Probing stellar binary black hole formation in galactic nuclei via the imprint of their center of mass acceleration on their gravitational wave signal

Kohei Inayoshi,[1] Nicola Tamanini,[2] Chiara Caprini,[3] and Zoltán Haiman[1,4]

[1]*Department of Astronomy, Columbia University, 550 W. 120th St., New York, New York 10027, USA*
[2]*Institut de Physique Théorique, CEA-Saclay, CNRS UMR 3681, Université Paris-Saclay, F-91191 Gif-sur-Yvette, France*
[3]*Laboratoire Astroparticule et Cosmologie, CNRS UMR 7164, Université Paris-Diderot, 10 rue Alice Domon et Léonie Duquet, 75013 Paris, France*
[4]*Department of Physics, New York University, New York, New York 10003, USA*

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Multifrequency gravitational wave (GW) observations are useful probes of the formation processes of coalescing stellar-mass binary black holes (BBHs). We discuss the phase drift in the GW inspiral waveform of the merging BBH caused by its center-of-mass acceleration. The acceleration strongly depends on the location where a BBH forms within a galaxy, allowing observations of the early inspiral phase of Laser Interferometer Gravitational Wave Observatory (LIGO)-like BBH mergers by the Laser Interferometer Space Antenna (LISA) to test the formation mechanism. In particular, BBHs formed in dense nuclear star clusters or via compact accretion disks around a nuclear supermassive black hole in active galactic nuclei would suffer strong acceleration, and produce large phase drifts measurable by LISA. The host galaxies of the coalescing BBHs in these scenarios can also be uniquely identified in the LISA error volume, without electromagnetic counterparts. A nondetection of phase drifts would rule out or constrain the contribution of the nuclear formation channels to the stellar-mass BBH population.

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I. INTRODUCTION

The advanced Laser Interferometer Gravitational Wave Observatory (LIGO) has so far detected gravitational waves from three stellar binary black hole (BBH) mergers [1–4]. Several scenarios for the origin of such massive compact BBHs have been proposed [5], through the evolution of isolated massive stellar binaries [6–10], dynamical formation in dense stellar clusters [11–14] or in active galactic nuclei (AGN) [15–17].

Recently, Ref. [18] pointed out that the early inspiral of GW150914-like BBHs can be measured by the space-based Laser Interferometer Space Antenna (LISA) [19]. The BBH coalescence rate inferred from the LIGO detections implies that $\sim$10–100 of such BBHs will be individually resolved by LISA and then merge in the LIGO band in $\lesssim 10$ yr. These LISA observations alone can determine the coalescence time with an accuracy of $\sim$10 s and the sky position to within < 1 deg^2, allowing advance planning of electromagnetic (EM) observations of the merger.

Multifrequency gravitational wave (GW) observations by LISA and LIGO are also useful to distinguish formation scenarios of stellar-mass BBHs (e.g. measurements of spin-orbit misalignments by LIGO [20–22]). LISA could detect nonzero eccentricities of the merging BBHs [23–25]. Measurable eccentricities are expected in formation channels involving dynamical interactions in dense stellar clusters [12], or as a result of the interaction with AGN accretion disks [15–17], but not if typical BBHs form via isolated binaries [9]. The predicted eccentricities are uncertain and alternative ways to distinguish formation channels remain useful.

Another important advantage of low-frequency GW observations by LISA (and/or by DECIGO [26]) is that the acceleration of the merging BBHs can produce a measurable phase drift in their GW inspiral waveform. In the cosmological context, the apparent acceleration caused by the time evolution of the Hubble expansion is weak, but if detected, it would allow us to measure the accelerated expansion directly [27,28]. However, the peculiar acceleration of the coalescing BBHs due to astrophysical processes could be much larger than that produced by the cosmic expansion [29,30].

In this paper, we discuss the possibility to distinguish the formation channels of merging BBHs, focusing on the binary motion inside the host galaxy. We show that BBHs located in dense nuclear star clusters or in compact accretion disks around a nuclear supermassive BH (SMBH) suffer strong acceleration, and produce a large phase drift measurable by LISA [31]. Since the acceleration effect strongly depends on the location where BBHs form within a galaxy, observations by LISA of stellar-mass BBH mergers offer a test of proposed formation scenarios.

II. PHASE DRIFT IN GRAVITATIONAL WAVES

We consider a coalescing BBH with redshifted chirp mass $M_{\rm cz}$ ($M_{\rm cz,40} \equiv M_{\rm cz}/40\ M_{\odot}$) in the LISA band with a signal-to-noise ratio (SNR) of $\rho > 8$ which will merge in the advanced LIGO/Virgo band in $\tau_c \lesssim 10$ yr [18]. The rest-frame GW frequency of the coalescing BBH is given by (see e.g. [32,33])

$$f \simeq 13.8 M_{\mathrm{cz},40}^{-5/8} \left(\frac{\tau_c}{4\ \mathrm{yr}}\right)^{-3/8} \mathrm{mHz}. \tag{1}$$

The SNR accumulated during the LISA observation time δt can be approximated as [23]

$$\rho \simeq 51 d_{L,100}^{-1} M_{\mathrm{cz},40}^{5/3} f_{14}^{2/3} \delta t_5^{1/2}, \tag{2}$$

where $d_{L,100} \equiv d_L/(100\ \mathrm{Mpc})$ is the luminosity distance to the GW source and $f_{14} \equiv f/(14\ \mathrm{mHz})$ is a representative frequency. Throughout the paper we assume a LISA configuration with six links, 2 million km arm length and mission duration of $\delta t = 5$ yr ($\delta t_5 \equiv \delta t/5$ yr). LISA's noise curve $S_{\mathrm{n}}(f)$ has been taken from [34]. In Eq. (2), we have set LISA's sensitivity to $\sqrt{S_{\mathrm{n}}(f)} = 7 \times 10^{-21}/\sqrt{\mathrm{Hz}}$ at $f \simeq 10$ mHz.

From the BBH merger rate inferred from the existing LIGO events, $R \simeq 9$–$240\ \mathrm{Gpc}^{-3}\ \mathrm{yr}^{-1}$ [3], the space density of BBHs inspiralling near $f \simeq 10^{-2}$ Hz at a given time can be estimated as

$$n_{\mathrm{m}} \equiv R \frac{f}{\dot{f}} = 10^{-6} f_{14}^{-8/3} M_{\mathrm{cz},40}^{-5/3} R_{100}\ \mathrm{Mpc}^{-3}, \tag{3}$$

where $R_{100} = R/(100\ \mathrm{Gpc}^{-3}\ \mathrm{yr}^{-1})$. Thus, below we require 60 Mpc $< d_L <$ 640 Mpc, in order to ensure that at least one event ($N_{\mathrm{m}} \equiv 4\pi n_{\mathrm{m}} d_L^3/3 > 1$, evaluated with $R_{100} = 1$) occurs in the local cosmic volume with a total SNR $\rho > 8$, during a $\delta t = 5$ yr LISA mission lifetime.

Next, we consider the impact of the center-of-mass (CoM) acceleration of a merging BBH. Over δt, the source will appear to change its redshift by an amount

$$\delta z_{\mathrm{acc}} \simeq \frac{a_{\mathrm{CoM}} \delta t}{c} \equiv 1.7 \times 10^{-7} \delta t_5 \left(\frac{\epsilon}{10^4}\right), \tag{4}$$

where we have expressed the acceleration $a_{\mathrm{CoM}} = v_{\mathrm{acc}}^2/r$ along the line of sight in terms of a characteristic velocity v_{acc} and distance r (interpreted below as the orbital velocity and distance from the barycenter of the host galaxy), and defined the dimensionless acceleration parameter ϵ

$$\epsilon \equiv 10^4 \left(\frac{v_{\mathrm{acc}}}{100\ \mathrm{km\,s^{-1}}}\right)^2 \left(\frac{r}{1\ \mathrm{pc}}\right)^{-1}. \tag{5}$$

We define a variable Y which accounts for the CoM acceleration of a merging BBH by

$$Y \equiv \frac{1}{2(1+z)} \cdot \frac{\delta z_{\mathrm{acc}}}{\delta t} \simeq 1.5 \times 10^{-8} \epsilon_{z,4}\ \mathrm{yr}^{-1}, \tag{6}$$

where $\epsilon/(1+z) \equiv 10^4 \epsilon_{z,4}$ [35]. The CoM acceleration causes a linear frequency drift $\delta f \propto Y \delta t$ and the corresponding phase drift in the GW inspiral waveform is expressed as [30]

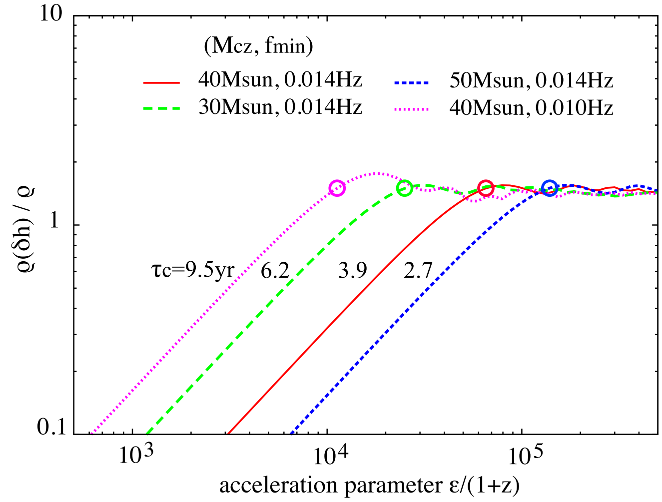

FIG. 1. The relative SNR of the deviation in the GW inspiral waveform caused by the CoM acceleration ϵ, for different combinations of the redshifted chirp mass M_{cz} and the frequency f_{min} when the LISA observation begins. The corresponding time to coalescence τ_c is indicated in the figure. The LISA observation is limited to the duration $\delta t = 5$ yr. The relative SNRs saturate at $\epsilon > \epsilon_{\mathrm{crit}}$ ($\delta\Psi_{\mathrm{acc}} \gtrsim 1$) shown by open circles.

$$\delta\Phi_{\mathrm{acc}} \simeq 1.0 \epsilon_{z,4} \delta t_5 M_{\mathrm{cz},40}^{-5/3} f_{14}^{-5/3}. \tag{7}$$

The total number of GW cycles without the acceleration is $\mathcal{O}(10^6)$ for stellar-mass binaries, and Eq. (7) gives about one extra cycle for the reference values of the parameters.

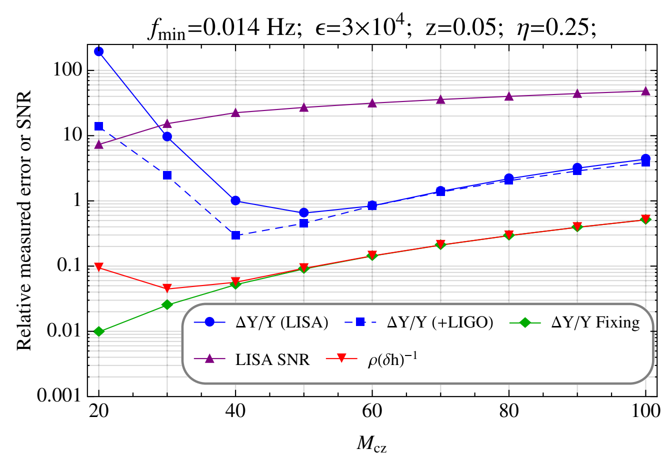

FIG. 2. The 1σ errors $\Delta Y/Y$ from LISA alone (blue solid) and LISA + LIGO (i.e., assuming that the coalescence time t_c has been fixed by LIGO; blue dashed). For our fiducial case ($f_{\mathrm{min}} = 0.014$ Hz, $\epsilon = 3 \times 10^4$, $z = 0.05$ and $\eta = 0.25$), nonzero acceleration can be detected, i.e., $\Delta Y/Y < 1$, for merging BBHs with 35 $\mathrm{M}_\odot \lesssim M_{\mathrm{cz}} \lesssim 63\ \mathrm{M}_\odot$. The LISA SNR (purple) and the inverse of the numerically computed $\rho(\delta h)$ due to the CoM acceleration (red) are shown. The green curve shows the Fisher error assuming all parameters but Y are fixed: for $M_{\mathrm{cz}} \gtrsim 35\ \mathrm{M}_\odot$ it coincides with the inverse of $\rho(\delta h)$, validating the Fisher analysis; for lower M_{cz} the Fisher analysis does not capture the saturation effect discussed below Eq. (9) and shown in Fig. 1.

To detect the phase drift in the GW inspiral waveform, the strain perturbation $\delta h(f) = h(f)[1 - e^{i\delta\Psi_{\rm acc}(f)}]$ must have a significant SNR [36,37], where $\delta\Psi_{\rm acc}(f)$ is the phase drift in frequency space [38]

$$\delta\Psi_{\rm acc}(f) \simeq -0.59\epsilon_{\rm z,4} M_{\rm cz,40}^{-10/3} f_{14}^{-13/3}. \tag{8}$$

Figure 1 shows the relative SNR of the perturbation $\rho(\delta h)/\rho = [\int_{f_{\rm min}}^{f_{\rm max}} df \frac{|\delta h(f)|^2}{S_n(f)} / \int_{f_{\rm min}}^{f_{\rm max}} df \frac{|h(f)|^2}{S_n(f)}]^{1/2}$ for different combinations of the redshifted chirp mass $M_{\rm cz}$ and the frequency $f_{\rm min}$ when the observation begins ($f_{\rm max}$ is the smaller between twice the inner-most-stable-circular-orbit frequency or the frequency reached in $\delta t = 5$ yr).

For small ϵ, the SNR of the perturbation is proportional to $|\delta\Psi_{\rm acc}|$ [37,39] and is given by

$$\rho(\delta h) \simeq 16\epsilon_{\rm z,4} d_{L,100}^{-1} \delta t_5^{1/2} M_{\rm cz,40}^{-5/3} f_{14}^{-11/3}. \tag{9}$$

For larger $\epsilon(> \epsilon_{\rm crit})$, when the phase drift approaches a full cycle, the relative SNR of $\rho(\delta h)/\rho$ saturates at a roughly constant value ($\simeq$1.5) because of de-phasing. Computing the relative SNR numerically, we found the critical accelerations to be $\epsilon_{\rm crit}/(1+z) \simeq 6.6 \times 10^4 M_{\rm cz,40}^{10/3} f_{14}^{13/3}$.

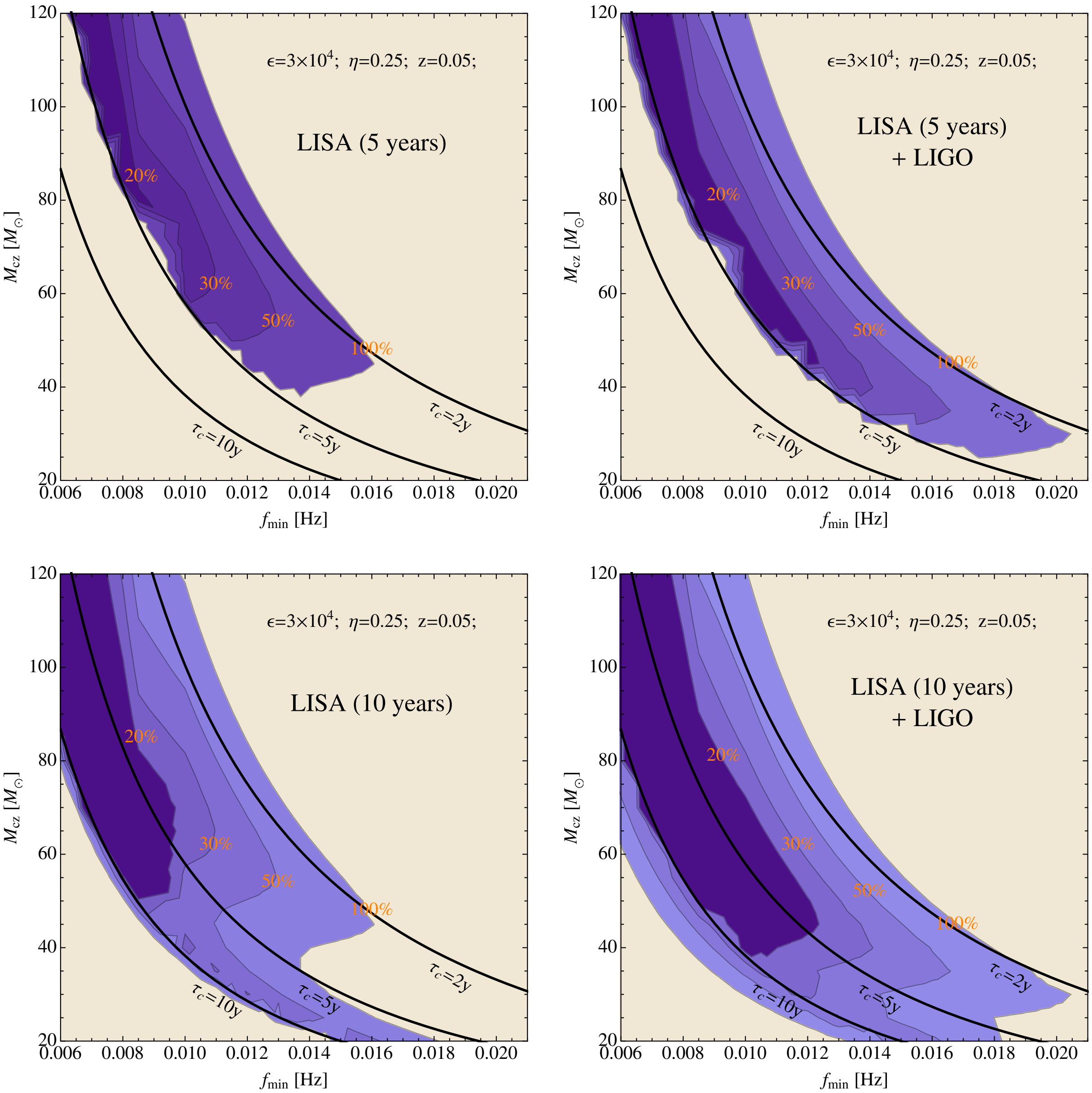

FIG. 3. Contours of the marginalized 1σ error $\Delta Y/Y$ in the $f_{\rm min}$–$M_{\rm cz}$ parameter space, provided by LISA alone (top left panel) and LISA + LIGO (top right panel) assuming a five year mission, and LISA alone (bottom left panel) and LISA + LIGO (bottom right panel) assuming a 10 year mission. The solid curves indicate constant times to coalescence. Merging BBHs with 2 yr $\lesssim \tau_c \lesssim 5$ (or 10) yrs provide the best combinations of $f_{\rm min}$ and $M_{\rm cz}$ to probe the CoM acceleration.

In order to estimate the LISA error on the acceleration parameter Y, including possible degeneracies with other system parameters, we perform a Fisher matrix analysis. We adopt the six parameters $M_{\rm cz}$, Φ_c, t_c, η, d_L and Y, where Φ_c is the phase at the coalescence time t_c, and η is the symmetric mass ratio. We further follow Ref. [30] and adopt the sky-averaged GW waveform of $h(f) = A(f)\exp[i\Psi(f)]$ with the amplitude $A(f)$ at Newtonian order, and the phase $\Psi(f)$ at 3.5PN order, plus the contribution of the CoM acceleration effect:

$$\Psi(f) = 2\pi f t_c - \frac{\pi}{4} - \Phi_c + \Psi_{\rm PN}(f,\eta) + \delta\Psi_{\rm acc}(f). \qquad (10)$$

The explicit form of $\Psi_{\rm PN}(f)$ is given in [40].

Figure 2 shows the marginalized 1σ error $\Delta Y/Y$ provided by LISA alone (solid blue) for our fiducial values of $\epsilon = 3\times10^4$, $f_{\rm min} = 0.014$ Hz, $z = 0.05$ ($d_L \simeq 200$ Mpc; well inside the horizon of both LIGO and LISA) and $\eta = 0.25$. The error is small enough to detect nonzero Y (i.e., $\Delta Y/Y < 1$) at $35 \lesssim M_{\rm cz}/{\rm M}_\odot \lesssim 63$ because the GW chirping helps break the degeneracies among the waveform parameters. When the binary frequency hardly evolves during the LISA observation, i.e., for lower $M_{\rm cz}$ (and/or $f_{\rm min}$), strong degeneracies remain and render the acceleration undetectable. For higher masses (and/or $f_{\rm min}$), the binary exits the LISA band more rapidly, diminishing the SNR.

LIGO observations during/after the LISA operation time can reduce parameter degeneracies, by detecting the merger event and fixing the coalescence time t_c. As shown by the dashed blue curve in Fig. 2, for low masses the error $\Delta Y/Y$ provided by LISA + LIGO (i.e., t_c fixed) is reduced by a factor of 3–10 from that by LISA alone. As a result, the best error estimate in this case is given by $\Delta Y/Y \simeq 0.3$ at $M_{\rm cz} = 40$ ${\rm M}_\odot$.

In Fig. 3, we show the measurement error $\Delta Y/Y$ as a function of $M_{\rm cz}$ and $f_{\rm min}$. In this case we also present the results for a LISA mission lasting 10 yr to show how they could improve: the nominal mission duration is four years but a duration in flight up to 10 years is conceivable [19]. In a suitable range of values for $f_{\rm min}$, for a five year LISA mission without any input from LIGO (top left), the acceleration effect can be detected with an error of $\Delta Y/Y < 0.5(1)$ only for relatively massive binaries with $M_{\rm cz} \gtrsim 50(40)$ ${\rm M}_\odot$. Fixing t_c with LIGO (top right) extends the same limits down to $M_{\rm cz} \gtrsim 35(25)$ ${\rm M}_\odot$. A 10 year LISA mission instead can provide errors of $\Delta Y/Y < 0.5$ for masses $M_{\rm cz} \gtrsim 25$ ${\rm M}_\odot$, even without detecting the merger with LIGO (bottom left), while for BBHs with higher masses the acceleration effect can be measured with more accuracy: $\Delta Y/Y < 0.2$ for $M_{\rm cz} \gtrsim 50$ ${\rm M}_\odot$. Fixing the merger time with LIGO (bottom right) improves these results yielding the possibility of detecting the acceleration effect with $\Delta Y/Y < 0.5$ for $M_{\rm cz}$ below 20 ${\rm M}_\odot$, while $\Delta Y/Y < 0.2$ can be reached for $M_{\rm cz} \gtrsim 40$ ${\rm M}_\odot$. Long-lived binaries with $\tau_c > 5$ (10) yrs do not chirp rapidly enough to break parameter degeneracies, whereas short-lived binaries with $\tau_c < 2$ yrs do not spend sufficient time in the LISA band to accumulate SNR. We conclude that merging BBHs with 2 yr $\lesssim \tau_c \lesssim 5(10)$ yr provide the best combinations of $f_{\rm min}$ and $M_{\rm cz}$ to probe CoM acceleration.

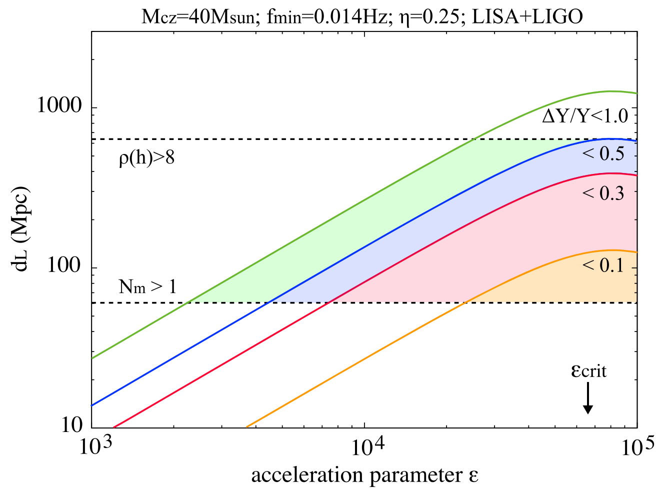

FIG. 4. GW phase drift detection conditions in the ϵ–d_L parameter space. The four solid curves mark marginalized 1σ errors $\Delta Y/Y < 0.1$ (orange), < 0.3 (red), < 0.5 (blue) and < 1.0 (green). The two horizontal dotted lines show a maximum distance ($D < 640$ Mpc) for a total SNR $\rho(h) \geq 8$ and a minimum distance ($D > 60$ Mpc) to find a BBH at $f \simeq 0.014$ Hz (i.e., $N_{\rm m} \equiv 4\pi n_{\rm m} d_L^3/3 > 1$). The maximum distance saturates and the Fisher analysis is invalid for large accelerations $\epsilon > \epsilon_{\rm crit}$, marked by an arrow [see below Eq. (9)].

Figure 4 presents the maximum distances out to which phase drifts can be measured with errors of $\Delta Y/Y < 0.1$–1.0. We consider equal-mass BBH mergers with $M_{\rm cz} = 40$ ${\rm M}_\odot$ and $f_{\rm min} = 0.014$ Hz. The horizontal dashed lines show the conditions 60 Mpc $< d_L <$ 640 Mpc discussed below Eq. (3). For $\epsilon > \epsilon_{\rm crit}$, the maximum distance does not increase linearly with ϵ because of the saturation of the relative SNR discussed below Eq. (9) and shown in Fig. 1. For merging BBHs located in the shaded region, the phase drifts in the GW inspiral waveform can be observed.

III. FORMATION SCENARIOS OF LIGO BBHS AND CORRESPONDING ACCELERATION

In this section, we review proposed stellar-mass BBH formation scenarios, from field binaries (A), dynamical formation in dense stellar systems (B) and in AGN accretion disks (C) and massive high-redshift binaries (D). We discuss the typical value of the acceleration parameter ϵ expected in each case, summarized in Table I.

A. Field binaries

A compact ($\lesssim$0.1 AU) massive stellar binary could form a BH remnant coalescing due to GW emission in a Hubble time. Such BBHs are formed in low-metallicity star

TABLE I. The first four columns show, in each BBH formation scenario, the expected center-of-mass orbital velocity (v) and radius (r), and the acceleration parameter (ϵ). Column 5 shows the maximum distance ($d_{L,\mathrm{obs}}$) at which the phase drift can be measured by LISA with an SNR of $\rho(\delta h) > 8$, corresponding to $\Delta Y/Y < 0.5$. In column 6, the number densities of the host objects are shown. Columns 7 and 8 show the number of host objects (n_{host}) and of GW events (n_{m}) in the LISA band, within the cosmic volume of $V_{\mathrm{eff}} = 4\pi d_{\mathrm{eff}}^3/3$, where $d_{\mathrm{eff}} = \min(d_{L,\mathrm{obs}}, 640\ \mathrm{Mpc})$ and $n_{\mathrm{m}} = Rf/\dot{f} \simeq 10^{-6}\ \mathrm{Mpc}^3$. Here we adopt our fiducial case: $M_{\mathrm{cz}} = 40\ \mathrm{M}_\odot$, $f_{\mathrm{min}} = 0.014$ Hz, and $\eta = 0.25$. In the three scenarios indicated by boldface, the acceleration is large and measurable.

scenario	v (km s^{-1})	r (pc)	ϵ	$d_{L,\mathrm{obs}}$ (Mpc)	n_{host} (Mpc^{-3})	$n_{\mathrm{host}}V_{\mathrm{eff}}$	$n_{\mathrm{m}}V_{\mathrm{eff}}$
Field binaries (A)							
formed at $z \simeq 0$	$\sim$200	$> 5 \times 10^3$	< 10	$\sim$0.2	$\sim 2 \times 10^{-2}$ [41]	$\ll 1$	$\ll 1$
formed at $z \simeq 3$	$\sim$300	$10^3 - 10^4$	10–100	0.2–2	$\sim 5 \times 10^{-4}$ [42]	$\lesssim$0.02	$\ll 1$
Dense stellar systems (B)							
globular clusters	$\sim$200	$\sim 5 \times 10^4$	$\sim$1	$\sim$0.02	$\sim$1 [43]	$\ll 1$	$\ll 1$
nuclear star clusters	30–100	$\sim$1	$\mathbf{10^3 - 10^4}$	20–200	$\sim$0.01 [44]	$\lesssim 3 \times 10^5$	$\lesssim$30
AGN disks (C)							
formed in disk	$\sim$200	$\sim$1	$\mathbf{\sim 10^4}$	$\sim$200	$\sim 10^{-5}$ [45]	$\sim$300	$\sim$30
captured or migrated in	$\sim$600	$\sim$0.1	$\mathbf{\sim 10^5}$	$\sim$950	$\sim 10^{-5}$	$\sim 10^4$	$\sim 10^3$
Very high-redshift (D)							
Population III	$\sim$200	$\lesssim 10^3$	10–100	0.2–2	$\sim 2 \times 10^{-2}$ [41]	$\lesssim$0.7	$\ll 1$

forming regions [5], possibly over an extended range of redshifts ($0 \lesssim z \lesssim 3$; e.g. Ref. [7]).

In the nearby universe, most star-formation occurs in disks of spiral galaxies, within their half-light radii of $\sim$5 kpc [46]. Assuming that the stars are orbiting around the center of the galaxy at the circular velocity $\sim$200 km s^{-1} of a typical disk galaxy, the acceleration parameter is $\epsilon \simeq 8$ [47]. However, LIGO BBHs are expected to arise from massive stellar binaries with $Z < 0.1\ \mathrm{Z}_\odot$ [5]. Since metallicities decrease farther out in the disk [48], BBH formation could occur preferentially at these larger radii, where the acceleration parameter is reduced to $\epsilon \sim O(1)$.

A large fraction of low-metallicity massive (binary) stars could form in high-redshift star-forming galaxies. Their host galaxies will undergo several mergers and most of these binaries may end up in the cores of massive elliptical galaxies. These old remnant BBHs would be located in the core with a typical size of a few kpc [49,50] and with the circular velocity of $v \sim 300$ km s^{-1} [51], resulting in somewhat larger accelerations of $\epsilon \simeq$ 10–100.

B. Dynamical formation in dense stellar systems

Two single BHs can be paired when they interact and form a bound binary in a dense stellar system, either through a chance close fly-by, or involving a third object. These processes likely occur in globular clusters (GCs), nuclear star clusters (NSCs) and around SMBHs in galactic nuclei.

Most GCs are in orbit inside dark matter (DM) halos with $M \simeq 10^{12}\ \mathrm{M}_\odot$, because galaxies in such a mass range contain most of the present-day stellar mass, and the number of GCs scales with their host galaxy mass [43]. The acceleration parameter is as low as $\epsilon \lesssim 1$, for the circular velocity of $v \simeq 200$ km s^{-1} at $r \sim 50$ kpc ($\sim$ half of the virial radius). On the other hand, the BBHs also orbit inside GCs, where the velocity dispersion is at most $\sim$10 km s^{-1} and half-light radii are $\sim$2–3 pc [52]. Since massive BBHs should have sunk to the center due to dynamical friction, the acceleration parameter could increase to $\epsilon \approx 100(r/\mathrm{pc})^{-1}$. However, many BBHs would be ejected from the GCs, reducing their acceleration back to values for orbits in the halo ($\epsilon \lesssim 1$).

LIGO binaries could also be formed in NSCs and/or in galactic nuclei due to mass segregation through dynamical friction [12,14]. Since the escape velocity from these systems is higher, a larger number of BBHs can remain within smaller radii of $r \sim 1$ pc with velocities of $\sim$30–100 km s^{-1}. The acceleration parameter for these binaries would be larger, $\epsilon \simeq 10^3(v/30\ \mathrm{km\,s^{-1}})^2(r/\mathrm{pc})^{-1}$.

C. Binary BH formation in AGN disks

It is possible to form BBHs, detectable by LIGO, with the help of AGN disks. They could form either from massive stellar binaries in the disk itself, at a few pc from the central SMBHs [17] or at migration traps located closer in [15]; pre-existing binaries in the 3D bulge can also be captured in the inner regions (< 1 pc) of the disk [16]. In these scenarios, SMBHs with masses of $10^{6-7}\ \mathrm{M}_\odot$ likely dominate, since they are the most numerous and most efficiently accreting SMBHs with the densest disks in the local universe [53,54]. At the location of the birth of the BBHs ($\sim$1 pc), their orbital velocity around the SMBH would be $\sim$200 km s^{-1}. The acceleration is already as high as $\epsilon \simeq 4 \times 10^3(M_\bullet/10^7\ \mathrm{M}_\odot)(r/1\ \mathrm{pc})^{-2}$. However, these binaries are expected to migrate inward through the accretion disk, and many of them may be located closer to the center when they enter the LISA band [15–17]. At a distance of 0.1 pc from the center, the Keplerian velocity increases and the acceleration parameter is $\epsilon \sim 10^5$.

D. Very high-z binaries

Finally, another scenario is massive BBH formation in extremely metal-poor environments at high redshift. The first generation of stars in the universe at $z > 10-20$, the so-called Population III (PopIII) stars, are typically as massive as ∼10–300 $M_\odot$ [55]. PopIII binaries form coalescing BBHs efficiently, which can contribute to the rate of detectable LIGO events [8], including the existing O1 detections [56]. PopIII remnants are expected to be located inside spiral galaxies like the Milky Way in the current Universe [57–59]. Cosmological N-body simulations have suggested that PopIII remnants are distributed in the bulge, with ∼0.1–1% of the remnants concentrated inside $r \lesssim 1$ kpc. In this case, the acceleration parameter is $\epsilon \simeq 10$–100.

IV. DISCUSSION AND IMPLICATIONS

Different formation scenarios of stellar-mass BBHs predict a wide range of typical acceleration parameters (see Table I). As pointed out in [30] and confirmed by our analysis, in the BBH formation scenarios with low values of $\epsilon < 100$, the effect of the CoM acceleration of merging BBHs is difficult to observe by LISA in the operation time of $\delta t \simeq 5$ yr. On the other hand, BBHs produced in NSCs and in AGN disks are expected to have large and measurable accelerations, with $\epsilon > 10^3$. Moreover, the number density of the NSCs ($n_{\rm NSC} \simeq 10^{-2}$ Mpc^{-3} [44]) and AGN ($n_{\rm AGN} \simeq 10^{-5}$ Mpc^{-3} [45]) are higher than the number density of merging BBHs at $f \simeq 14$ mHz, $n_{\rm m}(= Rf/\dot{f}) \simeq 10^{-6}$ Mpc^{-3}, inferred from the existing LIGO detections. Thus, LISA will likely observe phase drifts in the GW inspiral waveform if these scenarios contribute significantly to the total event rate.

Conversely, if no acceleration is detected among a total of $N_{\rm tot}$ events in the region of measurable parameter space, then this requires that other formation channels, not involving SMBHs are dominant. To illustrate this quantitatively, let us consider BBHs with intrinsic acceleration ϵ, and suppose that line-of-sight accelerations $\epsilon_{\rm los} = |\cos\theta|\epsilon \geq \epsilon_{\rm obs}$ are detectable. Assuming that θ, the angle between the line of sight to the BBH and the BBH's instantaneous acceleration vector, has an isotropic distribution, we would expect $N_{\rm det} = f(\epsilon)N_{\rm tot}[1-\epsilon_{\rm obs}/\epsilon]$ events with measurable acceleration (i.e., within double cones with $|\cos\theta| \geq \epsilon_{\rm obs}/\epsilon$), where $f(\epsilon)$ is the fraction of events with a 3D acceleration above ϵ. Setting $N_{\rm det} < 1$ yields the upper limit $f(\epsilon) < N_{\rm tot}^{-1}[1-\epsilon_{\rm obs}/\epsilon]^{-1}$. For example, if we have $N_{\rm tot} = 100$ events and the sensitivity was $\epsilon_{\rm obs} = 10^4$, then at most 3% of BBHs could have $\epsilon > 1.5 \times 10^4$, or be located within $\lesssim 0.5\ {\rm pc}(M_\bullet/10^7\ M_\odot)^{1/2}$ from SMBHs.

The sky position and the distance to merging BBHs for $\delta t > 2$ yr with a high SNR ($\rho \gtrsim 10$) can be estimated by LISA alone to a statistical accuracy of $\Delta\Omega_{\rm s} \simeq 1.2 f_{14}^{-2}(\rho/10)^{-2}$ deg^2 and $\Delta d_L/d_L \simeq 0.2(\rho/10)^{-1}$ [60,61]. The corresponding error volume is given by $\Delta V = d_L^2 \Delta d_L \Delta\Omega \simeq 9.6 \times 10^3 f_{14}^{-2}(\rho/10)^{-6}$ Mpc^3. Note that $\rho \simeq 10(d_L/510\ {\rm Mpc})^{-1}$. Since AGN are rare objects, with abundance of a few $\times 10^{-5}$ Mpc^3 [53,54], the number of random interloping AGN within the error volume is well below unity even for $\rho = 8$. This means that the AGN hosts can be identified uniquely from LISA observations alone, without EM counterparts. By comparison, the advanced LIGO-Virgo O3 observing run can achieve a 3D error volume of $\sim 10^5$ Mpc^3 or better only for $< 10\%$ of merging BBHs with $30+30\ M_\odot$ [62]. This still allows a secure identification of the connection with AGN hosts, but only statistically [63]. In the NSC scenario, the LISA error volume contains several candidate host galaxies even for relatively high SNR, $\rho > 15$ ($d_L < 340$ Mpc), so that one would have to resort to a statistical correlation between LISA events and NSCs.

The LISA data predict the coalescence time of BBHs with an error of < 10 s [18]. However, this prediction would be biased due to the CoM acceleration of the BBHs [30]. This bias has to be taken into account for any advance planning of follow-up EM observations of the merging BBHs. The phase drifts caused by the acceleration could be partially mimicked by a slight change in the mass ratio and time of coalescence. However, our Fisher analysis indicates that for sources with $35\ M_\odot \lesssim M_{\rm cz} \lesssim 63\ M_\odot$ which chirp inside the LISA band for 2–5 yrs, and especially for those whose eventual merger is detected by LIGO, these degeneracies are mitigated, and a measurement of the acceleration remains viable (see Figs. 3 and 4). Such a measurement will robustly test formation channels of coalescing stellar-mass BBHs involving a SMBH in a galactic nucleus.

The possibility of measuring the CoM acceleration of a merging BBH due to a nearby SMBH has been previously discussed for extreme mass ratio inspirals ($10^{5-6}\ M_\odot + 10\ M_\odot$) in the LISA [29] band, and for stellar-mass BBHs in the LIGO band [64]. In the latter case, detection of the phase drift of the BBH during the handful of orbits executed inside the LIGO band requires an extremely close separation between the BBH and the SMBH ($\sim 10^{11}$ cm); these rare cases of extremely close-in binaries would however provide the opportunity to measure several other relativistic effects.

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