$D \to Kl\nu$ semileptonic decay using lattice QCD with HISQ at physical pion masses

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Abstract.
The quark flavor sector of the Standard Model is a fertile ground to look for new physics effects through a unitarity test of the Cabbibo-Kobayashi-Maskawa (CKM) matrix. We present a lattice QCD calculation of the scalar and the vector form factors (over a large q^2 region including $q^2 = 0$) associated with the $D \to Kl\nu$ semi-leptonic decay. This calculation will then allow us to determine the central CKM matrix element, V_{cs} in the Standard Model, by comparing the lattice QCD results for the form factors and the experimental decay rate. This form factor calculation has been performed on the $N_f = 2 + 1 + 1$ MILC HISQ ensembles with the physical light quark masses.

1 Introduction

The flavour changing weak interactions between quarks via emission of W bosons can be parametrised in terms of the Cabbibo-Kobayashi-Maskawa (CKM) unitary matrix in the Standard Model given by [1, 2]

$$V_{CKM} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}. \quad (1)$$

Precise and independent determination of each of the CKM matrix elements is crucial to test the Standard Model stringently and any deviation from unitarity would signal the existence of physics beyond the Standard Model.

The uncertainties in the unitarity checks of the second row and second column of the CKM matrix are dominated by that of $|V_{cs}|$, the central CKM matrix element. This element is calculated from the studies of the leptonic and semileptonic meson decays involving charged flavour changing current from c to s by combining the experimental decay rate with the vector form factor calculated from lattice QCD [1, 2]. The best experimental result to date is achieved by combining the experimental data from BaBar [3], Belle [4], BES [5] and CLEO [6]. However, in the present scenario, the uncertainty in $|V_{cs}|$ is dominated by the lattice uncertainty in the form factors.

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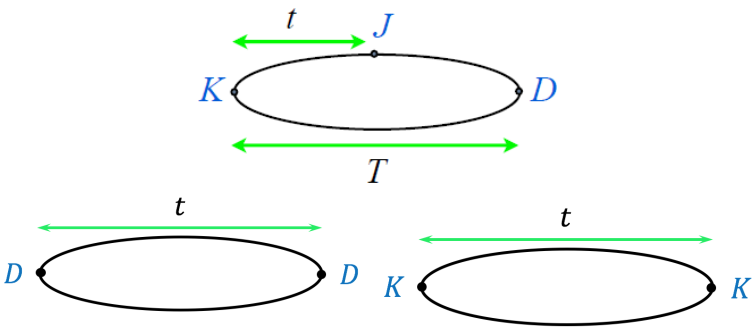
Here, we present a calculation of the semileptonic $D \to Kl\nu$ decay on the $N_f = 2 + 1 + 1$ lattices generated by MILC using highly improved staggered quark (HISQ) formalism [7], which is an improvement over HPQCD's previous work reported in [8, 9]. In contrast to the work done by Fermilab lattice and MILC collaboration in the reference [10], we have calculated both the scalar and vector form factors of this decay over the whole range of kaon momentum instead of only at the maximum kaon momentum. A similar study [11] has been recently done using twisted mass fermions.

Figure 1. The diagram represents the three-point correlator for the $D \to Kl\nu$ semileptonic decay (on the top) and the two-point correlators for the D and K mesons (at the bottom).



2 Formalism

The matrix element of the $D \to Kl\nu$ semileptonic decay via the charged electroweak current gets contribution only from the vector current. The vector current matrix element can be parametrised in terms of the scalar and vector form factors $f_0(q^2)$ and $f_+(q^2)$, and can be written as -

$$\begin{aligned} Z_{V,t} \times < K^- | V^\mu | D^0 > \quad &= \quad f_+^{D\to K(q^2)} [p_D^\mu + p_K^\mu - \frac{M_D^2 - M_K^2}{q^2} q^\mu] \\ &+ f_0^{D\to K(q^2)} \frac{M_D^2 - M_K^2}{q^2} q^\mu \end{aligned} \tag{2}$$

where, $q^\mu = p_D^\mu - p_K^\mu$ is the exchanged 4-momentum from D with a 4-momentum p_D^μ to K with a 4-momentum p_K^μ and carried away by the W boson. M_D amd M_K are the masses of the D and K mesons respectively. In our setup, we give momentum to the strange quark inside the K meson.

The local vector current using the HISQ formalism is not conserved, and therefore requires a renormalisation factor Z_V to obtain the continuum result. We determine this renormalisation by using the partially conserved vector current (PCVC) relation [8] and also the scalar current amplitude $< K^- | S | D^0 > = < K^- | \bar{\psi}_s \psi_c | D^0 >$.

The scalar current amplitude is parametrised as

$$< K^- | S | D^0 > = \frac{M_D^2 - M_K^2}{m_{0c} - m0s} f_0^{D\to K}(q^2) \tag{3}$$

Inverting this equation the scalar form factor for different q^2 can be extracted. The scalar current using HISQ is absolutely renormalised when multiplied by lattice quark mass [8].

Table 1. Sets of MILC configurations used with their $\beta = 10/g^2$ [13], w_0/a for $w_0 = 0.1715(9)$ fm fixed from f_π [14], L_s/a, L_t/a, number of configurations N_{conf}, number of independent time sources for each configuration t_0, multiple values of the source-sink separation T for each t_0, (HISQ) sea quark masses- m_l (at physical pion mass), m_s and m_c in lattice units [13].

Set	β	w_0/a	L_s/a	L_t/a	N_{conf}	t_0	T	am_l^{sea}	am_s^{sea}	am_c^{sea}
3	5.8	1.1367(5)	36	48	997	16	9, 12, 15, 18	0.00235	0.0647	0.831
8	6.0	1.4149(6)	48	64	998	16	12, 15, 18, 21	0.00184	0.0507	0.628
11	6.0	1.9518(7)	64	96	660	8	16, 19, 22, 25	0.00120	0.0363	0.432

For easier calculation of the Z_V factor and a better signal we have used only the local temporal vector current. For calculating $Z_{V,t}$ we consider the kinematics at $q^2 = q^2_{max} = (M_D - M_K)^2$ i.e. $\vec{p}_k = \vec{0}$. Plugging in the $f_0^{D\to K}(q^2)$ values from equation 3 into equation 2 we can extract $Z_{V,t}$. For different kinematic combinations we can extract the vector form factor $f_+^{D\to K}(q^2)$ by plugging in the $Z_{V,t}$ value and $f_0^{D\to K}(q^2)$ values from equation 3 into equation 2. In this way we can cover the full physical range of q^2 starting from q^2_{max} where the momentum exchange is maximum i.e. K meson is rest to $q^2 = 0$ where K gets the maximum possible momentum in the opposite direction to leptons.

The differential decay rate is dominated by the vector channel in the vanishing lepton mass limit [12] and we get

$$d\Gamma(q^2) = \frac{G_F^2 |V_{cs}|^2}{192\pi^3 m_D^3} dq^2 \lambda(q^2)^{3/2} |f_+(q^2)|^2 \tag{4}$$

Staggered quarks have four tastes running in the correlator loop. To get a non-zero expectation value of the scalar and vector current operator matrix elements, we need to choose correct combinations of operators at the source, sink and current insertion point such that the correlator becomes taste-singlet. For generating the scalar current amplitude, the current carries spin-taste $1 \otimes 1$. We keep the spin-taste at the K-meson annihilation point the same for both scalar and vector currents as we want to use the same strange propagators in both cases. This end has the spin-taste content $\gamma_5 \otimes \gamma_5$. To nullify the tastes, the simplest choice of operators for the D meson end is also the Goldstone pseudoscalar operator $\gamma_5 \otimes \gamma_5$.

For the local temporal vector current, at the current insertion point we have used the $\gamma_t \otimes \gamma_t$ operator. As mentioned before, the same K meson propagators are used in this case as well, therefore we have a spin-taste operator $\gamma_5 \otimes \gamma_5$ at this end. Now, to cancel the overall taste the simplest operator choice at the D meson end would be the local non-Goldstone operator $\gamma_5\gamma_t \otimes \gamma_5\gamma_t$ which generates a D meson with slightly different mass. To use these three-point correlators we need to make Goldstone and local non-Goldstone two-point D correlators and Goldstone two-point K correlators as well.

3 Lattice setup

We have used publicly available MILC HISQ $N_f = 2 + 1 + 1$ configurations with three different lattice spacing $\sim 0.09fm$ (fine), $\sim 0.12fm$ (coarse), $\sim 0.15fm$ (very coarse) and the physical values of all of the sea quark masses. The details of these configurations are given in Table 1.

The values of the time sources t_0 have been chosen randomly to reduce autocorrelation and for each configuration multiple values of t_0, uniformly placed on the lattice, have been used to get better statistics. To increase the statistics further, multiple values of the source-sink separation T have been used for each t_0 value. The valence light quark mass am_l^{val} is taken to be the same as the sea light quark mass am_l^{sea} whereas the valence strange quark mass $am_s^{val,tuned}$ is tuned [15] to give the mass of

the η_s meson to be 0.6885(22) GeV [15]. We also tuned the valence charm quark mass $am_c^{val,tuned}$ to get the mass of the η_c meson m_{η_c} = 2.9863(27) GeV [15].

The three-point correlation functions on the lattice have been generated using the "sequential technique" shown in Figure 1. In this set up, zero momentum D meson is created at time $t_0 + T$ on lattice, after it propagates to time t on lattice, the current (scalar or local temporal vector) is inserted at time t which changes the flavor c inside the D meson to flavor s to create a K meson and emits a W boson.

4 Fits and data analysis

The two point heavy-light D and K meson correlators have the following fit form -

$$G^{2pt}(t;\vec{p}) = \sum_n a_n^2(e^{-E_n t} + e^{-E_n(T-t)}) + (-1)^t \sum_{no} a_{no}^2(e^{-E_{no}t} + e^{-E_{no}(T-t)}). \tag{5}$$

Here, both mesons in staggered quark formalism have oscillation in the correlators; E_n represents the energy of the $n-$th excited state whereas E_{no} represents the energy of the n-th oscillating state. Similarly, a_n and a_{no} respectively give the non-oscillating and oscillating pieces of the amplitude for the n-th state of the meson. We have taken for simplicity $t_0 = 0$ by always shifting the source time in the correlators to the origin of the lattice. In the two-point correlators apart from ground state, other excited states are also present, but we are only interested in the mass, energy and amplitude of the ground state for this calculation.

The ground state probability of the D/K meson is extracted as

$$(a_o^D)^2 = \frac{|\langle 0|\chi_D|D\rangle|^2}{2M_D a^3}, \qquad (a_o^K)^2 = \frac{|\langle 0|\chi_K|K\rangle|^2}{2M_K a^3}. \tag{6}$$

Here, χ_D and χ_K are the interpolating operators for the D and the K mesons respectively; a is the lattice spacing.

The three-point correlators (for both the scalar and vector currents) have oscillations at both ends and can be written as -

$$\begin{aligned} G^{3pt}(t;T) \;=\; & \sum_{n_1,n_2} a_{n_1}^K a_{n_2}^D V_{n_1 n_2}^{nn}(e^{-E_{n_1}^K t} + e^{-E_{n_2}^D(T-t)}) \\ & (-1)^t \sum_{n_1 o,n_2} a_{n_1 o}^K a_{n_2}^D V_{n_1 o n_2}^{on}(e^{-E_{n_1 o}^K t} + e^{-E_{n_2}^D(T-t)}) \\ & (-1)^T \sum_{n_1,n_2 o} a_{n_1}^K a_{n_2 o}^D V_{n_1 n_2 o}^{no}(e^{-E_{n_1}^K t} + e^{-E_{n_2 o}^D(T-t)}) \\ & (+1)^{t+T} \sum_{n_1 o,n_2 o} a_{n_1 o}^K a_{n_2 o}^D V_{n_1 o n_2 o}^{oo}(e^{-E_{n_1 o}^K t} + e^{-E_{n_2 o}^D(T-t)}). \end{aligned} \tag{7}$$

Here, following a similar notation, "nn", "no", "on" and "oo" represent the non-oscillating/non-oscillating, non-oscillating/oscillating, oscillating/non-oscillating, and oscillating/oscillating states respectively.

We use multi-exponential Bayesian fitting methods [16] to simultaneously fit the two-point and three-point correlators for multiple Ts with all correlations among errors taken into account to extract the three-point amplitude V^{nn}.

The ground state nonoscillating-nonoscillating amplitude of the three-point function for any current J is

$$V_{00}^{nn} = \frac{< 0|\chi_K|K >< K|J|D >< D|\chi_D|0 >}{(2E_K a^3) a_0^K a_0^D}. \tag{8}$$

For the first sum in equation 5 (and for $\vec{p}_K = \vec{0}$) we have used the priors as (energy in the units of GeV)

$$\begin{aligned} \log(E_{(0)}^K) = \log(0.48(5)), \quad & \log(E_{(n)}^K - E_{(n-1)}^K) = \log(0.40(20)) \quad (n > 0) \\ \log(E_{(0)}^D) = \log(1.80(18)), \quad & \log(E_{(n)}^D - E_{(n-1)}^D) = \log(0.40(20)) \quad (n > 0) \\ & a_{(0)}^K = 0.01(1.0), \quad a_{(0)}^D = 0.01(1.0). \end{aligned} \tag{9}$$

We have assigned analogous priors for the second sum as well, but with

$$\log(Eo_{(0)}^{D/K}) = \log(E^D/K_{(0)} + (0.23, 0.12)). \tag{10}$$

For the three point amplitudes we assign

$$\begin{aligned} V_{\text{scalar}}^{nn} = 0.01(5.0), V_{\text{scalar}}^{on} = 0.01(15.0) \\ V_{\text{scalar}}^{no} = 0.01(15.0), V_{\text{scalar}}^{oo} = 0.01(15.0) \\ V_{\text{vector}}^{nn} = 0.01(10.0), V_{\text{vector}}^{on} = 0.01(10.0) \\ V_{\text{vector}}^{no} = 0.01(10.0), V_{\text{vector}}^{oo} = 0.01(10.0). \end{aligned} \tag{11}$$

The energy priors for other kaon momenta $\vec{p}_K$ are given following the dispersion relation $E^2 = \vec{p}_K^2 + m^2$.

The time range we have used for fitting two-point correlators is $[t_{\text{min}}, L_t - t_{\text{min}}]$, where t_{min} for very coarse, coarse and fine lattices are 3, 4 and 5 respectively. To fit the three point correlators on very coarse, coarse and fine lattices we have used time ranges $[3, T-3]$, $[4, T-4]$ and $[6, T-6]$ respectively. The fits are generally consistent within a range of t_{min} values.

5 Results

While extracting meson ground state properties, we have fitted starting from number of exponentials $n_{\text{exp}} = 2$ up to $n_{\text{exp}} = 7$ to get a stable fit with a $\chi^2/\text{dof} < 1$. For the D and K meson properties, we have achieved stable fit results from the 3rd exponential fits and hence, these results are taken as the final results. The behavior of these results with number of exponentials is shown in Figure 2.

We have also tested the taste-effects between the Goldstone and $\gamma_5\gamma_0$ non-Goldstone D mesons arising from the staggered formalism and as expected their mass difference became zero in the continuum, as shown in Figure 3.

We have tested the relativistic dispersion relation as it is only approximate on the lattice. We check the deviation of the square of the velocity of light

$$c^2(\vec{p}) = \frac{E_K^2(\vec{p}) - M_K^2}{\vec{p}^2} \tag{12}$$

from 1 for different kinematics and on all ensembles. Generally, on the lattice we expect to get violations $O(\alpha_S(pa)^2)$ for the HISQ formalism. Figure 4 shows that in our calculation we do not see

Figure 2. M_D vs. $n_{\rm exp}$ plot (left), and a_0^D vs. $n_{\rm exp}$ plot (right) on the physical coarse lattice.

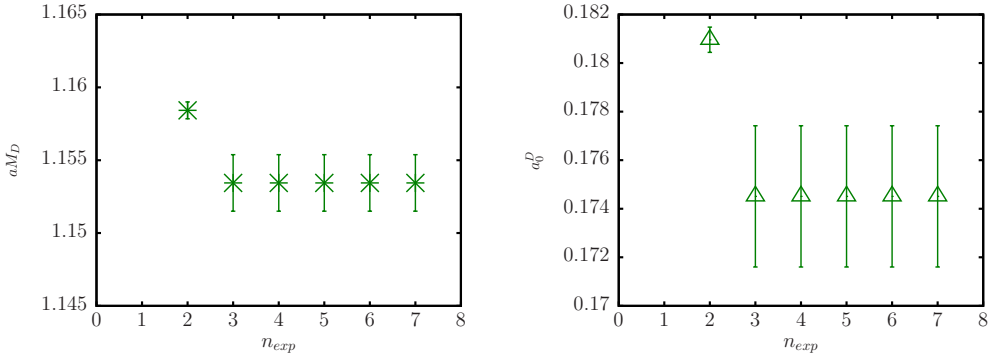


Figure 3. Mass difference of the Goldstone and $\gamma_5\gamma_0$ non-goldstone D mesons.

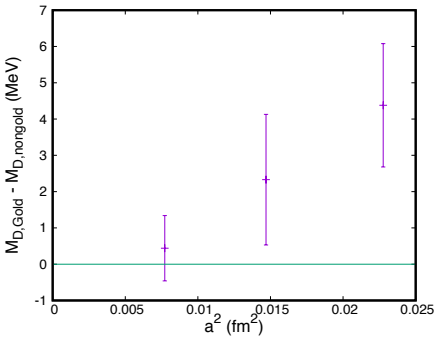


Figure 4. Check for relativistic dispersion relation on all lattice ensembles: the square of the speed of light in free space c^2 vs. the kaon momentum $\vec{p}_K$.

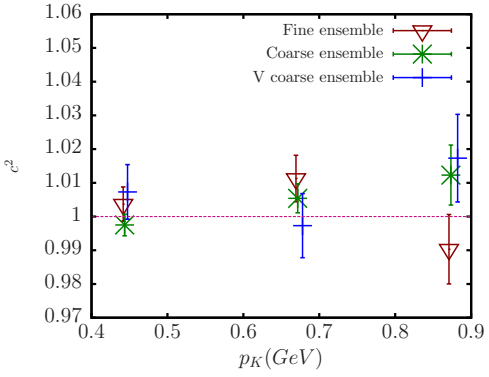
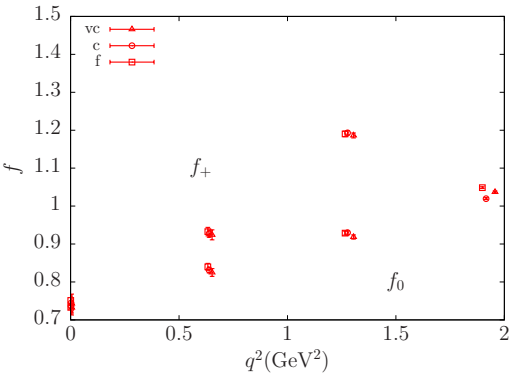


Figure 5. This plot shows the q^2 dependence of the form factors $f_0(q^2)$ and $f_+(q^2)$ with q^2 on all the lattice ensembles we have used. These results come from an uncorrelated fit and so is only preliminary at this stage.



any significant deviation and the relativistic dispersion relation holds within $1-2\%$ statistical deviation, which is within our expectation using the HISQ formalism. However the statistical uncertainties increase in the fitted results for the kaon energies with non-zero momenta.

The scalar and vector form factors are extracted from the simultaneous fits of all data - including two-point and three-point correlators for all q^2 values on each ensemble. Generally the vector current is noisier and hence the vector form factor $f_+(q^2)$. The results for the form factors and their q^2 dependence is shown in Figure 5. These results come from an uncorrelated fit and so is only preliminary at this stage.

The results we show here include u/d quarks with physical masses. We plan to extend the study to heavier u/d masses, however, in order to map out the light quark mass dependence. This may also improve our uncertainties somewhat since heavier u/d masses typically give smaller statistical errors. We will then fit our results to a power series expansion in z-space (converting from q^2 to z) following [9]. Taking the coefficients in the z-expansion to depend on lattice spacing and light quark mass allows a smooth connection to the continuum physical point where we can compare to experiment, q^2-bin by q^2-bin, to optimise the final uncertainty on V_{cs}.

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