left[\mathbf{g}_{\mathrm{RD}}^{T}\mathbf{x}_{\mathrm{R}}\right]^{\Re}$ for $\alpha > 0.5$ and minimizing $\left[\mathbf{g}_{\mathrm{RD}}^{T}\mathbf{x}_{\mathrm{R}}\right]^{\Re}$ for $\alpha < 0.5$. This completes the proof. ■

IV. Numerical Results

In this section, we compare our proposed algorithms with quantized linear precoding strategies. For quantized MRC/MRT, the transmit vector is given by $\mathbf{x}_{\mathrm{R}} = \mathcal{Q}\left(\mathbf{G}_{\mathrm{RD}}^{*}\mathbf{G}_{\mathrm{SR}}^{H}\mathbf{y}_{\mathrm{R}}\right)$, while for quantized ZFR/ZFT it is expressed as $\mathbf{x}_{\mathrm{R}} = \mathcal{Q}\left(\mathbf{G}_{\mathrm{RD}}^{*}\left(\mathbf{G}_{\mathrm{RD}}^{T}\mathbf{G}_{\mathrm{RD}}^{*}\right)^{-1}\left(\mathbf{G}_{\mathrm{SR}}^{H}\mathbf{G}_{\mathrm{SR}}\right)^{-1}\mathbf{G}_{\mathrm{SR}}^{H}\mathbf{y}_{\mathrm{R}}\right)$. We also define SNR $\triangleq p_{\mathrm{S}}$, and choose $p_{\mathrm{R}} = p_{\mathrm{S}}$. The large-scale fading parameters are calculated as $\beta_{\mathrm{AR},k} = z_k\left(r_{\mathrm{AR},k}/r_0\right)^{\alpha}$ and $\beta_{\mathrm{RB},k} = z_k\left(r_{\mathrm{BR},k}/r_0\right)^{\alpha}$, where z_k is a log-normal random variable with standard deviation 8 dB, $r_{\mathrm{AR},k}$ and $r_{\mathrm{RB},k}$ are the distances of the sources and destinations from the relay, $\alpha = 3.8$ is the path loss exponent, and r_0 denotes nearest possible distance between the users and the relay. The relay is assumed to be located at the center of a cell with a radius of 1000 meters and $r_0 = 100$ meters.

Fig. 2 studies the the impact of SNR and the number of relay antennas M on the BER for QPSK signals with $K = 10$. As can be observed, our proposed nonlinear schemes outperform quantized MRC/MRT and ZFR/ZFT precoding, especially for high SNRs and large M. Compared to our proposed nonlinear approaches, quantized ZFR/ZFT requires approximately 5 dB more power or approximately 2 times more antennas to achieve the same target BER $= 10^{-2}$ (see Fig. 2(a) when $M = 128$, and Fig. 2(b) when SNR $= 20$ dB). Furthermore, successive decoding achieves the same performance when M is moderate, while it outperforms the vector perturbation method when M is very large. The average number of iterations needed to obtain the optimal solution are respectively 8, 12, and 25 for $M = 32$, $M = 64$, and $M = 128$ when $\epsilon = 10^{-3}$ and $\mu = 10$. Thus, the improved performance comes at the cost of an increase in computational load.



(a) Impact of SNR (b) Impact of M

Fig. 2: Average BER for QPSK signaling with $K = 10$.

Fig. 3 illustrates the BER versus SNR for the single user case and BPSK signals, with quantized MRC/MRT precoding as a benchmark. For MRC/MRT, the transmit vector is given by $\mathbf{x}_{\mathrm{R}} = \mathcal{Q}\left(\mathbf{g}_{\mathrm{RD}}^{*}\mathbf{g}_{\mathrm{SR}}^{H}\mathbf{y}_{\mathrm{R}}\right)$, and we observe about a 1-2 dB improvement in BER for the nonlinear precoder.

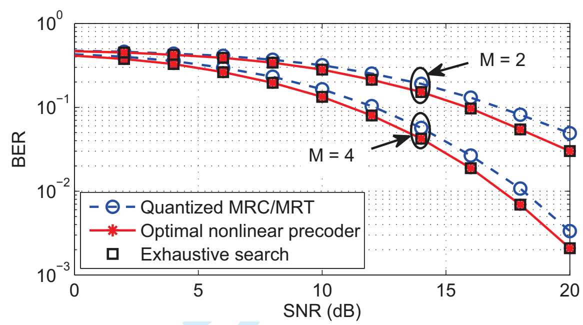


Fig. 3: Average BER versus SNR for single-user system with BPSK signaling

V. Conclusion

We have formulated a nonlinear precoding problem in terms of minimizing the BER for the multipair relaying system with one-bit quantization. The problem was reformulated to be convex by relaxing the one-bit constraint, and gradient projection was proposed to maximize the resulting cost function. Two approaches based on successive decoding and vector perturbation were then proposed to refine the approximate solution. Numerical results demonstrated that the proposed nonlinear precoding methods require approximately 5 dB less transmit power or only half the number of relay antennas to achieve the same BER as quantized ZFR/ZFT. For the single-user case, a closed-form solution was derived and shown to provide performance similar to that of an optimal exhaustive search.

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