One-Bit Digital Radar

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***Abstract*—This paper introduces a one-bit digital radar involving direct one-bit sampling with unknown dithering of the received radio frequency (RF) signal. Due to avoiding the analog mixer and the down-conversion of the RF signal, the digital radar can be energy-efficient and low-priced. The use of unknown dithering allows for the one-bit samples to be processed efficiently using conventional algorithms. A computationally efficient range-Doppler estimation method based on fractional Fourier transform (FRFT) and fast Fourier transform (FFT) is used for linear frequency modulated continuous wave (LFMCW) transmissions, and the CLEAN algorithm is used for target parameter estimation.**

Keywords*—*One-Bit Digital Radar, Range-Doppler Estimation, Bandpass Sampling, Fractional Fourier Transform.

I. Introduction

In recent years, the demand for low-cost, low-power radar systems has increased significantly [1]–[3]. For instance, automotive radars [1], [2] need to be as inexpensive as possible and Google's hand gesture recognition radars [3] must be small and have low power consumption. The direct one-bit RF sampling strategy employed in digital radar is a promising approach to meet the demand. The goal of a digital radar is to place the analog-to-digital converter (ADC) as early in the receiver as possible [4]. According to the theory of bandpass sampling [5], the required sampling rate can be much lower than the carrier frequency. Indeed, the sampling rate needs only to be slightly more than twice the signal bandwidth. The down-conversion of the RF signal can be automatically realized within the bandpass sampling process via intentional aliasing [4], [5].

In many cases, the transmitted signal occupies a quite wide bandwidth to achieve a high range resolution. Therefore, direct RF sampling in a digital radar requires a high sampling frequency [2], [3]. Since the conventional high precision quantization can be impractical at the required RF sampling rate due to the high cost and high power consumption concerns, a tradeoff between the sampling rate and quantization precision is encountered [6]–[9]. One-bit sampling, which is the most extreme form of quantization, can be a good alternative to the conventional high precision ADC. The main advantage of one-bit sampling is that it is very inexpensive and energy-efficient even at a very high sampling rate, allowing for an affordable digital radar system.

Problems involving one-bit sampling have been studied from many aspects [10]–[12]. Most of the recent works on one-bit sampling are focused on comparing the signal to zero, but then the signal energy cannot be recovered accurately [13]. In several recent papers, the one-bit sampling with a known time-varying threshold has been proposed to achieve accurate parameters, including amplitude, estimation of the original signal [6]–[9]. However, these methods solve complicated optimization problems and have high computational complexities. As early as in the 1990s, Francesschetti et al. have shown that the one-bit Gaussian dithered samples can be processed by using conventional efficient methods proposed for infinite precision samples under certain conditions [14], [15]. The dithering introduces unknown time-varying thresholds. Compared to known time-varying thresholds, the performance of using dithering is expected to be worse since dithering trades off higher uncertainty for lower bias. Yet unknown dithering allows for an acceptable performance at a low computational complexity.

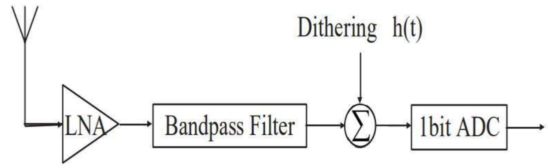

Fig. 1. RF sampling front end of one-bit digital radar with unknown dithering.

In this paper, we introduce the notion of one-bit digital radar with unknown dithering. The conventional radar measurement scheme [1], [2] is replaced by the direct one-bit RF sampling in the one-bit digital radar. The inexpensive and flexible receiver with minimal front-end components is shown in Fig. 1. We consider both Gaussian and uniform dithering whose probability density function is assumed known. The notion of one-bit digital radar is for arbitrary waveform transmissions. However, our range-Doppler estimation algorithms are based on LFMCW transmissions. Via transmitting periodic chirp sequences, radar targets can be resolved in both range and radial velocity simultaneously [1], [2]. Since the one-bit dithered RF samples can be approximated as a superposition of several scaled and Doppler shifted chirp sequences for this kind of LFMCW transmissions, we perform range-Doppler estimation using the FRFT and the FFT. The CLEAN algorithm [16]–[18] is used to estimate the parameters of each target. Numerical examples are used to demonstrate the performance of the proposed one-bit digital radar and the estimation algorithms.

Notation: $\lfloor a \rfloor$ denotes the largest integer less than or equal to a. $\langle b \rangle$ denotes the expectation of the random variable b. ${}_1F_1$ denotes the confluent hypergeometric function. $\mathbf{S} = [s(n,l)]$ means the $(n,\ l)$th element of $\mathbf{S}$ is $s(n,l)$.

II. Direct RF Sampling

As shown in Fig. 1, there is no de-chirping in a direct RF sampling approach. Rather, the down-conversion of the

RF signal is realized within the bandpass sampling process with intentional aliasing. Bandpass sampling is a technique of sampling the bandpass signal located between f_L and f_U at a sampling rate based on the information bandwidth [5] seatisfying the following condition,

$$\left\lfloor \frac{f_L}{F_s/2} \right\rfloor = \begin{cases} 2p & 0 < f_0 < \frac{F_s}{2} - B \\ 2p-1 & -\frac{F_s}{2} < f_0 < -B \end{cases} \tag{1}$$

where $f_0 = f_L - pF_s$, $B = f_U - f_L$, and p is an integer. Therefore, bandpass sampling can preserve the information with an attractive sampling rate which is not much higher than twice of the information bandwidth B.

III. One-bit Dithered Sampling

A. One-Bit Sampling without dithering

Conventional one-bit sampling methods is usually a comparator that compares the signal to zero. Assume that the received signal due to multiple target reflections can be expressed as [14], [15]:

$$y(t) = A(t)\cos\left[\omega t + \phi(t)\right], \tag{2}$$

where ω is an intermediate frequency and $A(t)$ varies slowly compared with $\phi(t)$ [14]. Then the one-bit samples obtained by using zero as threshold can be decomposed as a superposition of the odd harmonics $m\omega$ of the original signal [14], [15]:

$$\begin{aligned} v(t) &= \operatorname{sign}(y(t)) = \sum_{m=0}^{\infty} \varepsilon_m \frac{j^{m+1}}{\pi} A_m(t)\cos\left[m\omega t + m\phi(t)\right] \\ &\triangleq \sum_{m=0}^{\infty} v_m(t), \end{aligned} \tag{3}$$

where $\varepsilon_0 = 1$, $\varepsilon_m = 2$ for $m \neq 0$,

$$A_m(t) = \begin{cases} \operatorname{sign}(A(t))/m, & m \text{ is odd} \\ 0, & m \text{ is even.} \end{cases} \tag{4}$$

Apparently, the amplitude, including the relative amplitude, information is lost with one-bit sampling without dithering.

B. One-Bit Sampling with Gaussian Dithering

To recover the amplitude information from sign measurements, one-bit sampling with unknown dithering can be considered. As early as in 1991, Franceschtti et al. considered adding a Gaussian noise $n(t)$ with zero-mean and variance σ^2 to the original signal $y(t)$ before comparing the dithered signal to zero [14], [15]. Since $n(t)$ is a Gaussian random variable, the mean value of the one-bit samples $v(t) = \operatorname{sign}(y(t) + n(t))$ is [14], [15]:

$$\langle v(t)\rangle = \sum_{m=0}^{\infty} \varepsilon_m \frac{j^{m+1}}{\pi} \langle A_m(t)\rangle \cos[m\omega t + m\phi(t)]. \tag{5}$$

When $A(t)^2 \big/ 2\sigma^2 \ll 1$, for odd m,

$$\begin{aligned} \langle A_m(t)\rangle &= \frac{\Gamma\left(\frac{m}{2}\right)}{\Gamma(m+1)} \left(\frac{A(t)}{\sqrt{2}\sigma}\right)^m \cdot {}_1F_1\left(\frac{m}{2}, m+1, -\frac{A(t)^2}{2\sigma^2}\right) \\ &\approx c_m \left(\frac{A(t)}{\sigma}\right)^m, \text{ where } \mathrm{c_m} = \frac{2\sqrt{\pi}}{2^{\frac{3m}{2}} \mathrm{m} \left(\frac{\mathrm{m}-1}{\mathrm{m}}\right)!}, \end{aligned} \tag{6}$$

and for even m, $\langle A_m(t)\rangle = 0$.

Note that $c_1 = \sqrt{\frac{2}{\pi}}$, $c_3 = -\frac{c_1}{24}$, $c_5 = \frac{c_1}{640}$. It is clear that the harmonics decrease rapidly with the harmonic order m. Therefore, the mth ($m > 1$) harmonic can be neglected and the first harmonic contains the original signal with a scaling factor associated with the variance σ^2 of the Gaussian keep. The one-bit Gaussian dithered samples can be processed efficiently by using the conventional algorithm proposed for the infinite precision samples when σ^2 is sufficiently large.

C. One-Bit Sampling with Uniform Dithering

Assuming that $h(t)$ is a uniformly distributed random variable between $-b$ to b. When $b \geq \max_t \{y(t)\}$, the mean value of one-bit samples with uniform random dithering at time t can be expressed as follows:

$$\begin{aligned} \langle v(t)\rangle &= \langle \operatorname{sign}(y(t)+h(t))\rangle = \int_{-b}^{+b} \frac{1}{2b} \operatorname{sign}(y(t)+h(t))\,dh(t) \\ &= \frac{y(t)}{b}. \end{aligned} \tag{7}$$

Apparently, the original signal $y(t)$ can be recovered from the mean-value of the uniformly dithered one-bit samples for a sufficiently large b.

IV. Range-Doppler Estimation via One-Bit Digital Radar

A. Radar Signal Model

Periodic chirp sequences are widely used to measure the target range and radial velocity parameters [1], [2]. Fig. 2 shows such a periodic chirp sequence with period T. The coherent processing interval (CPI) is assumed to contain L periods. For the carrier frequency f_c and bandwidth B, one period of the transmitted signal can be expressed as:

$$s_t(t) = \cos\left[2\pi\left(\left(f_c - \frac{B}{2}\right)t + \frac{B}{2T}t^2\right)\right], 0 \leq t \leq T. \tag{8}$$

The received RF signal can be approximately described by the following equation:

$$s(t,l) \approx \sum_{k=1}^{K} g_k \cos\left[2\pi\left(\phi_k + \left(f_c - \frac{B}{2} - f_{B_k}\right)t + \frac{B}{2T}t^2 + f_{D_k} lT\right)\right], \tag{9}$$

where $f_{B_k} = f_{R_k} - f_{D_k} = \frac{2BR_k}{Tc} + \frac{2f_c v_k}{c}$ is the beat frequency related to the target range related frequency f_{R_k} and Doppler

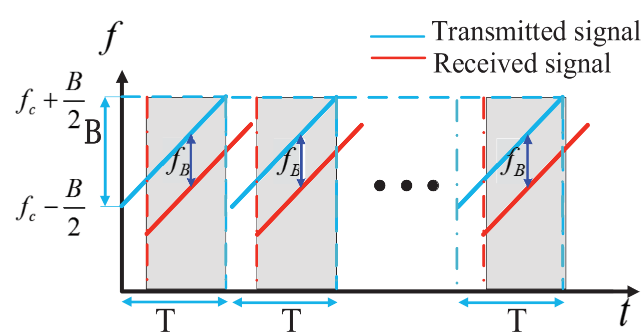

Fig. 2. Transmitted periodic chirp sequences.

frequency f_{D_k}, ϕ_k is the phase information, (R_k, v_k) and g_k are, respectively, the (range, radial velocity) and the reflection coefficient of the kth target, and K is the number of targets.

According to [14], the received signal takes the form of (2) approximately. The one-bit RF samples under Gaussian and uniform dithering, respectively, can be approximated by the first harmonic of its mean value and the mean value itself, with proper scaling, as:

$$s(n,l) \approx \sum_{k=1}^{K} \beta g_k \cos\left[2\pi\left(\phi_k + (f_0 - f_{B_k}) n T_s + \frac{B}{2T}(nT_s)^2 + f_{D_k} l T\right)\right], \tag{10}$$

where $f_0 = f_c - \frac{B}{2} - pF_s$, $p = \left\lfloor \frac{f_c - B/2}{F_s} + \frac{1}{2} \right\rfloor$, F_s is the sampling frequency, $T_s = 1/F_s$, and β is a scaling factor. The fast-time index n, $n = 0, 1, \cdots, N-1$, corresponds to the nth sample within a period, and the slow-time index l, $l = 0, 1, \cdots, L-1$, is the lth period. Due to Doppler shift, the received signal can have a bandwidth, say $\widetilde{B}$, slightly larger than the bandwidth B of the transmitted signal. Therefore, $\widetilde{B}$ should be used in lieu of the B in (1) to determine the proper F_s.

B. Range Doppler Estimation Based on FRFT and FFT

The signal model for the one-bit digital radar is approximated as a superposition of K chirp sequences. Thus, we consider a computationally efficient range-Doppler estimation method based on fractional Fourier transform (FRFT) [19] and FFT.

Fractional Fourier transform is an efficient method for chirp parameter estimation [19]. Following from [20], the energy of a chirp signal, whose chirp rate is μ, can be concentrated only in a proper fractional domain satisfying $\cot\left(\frac{a\pi}{2}\right) = -\mu$. Therefore, we choose the order $a = -\frac{\pi}{2}\text{acot}\left(\frac{B}{T}\right)$ and apply ath FRFT across the fast-time to obtain the beat frequency f_B by $f_B = f_0 - u\csc\frac{\pi a}{2}$, where u is the fractional Fourier domain. This FRFT can be implemented by Discrete FRFT (DFRFT) which computes the FRFT by FFT in $O(N\log N)$ complexity [21]. Then, the Doppler frequency f_D can be measured by applying FFT across the slow-time. Since N is always larger than L, the computational complexity of the proposed estimation method based on FRFT and FFT should be $O(NL\log N)$.

Apparently, the range and radial velocity can be calculated from the beat frequency f_B and Doppler frequency f_D as follows:

$$R = \frac{Tc}{2B}(f_B + f_D), \; v = -\frac{cf_D}{2f_c}. \tag{11}$$

The range resolution is determined by the bandwidth B and the velocity resolution is decided by the overall measurement time TL:

$$\Delta R = \frac{c\csc\frac{a\pi}{2}}{2B}, \; \Delta v = \frac{1}{LT}\frac{\lambda}{2}, \tag{12}$$

where $\lambda = c/f_c$. Since $\csc\left(\frac{a\pi}{2}\right)$ is very close to 1, the proposed method has a similar resolution to that of the conventional LFMCW radar.

The CLEAN technique can be used to accomplish the target parameter estimation following the steps similar to [16]–[18].

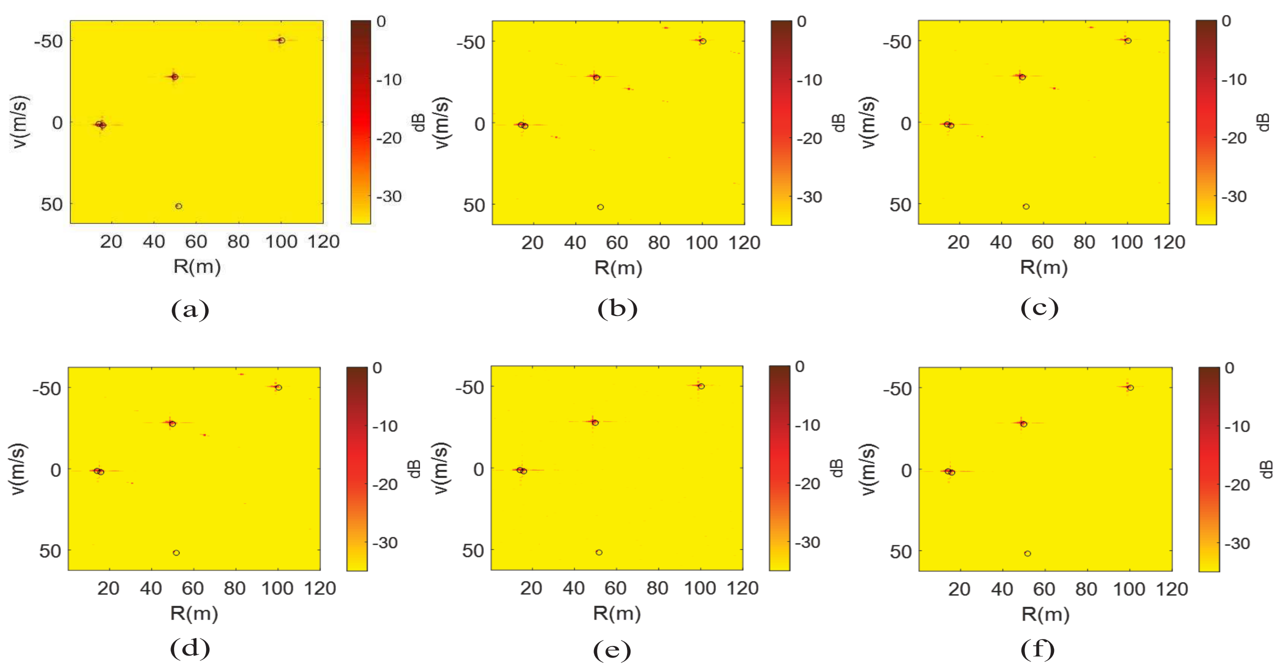

Fig. 3. The estimation results by FRFT+FFT: (a) infinite precision samples; (b) one-bit samples without dithering; (c) one-bit Gaussian dithered samples at SNR=10 dB; (d) one-bit uniformly dithered samples with $b = 3$; (e) one-bit Gaussian dithered samples at SNR=-10 dB; (g) one-bit uniformly dithered samples with $b = 12$.

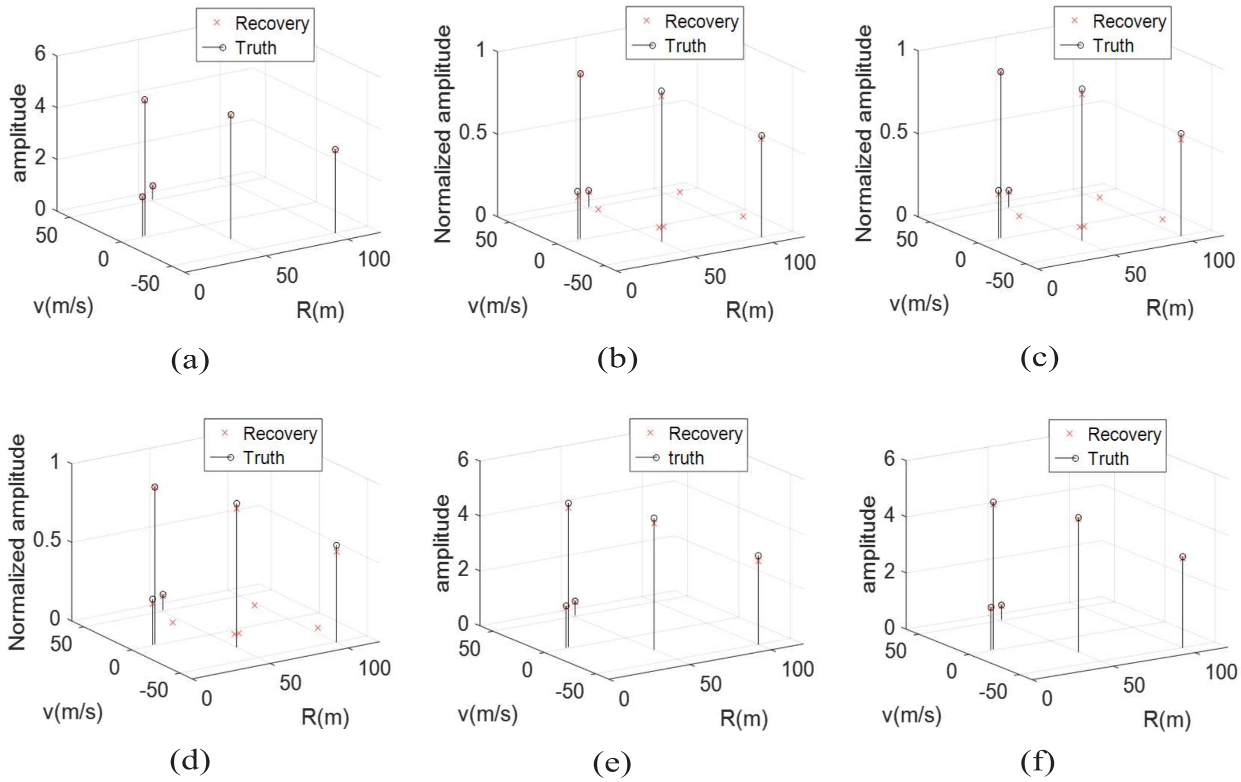

Fig. 4. The estimation results by the CLEAN technique: (a) infinite precision samples; (b) one-bit samples without dithering; (c) one-bit Gaussian dithered samples at SNR=10 dB; (d) one-bit uniformly dithered samples with $b = 3$; (e) one-bit Gaussian dithered samples at SNR=-10 dB; (g) one-bit uniformly dithered samples with $b = 12$.

V. Numerical Examples

In this section, we use numerical examples to evaluate the performance of the proposed one-bit digital radar system and the parameter estimation methods. Consider a scenario with five targets with ranges $d_1 = 10\sqrt{2}$ m , $d_2 = 16$ m, $d_3 = 50$ m , $d_4 = 50+\sqrt{3}$ m, $d_5 = 100.16$ m and with radial velocities $v_1 = \sqrt{2}$ m/s, $v_2 = 2$ m/s, $v_3 = -16\sqrt{3}$ m/s, $v_4 = 50+\sqrt{3}$ m/s , $v_5 = -50$ m/s. The amplitudes are $g_1 = 1.523$, $g_2 = 5.245$, $g_3 = 4.789$, $g_4 = 0.519$, $g_5 = 3.231$. The carrier frequency is f_c =60.09 GHz, the sweep bandwidth is $B = 180$ MHz, and the chirp duration is $T = 20$ μs. The chirp sequence has $L = 256$ periods and the sampling frequency is $F_s = 408$ MHz. The range resolution is 0.932 m (csc $\frac{a\pi}{2}$=-1.118) and the radial velocity resolution is 0.488 m/s. The range of unambiguous velocity [2] is $[-\frac{1}{T}\frac{\lambda}{4}, \frac{1}{T}\frac{\lambda}{4}] = [-62.5, 62.5]$ m/s, which is large enough for automotive radar applications. All results are obtained by an ordinary PC with an Intel Core i7 3.40 GHz CPU and 64.0 GB RAM.

Fig. 3 and Fig. 4 show the estimation results obtained via FRFT+FFT and CLEAN, respectively, using one-bit RF samples with Gaussian and uniform dithering. It is readily observed that the one-bit samples without dithering cannot be used to recover accurate amplitude information. Due to the presence of harmonics, many ghost targets exist. In contrast, the amplitudes of the targets can be accurately estimated from the one-bit Gaussian dithered samples at a low SNR when the variance σ is known. And the uniformly dithered one-bit samples can also detect all targets accurately when b is sufficiently large. Moreover, the CLEAN technique can be used to estimate the target parameters even for weak targets. The average CPU time for FRFT+FFT and CLEAN are, respectively, 2.7258s and 20.6474s.

VI. Conclusion

We have introduced a notion of one-bit digital radar that is energy-efficient and inexpensive. Through Gaussian or uniform dithering, the one-bit samples can be efficiently processed by using conventional methods proposed for infinite precision samples under certain conditions. A computationally efficient range-Doppler estimation method based on FRFT and FFT is considered and the CLEAN algorithm is utilized to estimate the parameters of each target. Numerical examples have been used to demonstrate the effectiveness of the proposed one-bit digital radar and the range-Doppler and target parameter estimation methods.

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