Experimental demonstration of loop state-preparation-and-measurement tomography

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Abstract: We have demonstrated, using qubits encoded in the polarization of heralded individual photons, that loop state-preparation-and-measurement tomography is capable of detecting correlated errors between the preparation and the measurement of a quantum system.
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1. Introduction

Quantum tomography is an important tool for characterizing small quantum systems, and is useful for quantum information processing applications. Quantum-state tomography (QST) estimates the state of a quantum system, while quantum-detector tomography (QDT) estimates the positive-operator-valued measure (POVM) that describes a detector. Here we describe loop (or non-holonomic) state-preparation-and-measurement (SPAM) tomography, which attempts to estimate both the state and measurement parameters in a self-consistent manner [1,2]. The Hilbert space dimension is assumed to be known, and measurements are performed as both the state and detector settings are varied. The data is analyzed to look for self-consistency, which allows one to determine if there are correlations between the state preparations and the measurements [1]. Finally, if no correlated SPAM errors are found it is then possible to estimate the states using information about the detectors, or vice versa.

We have performed experiments demonstrating that loop SPAM tomography is capable of detecting correlated errors in the preparation and measurement of qubits encoded in the polarizations of individual photons [2].

2. Theory

Suppose we have a source that can be prepared in states that are described by density operators $\hat{\rho}_a$, where the subscript labels the different possible state preparations. We also have a detector that is described by the POVM elements $\hat{\Pi}^i$, where the superscript labels the different possible measurements. The probability p_a^i of a detection is given by the Born rule $p_a^i = \mathrm{Tr}\left(\hat{\rho}_a \hat{\Pi}^i\right)$. This equation can be generalized to any observable (Hermitian operator) $\hat{\Sigma}^i$, as $S_a^i = \mathrm{Tr}\left(\hat{\rho}_a \hat{\Sigma}^i\right)$, where S_a^i is the expectation value of the observable. We will consider S_a^i to be the element in the a'th row and i'th column of matrix $\overline{S}$ (the overbar indicates a quantity expressed as a matrix).

In real experiments neither the state preparations $\hat{\rho}_a$, nor the observables $\hat{\Sigma}^i$ can be reproduced with perfect precision, and we must average over the fluctuations (denoted by < >). If there are no correlations between the state preparation and the observables, then there is no problem in using measurements of $\left\langle S_a^i \right\rangle$ and a tomographic inversion to estimate either $\left\langle \hat{\rho}_a \right\rangle$ or $\left\langle \hat{\Sigma}^i \right\rangle$. However, if there are correlations then

$$\left\langle S_a^i \right\rangle = \left\langle \mathrm{Tr}\left(\hat{\rho}_a \hat{\Sigma}^i\right) \right\rangle \neq \mathrm{Tr}\left(\left\langle \hat{\rho}_a \right\rangle \left\langle \hat{\Sigma}^i \right\rangle\right) , \tag{1}$$

so we cannot estimate $\left\langle \hat{\rho}_a \right\rangle$ or $\left\langle \hat{\Sigma}^i \right\rangle$ individually. The first question that loop SPAM tomography addresses is the detection of such correlated errors, with the only assumption being that the system dimensions are known.

Consider the case where $n = 3$ is the number of independent state and detector parameters. Measurements are performed with $M = 2n = 6$ different state preparations and $N = 2n = 6$ detector settings. The 6x6 matrix of expectation values $\overline{S}$ can be partitioned into corners consisting of 3x3 matrices as follows

$$\overline{S} = \begin{pmatrix} \overline{A} & \overline{B} \\ \overline{C} & \overline{D} \end{pmatrix} . \tag{2}$$

The rows of $\overline{S}$ refer to a fixed state preparation, while its columns refer to a fixed detector setting. The *n*x*n* matrix $\overline{A}$ consists of enough measurements to be tomographically complete, but because we don't know either the state preparations or the measurements, it is not possible to use tomography to uniquely determine the states or the detector settings. However, matrix $\overline{A}$ is connected to matrix $\overline{B}$ in the sense that they share a common set of state preparations, and the measured matrix elements of $\overline{B}$ must be consistent with that fact. Furthermore, matrices $\overline{C}$ and $\overline{D}$ share state and detector settings with $\overline{A}$ and $\overline{B}$, and they must be consistent with that fact.

Define the partial determinant of $\overline{S}$ as $\Delta\left(\overline{S}\right) \equiv \overline{A}^{-1}\overline{B}\overline{D}^{-1}\overline{C}$. It can be shown that the measured data are internally consistent as described above, and free of correlated SPAM errors under the condition that $\Delta\left(\overline{S}\right) = \overline{1}$, where $\overline{1}$ is the 3x3 identity matrix [1]. Thus, to determine if there are any correlated SPAM errors present we construct the matrix of expectation values $\overline{S}$ as given in Eq. (2), and then calculate the partial determinant $\Delta\left(\overline{S}\right)$. If $\Delta\left(\overline{S}\right) - \overline{1} = 0$, to within the statistical errors of the measurements, there is no evidence for correlated SPAM errors.

3. Experiments

We have prepared pure and mixed polarization states of heralded single photons produced by a parametric down conversion source. The different state preparations and measurement settings are implemented by rotating quarter- and half- wave plates. In Fig. 1(a)-(c) we show measurements for the mean and standard deviation of $\Delta\left(\overline{S}\right) - \overline{1}$ and the ratio of these two quantities, for an experiment in which we expect no SPAM correlations. Since all of the matrix elements of $\Delta\left(\overline{S}\right) - \overline{1}$ are 0 to within half of a standard deviation [Fig. 1(c)], no correlated SPAM errors were detected. Figure 1(d)-(f) shows measurements when a correlated error has been placed in matrix element S_1^1, and we see that $\Delta\left(\overline{S}\right) - \overline{1}$ differs from 0 by 48 standard deviations [Fig. 1(f)] and this error is detected.

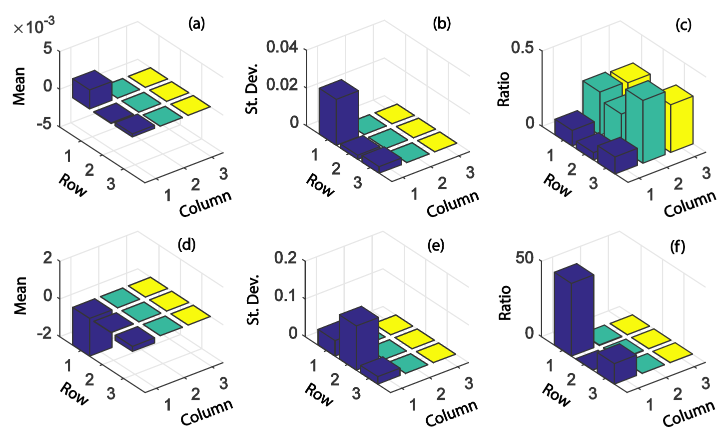

Fig. 1. (a) and (d) the mean of $\Delta\left(\overline{S}\right) - \overline{1}$, (b) and (e) the standard deviation of $\Delta\left(\overline{S}\right) - \overline{1}$, and (c) and (f) the absolute value of the ratio of these two quantities (mean divided by standard deviation). In (a)-(c) there are no SPAM correlations, while in (d)-(f) there are SPAM correlations.

Furthermore, we have performed experiments that demonstrate we can determine the location of a correlated error, not just its presence. We have also explored the precision with which we can detect correlated errors.

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4. References

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