Light meson form factors at high Q^2 from lattice QCD

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Abstract. Measurements and theoretical calculations of meson form factors are essential for our understanding of internal hadron structure and QCD, the dynamics that bind the quarks in hadrons. The pion electromagnetic form factor has been measured at small space-like momentum transfer $|q^2| < 0.3$ GeV^2 by pion scattering from atomic electrons and at values up to 2.5 GeV^2 by scattering electrons from the pion cloud around a proton. On the other hand, in the limit of very large (or infinite) $Q^2 = -q^2$, perturbation theory is applicable. This leaves a gap in the intermediate Q^2 where the form factors are not known.

As a part of their 12 GeV upgrade Jefferson Lab will measure pion and kaon form factors in this intermediate region, up to Q^2 of 6 GeV^2. This is then an ideal opportunity for lattice QCD to make an accurate prediction ahead of the experimental results. Lattice QCD provides a from-first-principles approach to calculate form factors, and the challenge here is to control the statistical and systematic uncertainties as errors grow when going to higher Q^2 values.

Here we report on a calculation that tests the method using an η_s meson, a 'heavy pion' made of strange quarks, and also present preliminary results for kaon and pion form factors. We use the $n_f = 2 + 1 + 1$ ensembles made by the MILC collaboration and Highly Improved Staggered Quarks, which allows us to obtain high statistics. The HISQ action is also designed to have small discretisation errors. Using several light quark masses and lattice spacings allows us to control the chiral and continuum extrapolation and keep systematic errors in check.

1 Introduction

The electromagnetic form factor of the meson parameterises the deviations from the behaviour of a point-like particle when hit by a photon. By determining the form factor at different values of the square of the 4-momentum transfer, Q^2, we can test our knowledge of QCD as a function of Q^2. Measurements of π and K form factors are key experiments in the new Jefferson Lab 12 GeV upgrade (experiments E12-06-101 [1] and E12-09-11 [2]). The pion form factor is known experimentally but with sizeable uncertainties up to $Q^2 < 2.45$ GeV^2, and the new experiment will extend the Q^2 range up to 6 GeV^2.

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Lattice QCD calculations have been done at small Q^2 (see [3] for a review) as doing a calculation at small momenta is easier because of deteriorating signal to noise at large momentum. In [4] we studied the pion form factor close to $Q^2 = 0$ and determined the charge radius of the pion. The goal of this study is to provide predictions of the form factors at high Q^2 ahead of experiments and to test the applicability of asymptotic perturbative QCD (PQCD). Here we use the η_s meson, a pseudoscalar meson made of strange quarks, as a "pseudo pion" to see how (and if) the form factor approaches the PQCD value. The strange quark is light ($m_s << \Lambda_{\text{QCD}}$) from the PQCD point of view, and the behaviour is expected to be qualitatively similar for η_s and π. The advantage is that strange quarks are computationally cheaper to simulate on the lattice, and the signal to noise ratio is better than for lighter quarks. We are now extending the study to pions and kaons, although the maximum Q^2 we can reach in the current calculations is not as high as for the η_s.

2 Lattice configurations

We use lattice ensembles generated by the MILC Collaboration, with 3 different lattice spacings (ranging from 0.15 fm to 0.09 fm) and different light quark masses to allow a reliable continuum and chiral extrapolation. The Higly Improved Staggered Quark (HISQ) action is used for both valence and sea quarks, with u/d, s and c quarks included in the sea. The strange quark mass has been tuned to the physical mass by using the η_s mass. The ensembles are listed in Table 1.

Table 1. Lattice ensembles used in this study: Set 1 is 'very coarse' ($a \sim 0.15$ fm), sets 2 and 3 'coarse' ($a \sim 0.12$ fm) and set 4 'fine' $a \sim 0.09$ fm. Lattice spacing is set using the Wilson flow parameter $w_0 = 0.1715(9)$ fm. am_q are the sea quark masses in lattice units and $L_s/a \times L_t/a$ gives the lattice size in spatial and time directions. M_π and n_{conf} are the pion mass and the number of configurations. The last column gives the time extent of the 3-point correlators. More details of the lattice ensembles can be found in [5, 6].

Set	β	w_0/a	am_l	am_s	am_c	$L_s/a \times L_t/a$	M_π	n_{conf}	T/a
1	5.8	1.1119(10)	0.01300	0.0650	0.838	16×48	300 MeV	1020	9, 12, 15
2	6.0	1.3826(11)	0.01020	0.0509	0.635	24×64	300 MeV	1053	12, 15, 18
3	6.0	1.4029(9)	0.00507	0.0507	0.628	32×64	220 MeV	1000	12, 15, 18
4	6.3	1.9006(20)	0.00740	0.0370	0.440	32×96	310 MeV	1008	15, 18, 21

3 Electromagnetic form factors on the lattice

The electromagnetic form factor is extracted from the 3-point correlation function depicted in Fig. 1, where a current V is inserted in one of the meson's quark propagators. We also need the standard 2-point correlation function of the meson that propagates from time 0 to time t. The 3-point correlation function gives

$$\langle P(p_f)|V_\mu|P(p_i)\rangle = F_P(Q^2) \cdot (p_f + p_i)_\mu, \tag{1}$$

where p_i and p_f are the initial and final momenta of the pseudoscalr meson P, respectively. We use a 1-link vector current V_μ in the time direction, and the Breit frame $\vec{p}_i = -\vec{p}_f$ to maximise Q^2 for a given momentum pa. This leads to the simple relation $Q^2 = |2\vec{p}_i|^2$. The form factor is normalised by requiring $F_P(0) = 1$.

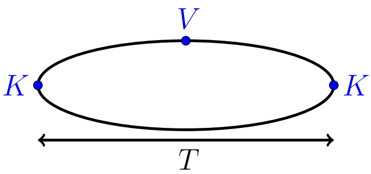

Figure 1. A 3-point correlation function. The meson, here a kaon, is created at time $t = t_0$, and destroyed at time $t = t_0 + T$. A vector current V is inserted at time t', where $t_0 < t' < t_0 + T$. We use multiple T values to fully map the oscillating states that are a feature of staggered fermions.

To extract the properties of the meson we use multi-exponential fits with Bayesian priors to fit both 2-point and 3-point correlators simultaneously. The fit functions are

$$
\begin{aligned}
C_{2\text{pt}}(\vec{p}) &= \sum_i b_i^2 f(E_i(p), t') + \text{o.p.t.}, \\
C_{3\text{pt}}(\vec{p}, -\vec{p}) &= \sum_{i,j} \left[b_i(p) f(E_i(p), t) J_{ij}(Q^2) b_j(p) f(E_j(p), T - t) \right] + \text{o.p.t.}, \\
f(E, t) &= e^{-Et} + e^{-E(L_t - t)},
\end{aligned} \tag{2}
$$

where E_i is the energy of the state i and $\vec{p}$ is the spatial momentum, and o.p.t. stands for the opposite parity terms. Note that E_i and the amplitudes b_i are common fit parameters for the 2-point and 3-point functions. We are interested in the ground state parameters E_o (the mass of the meson if momentum $p = 0$), b_0 (which is associated with the decay constant of the meson) and $J_{00}(Q^2)$, but use 6 exponentials to make sure that effects of the excited states are properly included in the error estimates. J_{00} gives the matrix element of the vector current that we need to extract the form factor. More details can be found in [7].

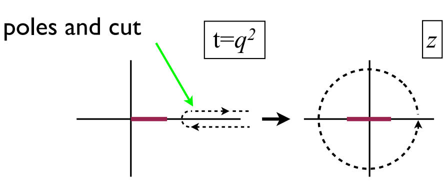

Figure 2. Mapping the domain of analyticity in $t = q^2$ onto the unit circle in z.

To determine the form factor F in the physical continuum limit we must extrapolate in the lattice spacing and u/d quark mass. We first remove the pole in $F_P(Q^2)$ by multiplying the form factor by $P_V(Q^2)$, where

$$
P_V^{-1}(Q^2) = \frac{1}{1 + Q^2/M_V^2}. \tag{3}
$$

The pole mass M_V is the mass of the vector meson that corresponds to the quarks at the current V. If the quarks are light quarks the mass is M_ρ, if the quarks are strange quarks the pole mass is M_ϕ. The product $P_V F$ has reduced Q^2-dependence because P_V^{-1} is a good match to the form factor at small Q^2. We then map the domain of analyticity in $t = q^2$ onto the unit circle in z — see Fig. 2:

$$
z(t, t_{\text{cut}}) = \frac{\sqrt{t_{\text{cut}} - t} - \sqrt{t_{\text{cut}}}}{\sqrt{t_{\text{cut}} - t} + \sqrt{t_{\text{cut}}}} \tag{4}
$$

and choose $t_{\text{cut}} = 4M_K^2$ for the η_s. Now $|z| < 1$ and we can do a power series expansion in z, and use a fit form

$$P_V F(z, a, m_{\text{sea}}) = 1 + \sum_i z^i A_i \left[1 + B_i (a\Lambda)^2 + C_i (a\Lambda)^4 + D_i \frac{\delta m}{10}\right], \tag{5}$$
$$\delta m = \sum_{u,d,s} (m_q - m_q^{\text{tuned}})/m_s^{\text{tuned}}, \ \Lambda = 1.0 \text{ GeV}.$$

The terms with B_i and C_i parametrise lattice discretisation effects and the last term takes into account possible mistunings of the sea quark masses.

4 Results

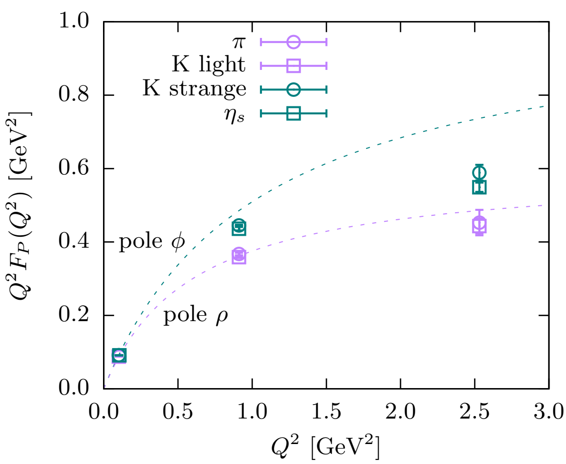

Figure 3. Pion, kaon and η_s form factors $Q^2 F_P$ on the coarse lattice (set 2). The form factors with a strange current are found to be very similar, and so are the form factors with a light current. The spectator quark has only very small effect to the form factor. The dashed lines show the corresponding pole forms $Q^2 P_V^{-1}(Q^2)$ (equation (3)) with pole masses M_ϕ and M_ρ respectively.

Figure 3 shows results for pion, kaon and η_s form factors $Q^2 F_P$. Let us start by noting how small the effect of the spectator quark is in the pseudoscalar meson electromagnetic form factor. The pion is made of two light quarks, whereas the η_s is made of two strange quarks. The K meson has one strange quark and one light quark, and the current can thus be either light or strange. Fig. 3 illustrates how the form factors can be grouped according to the flavor of the quarks at the current insertion: the η_s and the strange-current K form factors are very similar as are the pion and light-current K form factors. The form factors follow the pole form at small Q^2, but peel away from it when the momentum transfer grows larger.

In Figure 4 we plot the η_s form factor obtained on very coarse, coarse and fine lattices as a function of Q^2. We can reach $Q^2 \sim 6$ GeV^2 on the fine lattice, and the form factor multiplied by Q^2 is found to be almost flat in the Q^2 range 3 – 6 GeV^2. This can be compared to the asymptotic value marked with 'PQCD1'. At high Q^2 the electromagnetic form factor can be calculated using perturbative QCD, because the process in which the hard photon scatters from the quark or antiquark factorises from the distribution amplitudes which describe the quark-antiquark configuration in the meson, as is illustrated in figure 5 using a pion as an example. The asymptotic value is

$$F_P(Q^2) = \frac{8\pi\alpha_s f_P^2}{Q^2}, \tag{6}$$

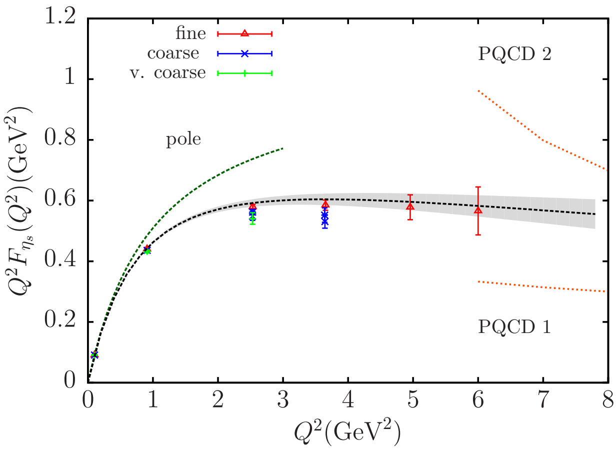

Figure 4. The η_s form factor $Q^2 F_{\eta_s}$ as a function of Q^2. At small Q^2 the form factor follows the pole form (with pole mass M_ϕ) as expected. The discretisation effects are very small. The grey band shows the continuum and chiral extrapolation (equation (5)). 'PQCD1' is the asymptotic value from perturbative QCD, and 'PQCD2' shows the perturbative value with corrections added to the asymptotic PQCD. We plot $Q^2 F_{\eta_s}$ rather than F_{η_s} to compare to the asymptotic value (eq. 6 multiplied by Q^2 gives $8\pi\alpha_s f_P^2$.)

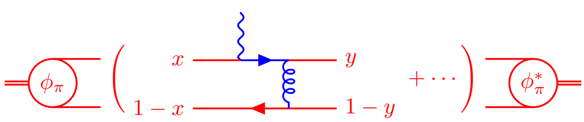

Figure 5. The perturbative QCD description of a meson electromagnetic form factor (here the pion is used as an example, but the calculation is analogous for the η_s). ϕ_π is the distribution amplitude and the blue colour marks the high momentum photon and gluon.

where f_P is the decay constant of the pseudoscalar meson (pion, kaon, η_s). The value we obtain for the η_s form factor is much higher than the asymptotic value at $Q^2 = 6\ \mathrm{GeV}^2$. On the other hand, the curve 'PQCD2' that includes non-asymptotic corrections to the distribution amplitude lies above the η_s form factor. More details can be found in [7].

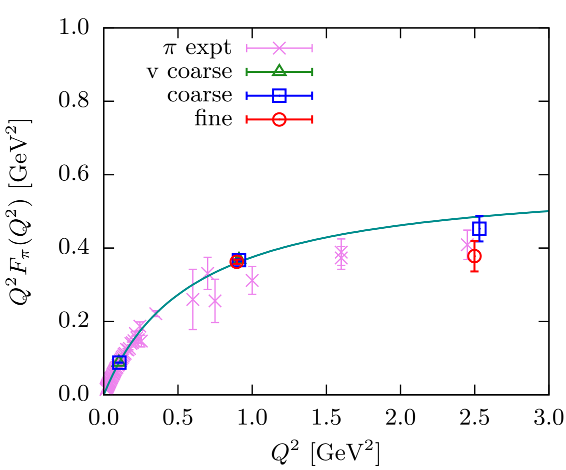

Figure 6. The pion form factor $Q^2 F_\pi$ as a function of Q^2. The agreement with experimental results at small Q^2 is excellent, and peeling away from the pole form (shown as the continuous line) is observed as expected. The results are preliminary as we are pushing to higher Q^2, and no continuum extrapolation is done at this time. Also smaller light quark masses have to be included in the study to do a reliable chiral extrapolation: the pion masses used here are $\sim$ 300 MeV. The experimental results are from [8–10].

In figures 6 and 7 we show our preliminary results for pion and kaon electromagnetic form factors as a function of Q^2. These are the first predictions of the K^0 and K^+ form factors from lattice QCD ahead of the Jefferson Lab experiment. The K^0 and K^+ form factors are calculated from the strange and light current K form factors by combining with the electric charges of the quarks: K^+ is $u\bar{s}$ and K^0 is $d\bar{s}$. Work is underway to go to higher Q^2 values and to study the dependence of the pion and kaon

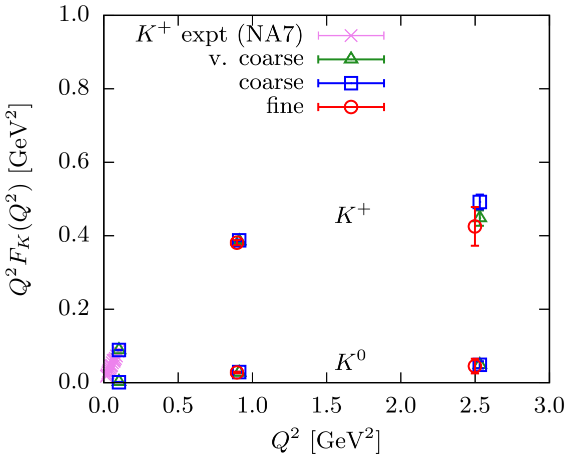

Figure 7. The kaon form factor $Q^2 F_K$ as a function of Q^2. The agreement with experimental results at small Q^2 is excellent. The results are preliminary as we are pushing to higher Q^2, and no continuum extrapolation is done at this time. Also smaller light quark masses have to be included in the study to do a reliable chiral extrapolation. The experimental results are from [11].

form factors on the light quark mass. The light quark masses used at this preliminary stage correspond to pion mass of $\sim$ 310 MeV. This has been studied in the case of the η_s form factor, where the effect is negligible, but smaller masses are needed to do the chiral extrapolation for the pion and kaon form factors. We plan to include results from physical light quarks in our final analysis. No continuum or chiral extrapolation is presented at this time for the pion and kaon form factors.

5 Conclusions and outlook

Our η_s form factor results indicate that asymptotic perturbative QCD is not applicable at $Q^2 \sim 6$ GeV^2 or below — much larger Q^2 are needed. Using strange quarks instead of light quarks allows us to get some qualitative knowledge of light pseudoscalar meson form factors (pion and kaon form factors) at high Q^2 ahead of the more lengthy calculations required for K and π. We can also probe higher Q^2 values with strange quarks than with light quarks. However, we can already provide first, preliminary predictions of the K^+ and K^0 form factors ahead of the upcoming Jefferson Lab experiment. The pion form factor is the most challenging. By gathering more statistics and pushing to higher Q^2 we will have good theoretical understanding of the form factors in the momentum range that the Jefferson Lab pion and kaon experiments will use.

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