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End-to-End Throughput in Multi-Hop Wireless Networks With Random Relay Deployment

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***Abstract*—This paper investigates the effect of relay randomness on the end-to-end throughput in multi-hop wireless networks using stochastic geometry. We model the nodes as Poisson Point Processes and calculate the spatial average of the throughput over all potential geometrical patterns of the nodes. More specifically, for problem tractability, we first start with the simple nearest neighbor (NN) routing protocol, and analyze the end-to-end throughput so as to obtain a performance benchmark. Next, note that the ideal equal-distance routing is generally not realizable due to the randomness in relay distribution, we propose a quasi-equal-distance (QED) routing protocol. We derive the range for the optimal hop distance, and select the relays to formulate a quasi-equidistant deployment. We analyze the end-to-end throughput both with and without intra-route resource reuse. Our analysis indicates that: (i) The throughput performance of the proposed QED routing can achieve a significant performance gain over that of the NN routing. As the relay intensity gets higher, the performance of QED routing converges to that of the equidistant routing. (ii) If the node intensity is a constant over the network, then intra-route resource reuse is always beneficial when the routing distance is sufficiently large. (iii) With randomly distributed relays, the communication distance can generally be extended. However, due to the uncertainty in relay distribution, long distance communication is generally not feasible with random relays. This implies that the existence of a reasonably defined infrastructure is critical in effective long distance communication. Our analysis is demonstrated through numerical examples.**

***Index Terms*—Multi-hop network, throughput, stochastic geometry, random relays.**

I. Introduction

Multi-hop communication with relay assistance has become a prominent scheme in today's hybrid network design. The main reason is that it can extend the communication distance in wireless networks without the deployment of wired backhaul facilities. In wireless networks, the geometric locations of the nodes play a key role in determining the signal to interference and noise ratio (SINR), and hence the probability of successful transmission. In large scale multi-hop wireless networks, the node locations, including the relay locations, are generally random. The spatial randomness in node locations raises significant challenges in network performance analysis.

An effective tool to characterize the spatial randomness in wireless networks is stochastic geometry, for which the basic idea is to model the nodes as Poisson Point Processes (PPPs) and calculate the spatial averages of network performance characteristics by averaging over all potential geometrical patterns of the nodes [1]–[5].

In literature, stochastic geometry modeling has been utilized to study multi-hop wireless networks. In [6], [7], the random access transport capacity, which was defined as the spatially normalized end-to-end data rate obtained by multi-hop relays, was evaluated and optimized with respect to hop number. In [8], the transport capacity was evaluated under delay constraints. In [9]–[12], the end-to-end delay of multi-hop wireless networks was characterized and optimized. In [13], the dependence of interference among the relays along a multi-hop route was discussed. It was shown that spatially and temporally correlated interference would increase both the mean and variance of the end-to-end delay. In most of these approaches, the source nodes were modeled as PPPs, however, the relay locations were assumed to be *deterministic and known*, and were often approximated as *equidistant*. Note that in large scale wireless networks, it is impractical to optimize the relay locations for each source destination pair as equidistant, and the overall relay distribution is generally random rather than deterministic, hence, for more reasonable performance evaluation, the relay randomness needs to be taken into account more accurately.

Assuming random relay distribution, in [14], [15], different hopping strategies were compared in terms of aggregate multi-hop information efficiency. These approaches focused on the efficiency of each individual hop in a multi-hop network, and the end-to-end performance of the network needs to be further exploited. Similarly, in [16]–[19], the performance analysis was also focused on individual hops. In [20], the cost of routing selection was evaluated under opportunistic geographic routing strategies in a Poisson network. In [21], the end-to-end delay was simulated in a Poisson multi-hop wireless network using the time-space opportunistic routing. In [22], limited random deviations of relays from their ideal locations in the equidistant deployment were introduced. This model was more practical than the equidistant one, but it required the relays to be deployed within a small range around the equidistant locations. In [23], the theoretical upper bounds were derived for the throughput that could be achieved by any routing algorithm assisted by dynamic routing selection. In [24], the relays were modeled as a linear PPP along the route, and the end-to-end delay was evaluated. While the randomness of relays was taken into account in [24], there was no the consideration on node stability or traffic overflow, and the end-to-end throughput was not explicitly evaluated. In addition, the results there only applied to the cases where the routing distances were sufficiently long so that asymptotic analysis could be utilized.

As an effort to further explore the effect of relay randomness on network performance, in this paper, we analyze the end-to-end throughput of a general multi-hop route in a wireless network with randomly located relays. In our analysis, we model the relays as a linear PPP between the source and destination following the TDMA medium access control (MAC) protocol, and model the external interferers as an independent PPP over the whole plane, following the ALOHA MAC protocol. We assume that multi-hop transmissions are performed under an interference limiting scenario, where the interference power is much more significant than the noise power.

More specifically, in this paper, *first*, for problem tractability, we start with a simple nearest neighbor routing protocol where each relay will select the nearest node along the direction to the destination as its next hop. We analyze the end-to-end throughput in a relatively *sparse* network so as to obtain a performance benchmark or lower bound. The throughput is evaluated under both conventional TDMA with fixed, uniform slot length, as well as TDMA with dynamic slot length or resource allocation; the optimal relay density is also discussed. *Next*, motivated by the observation that, while the ideal equal-distance routing generally provides the optimal network performance, it is generally not realizable due to the randomness in relay distribution, we propose a quasi-equal-distance (QED) routing protocol, where we derive the range for the optimal hop distance, and select the relays to formulate a quasi-equidistant deployment. We analyze the end-to-end throughput both with and without intra-route resource reuse. Our analysis indicates that, compared with the optimal end-to-end throughput of NN routing, the proposed QED routing obtains a significant performance improvement under the same relay intensity and routing distance.

The main contributions of this paper can be summarized as follows:

- First, to pave the way for throughput analysis, we derive the distribution of the longest hop distance L_m under NN routing for any given routing distance r. We formulate the distribution of L_m as a continuous auto-regression system, and solve it using the Laplace transform. It is shown that the mean of L_m scales with $\mathcal{O}(\ln r)$ as the routing distance $r \to \infty$, and the variance of L_m is bounded. This implies that the throughput vanishes as $r \to \infty$, hence multi-hop relaying with random relays is infeasible for long distance communication.
- Second, we derive the average end-to-end throughput of a multi-hop route under NN routing and TDMA MAC with both fixed and flexible slot length. By expressing the average end-to-end throughput as a function of the routing distance r, we obtain the Laplace transform of the throughput function. Under conventional TDMA with fixed slot length, we obtain a closed-form expression for the lower bound of the throughput, and derive the range for the optimal relay intensity. We maximize the throughput under TDMA, and show that the optimal slot length varies from hop to hop and is determined by the coverage probability of every hop. That is, TDMA with flexible, properly selected slot length can increase the system efficiency and lead to optimal throughput.
- Third, we propose a quasi-equal-distance (QED) routing protocol for throughput optimization with random relays. Under the proposed QED routing and conventional TDMA, we analyze the average end-to-end throughput with and without intra-route resource reuse, respectively. Note that accurate expression of the throughput is hard to derive, as an alternative, we obtain close approximations of the throughput under different scenarios. The optimal number of time slots is also analyzed when there is intra-route resource reuse. It is shown that the proposed QED routing protocol achieves a significant performance gain over NN routing. It is also observed that the effect of intra-route resource reuse depends on the network setup. If the node intensity is a constant over the network, then as expected, intra-route resource reuse is always beneficial when the routing distance r is sufficiently large (i.e. , as $r \to \infty$). However, if the source-destination pair density remains unchanged as the routing distance increases, then intra-route resource reuse is no longer beneficial for throughput improvement even if the routing distance $r \to \infty$.

Our results are demonstrated through numerical examples. Overall, our numerical results, together with the theoretical analysis, show that: (i) The throughput performance of the proposed QED routing can achieve a significant performance gain over that of the NN routing. For network with sparse random relays, compared with the ideal equidistant routing, the performance loss of QED routing due to relay randomness is not negligible. However, as the relay intensity gets higher, the performance of QED routing converges to that of the equidistant routing. (ii) If the node intensity is a constant over the network, then intra-route resource reuse can increase the network throughput when the routing distance r is sufficiently large. (iii) With randomly distributed relays, the communication distance can generally be extended. However, due to the uncertainty in relay distribution, long distance communication is generally not feasible with random relays. This implies that the existence of a reasonably defined infrastructure is critical for effective long distance communication. The results in this paper also echo our previous observations in [25]–[27] that future network design would reflect the convergence of centralized and ad hoc networks.

II. System Description

A. Network Model

We consider a source node $\boldsymbol{S}$, and a destination node $\boldsymbol{D}$ located at a distance of R. A linear relay pattern is studied, where the candidate relay nodes are distributed randomly along the line segment between $\boldsymbol{S}$ and $\boldsymbol{D}$. Without loss of generality, we assume $\boldsymbol{S}$ is at the origin and $\boldsymbol{D}$ is located at $(R, 0)$. Thus the candidate relay nodes formulate a 1D point process $\Phi = \{\boldsymbol{X}_i, i = 1, 2, ..., N\}$, where N is the random variable (RV) denoting the number of relays, and $\boldsymbol{X}_i$ is the location of the i-th relay along the line segment between $(0, 0)$ and $(R, 0)$. In the remaining part of this paper,

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we model Φ as a 1D homogeneous PPP (HPPP) of intensity λ. That is, for $i = 1, 2, ..., N$ and letting $\boldsymbol{X}_0 = \boldsymbol{S}$, the distances between successive nodes, $L_i = |\boldsymbol{X}_i - \boldsymbol{X}_{i-1}|$, are exponentially distributed independent RVs of mean $1/\lambda$ [24]. The locations of the relays would keep static during packet delivery. Considering a backlogged source $\boldsymbol{S}$ which has infinite packets to transmit, *we define the end-to-end throughput from* $\boldsymbol{S}$ *to* $\boldsymbol{D}$ *as the number of packets initiated from source* $\boldsymbol{S}$ *that are successfully received at destination* $\boldsymbol{D}$ *per time slot*.

The route selects a subset $\Phi' = \{\boldsymbol{X}'_1, \boldsymbol{X}'_2, ..., \boldsymbol{X}'_{N'}\} \subseteq \Phi$ as the actual relays to be used, following a specific routing protocol, where $\boldsymbol{X}'_i$ denotes the location of the i-th selected relay and N' the number of relays selected. Relay node $\boldsymbol{X}'_i$ transmits the packets originated from the source $\boldsymbol{S}$ to the next relay $\boldsymbol{X}'_{i+1}$ along the direction to $\boldsymbol{D}$ in a *decode-and-forward* manner. For tractable analysis, we assume that each relay node has an infinite transmission buffer, and each packet relayed is served in a first-in first-out fashion. The packet that fails in one transmission would go back to the head of the transmission queue, waiting for the opportunity of next transmission. The nodes on the route follow the TDMA MAC protocol, where each node will be assigned with at least one time slot in a TDMA cycle and is allowed to transmit signals only at the designated time slots.

We apply the *decoupling* technique in [24] to our network model, where all the other nodes that are not along the $\boldsymbol{S}$-$\boldsymbol{D}$ path are modeled as an independent 2D point process Ψ over $\mathbb{R}^2$ from Φ. Potentially, these nodes can be the external interferers to the relays we study when they transmit over the same spectrum and time slot. For the remaining part of this paper, we model Ψ as a 2D HPPP of intensity μ. We assume that the transmissions of the nodes in Ψ follow the ALOHA protocol, where each node would transmit at each time slot independently with a probability of p_a.

As can be seen, the network model adopted here is actually a combination of the models in [10] and [24]. More specifically, we combine the random relay model in [24] with the TDMA/ALOHA multi-hop network model in [10]. For the tractability of the problem, in this paper, we mainly consider the case where relays are deployed along the line segment between $\boldsymbol{S}$ and $\boldsymbol{D}$. However, the results obtained here actually provide an upper bound on the more practical scenario where relays are modeled as a 2D HPPP over an area surrounding segment $\boldsymbol{S}-\boldsymbol{D}$. For example, in Fig. 1, the relays are deployed randomly in a $R \times W$ rectangle $\mathcal{R}$ whose widths intersect $\boldsymbol{S}$ and $\boldsymbol{D}$. A simple routing protocol is that each relay would transmit to its nearest neighbor along the direction to $\boldsymbol{D}$ (the x-coordinate). By projecting the relays to the x-coordinate, we can find that the hop distances in the 2D case are lower bounded by the hop distances in the 1D case. Thus, the throughput in 1D case is an upper bound of the throughput in 2D case.

B. Channel Model

Both large-scale path-loss and small-scale fading are considered. The received power of a signal transmitted at a distance of x meters with transmit power P_T is [1]

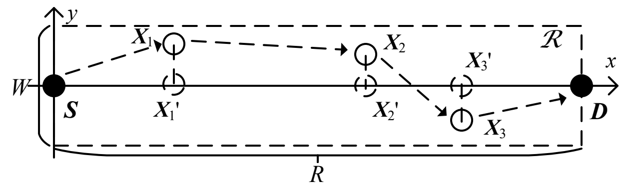

Fig. 1. An illustration of relays randomly deployed over a 2D area

$$P_R(x) = \frac{P_T \cdot H}{c \cdot x^{\beta}} , \tag{1}$$

where H denotes channel gain, β the path-loss exponent, and c a constant determined by the antenna gains and signal wavelength. H is an exponentially distributed random variable with mean 1, i.e., Rayleigh fading is considered. Independent small scale fading is assumed for different transmitter-receiver pairs in different time slots. The small scale fading from location $\boldsymbol{x}_1$ to $\boldsymbol{x}_2$ at time slot k is represented by $H^k_{\boldsymbol{x}_1,\boldsymbol{x}_2}$.

C. Routing Protocol

In this paper, we investigate two routing protocols: the nearest neighbor (NN) routing protocol and the proposed quasi-equal-distance (QED) routing protocol.

Nearest Neighbor Routing: For problem tractability, we start with the simple NN routing protocol to obtain a performance lower bound. In NN routing [14], each relay will select the nearest node in Φ along the direction to the destination D as its next hop. In this case, the selected relay set is the same as the candidate relay set, i.e., $\Phi' = \Phi$. So in the NN routing, we use Φ' and Φ interchangeably unless otherwise clarified.

The nearest neighbor routing protocol aims to guarantee the link quality of each single hop by utilizing all the available relays in Φ and minimizing the hop distances. When the node intensity λ is large, nearest neighbor routing will degrade the end-to-end throughput because of the extra delay and bandwidth it takes. A simple variation of the NN routing is to choose Φ' by independently thinning [28, Proposition 1.3.5] the original relay set Φ, which can generate an HPPP of intensity $\lambda^* < \lambda$. The throughput analysis in this case is the same as that of the original NN routing by replacing λ with λ^*.

Quasi-Equal-Distance (QED) Routing: As is well known [12], [29], equal-distance routing provides the optimal network performance. However, limited by the randomness in relay distribution, the ideal equal-distance routing generally cannot be realized in practical systems. Therefore, in this paper, we propose a QED routing protocol where we select the relays Φ' to be close to a equidistant relay deployment. In the QED protocol, given a selected relay, instead of choosing the nearest neighbor as the next hop, it will select the next hop to be the first node which is at least l_0 away along the direction to the destination, where l_0 is a parameter that can be tuned in the protocol. That is, the QED routing aims to make the hop distance of each hop close to l_0. In this case, except for the last hop, the hop distances should be at least l_0. Note that if

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l_0 is set to be 0, the QED routing will be reduced to the NN routing. The optimal value of l_0 can be obtained by optimizing the end-to-end throughput, which will be discussed in Section VI of this paper.

As the relay intensity $\lambda \to +\infty$, the distribution of the selected relays will converge to an equidistant deployment, and the performance of QED will converge to that of equidistant relays.

III. Problem Formulation

A fixed rate coding scheme is assumed in the physical layer, where a packet can be successfully received if and only if the received signal to interference and noise ratio (SINR) is above a given threshold $\theta > 0$. We consider an interference-limiting scenario, where the noise power is negligible compared with the interference power, so we use signal to interference ratio (SIR) and SINR interchangeably. Without loss of generality, we assume that each node in the network transmits with unit power. Let binary RV $B(\boldsymbol{X}'_{i-1}, k)$ indicate whether relay $\boldsymbol{X}'_{i-1}$ is allowed by the MAC protocol to transmit signals at time slot k. For $i = 1, 2, ..., N'+1$ and let $\boldsymbol{X}'_{N'+1} = \boldsymbol{D}$, given $B(\boldsymbol{X}'_{i-1}, k) = 1$, the received SIR at relay $\boldsymbol{X}'_i$ on time slot k can be expressed as

$$\text{SIR}(\boldsymbol{X}'_i, k) = \frac{H^k_{\boldsymbol{X}'_{i-1}, \boldsymbol{X}'_i} |\boldsymbol{X}'_i - \boldsymbol{X}'_{i-1}|^{-\beta}}{I_o(\boldsymbol{X}'_i, k) + I_{in}(\boldsymbol{X}'_i, k)} , \quad (2)$$

where $I_o(\boldsymbol{X}'_i, k)$ denotes the interference that the active external interferers in Ψ generates on relay $\boldsymbol{X}'_i$ at time slot k, and $I_{in}(\boldsymbol{X}'_i, k)$ denotes the intra-route interference generated by other relays in Φ' transmitting over time slot k.

Let $\mathbb{1}\{\mathcal{A}\}$ denote the indicator variable of event $\mathcal{A}$. So the local throughput of link $\boldsymbol{X}'_{i-1} \to \boldsymbol{X}'_i$ can be expressed as

$$T_{\text{local}}(\boldsymbol{X}'_i) = \lim_{K\to\infty} \frac{1}{K} \sum_{k=0}^{K-1} B(\boldsymbol{X}'_{i-1}, k) \mathbb{1}\{\text{SIR}(\boldsymbol{X}'_i, k) > \theta\} . \quad (3)$$

According to the stability analysis in queuing theory, the end-to-end throughput is determined by the hop with the lowest throughput [30]. So the end-to-end throughput can be expressed as

$$T_{\text{end}} = \min_{\boldsymbol{X}'_i \in \Phi' \cup \{\boldsymbol{D}\}} T_{\text{local}}(\boldsymbol{X}'_i) . \quad (4)$$

Our goal is to calculate the expectation of T_{end} over all the possible realizations of the relay distribution Φ' for any given routing distance $R = r$, $\mathbb{E}\{T_{end} \mid R = r\}$.

In general, the external interference $I_o(\boldsymbol{X}'_i, k)$ can be represented as

$$I_o(\boldsymbol{X}'_i, k) = \sum_{\boldsymbol{Y}_j \in \Psi} B(\boldsymbol{Y}_j, k) H^k_{\boldsymbol{Y}_j, \boldsymbol{X}'_i} |\boldsymbol{Y}_j - \boldsymbol{X}'_i|^{-\beta} , \quad (5)$$

where for any $\boldsymbol{Y}_j \in \Psi$, the binary RV $B(\boldsymbol{Y}_j, k)$ indicates whether the "external" node $\boldsymbol{Y}_j$ would transmit at time slot k. Under the ALOHA protocol, the distribution of external interferers in any given time slot can be viewed as an independent thinning of Ψ with a retention probability of p_a, i.e., an HPPP with intensity $\mu' = p_a \mu$. Following the same assumption in [10], we make the following approximation.

Approximation 1. *The distribution of external interferers are approximated as independent across different time slots.*

With the approximation above, for a given deployment of relays Φ', the distribution of $I_o(\boldsymbol{X}'_i, k)$ is independent of time slot k. So we discard the time index k for the external interference.

Given R and Φ', $B(\boldsymbol{X}'_{i-1}, k)$ and $I_{in}(\boldsymbol{X}'_i, k)$ will depend on the resource allocation scheme in the TDMA protocol. Here, we discuss the NN and the QED routing respectively.

A. NN Routing

For the NN routing, we assume that each time slot in a TDMA cycle will be allocated to at most one relay in Φ' (Φ). That is, no intra-route resource reuse is allowed in the NN routing and $I_{in}(\boldsymbol{X}'_i, k) = 0$ for all the possible i and k. This is because in the NN routing, relays can be quite close to each other, where a strong intra-route interference will possibly be generated. In this case, $\text{SIR}(\boldsymbol{X}'_i, k)$ is independent of time slot k given $B(\boldsymbol{X}'_{i-1}, k) = 1$. So we discard the time index k and let $\text{SIR}(\boldsymbol{X}'_i)$ denote the SIR at $\boldsymbol{X}'_i$ for an arbitrary time slot where $\boldsymbol{X}'_{i-1}$ is allowed to transmit signals.

Define $A_i \triangleq \lim_{K\to\infty} \frac{1}{K} \sum_{k=0}^{K-1} B(\boldsymbol{X}'_{i-1}, k)$ as the normalized slot length allocated to link $\boldsymbol{X}'_{i-1} \to \boldsymbol{X}'_i$. So the local throughput $T_{\text{local}}(\boldsymbol{X}'_i)$ can be rewritten as

$$T_{\text{local}}(\boldsymbol{X}'_i) = \lim_{K\to\infty} \frac{\sum_{k=0}^{K-1} B(\boldsymbol{X}'_{i-1}, k)}{K} \frac{\sum_{B(\boldsymbol{X}'_{i-1}, k)=1} \mathbb{1}\{\text{SIR}(\boldsymbol{X}'_i, k) > \theta\}}{\sum_{k=0}^{K-1} B(\boldsymbol{X}'_{i-1}, k)} . \quad (6)$$

From ergodicity, given the routing distance R and the relay set Φ', the local throughput

$$T_{\text{local}}(\boldsymbol{X}'_i) = A_i \Pr\{\text{SIR}(\boldsymbol{X}'_i) > \theta \mid R, \Phi'\} . \quad (7)$$

From the stationarity of HPPP, the distribution of $I_o(\boldsymbol{X}'_i)$ will be independent of the location of $\boldsymbol{X}'_i$. If we assume that the distribution of the small scale fading $H_{\boldsymbol{X}'_{i-1}, \boldsymbol{X}'_i}$ is independent of the location of $\boldsymbol{X}'_{i-1}$ and $\boldsymbol{X}'_i$, then it follows from (2) that the conditional probability $\Pr\{\text{SIR}(\boldsymbol{X}'_i) > \theta \mid R, \Phi'\}$ is a function of hop distance $|\boldsymbol{X}'_i - \boldsymbol{X}'_{i-1}|$. Define the *coverage probability* of a hop distance of l with only external interference as

$$P_s(l) \triangleq \Pr\left\{H l^{-\beta} / I_o > \theta\right\} , \quad (8)$$

where I_o denotes the external interference at an arbitrary location. So we have

$$\Pr\{\text{SIR}(\boldsymbol{X}'_i) > \theta \mid R, \Phi'\} = P_s(|\boldsymbol{X}'_i - \boldsymbol{X}'_{i-1}|) . \quad (9)$$

The coverage probability $P_s(l)$ can be calculated using the following lemma.

Lemma 1. *Given a hop distance l, the coverage probability for the hop is*

$$P_s(l) = \exp(-\kappa l^2) , \quad (10)$$

where $\kappa = 2\pi\mu' \frac{\pi}{\beta \sin(2\pi/\beta)} \theta^{2/\beta}$.

Proof. See [16]. □

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So the end-to-end throughput can be expressed as

$$T_{\text{end}} = \min_{\boldsymbol{X}_i' \in \Phi' \cup \{\boldsymbol{D}\}} A_i P_s(|\boldsymbol{X}_i' - \boldsymbol{X}_{i-1}'|) \ . \quad (11)$$

B. QED Routing

In the QED routing, we analyze the throughput both with and without intra-route resource reuse.

For the case without intra-route resource reuse, the analysis follows that of the NN case, where the expression of the end-to-end throughput is the same as (11).

Next, we consider the case with intra-route resource reuse. Assuming a TDMA scheme where the TDMA cycle consists of M time slots, indexed from 0 to $M-1$, the source node will be assigned with slot 0, and relay i will be assigned with slot $i \mod M$. The nodes assigned with the same time slot will transmit concurrently at the specified time slot, thus the nodes will be subject to the intra-route interference generated. The intra-route interference experienced by relay $\boldsymbol{X}_i'$ at time slot k, $I_{in}(\boldsymbol{X}_i', k)$, can be expressed as

$$I_{in}(\boldsymbol{X}_i', k) = \sum_{\boldsymbol{X}_m \in \Phi', \ m \neq i} B(\boldsymbol{X}_m, k) H^k_{\boldsymbol{X}_m, \boldsymbol{X}_i'} |\boldsymbol{X}_m - \boldsymbol{X}_i'|^{-\beta} \ . \quad (12)$$

Here, without loss of generality, we assume that each relay will always transmit signals at its designated time slots. Note that, given $B(\boldsymbol{X}_{i-1}, k) = 1$, $B(\boldsymbol{X}_m, k) = 1$ iff. $m = i - 1 + jM$ for some integer $j \neq 0$. So the intra-route interference is independent of time index k, which can be expressed as

$$I_{in}(\boldsymbol{X}_i') = \sum_{\substack{j \in \mathbb{Z} \setminus 0, \\ \boldsymbol{X}_{i-1+jM}' \in \Phi'}} H_{\boldsymbol{X}_{i-1+jM}', \boldsymbol{X}_i'} |\boldsymbol{X}_{i-1+jM}' - \boldsymbol{X}_i'|^{-\beta}. \quad (13)$$

Similar to the NN case, we use $\text{SIR}(\boldsymbol{X}_i')$ to denote the SIR at $\boldsymbol{X}_i'$ for an arbitrary time slot.

The introduction of intra-route resource reuse greatly complicates the throughput analysis because that: (i) the intra-route interference is correlated with the distribution of the relays; and (ii) the temporal correlation of intra-route interference results in the correlation of transmission success probability across time [31]. To make the problem tractable, we assume $M \geq 2$ and approximate the intra-route interference at $\boldsymbol{X}_i'$ as

$$\tilde{I}_{in}(\boldsymbol{X}_i') = \sum_{j \in \mathbb{Z}^-} H_j |jMl_0 + L_i'|^{-\beta} + \sum_{j \in \mathbb{Z}^+} H_j |(jM-1)l_0|^{-\beta} \ , \quad (14)$$

where $L_i' = |\boldsymbol{X}_i' - \boldsymbol{X}_{i-1}'|$ and H_j, $j = \pm 1, \pm 2, ...$, are independent exponential RVs of mean 1. As hop distance is lower bounded by l_0 except for the last hop in the QED routing, it follows that $\Pr\{\tilde{I}_{in}(\boldsymbol{X}_i') \geq x \mid \Phi'\} \geq \Pr\{I_{in}(\boldsymbol{X}_i') \geq x \mid \Phi'\}$ for any x.

Define the corresponding lower bound of $\text{SIR}(\boldsymbol{X}_i')$ as

$$\widetilde{\text{SIR}}(\boldsymbol{X}_i') = \frac{H_{\boldsymbol{X}_{i-1}', \boldsymbol{X}_i'} |\boldsymbol{X}_i' - \boldsymbol{X}_{i-1}'|^{-\beta}}{I_o(\boldsymbol{X}_i') + \tilde{I}_{in}(\boldsymbol{X}_i')}. \quad (15)$$

We use $\widetilde{\text{SIR}}(\boldsymbol{X}_i')$ instead of $\text{SIR}(\boldsymbol{X}_i')$ in the throughput analysis under intra-route resource reuse. Assuming M time slots per TDMA cycle, we have $A_i = \lim_{K\to\infty} \frac{1}{K} \sum_{k=0}^{K-1} B(\boldsymbol{X}_{i-1}', k) = 1/M$. Given the routing distance R and the relay set Φ', the local throughput of link $\boldsymbol{X}_{i-1}' \to \boldsymbol{X}_i'$ is

$$T_{\text{local}}(\boldsymbol{X}_i') = \frac{1}{M} \Pr\left\{ \widetilde{\text{SIR}}(\boldsymbol{X}_i') > \theta \mid R, \Phi' \right\} \ . \quad (16)$$

Except for the hop distance L'_i of link $\boldsymbol{X}_{i-1}' \to \boldsymbol{X}_i'$, $\tilde{I}_{in}(\boldsymbol{X}_i')$ is independent of the distribution of other relays in Φ. So following the definition of (8), we define the coverage probability of a hop distance l with intra-route resource reuse as

$$P_s'(l) \triangleq \Pr\left\{ Hl^{-\beta}/(I_o + \tilde{I}_{in}(l)) > \theta \right\} \ , \quad (17)$$

where

$$\tilde{I}_{in}(l) = \sum_{j \in \mathbb{Z}^-} H_j |jMl_0 + l|^{-\beta} + \sum_{j \in \mathbb{Z}^+} H_j |(jM-1)l_0|^{-\beta} \ . \quad (18)$$

Let function $\mathcal{L}_{I_o}(s)$ denote the Laplace transform of the PDF of the external interference I_o, and $\mathcal{L}_{\tilde{I}_{in}(l)}(s)$ the Laplace transform of the PDF of the intra-route inference $\tilde{I}_{in}(l)$. We have

$$P_s'(l) = \mathcal{L}_{I_o}(\theta l^\beta) \mathcal{L}_{\tilde{I}_{in}(l)}(\theta l^\beta) \ . \quad (19)$$

Since we assume that the locations of the active external interferers are independent across time, it follows that $\mathcal{L}_{I_o}(\theta l^\beta) = P_s(l) = \exp(-\kappa l^2)$ as defined in *Lemma 1*. $\mathcal{L}_{\tilde{I}_{in}(l)}(s)$ can be calculated as [32]

$$\prod_{k \in \mathbb{Z}^+} \frac{1}{s(kMl_0 + l)^{-\beta} + 1} \cdot \frac{1}{s[(kM-1)l_0]^{-\beta} + 1} \ . \quad (20)$$

Combining (19) and (20), we have the following lemma on the local throughput.

Lemma 2. *Define function $P_{in}(l)$ as*

$$P_{in}(l) \triangleq \prod_{k \in \mathbb{Z}^+} \frac{1}{\theta(kM\frac{l_0}{l} + 1)^{-\beta} + 1} \cdot \frac{1}{\theta[(kM-1)\frac{l_0}{l}]^{-\beta} + 1} \ . \quad (21)$$

In the QED routing with intra-route resource reuse, for a hop distance of l, the coverage probability is

$$P_s'(l) = P_s(l) P_{in}(l) \ , \quad (22)$$

where $P_s(l)$ is defined in Lemma 1. *A closed form lower bound of $P_{in}(l)$ can be calculated as*

$$\begin{aligned} P_{in}(l) \geq \exp\{ &-\frac{l}{Ml_0\theta^{-\frac{1}{\beta}}} \mathcal{B}(\frac{1}{\theta^{-1}(M\frac{l_0}{l}+1)^\beta + 1}; 1 - \frac{1}{\beta}, \frac{1}{\beta}) \\ &- \left(2 + \frac{l}{Ml_0}\right) \ln\left(\theta(M\frac{l_0}{l} + 1)^{-\beta} + 1\right) \\ &- \frac{l}{Ml_0\theta^{-\frac{1}{\beta}}} \mathcal{B}(\frac{1}{\theta^{-1}[(M-1)\frac{l_0}{l}]^\beta + 1}; 1 - \frac{1}{\beta}, \frac{1}{\beta}) \\ &- (2 - \frac{1}{M}) \ln\left(\theta[(M-1)\frac{l_0}{l}]^{-\beta} + 1\right)\} \ , \end{aligned} \quad (23)$$

where $\mathcal{B}(\cdot; \cdot, \cdot)$ is the incomplete beta function.

Proof. Note that $\ln P_{in}(l)$ can be expressed as

$$-\sum_{k=1}^{\infty} \ln\left(\theta(kM\frac{l_0}{l} + 1)^{-\beta} + 1\right) + \ln\left(\theta[(kM-1)\frac{l_0}{l}]^{-\beta} + 1\right).$$

Since $\ln\left(\theta(kM\frac{l_0}{l}+1)^{-\beta}+1\right)$ and $\ln(\theta[(kM-1)\frac{l_0}{l}]^{-\beta}+1)$ are both decreasing functions with respect to k for $k \geq 1$, (23) can be obtained by approximating the summation of series with the integral of the corresponding function. □

So the end-to-end throughput can be expressed as

$$T_{\text{end}} = \min_{\boldsymbol{X}'_i \in \Phi' \cup \{\boldsymbol{D}\}} \frac{1}{M} P'_s(|\boldsymbol{X}'_i - \boldsymbol{X}'_{i-1}|) \ . \tag{24}$$

In the rest of this paper, we first derive the average end-to-end throughput of the NN routing, followed by the case of the QED routing.

IV. Stochastic Analysis on Hop-Distance under NN Routing

As a preparation for further throughput analysis, in this section, we analyze the distribution of the longest hop distance in the NN routing, denoted by L_m.

The longest hop distance L_m of Φ is of special interest for two reasons. First, consider a simple TDMA where the relays in Φ are assigned with a fixed slot length, i.e., $A_i = 1/(N+1)$ for $i = 0, 1, ..., N$, with N being the number of relays. In this case, as the coverage probability $P_s(l)$ is a non-increasing function with respect to the hop distance l, the end-to-end throughput will be the local throughput of the hop with the longest hop distance. Second, for any MAC protocol or resource allocation scheme employed, the end-to-end throughput cannot exceed the coverage probability of the longest hop distance, $P_s(L_m)$. Thus, the stochastic analysis of L_m sheds light on the theoretical upper bound of the end-to-end throughput.

We have the following theorem on the distribution of L_m.

Theorem 1. *Given the routing distance between source and destination $R = r$, we have:*

1) *The conditional CDF of L_m, $\Pr\{L_m \leq l | R = r\} = 1$ for $l \geq r$. Moreover, $\Pr\{L_m = r | R = r\} = e^{-\lambda r}$ and $\Pr\{L_m < r | R = r\} = 1 - e^{-\lambda r}$.*
2) *Define $g(l, r) \triangleq \Pr\{L_m \leq l | R = r\}$ and denote the Laplace transform (LT) of $g(l, r)$ with respect to r by $G(l, s)$, then*

$$G(l, s) = \frac{1 - e^{-(\lambda+s)l}}{s + \lambda e^{-(\lambda+s)l}} \ . \tag{25}$$

Proof. 1) This part follows directly from the properties of PPP.

2) For $0 < l < r$, consider the conditional probability of L_m given that the first relay $\boldsymbol{X}_1$ is located at $(x, 0)$, $\Pr\{L_m \leq l | R = r, |\boldsymbol{X}_1| = x\}$. Since the distribution of the points of Φ in disjoint intervals are independent, basing on the Palm theory of PPP[1], given $|\boldsymbol{X}_1| = x$, the remaining relay nodes within the interval (x, r) is still a 1D PPP of intensity λ. The distribution of the longest hop distance for the relays within (x, r) should be the same as that for $R = r - x$. Since $|\boldsymbol{X}_1|$ is exponentially distributed for $x < r$, we have

$$\begin{aligned}
&\Pr\{L_m \leq l | R = r\} \\
&= \int_0^l f_{|\boldsymbol{X}_1|}(x) \Pr\{L_m \leq l | R = r, |\boldsymbol{X}_1| = x\} \mathrm{d}x \\
&= \int_0^l \lambda e^{-\lambda x} \Pr\{L_m \leq l | R = r - x\} \mathrm{d}x \ .
\end{aligned} \tag{26}$$

Consider the following integral

$$\begin{aligned}
&\int_l^{+\infty} \Pr\{L_m \leq l | R = r\} e^{-sr} \mathrm{d}r \\
&= \int_l^{+\infty} \int_0^l \lambda e^{-\lambda x} \Pr\{L_m \leq l | R = r - x\} e^{-sr} \mathrm{d}x \mathrm{d}r \\
&= \int_0^l \lambda e^{-\lambda x} \left(\int_l^{l+x} \Pr\{L_m \leq l | R = r - x\} e^{-sr} \mathrm{d}r \right. \\
&\quad \left. + \int_{l+x}^{+\infty} \Pr\{L_m \leq l | R = r - x\} e^{-sr} \mathrm{d}r \right) \mathrm{d}x \\
&= \frac{e^{-sl}}{s}(1 - e^{-\lambda l}) - \frac{\lambda e^{-sl}}{s(\lambda + s)}(1 - e^{-(\lambda+s)l}) \\
&\quad + \frac{\lambda}{\lambda + s}(1 - e^{-(\lambda+s)l})[G(l, s) - \frac{1 - e^{-sl}}{s}] \ .
\end{aligned} \tag{27}$$

Note that we also have

$$\int_l^{+\infty} \Pr\{L_m \leq l | R = r\} e^{-sr} \mathrm{d}r = G(l, s) - \frac{1 - e^{-sl}}{s} \ . \tag{28}$$

Following (27) and (28), we get $G(l, s) = \frac{1 - e^{-(\lambda+s)l}}{s + \lambda e^{-(\lambda+s)l}}$. □

Moreover, we have the following result about the region of convergence (ROC) of $G(l, s)$.

Corollary 1. *The ROC of $G(l, s)$ includes the imaginary axis. More specifically, $g(l, r)$ is absolutely integrable, i.e.,*

$$\int_{-\infty}^{+\infty} |g(l, r)| dr < +\infty \ . \tag{29}$$

Proof. See Appendix A in the supplementary file. □

Following *Theorem 1* and *Corollary 1*, the conditional CDF of L_m given $R = r$ can be computed numerically by calculating the inverse Fourier Transform (FT) of $G(l, j\omega)$.

To obtain a closed form expression for the CDF of L_m, instead of fixing the routing distance $R = r$, we can model R as an exponentially distributed RV of mean $\frac{1}{\nu}$. Basing on *Theorem 1*, the CDF of L_m can be calculated as

$$\Pr\{L_m \leq l\} = \frac{\nu(1 - e^{-(\lambda+\nu)l})}{\nu + \lambda e^{-(\lambda+\nu)l}} \ , \quad \forall l \geq 0 \ . \tag{30}$$

Basing on *Theorem 1*, we can also evaluate the mean and variance of L_m with respect to the routing distance.

Theorem 2. *For a fixed relay intensity λ, as r approaches $+\infty$, $\mathbb{E}\{L_m \mid R = r\} \sim \mathcal{O}(\ln(r))$. More specifically, for $r > 0$, let $m_{L_m}(r) \triangleq \mathbb{E}\{L_m \mid R = r\}$, we have*

$$m_{L_m}(r) = \int_0^r \frac{1 - e^{-\lambda x}}{\lambda x} dx \ , \tag{31}$$

[1] For a PPP, given that one node is located at a particular point, the conditional distribution of all other nodes is still a PPP, which is known as Slivnyak-Mecke Theorem [28, Theorem 1.4.5].

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whose LT is

$$M_{L_m}(s) = \frac{1}{s\lambda}\ln(\frac{\lambda}{s}+1) \ . \tag{32}$$

Moreover, for any given λ, the conditional variance of L_m under $R=r$, $\mathbb{D}\{L_m \mid R=r\}$, satisfies

$$\lim_{r\to\infty}\mathbb{D}\{L_m \mid R=r\} = \frac{\pi^2}{6\lambda^2} \ . \tag{33}$$

Proof. See Appendix B in the supplementary file. □

Remark 1. From *Theorem 2*, we can see that for a fixed relay intensity λ, the conditional mean of the longest hop distance $\mathbb{E}\{L_m \mid R=r\} \sim \mathcal{O}(\ln(r))$ as the routing distance $r \to +\infty$, and its variance is bounded. Unlike the case with evenly deployed relays where the per hop distance stays constant with respect to the routing distance, for randomly distributed relays, the longest hop distance would go to infinity as the routing distance approaches infinity, i.e., the throughput of the worst hop would approach zero. This shows that long distance communication is not feasible in randomly deployed networks.

V. The Average End-to-End Throughput under NN Routing

In this section, we derive the average end-to-end throughput under NN routing over all the possible realizations of relays Φ. We discuss two different resource allocation schemes: a conventional TDMA scheme with fixed slot length and a dynamic TDMA scheme with flexible slot length.

A. Throughput Analysis under NN Routing with fixed slot length

We consider a conventional TDMA resource allocation scheme where a TDMA cycle would consist of $N+1$ time slots, each of which would be allocated to one relay node or the source node. Following (11), the end-to-end throughput can be expressed as

$$T_{\text{end}} = \min_{\boldsymbol{X}_i'\in\Phi'\cup\{\boldsymbol{D}\}} \frac{P_s(|\boldsymbol{X}_i'-\boldsymbol{X}_{i-1}'|)}{N+1} = \frac{P_s(L_m)}{N+1} \ , \tag{34}$$

where the second equation follows from the fact that $P_s(\cdot)$ is a non-increasing function.

Given the coverage probability function $P_s(\cdot)$, the average coverage probability of the longest hop, $\mathbb{E}\{P_s(L_m)\}$, should depend on the intensity of the relays, λ, and the routing distance R. For this reason, we define $p(x,r) \triangleq \mathbb{E}\{P_s(L_m) \mid \lambda = x, R=r\}$. Then, we have the following theorem on the end-to-end throughput.

Theorem 3. *For a relay intensity λ, given the routing distance R, the average end-to-end throughput is given by*

$$\mathbb{E}\{T_{end} \mid R\} = \frac{e^{-\lambda R}}{\lambda}\int_0^{\lambda} e^{Rx}\ p(x,R)dx \ . \tag{35}$$

Proof. See Appendix C in the supplementary file. □

Note that the function $p(x,r)$ can be computed numerically, which only depends on the marginal distribution of L_m. Following *Theorem 3*, we can calculate $\mathbb{E}\{T_{\text{end}} \mid R\}$ without deriving the joint *probability density function* (PDF) of N and L_m explicitly. With the following Lemma, we can further reduce the computational complexity by calculating the Laplace transform of $\mathbb{E}\{T_{\text{end}} \mid R=r\}$ with respect to r.

Lemma 3. *Taking the relay intensity Λ as a random variable and let $f_{L_m|\Lambda,R}(l \mid x,r)$ denote the conditional PDF of the longest hop distance L_m given the relay intensity $\Lambda = x$ and routing distance $R=r$. Define*

$$q(l,\lambda,r) \triangleq \frac{e^{-\lambda r}}{\lambda}\int_0^{\lambda} e^{rx}\ f_{L_m|\Lambda,R}(l \mid x,r)\ dx \ , \tag{36}$$

then the Laplace transform of $q(l,\lambda,r)$ with respect to r is

$$Q(l,\lambda,s) = \frac{(s+\lambda)}{\lambda + e^{(s+\lambda)l}s} \ . \tag{37}$$

This lemma follows directly from the Laplace transform of $f_{L_m|\Lambda,R}(l \mid x,r)$ and we skip the proof for brevity. Basing on *Lemma 3*, we have the following result on $\mathbb{E}\{T_{\text{end}} \mid R=r\}$.

Proposition 1. *For a fixed relay intensity λ, define $T_{end}(r) \triangleq \mathbb{E}\{T_{end}|R=r\}$ as the average end-to-end throughput given routing distance $R=r$. The Laplace transform of $T_{end}(r)$, $\mathcal{T}_{end}(s)$, can be calculated as*

$$\mathcal{T}_{end}(s) = \int_0^{+\infty} P_s(l)\ \frac{s+\lambda}{\lambda+e^{(s+\lambda)l}s}\ dl \ . \tag{38}$$

Proof. Recall that $p(x,r) = \mathbb{E}\{P_s(L_m) \mid \lambda = x, R=r\}$, where $P_s(L_m)$ is the coverage probability of the longest hop, then $p(x,r)$ can be calculated as

$$p(x,r) = \int_0^{+\infty} P_s(l)\ f_{L_m|\Lambda,R}(l \mid x,r)\mathrm{d}l \ . \tag{39}$$

So $T_{\text{end}}(r)$ can be expressed as

$$\begin{aligned} T_{\text{end}}(r) &= \frac{e^{-\lambda r}}{\lambda}\int_0^{\lambda} e^{rx}\int_0^{+\infty} P_s(l)\ f_{L_m|\Lambda,R}(l \mid x,r)\mathrm{d}l\mathrm{d}x \\ &= \int_0^{+\infty} P_s(l)\ q(l,\lambda,r)\ \mathrm{d}l \ . \end{aligned} \tag{40}$$

Basing on *Lemma 3*, (38) can be obtained accordingly. □

Following *Proposition 1* and *Corollary 1*, the mean of the throughput can be computed numerically through the inverse Fourier transform of $T_{\text{end}}(j\omega)$.

As shown in *Proposition 1*, a closed-form expression of the end-to-end throughput is hard to derive. In order to further analyze the impacts of different network parameters on the network performance, we derive the following lower bound on end-to-end throughput.

Proposition 2. *For a given relay intensity λ and routing distance r, the end-to-end throughput $T_{end}(r)$ is lower bounded by $T_{end,L}(r)$, i.e., $T_{end}(r) \ge T_{end,L}(r)$, where*

$$\begin{aligned} T_{end,L}(r) = \frac{1-e^{-\lambda r}}{\lambda r}\exp(-\frac{\kappa\lambda^{-2}}{1-e^{-\lambda r}}[\ln^2(\lambda r)+2B(\lambda r)\ln(\lambda r) \\ +2C(\lambda r)+c-e^{-\lambda r}\left(A(\lambda r)^2+(2+c)\lambda r+c\right)]), \end{aligned} \tag{41}$$

with $c = \max_{t\in(0,1)}\left[\frac{\ln^2 t}{(1-t)^2}-\ln^2 t\right] \approx 1.51$, $A \triangleq \int_0^{+\infty}\frac{l^2e^{-l}}{1-e^{-l}}dl \approx 2.404$, $B(x) \triangleq \int_0^1 \frac{1-e^{-u}}{u}du - \int_1^x \frac{e^{-u}}{u}du$, $C(x) \triangleq \int_1^x \ln u\frac{e^{-u}}{u}du - \int_0^1 \ln u\frac{1-e^{-u}}{u}du$.

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For $\lambda r \gg 1$, the lower bound $T_{\mathit{end,L}}(r)$ approximates

$$T_{\mathit{end,L}}(r) \approx \frac{1}{\lambda r} \exp\left(-\kappa \lambda^{-2}[\ln^2(\lambda r) + 2B\ln(\lambda r) + 2C + c]\right), \quad (42)$$

where $B = \lim_{x\to+\infty} B(x) \approx 0.577$, $C = \lim_{x\to+\infty} C(x) \approx 0.989$.

Proof. See Appendix D in the supplementary file. □

In the following, we consider to optimize the lower bound $T_{\mathrm{end,L}}(r)$ with respect to the relay intensity λ.

Corollary 2. *For $\lambda r \gg 1$, the optimal relay intensity λ^* that maximizes $T_{\mathit{end,L}}(r)$ should satisfy $\lambda_1 < \lambda^* < \lambda_2$, where*

$$\lambda_1 = \frac{1}{r}\exp\left(\frac{1}{r\sqrt{2\kappa}}\exp\left(-W_{-1}(-\frac{1}{r\sqrt{2\kappa}})\right)\right), \quad (43)$$

$$\lambda_2 = \frac{1}{r}\exp\left(\frac{e^{-\sqrt{2C+c-B}}}{r\sqrt{2\kappa}}\exp\left(-W_{-1}(-\frac{e^{-\sqrt{2C+c-B}}}{r\sqrt{2\kappa}})\right)\right), \quad (44)$$

and $W_{-1}(\cdot)$ is the real branch of Lambert W function over $(-\infty, -1)$ [33].

Proof. See Appendix E in the supplementary file. □

B. Throughput Analysis under NN Routing with flexible slot length

In TDMA, it may be unwise to allocate equal time slots to each hop since the time resources may be wasted at the relay nodes whose arrival rates are much lower than their service rates. Recall that A_i denotes the normalized slot length allocated to hop i and L_i denotes its hop distance. Given the relay deployment Φ, we can formulate the resource allocation problem as

$$\begin{aligned} \text{Maximize} \quad & \min_i (A_i P_s(L_i)) \\ \text{s.t.} \quad & \sum_i A_i = 1 \ . \end{aligned} \quad (45)$$

It follows that the optimal $A_i = 1/\left[P_s(L_i)\sum_j 1/P_s(L_j)\right]$, and the optimized throughput is

$$T_{\mathrm{end}} = 1/\left(\sum_i 1/P_s(L_i)\right) \ . \quad (46)$$

Note that the end-to-end throughput (46) is a function of hop distances L_i for $i = 1, 2, ..., N+1$. Our goal is to evaluate the mean of (46) with respect to the distribution of Φ.

Define the RVs Y_i and Y as

$$Y_i \triangleq 1/P_s(L_i) \ , \quad Y \triangleq \sum_i Y_i = 1/T_{\mathrm{end}} \ . \quad (47)$$

We first derive the distribution of Y and then calculate the mean of its reciprocal, i.e., the average end-to-end throughput.

Define function $g_Y(y, r) \triangleq f_{Y|R}(y|r)$, the conditional PDF of Y given routing distance $R = r$. For a given location of the first relay, following a similar derivation as in the proof of *Theorem 1*, we have

$$\begin{aligned} & f_{Y|R}(y|r) = \Pr\{|\Phi| = 0\}\ f_{Y_1|L_1}(y \mid r) + \\ & \textstyle\int_0^r f_{L_1}(x) \int_0^y f_{Y_1|L_1}(\tau \mid x) f_{Y|R}(y-\tau|r-x)\mathrm{d}\tau\mathrm{d}x \ . \end{aligned} \quad (48)$$

Since Y_i is a deterministic function of L_i, we have $f_{Y_1|L_1}(y \mid x) = \delta(y - 1/P_s(x))$. The 2D Laplace transform of $g_Y(y, r)$ can be expressed as

$$G_Y(s_1, s_2) = \frac{F_{Y_1,L_1}(s_1, s_2)}{\lambda(1 - F_{Y_1,L_1}(s_1, s_2))} \ , \quad (49)$$

where

$$F_{Y_1,L_1}(s_1, s_2) = \int_0^{+\infty} \lambda e^{-(\lambda+s_2)r} e^{-\frac{s_1}{P_s(r)}} \mathrm{d}r \ , \quad (50)$$

is the 2D Laplace transform of $f_{Y_1,L_1}(y, x)$. We have the following theorem on the average end-to-end throughput:

Theorem 4. *Denote the average throughput with dynamic resource allocation given routing distance r, $\mathbb{E}\{T_{\mathit{end}} \mid R = r\}$, by function $T_{\mathit{end}}(r)$. The Laplace transform of $T_{\mathit{end}}(r)$ is*

$$\mathcal{T}_{\mathit{end}}(s) = \int_0^{+\infty} G_Y(u, s) du \ , \quad (51)$$

where $G_Y(u, s)$ is defined in (49).

This theorem follows directly from the property of Laplace transform.

VI. The Average End-to-End Throughput under QED Routing

In this section, we evaluate the average end-to-end throughput of the proposed QED routing under fixed-length TDMA, both with and without intra-route resource reuse.

A. Throughput Analysis under QED Routing without Intra-Route Resource Reuse

In this scheme, each relay in Φ' is allocated with a time slot of fixed length. Let L'_m denote the longest hop distance in the relay set Φ'. Following (34), the end-to-end throughput can be expressed by

$$T_{\mathrm{end}} = \frac{1}{N'+1} P_s(L'_m) \ . \quad (52)$$

where N' is the number of relays in Φ'. Unfortunately, as Φ' is not a PPP anymore, it is hard to derive an accurate expression of the end-to-end throughput for QED routing as we did for the NN routing. In order to make the end-to-end throughput analysis tractable, the following approximation is made.

Approximation 2. *Given routing distance $R = r$, the average end-to-end throughput is approximated by*

$$\mathbb{E}\{T_{\mathit{end}} \mid R = r\} = \frac{P_s(\mathbb{E}\{L'_m \mid R = r\})}{\mathbb{E}\{N' \mid R = r\} + 1} \ . \quad (53)$$

The validity of this approximation is verified by simulation, as shown in Section VII. In addition, we make the following approximation on $\mathbb{E}\{N' \mid R = r\}$.

Approximation 3. *Given routing distance $R = r$, the average number of relays selected by QED routing, $N' = |\Phi'|$ is approximated by*

$$\mathbb{E}\{N' \mid R = r\} = \frac{r}{l_0 + 1/\lambda} \ . \quad (54)$$

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The reason behind this approximation is that, if an infinite routing distance is assumed, the average routing distance per hop under QED routing is $l_0 + 1/\lambda$. Also, according to the ergodicity, N' will approach its mean almost surely as $r \to +\infty$.

We have the following theorem on the distribution of the longest hop distance L'_m for Φ'.

Theorem 5. *Define $g'(l,r) \triangleq \Pr\{L'_m \le l \mid R = r\}$ as the condition CDF of L'_m given routing distance $R = r$. Denote its Laplace transform with respect to r by $G'(l,s) = \int_0^{+\infty} g'(l,r)e^{-sr}dr$ for $l > l_0$, we have*

$$G'(l,s) = \frac{1}{s} - \frac{(\lambda+s)e^{-(s+\lambda)\cdot l + \lambda l_0}}{s\{\lambda \cdot [1 + e^{-(s+\lambda)\cdot l+\lambda l_0} - e^{-s\cdot l_0}] + s\}} \ . \tag{55}$$

As $r \to +\infty$, the conditional mean of L'_m given routing distance $R = r$, $\mathbb{E}\{L'_m \mid R = r\}$, satisfies

$$\lim_{r\to+\infty} \mathbb{E}\{L'_m \mid R = r\} - \left[\frac{1}{\lambda}\left(\ln\frac{\lambda r}{\lambda l_0 + 1} + B\right) + l_0\right] = 0 \ . \tag{56}$$

where constant $B \approx 0.577$ is defined the same as Proposition 2.

Proof. See Appendix F in the supplementary file. □

Basing on *Theorem 5*, we can make the following approximation on L'_m.

Approximation 4. *Then conditional mean of L'_m given $R = r \gg l_0$ is approximated by*

$$\mathbb{E}\{L'_m \mid R = r\} \approx \frac{1}{\lambda}\left(\ln\frac{\lambda r}{\lambda l_0 + 1} + B\right) + l_0 \ . \tag{57}$$

Basing on the approximations above, we have the following theorem on the end-to-end throughput of QED routing.

Theorem 6. *The average end-to-end throughput of QED routing for routing distance $r \gg l_0$, $\mathbb{E}\{T_{end} \mid R = r\}$, can be approximated by*

$$\frac{l_0 + 1/\lambda}{r + l_0 + 1/\lambda}\exp\left\{-\kappa\left[\frac{1}{\lambda}\left(\ln\frac{\lambda r}{\lambda l_0 + 1} + B\right) + l_0\right]^2\right\} . \tag{58}$$

Routing parameter l_0 can be optimized to maximize the end-to-end throughput (58) by calculating its derivative, by which the following Corollary is obtained.

Corollary 3. *For a routing distance $r \gg \max(l_0, 1/\lambda)$, the optimal l_0 satisfies $\underline{l_0} < l_0^* < \overline{l_0}$, where*

$$\underline{l_0} = \frac{\sqrt{(\ln\lambda r + B)^2 + \frac{2\lambda^2}{\kappa}} - (\ln\lambda r + B)}{2\lambda} \ , \tag{59}$$

$$\overline{l_0} = \frac{\sqrt{B^2 + \frac{2\lambda^2}{\kappa}} - B}{2\lambda} \ . \tag{60}$$

B. Throughput Analysis under QED Routing with Intra-Route Resource Reuse

Recall that in *Lemma 2*, we derived the coverage probability function $P'_s(l)$ for the intra-route resource reuse, which is a non-increasing function with respect to the hop distance l. Following (24), the end-to-end throughput of QED routing with the intra-route resource reuse can be expressed as

$$T_{\text{end}} = \min_{\boldsymbol{X}'_i \in \Phi' \cup \{\boldsymbol{D}\}} \frac{1}{M} P'_s(|\boldsymbol{X}'_i - \boldsymbol{X}'_{i-1}|) = \frac{1}{M} P'_s(L'_m) \ , \tag{61}$$

where M is the number of time slots in a TDMA cycle. Following *Approximation 2* where the RV L'_m is replaced by its mean, we have the following theorem on the average end-to-end throughput.

Theorem 7. *Given the routing distance r, the average end-to-end throughput of QED routing with intra-route resource reuse can be approximated by*

$$\mathbb{E}\{T_{end} \mid R = r\} = \frac{1}{M} P'_s(\mathbb{E}\{L'_m \mid R = r\}) \ , \tag{62}$$

where $\mathbb{E}\{L'_m \mid R = r\}$ is given in Approximation 4.

The effect of slot number M on the network performance is two fold. First, a smaller M allows a larger number of concurrent transmissions on the route, by which a larger equivalent bandwidth can be obtained, whereas a stronger intra-route interference is introduced. Second, consider a large scale wireless network which consists of a number of multi-hop links, the increase of concurrent transmissions on each route, which results from the decrease of M, will also generate a stronger inter-route interference on other routes. That is, the selection of M also affects the intensity of the active external interferers. Since the number of concurrent transmissions on a route is linearly proportional to $1/M$, we assume that the intensity of active external interferers μ' satisfies $\mu' = \mu'_1/M$, where μ'_1 denotes the intensity of active interferers for $M = 1$. For a given μ'_1, we discuss the selection of M for different routing distance r under two scenarios: 1) the node intensity is constant; and 2) the source-destination pair intensity is constant.

1) Constant node intensity: In this case, μ'_1 is assumed to be constant for different r, denoted by $\mu'_1 = \bar{\mu}$ for a constant $\bar{\mu}$. Then we have the following corollary on the optimal number of time slots M.

Corollary 4. *As the routing distance $r \to \infty$, the optimal number of time slots $M^*(r)$ for routing distance r that maximizes the end-to-end throughput scales with*

$$M^*(r) \sim \mathcal{O}(\bar{\kappa}\mathbb{E}^2\{L'_m \mid R = r\}) \ , \tag{63}$$

where $\bar{\kappa} = 2\pi\bar{\mu}\frac{\pi}{\beta\sin(2\pi/\beta)}\theta^{2/\beta}$.

Proof. See Appendix G in the supplementary file. □

2) Constant source-destination pair intensity: In this case, we consider a multi-hop wireless network formed by multiple source-destination pairs, where the intensity of the source-destination pairs is assumed to be a constant and their routing distances are the same as r. So for a given r, μ'_1 will be linearly proportional to the average hop number per route,

$\mathbb{E}\{N'+1 \mid R=r\}$, which is linearly proportional to r as shown in *Approximation 3*. For simplicity, with other parameters being fixed, we assume that $\mu_1' = \bar{\mu}r$ for a constant $\bar{\mu}$. So the intensity of the active external interferers can be denoted by $\mu' = \bar{\mu}r/M$.

Corollary 4 still holds for this case by replacing $\bar{\kappa}$ with $\bar{\kappa}r$, i.e., $M^*(r) \sim \mathcal{O}(\bar{\kappa}r\mathbb{E}^2\{L_m' \mid R=r\})$. However, since the average hop number $\mathbb{E}\{N'+1 \mid R=r\}$ is linearly proportional to r, for $r \to +\infty$, we have

$$M^*(r) > \mathbb{E}\{N'+1 \mid R=r\} \ . \quad (64)$$

That is, *the number of time slots in a TDMA cycle is greater than the number hops on the route,* which implies that each time slot will be allocated to at most one hop on the same route. Thus, we have the following corollary.

Corollary 5. *In a network with random relays and a finite relay intensity λ, for a fixed source-destination pair intensity, as the routing distance $r \to +\infty$, the intra-route resource reuse should not be adopted for the end-to-end throughput optimization.*

Remark 2. It is interesting to note that when the relay intensity $\lambda = \infty$, the intra-route resource reuse would still be beneficial even for fixed source-destination pair intensity. In fact, when $\lambda = \infty$, the relays selected by the QED routing converge to a equidistant deployment with a hop distance of l_0. From *Corollary 4* and the discussions above, the optimal number of time slots for the equidistant relays scales with $\mathcal{O}(\bar{\kappa}rl_0^2)$ as the routing distance $r \to +\infty$. Since the hop number of the equidistant relays is r/l_0, as long as $r/l_0 \gg \bar{\kappa}rl_0^2$, i.e., $\bar{\kappa}l_0^3 \ll 1$, the intra-route resource reuse is still beneficial. Comparing with the result in *Corollary 5*, we can see that this is another difference between equidistant relays and random relays.

VII. Numerical Results

In this section, we evaluate the end-to-end throughput performance with random relay deployment through numerical results. Unless otherwise clarified, we will use the following parameters: the interferer intensity $\mu = 5 \times 10^{-4}$ $/m^2$, the ALOHA access probability $p_a = 0.1$, the path-loss exponent $\beta = 4$, the SINR threshold for successful transmission $\theta = 10$ dB. We first start with the case of NN routing and then simulate the performance of QED routing.

A. Numerical Results of NN Routing

Example 1: End-to-end throuhgput of NN routing with fixed slot length In this example, we evaluate the end-to-end throughput with random relays using NN routing and fixed slot length. Fig. 2 shows the average end-to-end throughput versus relay intensity λ for different routing distance r. It can be observed that: even without an optimized deployment of relays, the end-to-end throughput can still be obviously improved compared with the case of direct connection. For example, if a minimum end-to-end throughput of 1×10^{-2} packets/slot is required, the maximum communication range is around 75 m without the relays, while the communication range expands to more than 250 m with multi-hop relays. It can be observed

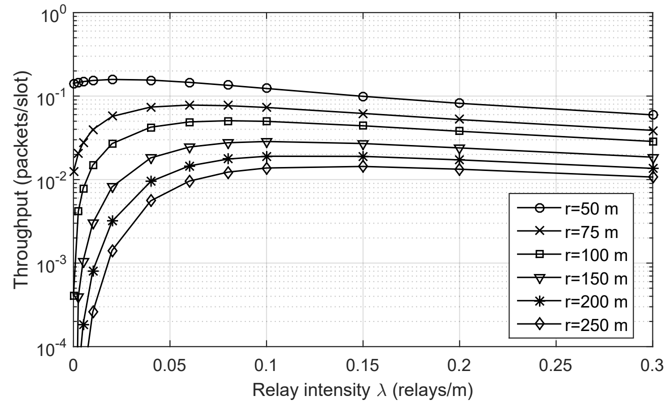

Fig. 2. Average end-to-end throughput versus relay intensity under NN routing with fixed slot length.

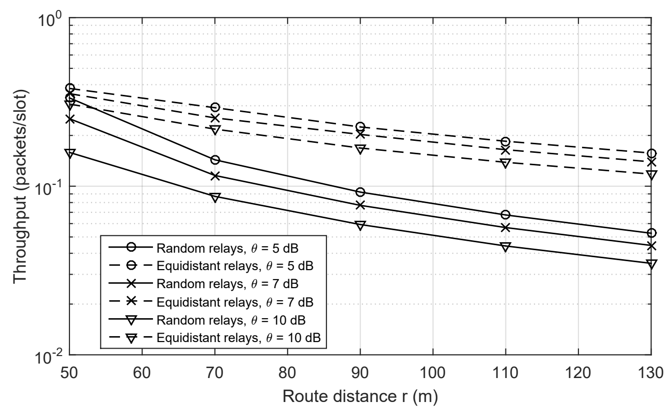

Fig. 3. Throughput comparison with fixed slot length: random relays under NN routing versus equidistant relays

that the optimal relay intensity λ increases as r increases, on the contrary to the equidistant relays where the optimal relay distance stays constant[2].

Example 2: Performance comparison with equidistant relays In this example, we compare the end-to-end throughput between equidistant and random relays using NN routing with fixed slot length. Fig. 3 shows the optimal end-to-end throughput of random relay deployment and that of equidistant relays under different routing distance r and SINR threshold θ. The relay intensity λ is optimized for each routing distance. The random relay deployment suffers a significant performance loss compared to the ideal case. For instance, with a SINR threshold θ of 10 dB under the network configuration, there is a 48% throughput loss at $r = 50$ m and a 70% performance loss at $r = 130$ m, which are not negligible for system evaluation.

Example 3: End-to-end throughput with flexible slot length In this example, we test the end-to-end throughput with flexible slot length in NN routing. Fig. 4 shows the end-to-end throughputs of different schemes under different relay intensities and routing distances. We can observe that a

[2]Here, we refer to the upper bound of end-to-end throughput for equidistant relays, $\frac{d}{r}P_{cov}(d)$, where d is the per hop distance, and the optimal d is independent of r.

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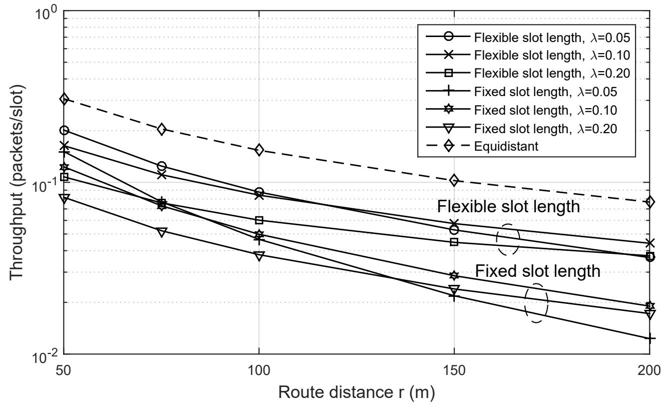

Fig. 4. Throughput comparison under NN routing: fixed slot length versus flexible slot length

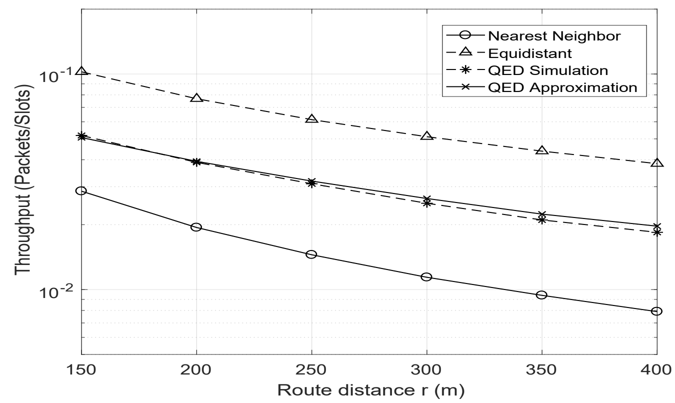

Fig. 5. Throughput comparison: QED versus NN

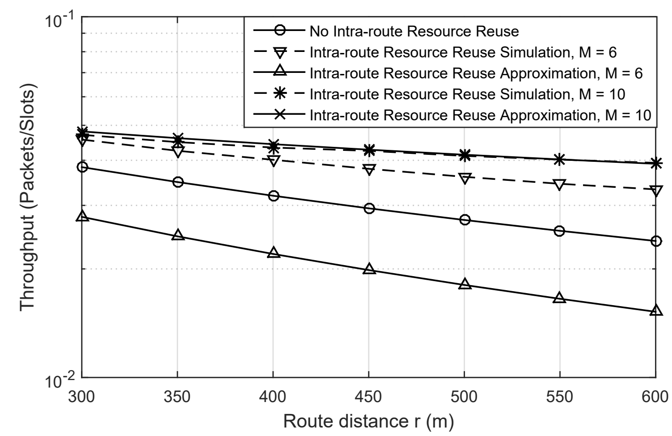

Fig. 6. The average end-to-end throughput under QED routing with the intra-route resource reuse for constant node intensity

significant performance improvement is achieved by dynamic resource allocation compared with the fixed slot length. Using a flexible slot length, the performance loss from the equidistant case also degrades much slower than the case of fixed slot length with the increase of routing distance.

B. Numerical Results of QED routing

Example 4: End-to-end throughput under QED routing without intra-route resource reuse In this example, we evaluate the average end-to-end throughput of the QED routing without intra-route resource reuse, and compare it with that of the NN routing as well as that of the equidistant relays. Fig. 5 shows the average end-to-end throughputs of the QED routing and the NN routing for different routing distances. For each routing distance, the relay intensity λ is the same for both the NN and the QED routing, which is optimized with respect to the average end-to-end throughput of the NN routing. In terms of the selection of parameter l_0 in the QED routing, we choose $l_0 = (\overline{l_0} + \underline{l_0})/2$ as defined in Corollary 3. First, it can be observed that the average end-to-end throughput derived in Theorem 6 provides a very close approximation to the simulation results, substantiating the validity of the approximations we made. In addition, a significant performance improvement can be achieved by the QED routing, compared with the NN routing. However, there is still a non-negligible performance loss compared with the equidistant relays for the selected relay intensities.

Example 5: End-to-end throughput under QED routing with intra-route resource reuse In this example, we evaluate the end-to-end throughput with the intra-route resource reuse for the QED routing. We set $l_0 = 15$ m and $\lambda = 0.2$ /m. As is discussed in the previous section, we consider both the case of constant node intensity and that of constant source-destination pair intensity. We use $\bar{\mu} = 5 \times 10^{-4}$ for the constant node intensity and $\bar{\mu} = 3 \times 10^{-6}$ for the constant source-destination pair intensity. The results are compared with the end-to-end throughput of the QED routing without the intra-route resource reuse, where we set the intensity of the interferers the same as the case of $M = \mathbb{E}\{N' + 1 \mid R = r\}$. The numerical results of the two cases are shown in Fig. 6 and Fig. 7 respectively. First, it can be observed that there is a gap between the end-to-end throughput derived in *Theorem 7* and the simulation results for small M's. This is because we approximate the intra-route interference by a very conservative upper bound of it. However, as M becomes larger and the intra-route interference becomes less significant, the approximation values become very close to the simulation results. Second, the numerical results verify our theoretical analysis on the effect of intra-route resource reuse. That is, generally, intra-route resource reuse can lead to an increase in the throughput; however, if the source-destination pair density is approximately time-invariant or does not change significantly, then it is not beneficial to apply intra-route resource reuse for long distance routing.

VIII. Conclusions & Discussions

In this paper, we investigated the effect of relay randomness on the end-to-end throughput in multi-hop wireless networks using stochastic geometry. We modeled the relays as a linear Poisson Point Process between the source and destination, and the external interferers as an independent Poisson Point Process. We first evaluated the end-to-end throughputs under nearest neighbor routing and TDMA MAC with fixed and flexible slot length, respectively. Then we proposed a quasi-equal-distance (QED) routing protocol, and analyzed its end-to-end

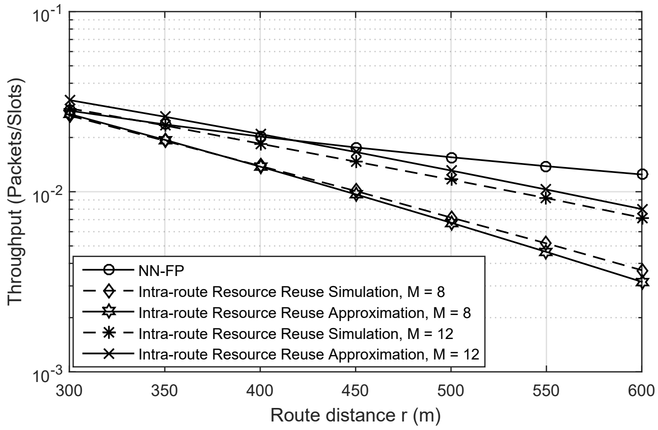

Fig. 7. The average end-to-end throughput under QED routing with the intra-route resource reuse for constant source-destination pair intensity

throughputs with and without intra-route resource reuse. The optimal number of time slots was also analyzed for intra-route resource reuse. The analysis was further demonstrated through numerical examples. Both the theoretic and numerical results indicated that: (i) The throughput performance of the proposed QED routing can achieve a significant performance gain over that of the NN routing. For network with sparse random relays, compared with the ideal equidistant routing, the performance loss of QED routing due to relay randomness is not negligible. However, as the relay intensity gets higher, the performance of QED routing converges to that of the equidistant relays; (ii) The effect of intra-route resource reuse depends on the network setup. If the node intensity is a constant over the network, then as expected, intra-route resource reuse is always beneficial when the routing distance r is sufficiently large. (iii) With randomly distributed relays, the communication distance can generally be extended. However, due to the uncertainty in relay distribution, long distance communication is generally not feasible with random relays. This implies that the existence of a reasonably defined infrastructure needs to be ensured for effective long distance communication. The results in this paper also echoed our previous observation in [25]–[27] that future network design would reflect the convergence of centralized and ad hoc networks.

References

[1] M. Haenggi, J. Andrews, F. Baccelli, O. Dousse, and M. Franceschetti, "Stochastic geometry and random graphs for the analysis and design of wireless networks," *IEEE Journal on Selected Areas in Communications*, vol. 27, no. 7, pp. 1029–1046, September 2009.

[2] M. Haenggi and R. K. Ganti, "Interference in large wireless networks," *Foundations and Trends in Networking*, vol. 3, no. 2, pp. 127–248, February 2009. [Online]. Available: http://dx.doi.org/10.1561/1300000015

[3] J. Andrews, F. Baccelli, and R. Ganti, "A tractable approach to coverage and rate in cellular networks," *IEEE Transactions on Communications*, vol. 59, no. 11, pp. 3122–3134, November 2011.

[4] H. ElSawy, E. Hossain, and M. Haenggi, "Stochastic geometry for modeling, analysis, and design of multi-tier and cognitive cellular wireless networks: A survey," *IEEE Communications Surveys Tutorials*, vol. 15, no. 3, pp. 996–1019, Thirdquarter 2013.

[5] M. Haenggi, *Stochastic Geometry for Wireless Networks*, 1st ed. New York, NY, USA: Cambridge University Press, 2012.

[6] J. G. Andrews, S. Weber, M. Kountouris, and M. Haenggi, "A simple upper bound on random access transport capacity," in *Communication, Control, and Computing, 2009. Allerton 2009. 47th Annual Allerton Conference on*, Sept 2009, pp. 849–856.

[7] ——, "Random access transport capacity," *IEEE Transactions on Wireless Communications*, vol. 9, no. 6, pp. 2101–2111, June 2010.

[8] R. Vaze, "Throughput-delay-reliability tradeoff with arq in wireless ad hoc networks," *IEEE Transactions on Wireless Communications*, vol. 10, no. 7, pp. 2142–2149, July 2011.

[9] K. Stamatiou, F. Rossetto, M. Haenggi, T. Javidi, J. R. Zeidler, and M. Zorzi, "A delay-minimizing routing strategy for wireless multi-hop networks," in *Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks, 2009. WiOPT 2009. 7th International Symposium on*, June 2009, pp. 1–6.

[10] K. Stamatiou and M. Haenggi, "The delay-optimal number of hops in poisson multi-hop networks," in *2010 IEEE International Symposium on Information Theory*, June 2010, pp. 1733–1737.

[11] ——, "Optimal spatial reuse in poisson multi-hop networks," in *Global Telecommunications Conference (GLOBECOM 2010), 2010 IEEE*, Dec 2010, pp. 1–6.

[12] ——, "Delay characterization of multihop transmission in a poisson field of interference," *IEEE/ACM Transactions on Networking*, vol. 22, no. 6, pp. 1794–1807, Dec 2014.

[13] A. Crismani, U. Schilcher, S. Toumpis, G. Brandner, and C. Bettstetter, "Packet travel times in wireless relay chains under spatially and temporally dependent interference," in *2014 IEEE International Conference on Communications (ICC)*, June 2014, pp. 2002–2008.

[14] P. H. J. Nardelli, P. Cardieri, and M. Latva-aho, "Efficiency of wireless networks under different hopping strategies," *IEEE Transactions on Wireless Communications*, vol. 11, no. 1, pp. 15–20, January 2012.

[15] M. J. Farooq, H. ElSawy, Q. Zhu, and M. S. Alouini, "Optimizing mission critical data dissemination in massive iot networks," in *2017 15th International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOpt)*, May 2017, pp. 1–6.

[16] F. Baccelli, B. Blaszczyszyn, and P. Muhlethaler, "An aloha protocol for multihop mobile wireless networks," *IEEE Transactions on Information Theory*, vol. 52, no. 2, pp. 421–436, Feb 2006.

[17] E. S. Sousa and J. A. Silvester, "Optimum transmission ranges in a direct-sequence spread-spectrum multihop packet radio network," *IEEE Journal on Selected Areas in Communications*, vol. 8, no. 5, pp. 762–771, Jun 1990.

[18] S. P. Weber, X. Yang, J. G. Andrews, and G. de Veciana, "Transmission capacity of wireless ad hoc networks with outage constraints," *IEEE Transactions on Information Theory*, vol. 51, no. 12, pp. 4091–4102, Dec 2005.

[19] P. H. Nardelli, M. de Castro Tom, H. Alves, C. H. de Lima, and M. Latva-aho, "Maximizing the link throughput between smart meters and aggregators as secondary users under power and outage constraints," *Ad Hoc Networks*, vol. 41, no. Supplement C, pp. 57 – 68, 2016, cognitive Radio Based Smart Grid The Future of the Traditional Electrical Grid. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S1570870515002802

[20] C. H. M. de Lima, P. H. J. Nardelli, H. Alves, and M. Latva-aho, "Contention-based geographic forwarding strategies for wireless sensors networks," *IEEE Sensors Journal*, vol. 16, no. 7, pp. 2186–2195, April 2016.

[21] F. Baccelli, B. Blaszczyszyn, and P. Muhlethaler, "On the performance of time-space opportunistic routing in multihop mobile ad hoc networks," in *2008 6th International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks and Workshops*, April 2008, pp. 307–316.

[22] H. Feng and L. J. Cimini, "On the optimum number of hops in a multi-hop linear network with randomly located nodes," in *2012 IEEE International Conference on Communications (ICC)*, June 2012, pp. 2329–2333.

[23] Y. Chen and J. G. Andrews, "An upper bound on multihop transmission capacity with dynamic routing selection," *IEEE Transactions on Information Theory*, vol. 58, no. 6, pp. 3751–3765, June 2012.

[24] B. Blaszczyszyn and P. Mhlethaler, "Random linear multihop relaying in a general field of interferers using spatial aloha," *IEEE Transactions on Wireless Communications*, vol. 14, no. 7, pp. 3700–3714, July 2015.

[25] M. Abdelhakim, Y. Liang, and T. Li, "Mobile access coordinated wireless sensor networks - design and analysis," *IEEE Transactions on Signal and Information Processing over Networks*, vol. 3, no. 1, pp. 172–186, March 2017.

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[26] ——, “Mobile coordinated wireless sensor network: An energy efficient scheme for real-time transmissions,” *IEEE Journal on Selected Areas in Communications*, vol. 34, no. 5, pp. 1663–1675, May 2016.

[27] T. Li, M. Abdelhakim, and J. Ren, “N-hop networks: a general framework for wireless systems,” *IEEE Wireless Communications*, vol. 21, no. 2, pp. 98–105, April 2014.

[28] F. Baccelli and B. Blaszczyszyn, “Stochastic geometry and wireless networks, volume 1: Theory,” *Foundations and Trends in Networking*, vol. 3, no. 3-4, pp. 249–449, 2009. [Online]. Available: http://dx.doi.org/10.1561/1300000006

[29] J. Lee, H. Shin, I. Lee, and J. Heo, “Optimal linear multihop system for df relaying in a poisson field of interferers,” *IEEE Communications Letters*, vol. 17, no. 11, pp. 2029–2032, November 2013.

[30] M. Sikora, J. N. Laneman, M. Haenggi, D. J. Costello, and T. E. Fuja, “Bandwidth- and power-efficient routing in linear wireless networks,” *IEEE Transactions on Information Theory*, vol. 52, no. 6, pp. 2624–2633, June 2006.

[31] M. Haenggi and R. Smarandache, “Diversity polynomials for the analysis of temporal correlations in wireless networks,” *IEEE Transactions on Wireless Communications*, vol. 12, no. 11, pp. 5940–5951, November 2013.

[32] M. Haenggi, “Outage, local throughput, and capacity of random wireless networks,” *IEEE Transactions on Wireless Communications*, vol. 8, no. 8, pp. 4350–4359, August 2009.

[33] R. M. Corless, G. H. Gonnet, D. E. G. Hare, D. J. Jeffrey, and D. E. Knuth, “On the lambertw function,” *Advances in Computational Mathematics*, vol. 5, no. 1, pp. 329–359, 1996. [Online]. Available: http://dx.doi.org/10.1007/BF02124750