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To cite this article: T Langen *et al* 2018 *New J. Phys.* **20** 023034

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New Journal of Physics

The open access journal at the forefront of physics

Deutsche Physikalische Gesellschaft 

IOP Institute of Physics

Published in partnership with: Deutsche Physikalische Gesellschaft and the Institute of Physics



PAPER

OPEN ACCESS

RECEIVED
19 September 2017

REVISED
18 December 2017

ACCEPTED FOR PUBLICATION
25 January 2018

PUBLISHED
15 February 2018

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Double light-cone dynamics establish thermal states in integrable 1D Bose gases

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Keywords: thermalization, non-equilibrium dynamics, spin-charge coupling, one-dimensional Bose gases, light-cone dynamics, generalized Gibbs ensemble, atom chips

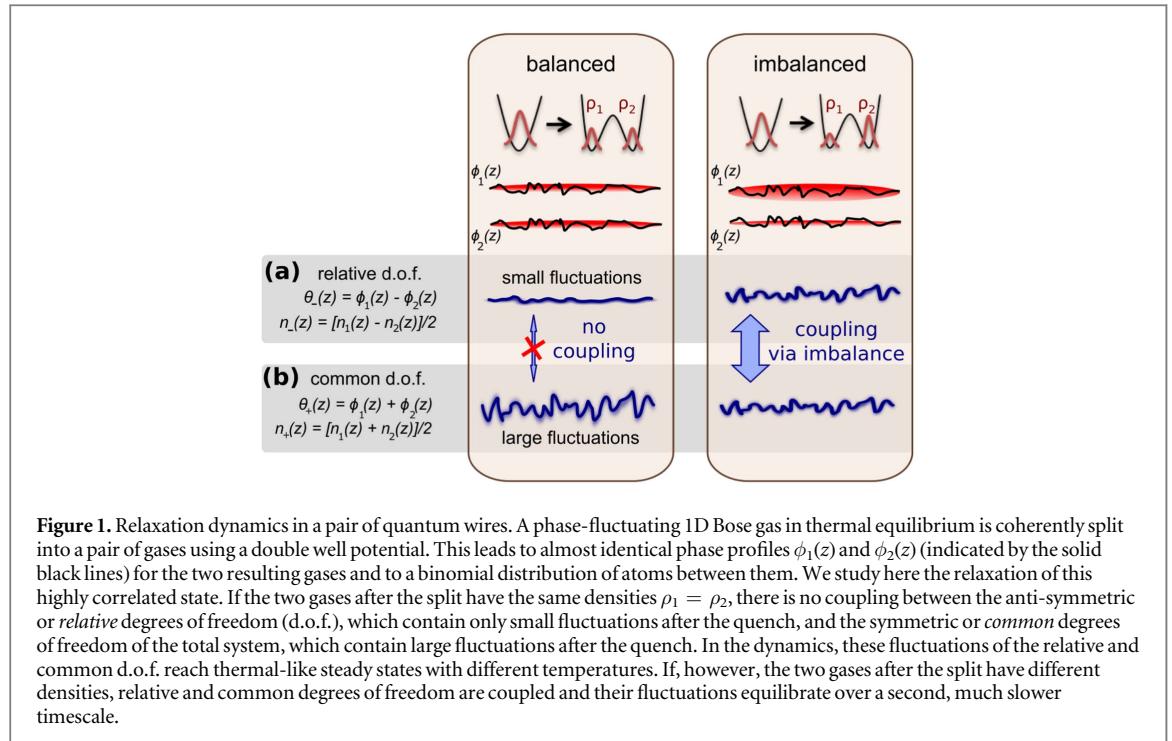
Abstract

We theoretically investigate the non-equilibrium dynamics in a quenched pair of one-dimensional Bose gases with density imbalance. We describe the system using its low-energy effective theory, the Luttinger liquid model. In this framework the system shows strictly integrable relaxation dynamics via dephasing of its approximate many-body eigenstates. In the balanced case, this leads to the well-known light-cone-like establishment of a prethermalized state, which can be described by a generalized Gibbs ensemble. In the imbalanced case the integrable dephasing leads to a state that, counter-intuitively, closely resembles a thermal equilibrium state. The approach to this state is characterized by two separate light-cone dynamics with distinct characteristic velocities. This behavior is a result of the fact that in the imbalanced case observables are not *aligned* with the conserved quantities of the integrable system. We discuss a concrete experimental realization to study this effect using matterwave interferometry and many-body revivals on an atom chip.

1. Introduction

Non-equilibrium dynamics of isolated quantum systems play a central role in many fields of physics [1]. An important question in this context is whether the unitary evolution can lead to the emergence of thermal properties. For example, the eigenstate thermalization hypothesis conjectures that dephasing can lead to thermalization in systems with a chaotic classical limit [2–5]. On the other hand, integrable or many-body localized systems are expected not to thermalize at all [6], but instead relax to generalized thermodynamical ensembles [7–10]. An important role in both cases is played by the signal propagation during the non-equilibrium dynamics. It has been shown for many systems [11–13] that this propagation follows a *light-cone-like* linear evolution in time with a characteristic velocity. This behavior has important consequences for the growth of entanglement in such systems [5, 14].

In this manuscript, we study the dynamics in a pair of bosonic 1D quantum gases with number imbalance using the Luttinger liquid formalism. The dynamics of such *quantum wires* has recently been studied in great detail using atom chips [10, 13, 15–17] or in optical lattices [18–22]. In particular, 1D Bose gases have been established as a prime experimental realization of a nearly integrable system with strongly suppressed thermalization [6, 15, 21]. So far, the consequences of this integrable behavior have mainly been studied for individual gases or sets of nearly identical gases. Here, we show that for imbalanced pairs of gases, i.e. gases that differ in their mean density, integrable dephasing can establish a state that closely resembles thermal equilibrium. The dynamics towards this state are found to be exceptionally rich, including a metastable thermal-like state described by a generalized Gibbs ensemble and two distinct light-cone dynamics, each exhibiting their individual characteristic velocity. Beyond the fundamental interest in thermalization dynamics, our study is of



high relevance for ongoing experiments, where density imbalances are often unavoidable and can thus fundamentally affect the interpretation of the results.

2. Model

In the following we consider an initial 1D Bose gas in thermal equilibrium that is coherently split into two gases (figure 1). This scenario is motivated by experiments with 1D Bose gases on atom chips, which have recently been established as an important model system to study non-equilibrium dynamics [10, 13, 15, 23, 24]. In these experiments gases are confined in two radial directions using a strong potential characterized by the trapping frequency ω_\perp , such that $\mu, k_B T < \hbar\omega_\perp$ and they behave effectively one-dimensional. Here, μ is their chemical potential and T their temperature.

The effective low-energy description for each of the gases is given by the Luttinger liquid Hamiltonian

$$\hat{H}_i = \int dz \left[\frac{\hbar^2 \rho_i}{2m} (\nabla \hat{\phi}_i(z))^2 + \frac{g}{2} (\hat{n}_i(z))^2 \right]. \quad (1)$$

Here, the index $i = 1, 2$ labels the individual gases with mean densities ρ_i independent of z (homogeneous case). We denote the density fluctuations around this mean densities by $\hat{n}_i(z)$, the phase fluctuations by $\hat{\phi}_i(z)$. Phase and density fluctuations represent conjugate variables. The 1D interaction strength is characterized by $g = 2\hbar a_s \omega_\perp$, with the 3D scattering length a_s .

The resulting total Hamiltonian of the system is given by $\hat{H} = \sum_{i=1,2} \hat{H}_i$. To analyze this Hamiltonian we transfer it into a basis formed by the symmetric and anti-symmetric superpositions of the individual fluctuations:

$$\begin{aligned} \hat{n}_\pm(z) &= [\hat{n}_1(z) \pm \hat{n}_2(z)]/2 \\ \hat{\theta}_\pm(z) &= [\hat{\phi}_1(z) \pm \hat{\phi}_2(z)]. \end{aligned} \quad (2)$$

This basis transformation allows us to directly connect our results to experiments, which probe the relative phase $\hat{\theta}_-(z)$ (i.e. the anti-symmetric degrees of freedom) between the gases through matterwave interference [13, 15, 25]. After this basis transformation we can write the total Hamiltonian as [26]

$$\hat{H} = \hat{H}_- + \hat{H}_+ + \hat{H}_c, \quad (3)$$

where \hat{H}_- and \hat{H}_+ are again LL Hamiltonians

$$\hat{H}_\pm = \int dz \left[\frac{\hbar^2(\rho_1 + \rho_2)}{8m} (\nabla \hat{\theta}_\pm(z))^2 + g(\hat{n}_\pm(z))^2 \right]. \quad (4)$$

We note the change in pre-factors, i.e. the factor 1/2 that appears in addition to the mean density $(\rho_1 + \rho_2)/2$ in the first term and the appearance of g instead of $g/2$ in the second term of H_\pm , as compared to the original Luttinger Hamiltonians describing the individual gases (equation (1)). These changes arise due to our specific definition of the symmetric and anti-symmetric degrees of freedom in equation (2). Coupling between the new degrees of freedom is mediated by

$$\hat{H}_c = \int dz \left[\frac{\hbar^2(\rho_1 - \rho_2)}{4m} (\nabla \hat{\theta}_+(z) \nabla \hat{\theta}_-(z)) \right]. \quad (5)$$

All these Hamiltonians are still quadratic and integrable and we thus do not expect any thermalization.

At this point there are two distinct cases to study. If the mean densities ρ_i of the two wires are identical, i.e. there is *no imbalance* $\Delta = (\rho_1 - \rho_2)/(\rho_1 + \rho_2) = 0$, we have $\hat{H}_{\text{mix}} = 0$ and symmetric and anti-symmetric degrees of freedom decouple. In this case, fluctuations in the individual gases evolve in exactly the same way. However, if the mean densities of the two wires are different ($\Delta \neq 0$), we have $\hat{H}_c \neq 0$, which leads to a coupling between symmetric and anti-symmetric degrees of freedom. In the latter case, the fluctuations in the two condensates evolve differently causing a relative dephasing of the two gases over time.

We note in passing that this scenario holds promise as a platform to study spin-charge physics within the Luttinger liquid framework [27, 28], with \hat{H}_\pm describing the spin and charge degrees of freedom, respectively. In experiments the relevant parameters can be made fully tunable by imbalancing both the densities and also the coupling constants (see appendices A and B).

Previously, the corresponding dynamics were obtained by diagonalizing \hat{H}_- [26, 29–31]. However, this is not sufficient to capture the full dynamics if $\hat{H}_{\text{mix}} \neq 0$. Instead we diagonalize \hat{H}_1 and \hat{H}_2 individually. This is possible, as their excitations are conserved independently of the imbalance. Assuming periodic boundary conditions for the gases of length L , we can expand the phase and density fluctuations into their Fourier components

$$\begin{aligned} \hat{\phi}_k &= \frac{1}{\sqrt{L}} \int_0^L dz e^{-ikz} \hat{\phi}(z) \\ \hat{n}_k &= \frac{1}{\sqrt{L}} \int_0^L dz e^{-ikz} \hat{n}(z) \end{aligned} \quad (6)$$

leading to

$$\hat{H}_i = \sum_k \frac{\hbar^2 k^2}{2m} \rho_i \hat{\phi}_{i,k}^\dagger \hat{\phi}_{i,k} + \frac{g}{2} \hat{n}_{i,k}^\dagger \hat{n}_{i,k}. \quad (7)$$

Using this Hamiltonian to solve the equation of motion for the phase operator we find

$$\hat{\phi}_{i,k}(t) = \hat{\phi}_{i,k}(t=0) \cos(c_i kt) - \frac{g}{\hbar c_i k} \hat{n}_{i,k}(t=0) \sin(c_i kt). \quad (8)$$

Here, the speed of sound in the individual gases is given by $c_i = \sqrt{g\rho_i/m}$, while $\phi_{i,k}(t=0)$ and $\hat{n}_{i,k}(t=0)$ denote the initial values for the phase and density fluctuations, respectively. We can transfer this result into the familiar symmetric/anti-symmetric basis using $\hat{\theta}_{\pm,k} = \hat{\phi}_{1,k} \pm \hat{\phi}_{2,k}$.

Our aim is to investigate the dynamics after a single 1D gas in thermal equilibrium (with temperature T_{in}) is split into two parts. The Hamiltonian of this initial gas is of the form given by equation (7) with 1D density $\rho = \rho_1 + \rho_2$. The thermal expectation values for the second moments in classical field approximation are therefore

$$\begin{aligned} \langle \hat{n}_k \hat{n}_{k'} \rangle_{\text{th}} &= \frac{k_B T_{\text{in}}}{g_{\text{in}}} \cdot \delta_{k,-k'} \\ \langle \hat{\phi}_k \hat{\phi}_{k'} \rangle_{\text{th}} &= \frac{m k_B T_{\text{in}}}{\hbar^2 k^2 (\rho_1 + \rho_2)} \cdot \delta_{k,-k'} \\ \langle \hat{n}_k \hat{\phi}_{k'} \rangle_{\text{th}} &= 0, \end{aligned} \quad (9)$$

where we have used $\hat{\phi}_k^\dagger = \hat{\phi}_{-k}$ and $\hat{n}_k^\dagger = \hat{n}_{-k}$. All first moments vanish. Note that experimentally the splitting often results in a change in radial trapping frequency, such that the gases before and after splitting will differ in their 1D interaction strength $g \sim \omega_\perp$. In the following, we will denote the interaction strength before splitting with g_{in} and after splitting with g_{f} to include this possibility in our calculations.

In the limit of fast splitting no information can propagate along the axis of the gases and we can assume that they both have the same phase profile after the splitting, which is identical to the profile of the initial gas. Moreover, we want to assume that in this case the probability for each atom to go into well 1 is $\rho_1/(\rho_1 + \rho_2)$ independently from where the other atoms go (binomial splitting) [26]. With these assumptions, we find for the fluctuations of the individual gases right after splitting

$$\langle \hat{n}_{i,k} \hat{n}_{j,k'} \rangle(t = 0) = \left[(\rho_1 + \rho_2) \cdot \frac{\rho_1 \rho_2}{(\rho_1 + \rho_2)^2} \cdot (2\delta_{i,j} - 1) + \frac{k_B T_{\text{in}}}{g_{\text{in}}} \cdot \frac{\rho_i \rho_j}{(\rho_1 + \rho_2)^2} \right] \cdot \delta_{k,-k'}, \quad (10)$$

$$\langle \hat{\phi}_{i,k} \hat{\phi}_{j,k'} \rangle(t = 0) = \langle \hat{\phi}_k \hat{\phi}_{k'} \rangle_{\text{th}} = \frac{m k_B T_{\text{in}}}{\hbar^2 k^2 (\rho_1 + \rho_2)} \cdot \delta_{k,-k'}, \quad (11)$$

$$\langle \hat{\phi}_{i,k} \hat{n}_{j,k'} \rangle(t = 0) = 0. \quad (12)$$

The first term in equation (10) represents the shot noise from the binomial splitting process, which is anti-correlated, as expressed by the factor $2\delta_{i,j} - 1$. The second term in equation (10), as well as equation (11), stem from the thermal fluctuations of the initial condensate, and describe correlated fluctuations. Again, all first moments vanish. Assuming Gaussian fluctuations, the second moments are sufficient to fully describe the system. This assumption is justified for long enough length scales containing a large number of particles. Note that the same assumption has to be made for the validity of the Luttinger liquid model and, also, typically only such length scales are accessible in experiments.

3. Results

We can now investigate the dynamics by combining equations (10)–(12) with equation (8). To clearly reveal the relevant dynamics we use the approximation of small imbalance ($\Delta \ll 1$) between the two gases, and obtain

$$\langle |\theta_{-,k}(t)|^2 \rangle = \frac{m k_B T_{\text{in}}}{\hbar^2 k^2 (\rho_1 + \rho_2)} \sin^2(c_- k t) \left[2 + \frac{g_f}{g_{\text{in}}} - \left(2 - \frac{g_f}{g_{\text{in}}} \right) \cos(2c_+ k t) \right] + \frac{2m g_f}{\hbar^2 k^2} \sin^2(c_+ k t) \cos^2(c_- k t), \quad (13)$$

with the average velocity $c_+ = (c_1 + c_2)/2$ and the velocity difference $c_- = (c_1 - c_2)/2$. For a derivation without the small imbalance approximation see appendix C.

In the following discussion of the dephasing dynamics we first focus on the case when the length scales under consideration are much smaller than the system size (large/infinite system limit). Finite system sizes and the occurrence of revivals are discussed in section 4.

If there is no imbalance (i.e. $\Delta = 0$ and $c_- = 0$, $c_+ = c_1 = c_2 = c$) the terms proportional to $\sin^2(c_- k t)$ vanish and we obtain the well known dephasing dynamics

$$\langle |\theta_{-,k}|^2 \rangle = \frac{2m g_f}{\hbar^2 k^2} \sin^2(c k t), \quad (14)$$

where thermal correlations are established with a light-cone [31]. In this process thermal correlations instantaneously emerge locally within a certain horizon, while they remain non-thermal outside of the horizon. This horizon spreads through the system with a characteristic velocity that is given by $c = c_1 = c_2$ [13, 30, 31]. One can see this from equation (14) by realizing that at a certain time t all modes down to a lower bound given by $2ct \cdot k_{\text{lower}} = 2\pi$ have dephased (note that the factor 2 comes from the square of the sine). The bound k_{lower} therefore corresponds to the length-scale $2ct$. The result of the dephasing dynamics is a prethermalized state with a temperature

$$T_{\text{eff}}^{(-)} = \frac{g_f (\rho_1 + \rho_2)}{4k_B}. \quad (15)$$

This temperature can be identified directly from equation (13) in the dephased limit, i.e. by averaging over kt and comparing to the result for a pair of gases (each with 1D density $(\rho_1 + \rho_2)/2$) in thermal equilibrium $\langle |\theta_k|^2 \rangle_{\text{th}} = \frac{4m k_B T}{\hbar^2 k^2 (\rho_1 + \rho_2)}$ (classical field approximation). It corresponds to the energy $k_B T_{\text{eff}}^{(-)}$ that is added to the relative degrees of freedom during the splitting quench [15, 29].

The corresponding expression to equation (13) for the common phase variance $\langle |\theta_{+,k}(t)|^2 \rangle$ is derived in appendix C. One observes that the symmetric degrees of freedom exhibit the temperature

$$T_{\text{eff}}^{(+)} = \frac{T_{\text{in}}}{2} \left(1 + \frac{1}{2} \cdot \frac{g_f}{g_{\text{in}}} \right), \quad (16)$$

which is coming from the initial thermal fluctuations. However, the initial temperature is decreased through an interaction quench (second term in the brackets). In the splitting not only the density but also the density fluctuations are halved [32–34]. As they enter quadratically in the Hamiltonian, this leads to a decrease of a factor 2 in energy, which can be further modified by the aforementioned change in the interaction constant g .

While the individual correlation functions of relative and common degrees of freedom are thus thermal, the state of the total system is non-thermal and has to be described by a generalized Gibbs ensemble with the two temperatures $T_{\text{eff}}^{(\pm)}$, respective [10, 15].

With *imbalance* ($|\Delta| > 0$) the dephasing dynamics changes significantly. The aforementioned light-cone dynamics to the prethermalized state still proceeds with the average velocity c_+ . In addition, examining the additional terms in equation (13) we identify a second dephasing timescale characterized by the slower velocity c_- . After complete dephasing (with the fast as well as the slow velocity), we end up with a second thermal-like state. Following the same procedures as before we can identify the temperature of this state to be identical for both relative and common degrees of freedom and given by⁴

$$T_f^{(\pm)} = \frac{T_{\text{in}}}{4} \left(1 + \frac{1}{2} \frac{g_f}{g_{\text{in}}} \right) + \frac{T_{\text{eff}}^{(-)}}{2}. \quad (17)$$

Again, this result can be interpreted intuitively in terms of the corresponding energies $k_B T_f^{(\pm)}$. The first term corresponds to half the energy that is initially contained in the common degrees of freedom (equation (16)), the second term to half the energy introduced to the relative degrees of freedom during the quench (equation (15)). Equation (17) hence describes an equipartition of energy that is dynamically established by the coupling term H_c .

Note that equation (17) remains true, even without the assumption of small imbalance⁵, which was used to obtain equation (13). For typical parameters in atom chip microtraps the change in confinement leads to $\frac{g_f}{g_{\text{in}}} \approx 1/\sqrt{2}$ and we find $T_f \approx \frac{T_{\text{in}}}{3} + \frac{T_{\text{eff}}^{(-)}}{2}$.

To visualize the corresponding dephasing dynamics leading to this equipartition in detail, we calculate the two-point phase correlation function [13]

$$\mathcal{PCF}(z, z') = \langle \cos(\hat{\theta}(z) - \hat{\theta}(z')) \rangle. \quad (18)$$

This function measures the correlation between the relative phases $\theta(z)$ at two arbitrary points z and z' along the length of the system and can directly be measured in the experiments [13]. An alternative but related probe, the interference contrast, is discussed in appendix D.

As discussed in the previous section, the initial fluctuations in our model are Gaussian and of course remain so during the evolution with the quadratic Hamiltonian. Therefore, the phase correlation function can be rewritten in the form $\mathcal{PCF}(z, z') = e^{-\frac{1}{2} \langle [\hat{\theta}(z) - \hat{\theta}(z')]^2 \rangle}$. In the limit of an infinitely large system this leads to

$$\mathcal{PCF}(z, z') = \exp \left[- \int_0^\infty \frac{dk}{\pi} \langle |\theta_{-,k}(t)|^2 \rangle (1 - \cos k(z - z')) \right]. \quad (19)$$

In figure 2 we plot equation (19) for increasing evolution times, revealing a double light-cone. First, the system relaxes to the prethermalized state with exponentially decaying (thermal) correlations. For longer evolution times, the system relaxes further to the second thermal-like steady state. As in the previous light-cone-like relaxation to the prethermalized state, the system reaches this new final state for a given time only up to a certain horizon, but then follows a different shape beyond that point. The position of this horizon moves with a second characteristic velocity that is given by the (typically small) velocity difference c_- of the individual gases.

While both symmetric and anti-symmetric degrees of freedom reach a thermal-like state with temperature $T_f^{(\pm)}$, the complete system still differs from the thermal equilibrium of two condensates with equal density $(\rho_1 + \rho_2)/2$ in the aspect that cross-correlations $\langle \theta_{+,k} \theta_{-,k} \rangle$ between symmetric and anti-symmetric degrees of freedom do not vanish (see appendix C). Note that for the thermal equilibrium of two gases with unequal densities, the cross-correlations between common and relative degrees of freedom also do not vanish. However, they are still of different magnitude than in the completely dephased case.

Another way to discuss the question of whether the system dephases to thermal equilibrium is to have a look at the quantities for the individual gases. These could e.g. be studied in experiments using density fluctuations in time of flight [35] or by probing the density fluctuations *in situ* [36]. The phase variance of the individual gases is given by

⁴ Note that this temperature is obtained by comparison with the thermal equilibrium of two gases of equal density $(\rho_1 + \rho_2)/2$. When comparing to two gases with unequal density ρ_1 and ρ_2 , the effective temperature is given by $4\rho_1\rho_2/(\rho_1 + \rho_2)^2 \times T_f^{\pm}$.

⁵ This can be best seen from the dephased state for the phase variances of the individual condensates (equation (20)) and the crossterms dephasing to 0 (equation (25)).

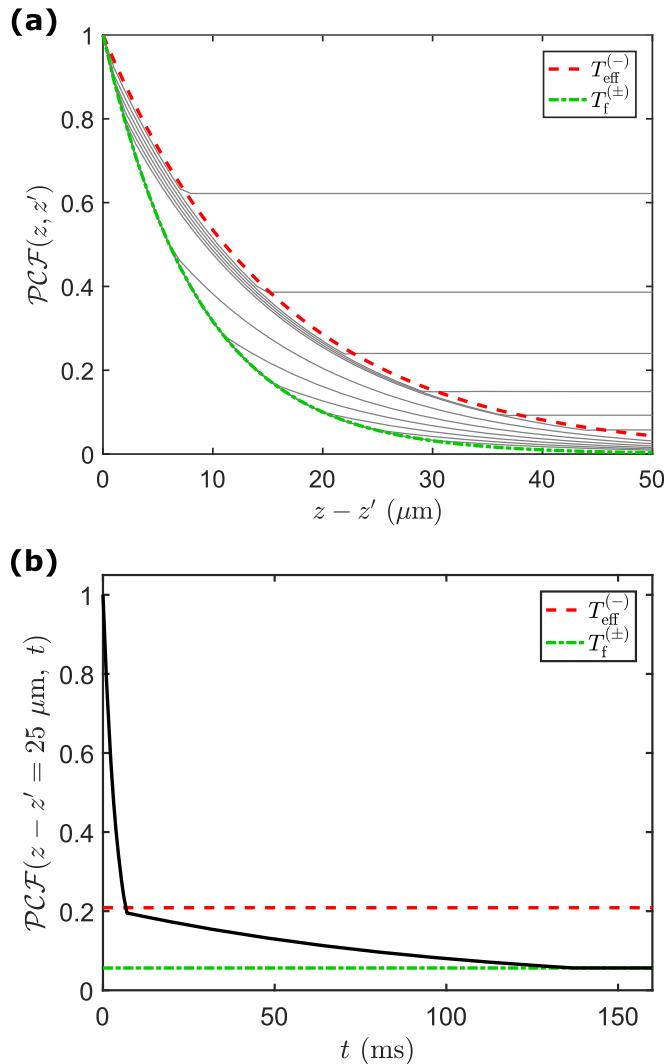


Figure 2. Dynamics of the two-point phase correlation function in the theoretical model. (a) The two-point phase correlation function reveals a fast light-cone-like decay with the average velocity c_+ to the prethermalized state (dashed red line), followed by a much slower second light-cone-like decay with the difference velocity c_- to the final relaxed state (dash-dotted green line). Evolution times increase in steps of 2 ms from top to bottom until the prethermalized state is reached, followed by steps of 25 ms in the approach to the final state. Parameters are $a_s = 5.24$ nm and $m = 1.44 \times 10^{-25}$ kg for ^{87}Rb , $\omega_{\perp,\text{in}} = 2\pi \times 2.1$ kHz, $\rho_1 + \rho_2 = 100 \mu\text{m}^{-1}$, corresponding to an initial Luttinger parameter $K = \hbar\pi\sqrt{(\rho_1 + \rho_2)/mg} \sim 74$. The imbalance is $\Delta = (\rho_1 - \rho_2)/(\rho_1 + \rho_2) = 0.1$ and $T_{\text{in}} = 70$ nK. The trap frequency after splitting is $\omega_{\perp,\text{f}} = 2\pi \times 1.4$ kHz. (b) Phase correlation function evaluated for a distance of $z - z' = 25 \mu\text{m} \sim 105 \times \xi_h$ as a function of time, highlighting the distinct timescales of the relaxation to the prethermalized and final states, respectively. Here, $\xi_h = \hbar/mc$ is the healing length.

$$\langle |\phi_{i,k}|^2 \rangle = \frac{mk_B}{\hbar^2 k^2 \rho_i} \left[T_{\text{in}} \frac{\rho_i}{\rho_1 + \rho_2} \left(\cos^2(c_i kt) + \frac{g_f}{g_{\text{in}}} \frac{\rho_i}{\rho_1 + \rho_2} \sin^2(c_i kt) \right) + \frac{\rho_i g_f}{k_B} \left(1 - \frac{\rho_i}{\rho_1 + \rho_2} \right) \sin^2(c_i kt) \right] \quad (20)$$

which describes a relaxation towards a temperature

$$T_f^{(i)} = \frac{T_{\text{in}}}{2} \frac{\rho_i}{\rho_1 + \rho_2} \left(1 + \frac{g_f}{g_{\text{in}}} \frac{\rho_i}{\rho_1 + \rho_2} \right) + \frac{\rho_i g_f}{2k_B} \left(1 - \frac{\rho_i}{\rho_1 + \rho_2} \right). \quad (21)$$

This expression is different from the results for the symmetric/anti-symmetric basis, highlighting how the observed dynamics and, in particular, also their timescales, are indeed intimately connected to the choice of observable.

In detail, the timescale for the dynamics within a single gas is, as expected, given by their speed of sound c_i . However, the cross-correlations of the form $\langle \phi_{1,k} \phi_{2,k} \rangle$ dephase to zero with the slow velocity c_- (see appendix C). After complete dephasing we therefore end up with two independent gases, which independently appear to be in thermal equilibrium with their respective temperatures $T_f^{(i)}$. However, for all the dynamics described only dephasing and no true thermalization has taken place. The intuitive reason for this complex behavior is that the two imbalanced gases are non-identical and dephase with respect to each other. Therefore, their individual

excitations are still conserved, but the symmetric and anti-symmetric modes are no longer connected to these conserved quantities.

As an example, for the parameters used in figure 2 ($\Delta = 0.1$, ^{87}Rb atoms with $T_{\text{in}} = 70$ nK and $\rho_1 + \rho_2 = 100 \mu\text{m}^{-1}$) these final temperatures are $T_{\text{f}}^{(1)} = 43.8$ nK and $T_{\text{f}}^{(2)} = 35.1$ nK. Note that in the limit of vanishing imbalance we have $\rho_i/(\rho_1 + \rho_2) \sim 1/2$ and the difference between the $T_{\text{f}}^{(i)}$ tends to zero and they approach the final temperature of the symmetric and anti-symmetric degrees of freedom given by equation (17). Also, already in the approach of this limit the small difference in final temperatures can be challenging to measure in an experiment. In both cases the system would thus appear completely thermalized independent of the choice of basis, with symmetric, anti-symmetric and individual degrees of freedom all exhibiting the same temperature. However, with imbalance going to zero, this approach of the final temperature would become infinitely slow.

4. Influence on many-body revivals

While the double light-cone dynamics are clearly visible in the correlation functions calculated for an infinite system, observing them directly in an experiment with a finite size system, in particular when a typically harmonic longitudinal confinement is present, is challenging. In particular, due to the nonlinear excitation spectrum in harmonic traps [30] the effect is severely scrambled by highly irregular many-body revivals. Examples of this behavior are shown in the appendix E.

However, regular and well controlled many-body revivals have recently been observed for the first time in homogeneous trapping potentials [24], demonstrating their power to probe the dephasing and higher-order interactions of phonon modes. In the following we thus illustrate the influence of our effect on such many-body revivals.

To this end, we first repeat our calculations for a system with periodic boundary conditions, but with a typical experimental finite size of $100 \mu\text{m} \sim 400 \times \xi_h$. In figure 3(a) we show the corresponding results for the phase correlation function. Due to the finite number of momentum modes they show clear rephasing behavior, as experimentally observed in [24]. However, due to the imbalance the two velocities in the system can be observed through the presence of two different types of revivals—slow revivals resulting from c_- and fast revivals coming from c_+ . The value of the phase correlation function reached in the slow revivals depends on how well slow and fast revivals coincide.

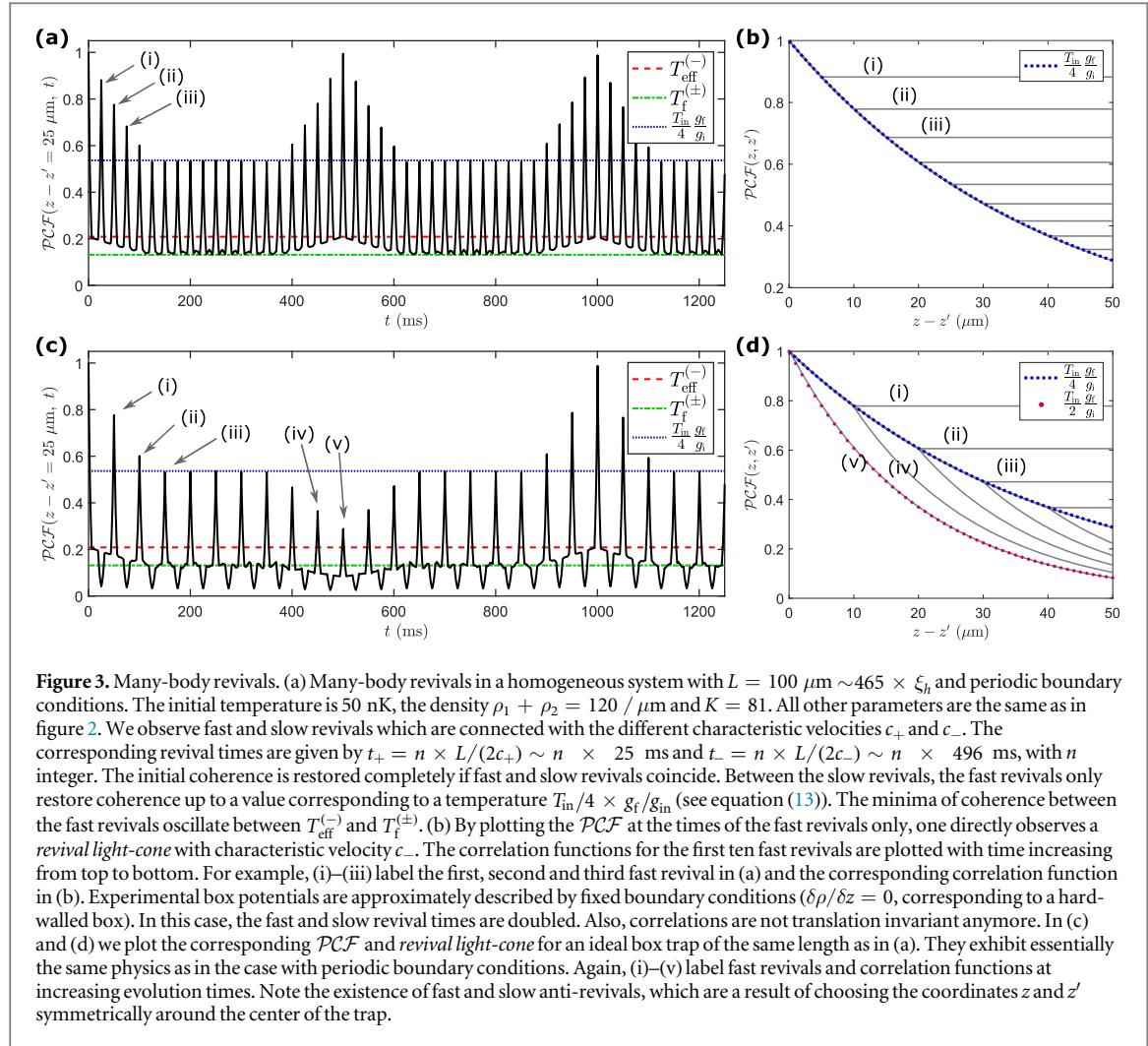
A scenario more relevant for an experimental realization is the one of fixed boundary conditions ($\partial\phi/\partial z = 0$), which corresponds to the case of a hard walled box. This boundary conditions guarantee that the particle current at the box walls vanishes. The corresponding results are shown in figures 3(c), (d). Again, a clear distinction between slow and fast revivals can be observed, which can directly be connected to the two characteristic velocities.

Our observations have important practical consequences for the experimental study of integrability breaking in 1D Bose gases [37–42]. As the effect described here and true thermalization through integrability breaking would essentially lead to the same experimental signatures (i.e. thermal correlations corresponding to a temperature given by equation (17)) they would be very challenging to disentangle from measurements of correlation functions alone. In particular, any experimental effort clearly has to take both effects into account simultaneously. The many-body revivals presented in figure 3 provide additional tools for such studies.

5. Discussion

We have observed how the dephasing of an imbalanced pair of 1D Bose gases can result in states which are, for all practical purposes, indistinguishable from thermal equilibrium. This is due to a coupling of the relative and common degrees of freedom that is mediated by the relative dephasing of the individual gases. It is important to note that this observation of an apparent thermalization relies on the thermal-like initial conditions that were imposed on the system by the coherent splitting process. The system always retains a strong memory of the initial conditions and thus has not truly reached global thermal equilibrium. For example, if the system was initialized with other non-thermal initial conditions like the ones demonstrated in [10], it would equilibrate, but never appear thermal in its correlation functions [43].

Interestingly, the observed dynamics are closely related to the measurement process. In experiments, fluctuations of the anti-symmetric degrees of freedom are probed. These degrees of freedom exhibit a rapid relaxation with a single timescale if there is no imbalance, and a relaxation with two distinct timescales if there is imbalance. The same timescales govern the relaxation of the symmetric degrees of freedom. In contrast to that, if the properties of a single gas were accessible in experiment, their individual correlations would already look completely relaxed after the first, rapid timescale. This highlights how even in integrable systems, observables



need to be properly *aligned* with (i.e. chosen such that they are sensitive to) the integrals of motion to reveal the integrable nature of the complex many-body dynamics. We note that the experiment in [24] recently revealed related behavior, where many-body revivals could be observed in certain correlation functions but not in others. This points to a general connection between the choice of measurement basis and the observed relaxation dynamics and will thus be an interesting topic for future research.

Acknowledgments

We acknowledge discussions with I Mazets, T Gasenzer, S Erne and B Rauer. This work is supported by the SFB 1225 ISOQUANT financed by the German Research Foundation (DFG) and the Austrian Science Fund (FWF). TL acknowledges support from the Alexander von Humboldt Foundation through a Feodor Lynen Fellowship, the EU through a Horizon2020 Marie Skłodowska-Curie IF (746525 coolDips), the Baden-Württemberg Foundation and the Center for Integrated Quantum Science and Technology (IQST). TS acknowledges support by the FWF through the Doctoral Programme CoQuS (W1210). ED acknowledges support from Harvard-MIT CUA, NSF Grant No. DMR-1308435, AFOSR Quantum Simulation MURI, ARO MURI on Atomtronics, ARO MURI Qusim program, and AFOSR MURI Photonic Quantum Matter. JS acknowledges support through the ERC advanced grant QuantumRelax.

Appendix A. Spin-charge coupling in 1D Bose gases

Our scenario can also be interpreted as a possible experimental platform to explore spin-charge physics within the Luttinger liquid framework [27, 28]. In this case, the symmetric degrees of freedom can be identified with the charge degrees of freedom of a fermionic spin chain, while the anti-symmetric degrees of freedom play the role of the spin. If the two gases are prepared with identical mean atom numbers, spin and charge degrees of freedom

are separated. The mixing in the imbalanced case, on the other hand, can be identified as a coupling between spin and charge.

For the system of two spatially separate 1D Bose gases the characteristic velocities $c_{s,c} = \sqrt{g_{s,c}(\rho_1 + \rho_2)/2m}$ of spin and charge degrees of freedom are identical, as $g_s = g_c \equiv g$, where g is the 1D interaction strength. Different tunable velocities for spin and charge can be achieved by replacing the two wells employed in this work by two internal atomic states $|1\rangle$ and $|2\rangle$ with different interaction constants g_{11}, g_{22} and g_{12} [18, 26, 44, 45]. This situation would lead to $g_{s,c} = g_{11} + g_{22} \mp 2g_{12}$ and thus different velocities for spin and charge. These velocities could be studied experimentally by probing the propagation of the *in situ* density fluctuations after a quench of the radial confinement [36, 46].

Appendix B. Details of the proposed experimental realization

The splitting quench can be realized by applying near-field RF radiation via two wires on an atom chip [25, 47, 48]. Previous experiments investigating a balanced splitting process [10, 13, 15, 31, 49] have demonstrated this to be a powerful scenario for non-equilibrium physics. The splitting process can be made much faster than the speed of sound in the system, realizing the binomial distribution of atoms that is discussed in the main text. In this case, no information about the quench can propagate along the system, leading to almost perfectly correlated phase profiles of the two gases after the splitting [13]. The relative amplitude and phase of the two RF currents defines the polarization of the RF radiation and therefore the orientation of the double well potential [25, 47]. A small imbalancing of in-phase RF currents in these wires creates a tilted double well potential and thus a small atom number imbalance after the splitting process. For the trap parameters used in the experiments we estimate that offsets below 250 Hz between the two minima of the tilted double well are sufficient to realize the scenario of small imbalances (i.e. up to $\Delta = 0.1$) discussed in the main text. In this case, the trapping potential provided by the two wells can still be assumed to be identical. Residual small collective excitations can be efficiently removed using optimal control [50]. For higher imbalances, the wells become increasingly distorted until eventually also tunneling from one well across the barrier into excited states of the other well becomes possible.

Appendix C. Evolution of the phase variance

Combining equations (10)–(12) with equation (8) we find for the time evolution of the relative phase variance

$$\begin{aligned} \langle |\theta_{-,k}(t)|^2 \rangle &= \frac{mk_B T_{\text{in}}}{\hbar^2 k^2 (\rho_1 + \rho_2)} \left([\cos(c_1 kt) - \cos(c_2 kt)]^2 + \frac{g_f}{g_{\text{in}}} \left[\sqrt{\frac{\rho_1}{\rho_1 + \rho_2}} \sin(c_1 kt) + \sqrt{\frac{\rho_2}{\rho_1 + \rho_2}} \sin(c_2 kt) \right]^2 \right) \\ &+ \frac{mg_f}{\hbar^2 k^2} \left[\sqrt{\frac{\rho_2}{\rho_1 + \rho_2}} \sin(c_1 kt) + \sqrt{\frac{\rho_1}{\rho_1 + \rho_2}} \sin(c_2 kt) \right]^2. \end{aligned} \quad (22)$$

For small imbalances we can assume $\rho_{1,2}/(\rho_1 + \rho_2) \approx 1/2$, which gives

$$\begin{aligned} \langle |\theta_{-,k}(t)|^2 \rangle &= \frac{mk_B T_{\text{in}}}{\hbar^2 k^2 (\rho_1 + \rho_2)} \left([\cos(c_1 kt) - \cos(c_2 kt)]^2 + \frac{g_f}{g_{\text{in}}} \frac{1}{2} [\sin(c_1 kt) + \sin(c_2 kt)]^2 \right) \\ &+ \frac{mg_f}{\hbar^2 k^2} \frac{1}{2} [\sin(c_1 kt) + \sin(c_2 kt)]^2. \end{aligned} \quad (23)$$

Simplifying this expression yields equation (13). Note that even for large imbalances the difference between the approximation and the full formula is only quantitative, qualitatively they still give the same results. Similarly we can obtain an expression for the phase variance of the symmetric degrees of freedom. Assuming small imbalances we get the approximate result

$$\langle |\theta_{+,k}(t)|^2 \rangle = \frac{mk_B T_{\text{in}}}{\hbar^2 k^2 (\rho_1 + \rho_2)} \cos^2(c_- kt) \left[2 + \frac{g_f}{g_{\text{in}}} + \left(2 - \frac{g_f}{g_{\text{in}}} \right) \cos(2c_+ kt) \right] + \frac{2mg_f}{\hbar^2 k^2} \sin^2(c_- kt) \cos^2(c_+ kt). \quad (24)$$

In analogy to equation (20) for the phase variances of the individual condensates, one can also calculate the evolution of the crossterms $\langle \hat{\phi}_{1,k} \hat{\phi}_{2,-k} \rangle$. From equations (8) and (12) we find that they are of the form

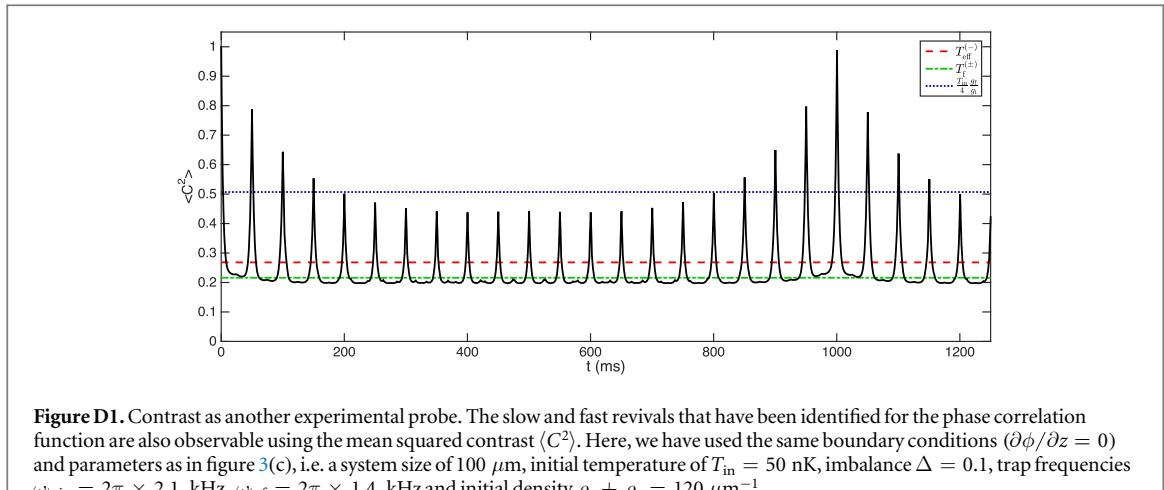


Figure D1. Contrast as another experimental probe. The slow and fast revivals that have been identified for the phase correlation function are also observable using the mean squared contrast $\langle C^2 \rangle$. Here, we have used the same boundary conditions ($\partial\phi/\partial z = 0$) and parameters as in figure 3(c), i.e. a system size of $100 \mu\text{m}$, initial temperature of $T_{\text{in}} = 50 \text{ nK}$, imbalance $\Delta = 0.1$, trap frequencies $\omega_{\perp,\text{in}} = 2\pi \times 2.1 \text{ kHz}$, $\omega_{\perp,\text{f}} = 2\pi \times 1.4 \text{ kHz}$ and initial density $\rho_1 + \rho_2 = 120 \mu\text{m}^{-1}$.

$$\begin{aligned} \langle \hat{\phi}_{1,k} \hat{\phi}_{2,-k} \rangle &= \mathcal{C}_1 \cdot \cos(c_1 kt) \cos(c_2 kt) + \mathcal{C}_2 \cdot \sin(c_1 kt) \sin(c_2 kt) \\ &= \frac{\mathcal{C}_1}{2} \cdot (\cos(2c_- kt) + \cos(2c_+ kt)) + \frac{\mathcal{C}_2}{2} \cdot (\cos(2c_- kt) - \cos(2c_+ kt)), \end{aligned} \quad (25)$$

where $\mathcal{C}_{1,2}$ are time-independent constants. This expression describes a dephasing of the cross terms to 0 with the velocity difference c_- .

Appendix D. Interference contrast

In experiments the expectation value in the definition of the phase correlation function is realized through an average over many experimental runs. We have previously demonstrated [13] that the number of runs that is required for a statistically meaningful determination of the correlation functions can dramatically increase for evolution times $t \gg 10 \text{ ms}$. However, for the parameters used in this work, the fully thermal state is not expected before $t \sim 100 \text{ ms}$ or more. For longer evolution times it could thus be beneficial to probe the level of coherence between the two gases using the mean squared interference contrast $\langle C^2 \rangle$ of the matterwave interference pattern in time-of-flight [13, 15, 49]. It is well known from one of our previous experiments [49] that this procedure involves an unknown factor that describes the finite resolution and other spurious experimental effects. We thus suggest to extract the relative phase $\theta(z) = \phi_L(z) - \phi_R(z)$ from every longitudinal position z of the interference pattern and calculate the contrast via the identity

$\langle C^2(L) \rangle = \int_{-L/2}^{L/2} \mathcal{PCF}(z_1, z_2) dz_1 dz_2$. Here, \mathcal{PCF} is the two-point phase correlation function discussed in the main text (equation (19)) and L is a length scale over which the interference pattern is integrated. Because of the integration this procedure can be more robust against statistical fluctuations than the phase correlation function alone. With the identity given above, it is straight forward to generalize our predictions for the dynamics to the contrast. An example is shown in figure D1.

Appendix E. Harmonic traps

The Luttinger Hamiltonian as written in equation (1) stays valid for inhomogeneous density profiles $\rho_i(z)$. For the calculation in the harmonic trap, we assume a Thomas–Fermi profile for the density distribution before splitting, and rescaled density distributions for the evolution of the fluctuations after the quench. For the latter, the initial density profile depending on the total atom number $N_1 + N_2$ and on the trap frequencies ω_z (longitudinal) and $\omega_{\perp,\text{in}}$ (radial) is simply multiplied by the factor $N_i/(N_1 + N_2)$.

With this density distributions the Hamiltonian can be diagonalized with the help of Legendre-Polynomials [30, 51]. Note that due to the density dependence of the shot noise fluctuations introduced in the splitting process, the initial density fluctuations expanded in Legendre-Polynomials are not diagonal anymore [52].

The results of the calculation are shown in figure E1. The incommensurate excitation energies of the trapped system lead to very complex dephasing and rephasing dynamics. As already discussed in the main text, such complex dynamics make it very challenging to experimentally disentangle different competing integrable (such as the one presented here) and non-integrable (such as thermalization) mechanisms.

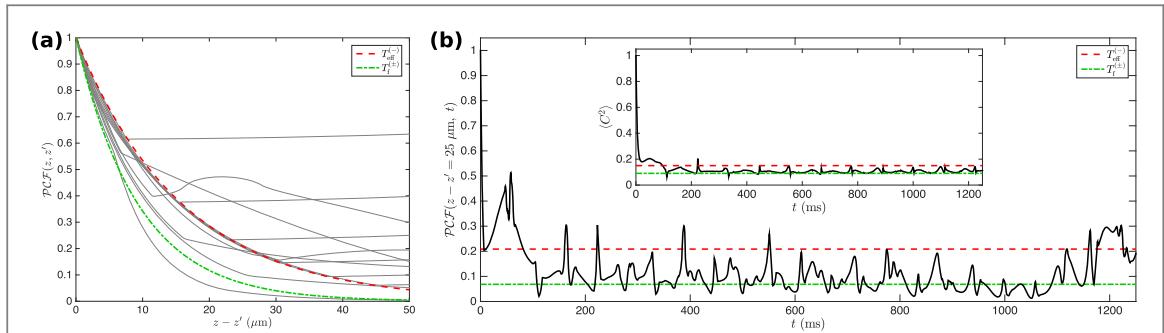


Figure E1. Dynamics in a harmonic trapping potential. (a) Dynamics of the phase correlation function, with time increasing in steps of 2 ms from 0 ms to 12 ms, and in steps of 25 ms from 35 ms to 160 ms. (b) The incommensurate mode energies of the trapped system lead to complex dephasing dynamics with many irregularly spaced partial revivals. We have used 10000 ^{87}Rb atoms with an initial temperature of $T_{\text{in}} = 70$ nK, in an initial trap with frequencies $\omega_z = 2\pi \times 11$ Hz and $\omega_{\perp, \text{in}} = 2\pi \times 2.1$ kHz. The atom number imbalance for the evolution is $\Delta = (N_1 - N_2)/(N_1 + N_2) = 0.1$, the perpendicular trap frequency is $\omega_{\perp, \text{f}} = 2\pi \times 1.4$ kHz. Note that we do not need to specify a longitudinal trap frequency for the evolution after the quench as we simply assume rescaled density profiles. The inset shows the corresponding evolution of the interference contrast. The coordinates z and z' have been chosen symmetrically around the center of the trap in both (a) and (b).

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