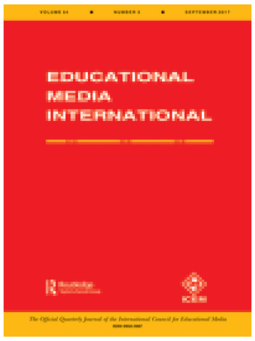

Educational Media International

ISSN: 0952-3987 (Print) 1469-5790 (Online) Journal homepage: http://www.tandfonline.com/loi/remi20

Counterintuitive effects of online feedback in middle school math: results from a randomized controlled trial in ASSISTments

Patrick McGuire, Shihfen Tu, Mary Ellin Logue, Craig A. Mason & Korinn Ostrow

EDUCATIONAL MEDIA INTERNATIONAL, 2017
VOL. 54, NO. 3, 231–244
https://doi.org/10.1080/09523987.2017.1384161

Counterintuitive effects of online feedback in middle school math: results from a randomized controlled trial in ASSISTments

Patrick McGuire[a], Shihfen Tu[b], Mary Ellin Logue[b], Craig A. Mason[b] and Korinn Ostrow[c]

[a]Department of Teaching and Learning, University of Colorado Colorado Springs, Colorado Springs, CO, USA; [b]College of Education and Human Development, University of Maine, Orono, ME, USA; [c]Learning Sciences and Technologies Program, Worcester Polytechnic Institute, Worcester, MA, USA

ABSTRACT

This study compared the effects of three different feedback formats provided to sixth grade mathematics students within a web-based online learning platform, ASSISTments. A sample of 196 students were randomly assigned to one of three conditions: (1) text-based feedback; (2) image-based feedback; and (3) correctness only feedback. Regardless of condition, students solved a set of problems pertaining to the division of fractions by fractions. This mathematics content was representative of challenging sixth grade mathematics Common Core State Standard (6.NS.A.1). Students randomly assigned to receive text-based feedback (Condition A) or image-based feedback (Condition B) outperformed those randomly assigned to the correctness only group (Condition C). However, these differences were not statistically significant ($F(2,108) = 1.394$, $p = .25$). Results of this study also demonstrated a completion-bias. Students randomly assigned to Condition B were less likely to complete the problem set than those assigned to Conditions A and C. To conclude, we discuss the counterintuitive findings observed in this study and implications related to developing and implementing feedback in online learning environments for middle school mathematics.

ARTICLE HISTORY
Received 2 August 2017
Accepted 12 September 2017

KEYWORDS
Middle school mathematics; differential effects of online feedback; intelligent tutoring systems; randomized controlled trial

Introduction

With the recent popularity of web-based learning platforms over the past decade, more and more K-12 students around the world are learning mathematics in online environments. A recent study conducted by SRI International estimated that Khan Academy, a popular online learning platform (www.khanacademy.org), had over 10 million unique users in just one month, with more than 365 million video views

CONTACT Patrick McGuire pmcguire@uccs.edu

and over 1.8 billion math problems solved (Murphy, Gallagher, Krumm, Mislevy, & Hafter, 2014). Hundreds, perhaps even thousands, of other web-based mathematics tutoring platforms and websites are simultaneously growing, indicative of the booming modern marketplace for online learning. These next generation, web-based, customizable learning environments not only allow students to engage with content at their own pace where appropriate to the curriculum, but can also provide teachers and researchers with valuable student performance data which can support individualized instruction, address common misconceptions, and help form targeted groups for differentiated instruction.

Although there is no debate about the usage of online learning platforms in K-12 learning environments, there have been mixed results regarding their effectiveness and impact on student learning. One recent meta analysis of empirical studies indicated generally positive effects that these programs have on student learning outcomes in K-12 mathematics classrooms (see Cheung & Slaven, 2013 for a meta-analysis involving 74 studies involving more than 56,000 students). Specifically, these platforms provide teachers and students with opportunities to engage in differentiated instruction within personalized learning environments (Schaffert & Hilzensauer, 2008; Swan, 2003), facets of teaching and learning that are not easily achievable through traditional paper and pencil formats. However, other researchers (see Steenbergen-Hu & Cooper, 2013) conducting similar meta-analyses involving intelligent tutoring systems (ITS) arrived at different conclusions. Among the major findings of Steenbergen-Hu & Cooper are that although most ITS have no negative effects on student learning, those with positive effects on K-12 students' mathematical learning, as indicated by the average effect sizes ranging from $g = .01$ to $g = .09$, are negligible.

In the present work, we describe how one such online learning platform, ASSISTments, was leveraged to develop and implement a randomized controlled trial investigating the effects of feedback provided to sixth grade mathematics students. This study was rooted in the goal of answering the following research question: How do three different feedback conditions (e.g., text-based, image-based, or correctness only) provided in ASSISTments influence students' conceptual understanding of division of fractions?

Literature review

Feedback

The advantages of immediate feedback have been well-documented by several studies across multiple grade levels and in different content areas such as chemistry (see Cole & Todd, 2003), with seminal papers summarizing the topic (e.g., Kluger & DeNisi, 1996; Shute, 2008). In mathematics-specific learning environments, immediate feedback has been shown to result in learning gains, particularly in web-based settings (Kelly et al., 2013; Mendicino, Razzaq, & Heffernan, 2009;

Nguyen & Kulm, 2005). Similar studies of web-based mathematics assessments involving feedback have suggested growth in students' learning attitudes and a positive effect on problem solving skills (Nguyen, Hsieh, & Allen, 2006), as well as heightened confidence in solving mathematics problems and reductions in math anxiety (Jansen et al., 2013). However, despite decades of research on feedback and the notion that it generally results in learning gains and attitudinal improvements for students, some in the field have pushed back, calling for a more systematic investigation into the efficacy of different types of feedback (Hattie & Gan, 2011; Hattie & Timperley, 2007). Others suggest a more careful consideration of the cognitive demands associated with differing feedback formats (Booth & Koedinger, 2012; Fyfe, DeCaro, & Rittle-Johnson, 2015). As such, context specific questions regarding when and how to provide feedback remain popular.

Concrete-representational-abstract feedback

The theoretical framework used to develop the three unique feedback conditions tested in the present work was predicated upon the Concrete – Representational – Abstract (CRA) framework, an instructional model that has been cited as a high impact instructional strategy (Institute of Education Sciences, 2009). CRA is a three-part instructional framework, with each part compounding previous instruction to promote student learning and retention. At the elementary level, it has been used to teach four basic operations, time, money, fractions, and beginning algebra. At the first stage, students use manipulatives to master concepts. In the second stage, they use semi-concrete representations (e.g., tally marks, pictures, etc.) to solve problems. When students can successfully use representations from stage 2, they are introduced to abstract problem solving with numbers and symbols. Extensive research on CRA suggests this is an effective framework for students in mathematics, including those with learning disabilities in mathematics (Butler, Miller, Crehan, Babbit, & Pierce, 2003; Flores, 2010; Mercer, Jordan, & Miller, 1996; Peterson, Mercer, & O'Shea, 1988; Witzel, Mercer, & Miller, 2003) and those without learning disabilities (Baroody, 1987; Jordan, Miller, & Mercer, 1999; Sousa, 2008; Van de Walle, Karp, & Bay-Williams, 2009). However, some researchers urge caution in applying the CRA framework to instruction without an understanding how students may interpret the representation presented in the scaffolds (Stampfer & Koedinger, 2013).

Despite the fact that mastery of the division of fractions is part of the Common Core, research on student learning outcomes related to *division* of fractions by fractions has not been studied in comprehensive detail or through gold standard, randomized controlled trials. The present study focused on the last two levels of the CRA framework, comparing feedback that is representational vs. abstract when students are required to solve problems where a fraction is divided by another fraction.

The present work focused on this Common Core standard (CCSS-M, 2010; 6.NS.A.1) because both teachers (anecdotally) and researchers (empirically) have indicated a significant number of students possess misconceptions when dividing fractions by fractions (Fendel, 1987; Tirosh, 2000). Other related studies involving the addition of fractions has suggested that fifth grade students, despite being provided meaningful visual scaffolds (e.g., fraction bars), struggled to make sense of abstract concepts (Stampfer & Koedinger, 2013) and that despite the expectation that fractions be taught and learned in fourth grade, evidence suggests that many eighth grade students do not possess the basic understanding required to master this task in the sixth grade curriculum (The National Mathematics Advisory Panel, 2008). Further, because the division of fractions by fractions is an abstract concept, many teachers struggle to succinctly describe this concept to students without resorting to procedural tricks (Ball, 1990; Borko et al.,1992). It has also been argued that students' learning of fraction division should extend beyond rote memorization (e.g., the copy-dot-flip algorithm), with teachers carefully considering alternative ways to introduce students to the conceptual underpinnings of the algorithm (Li, 2008) using multiple representations. This is made more complex when teachers share students' confusion about the process. Baek et al. (2017) suggests that preservice teachers often have trouble conceptualizing fractions and how to operate on them because the required reasoning runs contrary to their own learning, which was marked by misconceptions and memorized formulas, rather than conceptual understanding (Osana & Royea, 2011). Therefore, the CRA instructional framework for providing feedback, together with a historically challenging Common Core mathematics fraction division standard, presented a unique opportunity to study the effects of differential feedback conditions on middle school students using a web-based platform, ASSISTments. Considering past work, we hypothesized that students randomly assigned to the image-based feedback condition would learn more than those assigned to the non-image conditions, as measured by a brief post-test immediately following the items included in the learning phase of the problem set

Methods

Research context

The authors worked closely with the ASSISTments team to design and implement this study. The ASSISTments team was responsible for recruiting teachers to participate in this study, and for de-identifying all student and teacher-level data prior to analysis.

Participants

This randomized controlled trial involved 196 sixth grade (middle school level) students using the ASSISTments platform to support their mathematics learning. Students were collected via the established network of ASSISTments users using the process of creating certified material through collaboration with the TestBed (www.ASSISTmentsTestBed.org).

ASSISTments system

The feedback conditions described in this paper were created within ASSISTments (www.assistments.org), a free, university supported, online learning platform. ASSISTments was originally designed to provide students with immediate feedback (**assist**ance) and teachers with real-time data on student performance (formative assess**ment**). For students, ASSISTments offers dynamic tutoring feedback delivered using hint messages or scaffolds that break tough problems down into steps when a student generates an incorrect answer. This assistance is intended to improve learning and reduce frustration, allowing students to work through an assignment and better understand their errors without a delay in assessment. For teachers, the system provides in-depth reports on student and class performance that can be used to evaluate and monitor students' progress (see Figure 1). These reports range from summarized overviews of students, skills, or classes, to fine-grained logs of a student's performance.

ASSISTments is maintained as a free public service of Worcester Polytechnic Institute by a team of faculty researchers and graduate students. In 2014, over 600 teachers from 41 states and 6 countries used the ASSISTments platform. Their students solved over 10 million problems. Coupled with its popularity, ASSISTments recently procured grant funding to allow researchers around the country to use its mathematics content to conduct student-level randomized controlled trials. The processes we followed in conducting this work, and methods for going about similar research, can be found at www.ASSISTmentsTestBed.org.

There are currently over 35,000 unique problems in ASSISTments (most covering mathematics topics) that have been extensively vetted, tagged to cognitive learning models, and marked as certified content. These problems are then grouped to form problem sets that align to the Common Core State Standards, giving teachers an easy way to organize and assign materials. Teachers can also pick and choose problems to assign, developing their own unique problem sets. In addition, the system supplements more than 20 of the nation's top mathematics textbooks, making homework easy to integrate with extra practice. Traditionally, individual math problems are grouped to create one of two types of problem sets: (1) "static" problem sets requiring that students solve a predefined number of questions (i.e., a worksheet), and (2) "Skill Builders" or mastery-based assignments that require

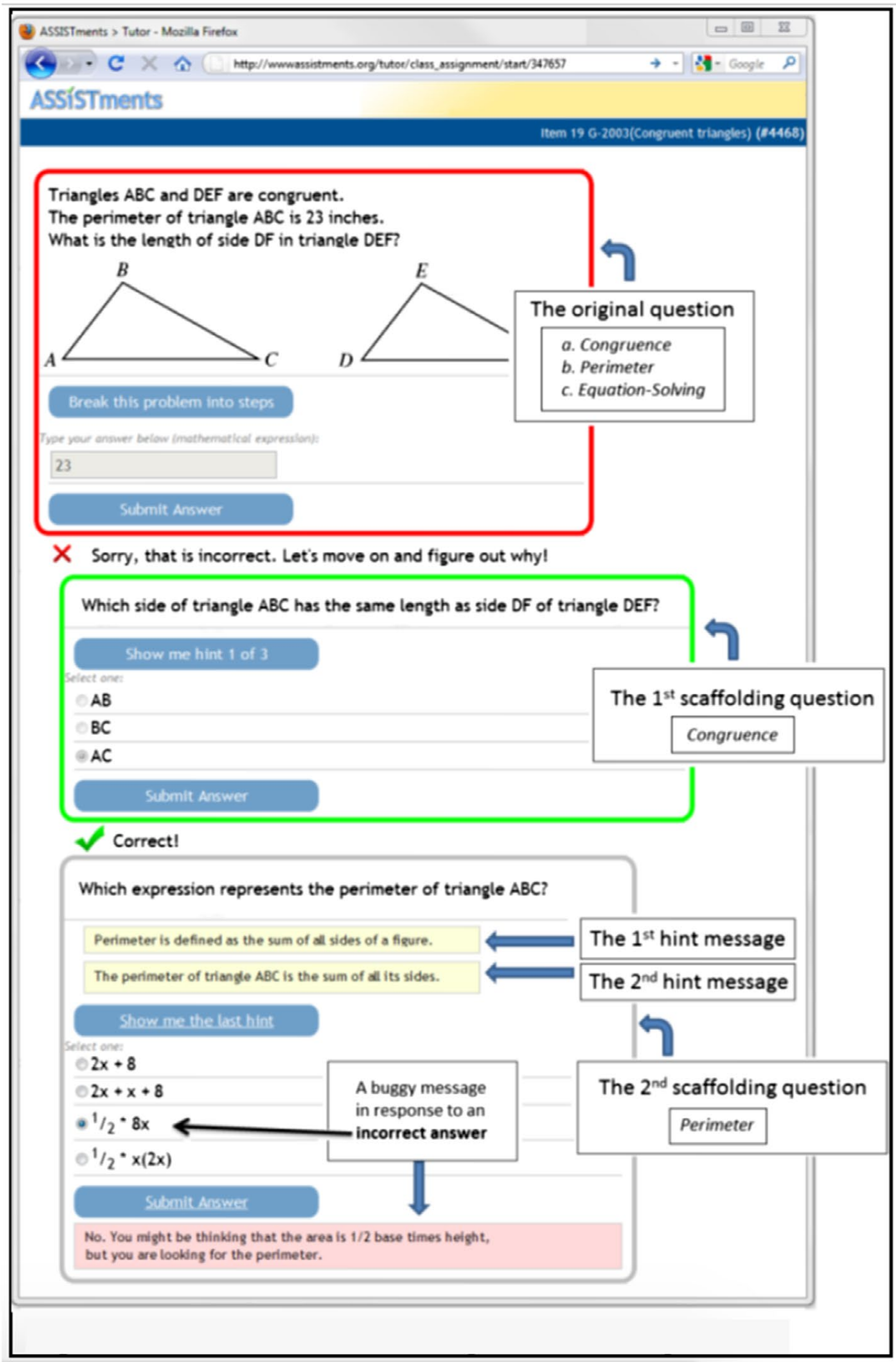

Figure 1. An ASSISTment showing a student being tutored.

students to solve problems until reaching a predefined threshold for completion (i.e., three right in a row).

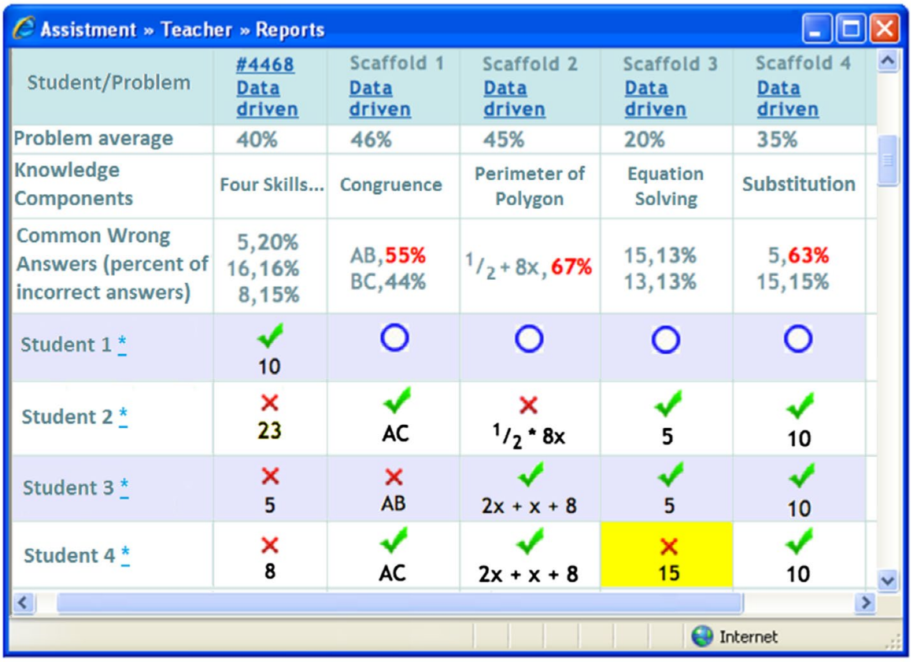

Figure 2. ASSISTments item report.
Notes: This item report shows the results of four students from a larger class. The problem included one original problem (#4468) and four scaffold problems (columns 2–5). ASSISTments generates the overall problem average, tags each item with knowledge components, and provides common wrong answers.

ASSISTments problems can be designed to include three primary types of feedback: (1) "scaffolds" that serve as conceptual guidance or break a similar problem into steps, often after the student has already provided an incorrect response, (2) "hints" that comprise a series of increasingly specific reminders that are meant to help the student solve the given problem, and (3) "correctness feedback" indicating whether the student's answer was correct, incorrect, or worth partial credit. Alternatively, problems can be assigned without feedback through the use of a "test mode" that simply records the student's first response and moves on to the next problem. Collectively, the feedback available in ASSISTments helps to create a relatively simple infrastructure that can adapt to complex student behaviors (Razzaq et al., 2009).

Study conditions and random assignment

Problem set

Prior to the start of the study, the research team created a static problem set consisting of eight problems(see Table 1 for problem content). Items 1–3 of the problem set, what we will refer to as the "learning phase," were offered using one of three feedback conditions (described below). Students were randomly assigned to condition. Items 4–8, what we will refer to as the "testing phase," served as a

Table 1. Problem set overview and feedback conditions.

Phase	Problem	Feedback
Learning Phase	1. There are 1 ½ pizzas to divide evenly among 4 friends. What fraction of one pizza will each friend get?	Condition A, B, or C (based on random assignment)
	2. A serving size for a dog's dinner is 2 ½ cups. If you only have a ½ measuring cup, how many scoops of the ½ measuring cup will it take to give the dog one full serving of food?	Condition A, B, or C (based on random assignment)
	3. Pat wants to run 9/4 of a mile in total distance on a track around a lake. If one lap on the track around the lake is 1/2 mile, then how many laps will Pat need to run? Enter your answer as a fraction or mixed number in lowest terms	Condition A, B, or C (based on random assignment)
Testing Phase	4. Brian purchased a pizza and shared 3/4 of the pizza with his friends. Each friend received 1/4 of the pizza. With how many friends did Brian share his pizza? Be sure to show your answer in simplest form	Correct answer only (no feedback)
	5. The coach of the track and field team is putting together groups of runners to run relays. Each group will need to run a race that is 1/2 of a mile long. Each runner has to run 1/8 of a mile in the race. How many runners will the coach need for each relay race group?	Correct answer only (no feedback)
	6. Jesse is planting a vegetable garden. The area of the garden is 16 square yards. Each vegetable plant will need 1/9 of a square yard to grow in. How many vegetable plants will Jesse be able to plant in her garden?	Correct answer only (no feedback)
	7. Sandy spent 1/4 of an hour waiting to see the dentist which was 1/6 of the total time she spent at the dentist's office. What was the total amount of time Sandy spent at the dentist's office?	Correct answer only (no feedback)
	8. One lap around the Indianapolis 500 racetrack is 2 and 1/2 miles long. Speedy Travers has driven 3/4 of a mile so far in the race. How many laps has he completed?	Correct answer only (no feedback)

post-test offering identical delivery to all students regardless of condition assignment. These problems were delivered in test mode and did not include any feedback except for the correct answer. All items aligned to Common Core standard 6.NS.A.1, which calls for students to "Interpret and compute quotients of fractions, and solve word problems involving division of fractions, e.g., using fraction models and equations to represent the problem" (CCSS-M, 2010).

Condition A: text-based feedback

Upon requesting a hint message for a problem (or answering incorrectly), students in Condition A were provided a series of hint messages involving text. Text-based feedback provided a one or two sentence description corresponding to the concept of fraction division covered in the problem.

Condition B: image-based feedback

Upon requesting a hint message for a problem (or answering incorrectly), students in Condition B were provided a series of scaffolded feedback messages involving images. Visually-based hints included pictorial representations corresponding to the concept of fraction division covered in the problem.

Condition C: correctness only

Students randomly assigned to Condition C, correctness only, were provided neither visual nor text-based feedback during the learning phase. If a student in Condition C entered an incorrect response during the learning phase, they were simply provided the correct answer before beginning the next problem.

Sample Problem

Below is the text of a sample problem provided to students in the learning phase of the study. We have also included links to dynamic versions of the corresponding ASSISTment problems developed for each feedback condition. Following each link will reveal differences in feedback provided with regard to the original problem.

Problem text: There are 1 ½ pizzas to divide evenly among 4 friends. What fraction of one pizza will each friend get? Enter your answer in lowest terms.

- Condition A (Text-Based Feedback) – Dynamic Link: https://goo.gl/9u97aa
- Condition B (Image-Based Feedback) – Dynamic Link: https://goo.gl/qTwCKp
- Condition C (Correctness Only) – Dynamic Link: https://goo.gl/dmC8BX

Random assignment

Random assignment occurred at the student-level (i.e., students within the same classroom were randomly assigned into different conditions). Of the 196 students, 71 were randomized into Condition A, 58 were randomized into Condition B, and 67 were randomized into Condition C. Ultimately, 47 of the 196 students did not complete the assignment and were excluded from subsequent analyses. Furthermore, an additional 37 students accurately answered all questions in the "Learning Phase," without ever experiencing feedback, and were therefore, also excluded from subsequent analyses. This resulted in a final sample of 112 students: 37 in Condition A, 27 in Condition B, and 48 in Condition C.

Results

All participating students were in the sixth grade, and were evenly divided by gender (50.5% female). Among the final set of participants –all of whom made at least one error in the learning phase – 42% made one error during the learning phase, 39.3% made two errors, and 18.8% made three errors. Students showed a range of performance during the testing phase, with a mean score of 2.35 correct (SD = 1.26) out of five possible questions.

An analysis of variance was conducted predicting testing-phase scores based on condition with the number of errors made during the learning-phase included as a covariate. While students in both the text-based (mean = 2.56) and visual feedback (mean = 2.51) conditions during the learning phase performed better in the testing phase than those who had received correctness-answer only feedback (mean = 2.10), this difference was not statistically significant ($F(2, 108) = 1.394$, $p = .253$).

Interestingly, follow-up chi-squared analyses indicated a completion-bias was present, in which students randomly assigned to the image-based feedback group for the learning phase were less likely to complete the assignment (62.1%) when compared to students receiving text-based feedback (77.5%) or correctness-answer only feedback (86.6%) during the learning phase ($\chi^2(2)=10.36$, $p = .006$).

Implications

The implications of this study are twofold. First, results suggest students randomly assigned to the image-based feedback condition during the learning phase performed lower (although not significantly lower) on the post-test compared to students randomly assigned to the text-based condition. At first glance this is counterintuitive and was somewhat disconcerting to the research team because it was totally incongruent with our initial hypothesis that students would learn more by receiving image-based feedback; however, upon further exploration we have concluded the image-based feedback may have actually confused students more. Perhaps additional challenges and confusion were placed on students in the image-based feedback condition. This resonates with seminal literature on cognitive load theory (Kalyuga, Ayres, Chandler, & Sweller, 2003; Pass, Renkl, & Sweller, 2003; Sweller, 1994; Van Merriënboer & Sweller, 2010) indicating learners can become overburdened with too many stimuli, thus undermining the original intent of the feedback to support student learning . It also suggests that despite the feedback conditions being "informationally equivalent" their computational efficiency depends on the information processing operators that act on them (Larken & Simon, 1987).

In more math-specific learning environments, our result is also consistent with a previous study involving fractions with fifth graders suggesting that students often fail to connect symbolic representations with procedural steps, due to a lack of qualitative reasoning skills or under-developed conceptual understanding of the topic (Stampfer & Koedinger, 2013). Another possible explanation is that the researcher/teacher-imposed representation confused students who may have been successful if they created their own representation like the preservice teachers in the Baek et al., 2017 study involving fractions. Baek and colleagues collected 93 different pictorial strategies, 81% of which were correct. The issue may not be images-or-no images but rather, images that match students' stage of thinking. Therefore, our first result has challenged us, as mathematics educators, to carefully consider how images embedded in media are used to support students' conceptual understanding. This implication is particularly noteworthy for teachers, researchers, curriculum developers, and instructional designers who use images to teach traditionally challenging material such as fraction division.

A second result of educational importance found by analyzing the completion rates of students across the three conditions. Interestingly, the percentage of students who completed the entire problem set (all eight items) were significantly different across the three learning phase conditions. Condition C (correct answer only) had the highest student completion rate, followed by Condition A (text-based feedback) and finally Condition B (image-based feedback). Again, this result is somewhat counterintuitive. This disproportional completion rate suggests students randomly assigned to the image phase perhaps became bored or confused working through problems. Given the Common Core's focus on supporting students in their ability to persevere and persist when solving problems, this result is important, as it suggests image-based feedback may actually be counterproductive for supporting this skill in fraction division word problems for some students, particularly if the images are teacher-created ones.

Limitations

There are at least three limitations related to this study. First, the study only explored the effects of the last two levels of CRA, due to constraints of web-based system limiting ability to provide "concrete" feedback in an online environment. Therefore, it may be interesting to replicate a study in the classroom using similar problem sets and actual manipulatives so the concrete level of CRA can be taken into consideration. Second, only one grade level of students in one state participated in the study. It is uncertain if similar results would be obtained if students were recruited from different grade levels (e.g., Grade 5 or Grade 7) or across different states due to their potential different learning trajectories and/or state standards. Third, the relatively small number of items in the problem set ($n = 8$ items; 3 in the learning phase and 5 in the testing phase), focusing on one standard, limits our ability to make generalizable statements about the effects of feedback for other Common Core standards. Establishing reliability of the problem set, especially the five items that were used in the testing phase, would further validate the results.

Future research

Our initial data for this study raise other more general pedagogical and research questions related to providing high quality feedback using online educational media. This is somewhat consistent with previous research suggesting there are in fact tradeoffs between different types of mathematical representations (see Koedinger, Alibali, & Nathan, 2008) and with the case with feedback involving fractions, are context-based (Stampfer & Koedinger, 2013). In future research and studies, we will consider the following questions to address both the tradeoffs and relevant contextual factors: Is there a developmental pathway to symbolic reasoning that we have overlooked in our beliefs about "best practice?" (i.e., do students need a solid procedural understanding before images are helpful?) Are the different types of feedback helpful or distracting to different groups of students (e.g., do remedial students benefit from one approach over another while accelerated students benefit from a different approach?) How can technology better support teachers in forming instructional groupings that take into consideration the cognitive load of particular tasks along with students' ability to work with abstract processes? This may involve cognitive labs and/or think-aloud studies with students who have been randomly assigned into feedback conditions. Follow-up discussion would provide more information about how students are interpreting the feedback provided and may help identify the underlying reasons for their performance and lack of persistence (e.g., boredom, distraction, confusion, etc.). Web-based platforms such as ASSISTments offer promise for researchers and teachers alike to unpack the assumptions about effective instruction and better match instruction to the needs of individual learners.

Acknowledgements

Our study was conducted within the context of a larger, National Science Foundation Grant ("Adding Research Accounts to the ASSISTments' Platform: Helping Researchers Do Randomized Controlled Studies with Thousands of Students").

Disclosure statement

No potential conflict of interest was reported by the authors.

References

Baek, J. M., Wickstrom, M. H., Tobias, J. M., Miller, A. L., Safak, E., Wessman-Enzinger, N., & Kirwan, J. V. (2017). Preservice teachers' pictorial strategies for a multistep multiplicative fraction problem. *The Journal of Mathematical Behavior, 45*, 1–14.

Ball, D. L. (1990). Prospective elementary and secondary teachers' understanding of division. *Journal for Research in Mathematics Education, 21*, 132–144.

Baroody, A. (1987). *Children's mathematical thinking*. New York, NY: Teachers College Press.

Booth, J. L., & Koedinger, K. R. (2012). Are diagrams always helpful tools? Developmental and individual differences in the effect of presentation format on student problem solving. *British Journal of Educational Psychology, 82*(3), 492–511.

Borko, H., Eisenhart, M., Brown, C. A., Underhill, R. G., Jones, D., & Agard, P. C. (1992). Learning to teach hard mathematics: Do novice teachers and their instructors give up too easily? *Journal for Research in Mathematics Education, 23*, 194–222.

Butler, F. M., Miller, S. P., Crehan, K., Babbit, B., & Pierce, T. (2003). Fraction instruction for students with mathematics disabilities: Comparing two teaching sequences. *Learning Disabilities Research and Practice, 18*(2), 99–111.

Cheung, A. C. K., & Slaven, R. E. (2013). The effectiveness of educational technology applications for enhancing mathematics achievement in K-12 classrooms: A meta-analysis. *Educational Research Review, 9*, 88–113.

Cole, R. S., & Todd, J. B. (2003). Effects of web-based multimedia homework with immediate rich feedback on student learning in general chemistry. *Journal of Chemical Education, 80*(11), 1338–1343.

Common Core State Standards for Mathematics (CCSS-M). (2010). *National governors association center for best practices & council of chief state school officers*. Washington, DC: Authors.

Fendel, D. M. (1987). *Understanding the structure of elementary mathematics*. Newton, MA: Allyn & Bacon.

Flores, M. M. (2010). Using the concrete-representational-abstract sequence to teach subtraction with regrouping to students at risk for failure. *Remedial and Special Education, 31*, 195–207.

Fyfe, E. R., DeCaro, M. S., & Rittle-Johnson, B. (2015). When feedback is cognitively-demanding: The importance of working memory capacity. *Instructional Science, 43*(1), 73–91.

Hattie, J., & Gan, M. (2011). Instruction based on feedback. In R. Mayer & P. Alexander (Eds.), *Handbook of research on learning and instruction* (pp. 249–271). New York, NY: Routledge.

Hattie, J., & Timperley, H. (2007). The power of feedback. *Review of Educational Research, 77*(1), 81–112.

Institute of Education Sciences. (2009). IES practice guide – assisting students struggling with mathematics: Response to Intervention (RtI) for Elementary and Middle Schools. Retrieved from http://ies.ed.gov/ncee/wwc/pdf/practice_guides/rti_math_pg_042109.pdf

Jansen, B. R. J., Louwerse, J., Straatemeier, M., Van der Ven, S. H. G., Klinkenberg, S., & Van der Maas, H. L. J. (2013). The influence of experiencing success in math on math anxiety, perceived math competence, and math performance. *Learning and Individual Differences, 24*, 190–197. doi:10.1016/j.lindif.2012.12.014

Jordan, L., Miller, M. D., & Mercer, C. D. (1999). The effects of concrete to semi-concrete to abstract instruction in the acquisition and retention of fraction concepts and skills. *Learning Disabilities: A Multidisciplinary Journal, 9*, 115–122.

Kalyuga, S., Ayres, P., Chandler, P., & Sweller, J. (2003). The expertise reversal effect. *Educational Psychologist, 38*(1), 23–31.

Kelly, K., Heffernan, N., Heffernan, C., Goldman, S., Pellegrino, G. & Soffer, D. (2013). Estimating the Effect of Web-Based Homework. In H. Lane, K. Yacef, J. Motow, & P. Pavlik (Eds.), *The artificial intelligence in education conference* (pp. 824–827). Springer-Verlag.

Kluger, A. N., & DeNisi, A. (1996). Effects of feedback intervention on performance: A historical review, a meta-analysis, and a preliminary feedback intervention theory. *Psychological Bulletin, 119*, 254–284.

Koedinger, K. R., Alibali, M. W., & Nathan, M. J. (2008). Trade-offs between grounded and abstract representations: Evidence from algebra problem solving. *Cognitive Science, 32*(2), 366–397.

Larken, J. H., & Simon, H. A. (1987). Why a diagram is (sometimes) worth ten thousand words. *Cognitive Science, 11*, 65–99.

Li, Y. (2008). What do students need to learn about division of fractions? *Mathematics Teaching in the Middle School, 13*, 546–552.

Mendicino, M., Razzaq, L., & Heffernan, N. T. (2009). A comparison of traditional homework to computer-supported homework. *Journal for Research on Technology Education, 41*(3), 331–359.

Mercer, C. D., Jordan, L., & Miller, S. P. (1996). Constructivistic math instruction for diverse learners. *Learning Disabilities Research and Practice, 11*, 147–156.

Murphy, R., Gallagher, L., Krumm, A., Mislevy, J., & Hafter, A. (2014). *Research on the use of khan academy in schools*. Menlo Park, CA: SRI Education.

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the national mathematics advisory panel*. Washington, DC: U. S. Department of Education.

Nguyen, D. M., Hsieh, Y. J., & Allen, G. D. (2006). The impact of web-based assessment and practice on students' mathematics learning attitudes. *Journal of Computers in Mathematics and Science Teaching, 25*(3), 251–279.

Nguyen, D. M., & Kulm, G. (2005). Using web-based practice to enhance mathematics learning and achievement. *Journal of Interactive Online Learning, 3*(3), 1–16.

Osana, H. P., & Royea, D. A. (2011). Obstacles and challenges in preservice teachers' explorations with fractions: A view from a small-scale intervention study. *Journal of Mathematical Behavior, 30*(4), 333–352.

Pass, F., Renkl, A., & Sweller, J. (2003). Cognitive load theory and instructional design: Recent developments. *Educational Psychologist, 38*(1), 1–4.

Peterson, S. K., Mercer, C. D., & O'Shea, L. (1988). Teaching learning disabled students place value using the concrete to abstract sequence. *Learning Disabilities Research, 4*, 52–56.

Razzaq, L., Patvarczki, J., Almeida, S. F., Vartak, M., Feng, M., Heffernan, N. T., & Koedinger, K. R. (2009). The ASSISTment builder: Supporting the Life cycle of tutoring system content creation. *IEEE Transactions on Learning Technologies, 2*(2), 157–166.

Schaffert, S., & Hilzensauer, W. (2008). On the way towards personal learning environments: Seven crucial aspects. *eLearning Papers (9)*, 1–10.

Shute, V. J. (2008). Focus on formative feedback. *Review of Educational Research, 78*(1), 153–189.

Sousa, D. A. (2008). *How the brain learns mathematics*. Thousand Oaks, CA: Corwin Press.

Stampfer, E., & Koedinger, K. R. (2013). When seeing isn't believing: Influences of prior conceptions and misconceptions. In M. Knauff, N. Sebanz, M. Pauen, & I. Wachsmuth (Eds.), *Proceedings of the 35th Annual Conference of the Cognitive Science Society* (pp. 1384–1389). Austin, TX: Cognitive Science Society.

Steenbergen-Hu, S., & Cooper, H. (2013). A meta-analysis of the effectiveness of intelligent tutoring systems on K-12. *Journal of Educational Psychology, 105*(4), 970–987.

Swan, K. (2003). Learning effectiveness: What the research tells us. In J. Bourne & J. C. Moore (Eds.), *Elements of quality online education, practice and direction* (pp. 13–45). Needham, MA: Sloan Center for Online Education.

Sweller, J. (1994). Cognitive load theory, learning difficulty, and instructional design. *Learning and Instruction, 4*(4), 295–312.

Tirosh, D. (2000). Enhancing prospective teachers' knowledge of children's conceptions: The case of division of fractions. *Journal for Research in Mathematics Education, 31*(1), 5–25.

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2009). *Elementary and middle school mathematics: Teaching developmentally* (7th ed.). Boston, MA: Allyn & Bacon.

Van Merriënboer, J. J., & Sweller, J. (2010). Cognitive load theory in health professional education: Design principles and strategies. *Medical Education, 44*(1), 85–93.

Witzel, B., Mercer, C. D., & Miller, D. M. (2003). Teaching algebra to students with learning difficulties: An investigation of an explicit instruction model. *Learning Disabilities Research and Practice, 18*(2), 121–131.