Improving ADC Figure-of-Merit in Wideband Antenna Array Receivers using Multidimensional Space-Time Delta-Sigma Multiport Circuits

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***Abstract*—This paper describes recent work on the topic of multi-dimensional (MD) spatio-temporal noise and distortion shaping for radio-frequency (RF) antenna arrays with applications in wireless communications, phased-array radar sensing, microwave/mm-wave imaging, and radio astronomy instrumentation. The MD spectral properties of propagating plane-waves that arise from electromagnetics are combined with MD circuits and signal processing theory based on passive resistively-terminated 2-D filters. The result, for the first time in the literature, is MD multi-port extensions of the analog and digital electronics found within wireless transceivers, including RF amplifiers, mixers, and data converters. In particular, a multi-port extension of conventional analog-to-digital converters (ADCs) is proposed in which the distortion that is generated from non-linear operations (such as quantization) is spectrally shaped in multiple spatio-temporal dimensions such that the region of support (ROS) of the desired RF signals corresponding to desired planar waves is ideally mutually exclusive with that of these undesired components. Because of spectral shaping, both distortion and noise are non-overlapping with the signal of interest, and can be filtered out after sampling by using a MD digital filter. This theoretical advance in MD noise and distortion shaping across both space and time domains ensures that both electronic noise and non-linear distortion arising from coarse low-complexity quantization do not appreciably reduce the signal to noise and distortion ratio (SNDR) of antenna array receivers. The proposed method is an extension of $\Delta - \Sigma$ modulation that is commonly used in conventional single-input single-output ADCs, but does not require either temporal or spatial over-sampling. Moreover, to the best of our knowledge this paper is the first to combine 2-D analog filters derived from resistively-terminated classical passive low-pass filter prototypes with active analog feedback control in multiple dimensions (space, time) to realize 2-D analog-digital mixed-signal electronics for RF array processing applications.**

I. Introduction

Higher carrier frequencies and bandwidths are desirable for maximizing the performance metrics of wireless systems, including channel capacity for wireless communications and range resolution for radar. However, the figures of merit (FOMs) of critical system components such as low-noise amplifiers (LNAs), power amplifiers (PAs), and ADCs degrade as the center frequency and/or bandwidth increase; important FOMs include noise figure (NF), power consumption, linearity, power efficiency, and resolution. Thus, fundamental trade-offs exist between the NF, bandwidth, and power consumption of the LNAs; the linearity and efficiency of the PAs; and the energy efficiency and resolution of the ADCs. As an example, consider transmit beamformers for digital wideband phased-array (or true-time delay) apertures, which are a key component of communications and radar systems. A modern digital transmit aperture consists of a dedicated digital to analog converter (DAC) and transmit chain (filters, PAs, antenna) at each location in the array. A typical system consists of digital beamforming algorithms followed by dedicated transmit and receive chains in the array to allow for maximum flexibility and lowest cost. The LNA outputs in each receive chain are down-converted in frequency using mixers and converted to the digital domain by ADCs. Lower NF and higher linearity in the LNAs comes at the cost of higher power consumption in the receiver elements, while the ADCs trade-off area, power consumption, quantization noise, resolution, and sampling frequency. Process scaling by itself is not sufficient to resolve such trade-offs; it must be coupled with fundamental innovations in circuit and system architectures in order to deliver next-generation performance. We have recently published a new approach based on both spatial oversampling combined with multi-dimensional (MD) noise-shaping using MD signal processing (MDSP) theory for significantly improving the FOM of critical components within array transceivers [1]–[3]. The improvements include reductions in both the thermal noise and non-linear distortion of LNAs, which enables highly-sensitive instruments with unprecedented capabilities for detecting signals from faint radio sources in the presence of strong interference and intentional jamming. Moreover, it also enables the use of MD filters (beamformers) to reduce non-linear distortion caused by jamming, thus effectively improving the linearity of phased-array receiver front-ends. Another advantage is higher resolution in the ADCs that eventually digitize the amplified signals. This paper extends our prior work by eliminating the need for spatial oversampling.

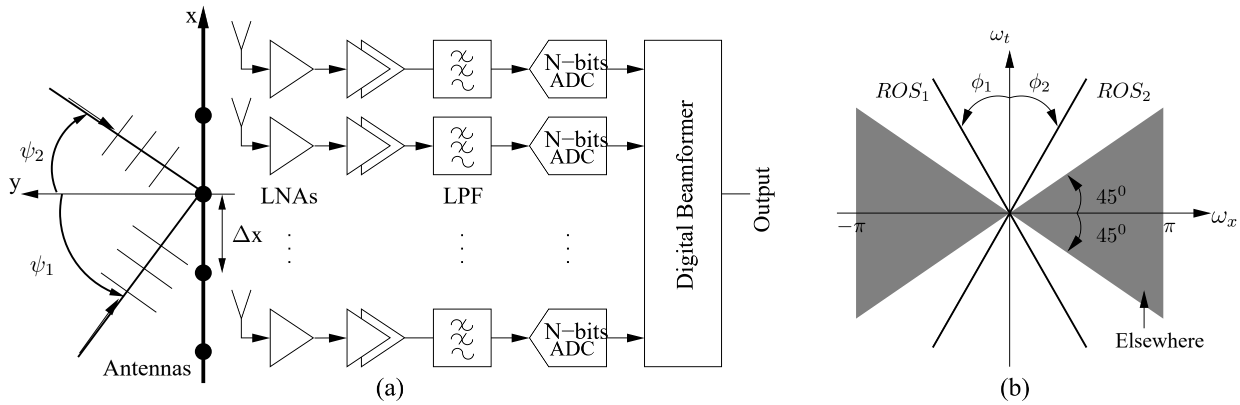

Fig. 1. (a) Far-field waves having several directions of arrival (DOAs) $\psi_k, k = 1, 2, ...$ are received by a uniform linear array (ULA) of antenna elements; (b) regions of support (ROS) of propagating waves and the "Elsewhere" region outside Einstein's light-cone (shown shaded) where no electromagnetic waves can exist in Nature.

II. Review of Spatio-Temporal MD Spectra of Propagating Plane-Waves

Electromagnetic waves behave as plane waves in the far-field (at distances > 20 wavelengths from an omni-directional transmit antenna). Such plane waves are source-free solutions of the wave equation and can be directly derived from Maxwell's Equations in free space. In the simplest 2-D case consisting of a single spatial dimension $x \in \mathbb{R}$ and time $t \in \mathbb{R}$, wave propagation in the far-field region can be modeled using functions of the form $w(x, ct) = p(-x\sin(\psi) + ct)$ where ψ is the direction of arrival (DOA) of the plane-wave measured counter-clockwise from the array broadside, x is the linear spatial distance along the array, t is time, and $c \approx 3 \cdot 10^8$ m/s is the speed of light. This can be thought of as a 1-D function $p(\chi)$ describing the waveform that is propagating in 2-D spacetime $(x, ct) \in \mathbb{R}^2$ such that a linear transform maps space and time to $\chi \in \mathbb{R}$ via $\chi = -x\sin\psi + ct$. From Einstein's Special Theory of Relativity, there exists a region in (x, ct) that is bounded by the "Light Cone" outside which no photons/waves can exist. This region is known as "Elsewhere". Propagating waves have regions of support (ROS) inside the light-code region. This is shown in Fig. 1(a) for the case of two wideband plane-waves having DOAs ψ_1 and ψ_2 in the spatial $x - y$ plane. These waves are sampled using a uniformly spaced linear array having inter-antenna spacing Δx chosen such that $\Delta x \leq \lambda_{min}/2$ where λ_{min} is the free-space wavelength of the highest frequency component of $p(\chi)$. The frequency domain ROS of a propagating plane-wave takes the form of a line-shaped region through the frequency origin. The orientation of each line-shaped ROS is given by $\tan^{-1}(\sin\psi)$. The light-cone also imposes a corresponding "Elsewhere" region in the 2-D spatio-temporal frequency domain where no frequency component exists, as shown in Fig. 1(b). This "Elsewhere" region is not occupied by wave energy but can still support electronic noise, distortion, and other artifacts from the electronics that do not correspond to free-space solutions of the wave equation such as thermal noise in the amplifiers used in the array.

The Gaussian pulses have the form $p(\chi_k) = e^{-A_k\chi_k^2}$, $A_k > 0 \in \mathbb{R}$ where $k = 1, 2$. While they are wideband, their energy is constrained to lie on two distinct line-shaped ROSs in the 2-D spectral domain. In this example, the bandwidths are chosen as $A_1 = 0.25\pi$ and $A_2 = 0.175\pi$ and the wave DOAs are $\psi_1 = 10°$ and $\psi_2 = 40°$, respectively. In other words, the 2-D spectra of these two wideband plane-waves are sparse. We will exploit this sparsity (in the MD sense) to shape both the noise and distortion of array receivers to lie outside the line-shaped ROS of the desired plane waves.

III. Review of 1-D Delta-Sigma (Δ-Σ) ADC Concepts

In a conventional delta-sigma (Δ-Σ) ADC, noise- and distortion-shaping is used along with temporal over-sampling and digital 1-D low-pass filtering to improve resolution, which is usually quantified using the effective number of bits (ENOB) [4], [5]. In the typical case, a feedback loop is used to present i) a low-pass response to the analog signal being sampled; and ii) a high-pass response to the quantization noise injected by the quantizer into its digital outputs which is known as noise-shaping. A low-resolution quantizer is used to improve hardware efficiency, which results in a highly non-linear quantization process that injects a significant amount of harmonic distortion into the quantized samples. Fortunately, the Δ-Σ feedback loop also reduces the non-linearity of the system, which is known as distortion-shaping. As a result, Δ-Σ modulation allows low-precision quantizers to generate digital outputs with low levels of quantization noise and harmonic distortion.

This paper improves and builds upon MD extensions of Δ-Σ algorithms for new applications in array processing, that were first proposed in [1]–[3]. Moreover, although used for ADCs, the mathematics of Δ-Σ conversions are *not necessarily limited to ADC circuits*. In fact, we have applied the Δ-Σ modulation technique to a variety of spatio-temporal MD circuits and systems found in antenna arrays, including LNAs. We have shown that all types of electronic noise and non-linear

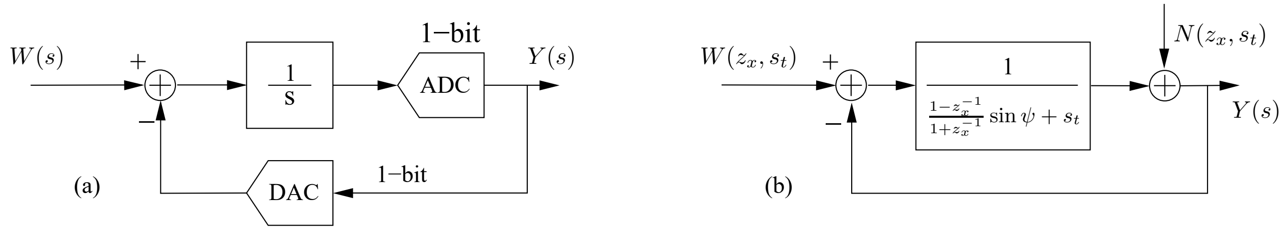

Fig. 2. a) Feedback circuit for a conventional first-order Δ-Σ ADC; b) mixed-domain 2-D $\Delta-\Sigma$ loop where the bilinear transform $s_x = \frac{1-z_x^{-1}}{1+z_x^{-1}}$ is applied.

distortion can be significantly reduced using spatio-temporal extensions of the Δ-Σ modulation principle.

The use of spatial over-sampling described in [1]–[3], [6] leads to dense antenna arrays, which in turn, leads to MD spatio-temporal wave spectra that are artificially compressed into a smaller area in the 2-D Nyquist region. The approach described in this paper completely removes the need for spatial over-sampling, and therefore, the need to increase the number of antenna and transceiver circuits in the array processor. The proposed approach therefore is highly efficient in terms of radio frequency (RF) and digital hardware since the number of array elements and transceiver circuits does not need to be increased in order to benefit from noise and distortion shaping.

To develop a model for Δ-Σ arrays, we begin by considering the (frequency- and bandwidth-normalized) temporal first-order Δ-Σ loop in the Laplace domain $s \in \mathbb{C}$. The corresponding block diagram is shown in Fig. 2(a). The temporally-oversampled analog input $X(s)$, digital output $Y(s)$, and quantization noise $N(s)$ (assumed to be additive and wideband) in the loop are related via $Y(s) = X(s)\frac{1}{1+s} + N(s)\frac{s}{1+s} \in \mathbb{C}$. This is the simplest possible Δ-Σ ADC; it has i) a first-order low-pass transfer function $\frac{1}{1+s}$ for $X(s)$, and ii) a first-order high-pass transfer function $\frac{s}{1+s}$ for $N(s)$. Performance can be further improved by using more complex loops that apply higher-order low-pass and high-pass transfer-function pairs to the signal and quantization noise, respectively. Once sampled and quantized, the signal of interest is recovered using digital low-pass filtering followed by temporal down-sampling (decimation).

The key point is that **the mathematics of non-linear distortion/noise shaping are independent of the physics of its source.** In our proposed array processing approach, the same Δ-Σ loop can be applied for any source of noise, whether it is ADC quantization noise, LNA thermal noise, mixer noise and intermodulation products, or PA distortion products. By recognizing the fact that the Δ-Σ concept is agnostic to the source of noise, we are able to extrapoloate it to the *entire transceiver electronics for array apertures, covering LNA, mixer, PA, ADC, and DACs* by using MD filtering and signal processing concepts. In our proposed approach, the conventional temporal-only Δ-Σ loop is expanded to both space and time dimensions, and realized using MD circuits and systems concepts. This is a fundamentally new concept in antenna array aperture design, and has the potential to offer performance improvements under certain conditions.

IV. Nyquist-Sampled MD Δ-Σ Algorithms for Array Transceivers

The extension of the Δ-Σ concept to antenna arrays for improving the noise and distortion of the array transceiver electronics requires two important distinctions from the well-known 2-port (input-output) ADC case: i) the Δ-Σ algorithm now encompasses both spatial and temporal dimensions, and is therefore MD in nature; and ii) while over-sampling is required for 2-port Δ-Σ ADCs, when the principle is extended to the MD spatio-temporal case, causality conditions for propagating electromagnetic waves imply that the spatio-temporal spectra of the signals of interest have a well-defined ROS. In fact, the spectral ROS of each wave is *extremely sparse* in the MD frequency domain pertaining to direction and frequency variables (space and time). Such sparsity allows us to employ Δ-Σ in multi-dimensional SFGs while removing the need for spatial and temporal over-sampling. The whole point of over-sampling is to make the signal of interest "low pass" compared to the "shaped noise", i.e., separate them in the spectral domain. In the spatio-temporal domain pertaining to aperture arrays, we can "shape" the noise/distortion in spatio-temporal dimensions such that they are mutually exclusive (do not overlap) with the sparse ROS of the desired wave signals. The first step in mapping a Δ-Σ modulator to a digital phased-array aperture is to include both time and space dimensions in the mathematical model. We can achieve this by adopting a linear transform of the type $s = s_x \sin\psi + s_t$ where $(s_x, s_t) \in \mathbb{C}^2$ are Laplace variables pertaining to spatial frequency ω_x and temporal frequency ω_t, respectively, and the direction of propagation (launch angle) of the transmitted waves is given by $\psi \leq 90^o$ measured counter-clockwise from array broadside. The proposed Laplace mapping leads to a 2-D spatio-temporal realization of the Δ-Σ modulator, given by

$$Y(s_x, s_t) = X(s_x, s_t)\frac{1}{1 + s_x \sin\psi + s_t} + N(s_x, s_t)\frac{s_x \sin\psi + s_t}{1 + s_x \sin\psi + s_t}. \quad (1)$$

The realization of the proposed 2-D spatio-temporal Δ-Σ modulator requires computing the algorithms only at the locations where the antennas and transceivers are spatially located. We achieve the spatial discretization by employing the bilinear transform $s_x = \frac{1-z_x^{-1}}{1+z_x^{-1}} \in \mathbb{C}$. The resulting system (see Fig. 2(b)) is a 2-D mixed-domain multi-dimensional spatio-

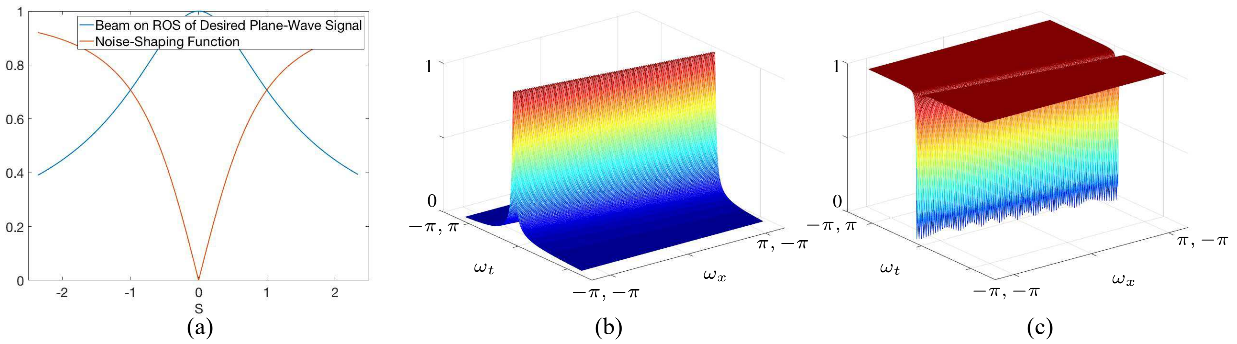

Fig. 3. (a) 1-D signal (low-pass) and noise (high-pass) transfer functions; (b) 2-D space-time transfer function for the desired plane-wave signal; (c) 2-D space-time transfer function for the unwanted quantization noise.

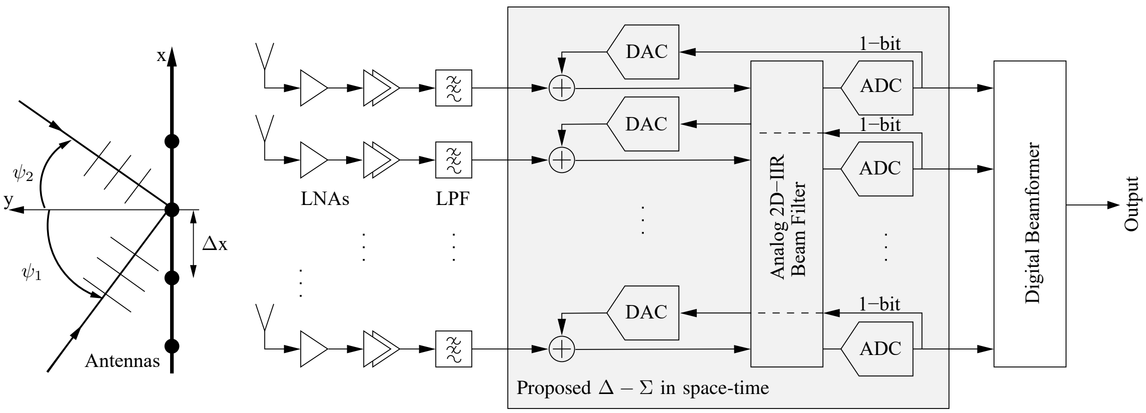

Fig. 4. Conventional $N-$bit ADCs in the array receiver are replaced by spatio-temporal MD $\Delta-\Sigma$ ADCs. These form a multi-port analog-digital mixed-signal control system in which the forward path consists of a 2D-IIR analog beam filter that provides the 2-D version of an ideal integrator $1/s$. Quantization noise is thus removed from the passband beam that falls on the resonance region $s = 0$. A high-resolution beam can then be recovered by digital beamforming; quantization noise is shaped into the side-lobes of the beamformer and thus attenuated.

temporal Δ-Σ modulator having the transform equation

$$\begin{aligned} Y(z_x, s_t) =& X(z_x, s_t)\frac{1}{1+\frac{1-z_x^{-1}}{1+z_x^{-1}}\sin\psi + s_t} + \\ & N(z_x, s_t)\frac{\frac{1-z_x^{-1}}{1+z_x^{-1}}\sin\psi + s_t}{1+\frac{1-z_x^{-1}}{1+z_x^{-1}}\sin\psi + s_t}. \end{aligned} \tag{2}$$

This equation describes an aperture array that shapes away non-linearity/noise generated by the analog electronics. Because the proposed Δ-Σ algorithm is recursive in nature in both the spatial and temporal dimensions, the stability of the system is important. We utilize the theory of practical bounded-input bounded-output (p-BIBO) stability of MD filters [7] to show that the system is stable for $0 \leq \psi < 90°$. To check p-BIBO stability [7], we first set $z_x = 1$ and test the 1-D BIBO stability of the resulting system given by $Y(1, s_t) = X(1, s_t)\frac{1}{1+s_t} + N(1, s_t)\frac{s_t}{1+s_t}$. The poles are on the left half of the s-plane, and therefore, BIBO stable. Then, we set $s_t = 0$ and check the stability of the resulting 1-D discrete system.

$$Y(z_x, 0) = \frac{X(z_x, 0)}{1+\frac{1-z_x^{-1}}{1+z_x^{-1}}\sin\psi} + N(z_x, 0)\frac{\frac{1-z_x^{-1}}{1+z_x^{-1}}\sin\psi}{1+\frac{1-z_x^{-1}}{1+z_x^{-1}}\sin\psi}. \tag{3}$$

The pole $z_x = \frac{\sin\psi - 1}{1+\sin\psi}$, $0 \leq \psi < 90^o$ is real-valued and lies inside the unit circle $|z_x| \leq 1$ ensuring stability. Therefore, the 2-D system is guaranteed to be practical-BIBO stable. In addition, the proposed Δ-Σ algorithm can be thought of a 2-D p-BIBO stable extension of a classical resistively-terminated passive first-order inductance-resistance network [8]. The shaping of both additive white Gaussian noise (AWGN) and non-linear distortion by the proposed algorithm is explained using examples in Figs. 5(a),(b). The AWGN in a conventional array occupies the entire frequency space in 2-D (or in 3-D, for rectangular arrays). However, plane waves that fall on the array are sparse in the 2-D/3-D frequency domain. Non-linear distortion generated by the electronics, especially due to quantization in the ADCs, lead to additional waves that overlap with the MD frequency-domain ROS of the input plane wave. Such overlap makes is impossible to use linear filtering to remove the undesired distortion components. Fortunately, noise shaping can greatly

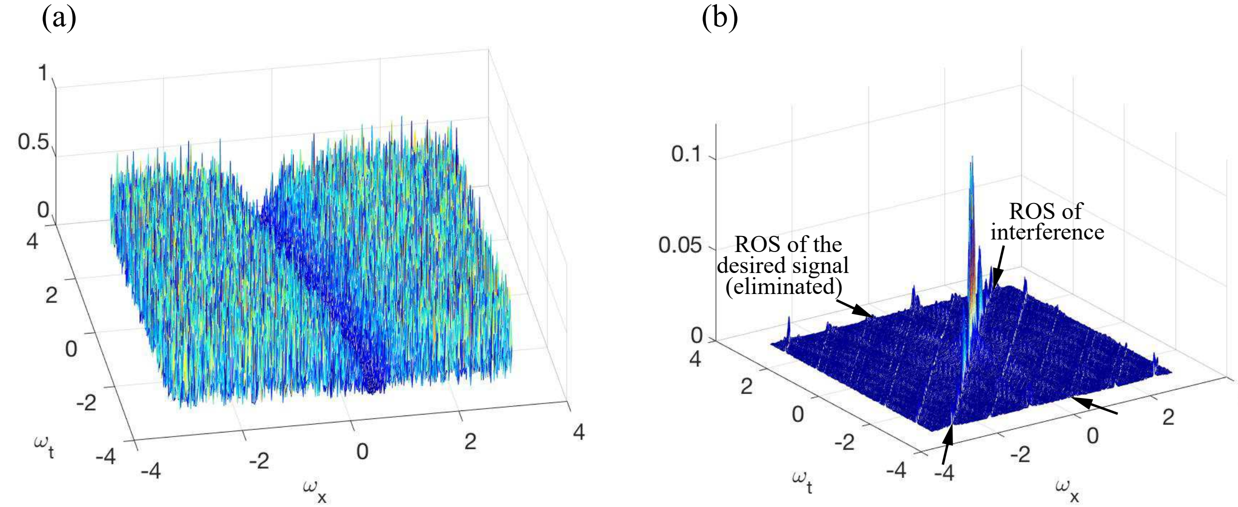

Fig. 5. (a) The inclusion of the proposed 2-D spatio-temporal $\Delta - \Sigma$ loop shapes the 2-D noise such that the PSD of AWGN is significantly reduced along the line-shaped ROS of the desired RF signal; (b) in the proposed 2-D $\Delta - \Sigma$ loop, the distortion components are shaped outside the beam shaped passband of interest.

improve the signal-to-noise ratio of the array receiver. In particular, the shaped noise PSD of the proposed 2-D $\Delta - \Sigma$ loop is close to zero along the chosen beam axis, as shown in Fig. 5(a). Similarly, distortion components are also greatly reduced along this axis (see Fig. 5(b)). Thus, the proposed technique can effectively remove ADC non-linearities from the input plane wave of interest as long as it is aligned with the chosen beam axis. This concept is, to our knowledge, completely new to the signal processing literature.

V. Proposed MD Δ-Σ SFG

Implementation of the proposed MD Δ-Σ algorithm is best understood by considering the first-order 2-D case. An antenna array processor realization of such a system based on receivers with dedicated direct-conversion electronics is shown in Fig. 4; the algorithm has been applied to the ADCs. The use of Nyquist spacing in the spatial sampling operation requires the inter-element distance between antennas to be equal to $\Delta x = \lambda_{min}/2$ where λ_{min} is the free-space wavelength of the highest frequency wave that is expected to be received by the array. The necessary 2-D analog beam filter consists of an active RF implementation denoted as $H(z_x, s_t)$. We now explain how a suitable infinite impulse response (IIR) beam filter can be designed in a manner that can be implemented in high-speed integrated circuit (IC) technologies.

The transfer-function of the 2-D beam filter used in the forward path of the 2-D Δ-Σ modulator takes a direct-form mixed $z_x - s_t$ domain representation given by eqn. (2). However, this representation can be complex and difficult to realize in analog RF-IC form. To address this problem, we propose the use of mixed differential-direct-form mixed-domain analog circuits, where the spatial interconnections between the different array processors are based on a differential-form operator and the temporal operations are maintained as direct-form operations. The spatial differential-form operator takes the mathematical form $z_D = \frac{z_x^{-1}}{1+z_x^{-1}} \in \mathbb{C}$ [9]. The forward beam transfer function can be written in the form $H(z_x, s_t) \equiv \frac{Y(z_x,s_t)}{W(z_x,s_t)} = \frac{1}{1+\alpha z_D + s_t/B}$, where $W(z_x, s_t)$ denotes the input signals from the antenna array, $\alpha = f(\sin\psi, R, T) \in \mathbb{R}$ with $f(.,.,.)$ being a closed form rational function, and $B \in \mathbb{R}$ is the bandwidth of a first-order low-pass filter. To find the corresponding SFG, we have to change the z_x transform variable back to the discrete spatial domain by computing an inverse spatial z-transform under zero initial conditions, while maintaining the Laplace nature of the 2-D filter (corresponding to the time dimension). Using well-known MD systems theory [8], we compute the inverse z-transform to obtain an array processor that uses a parallel realization of locally interconnected 2-input-2-output analog (that is, non-quantized and non-sampled) modules to yield the final transfer function as shown in Fig. 6(b). Thus, the 2-D filtering operation is achieved by a locally interconnected array of 2-D analog modules (2DAMs) described using $y_D(n_x, t) = y(n_x-1, t) - y_D(n_x-1, t) \in \mathbb{R}$, where $y_D(n_x, s_t)$ is the continuous-time domain quantity corresponding to the output of the differential operation in space given in the z_x-domain as z_D. Taking the temporal Laplace transform of the above spatial differential operator, we obtain a 2-D mixed-domain quantity $Y_D(n_x, s_t) = Y(n_x-1, s_t) - Y_D(n_x-1, s_t)$ that can be used to describe the primary signal flow path of the 2DAM using

$$Y(n_x, s_t) = \frac{W(n_x, s_t) - \alpha Y_D(n_x, s_t)}{1 + \frac{s_t}{B}}. \quad (4)$$

A full description of the mathematical and circuit theoretical aspects of deriving the SFG can be found in [10]. A circuit realization of the SFG in the forward path provides the 2-D spatio-temporal feedback necessary for MD noise and distortion shaping (see Fig. 4). In particular, the forward path consists of a linear combination of two quantities followed by a first order low-pass filtering operation in the analog domain, as shown in Fig. 6(a). It can be represented in the 2-D mixed z-Laplace domain using a 2-D extension of an ideal integrator, given by $\left(\frac{1-z_x^{-1}}{1+z_x^{-1}}\sin\psi+\frac{s_t}{B}\right)^{-1}$ where the bandwidth B can reach several GHz for modern antenna arrays.

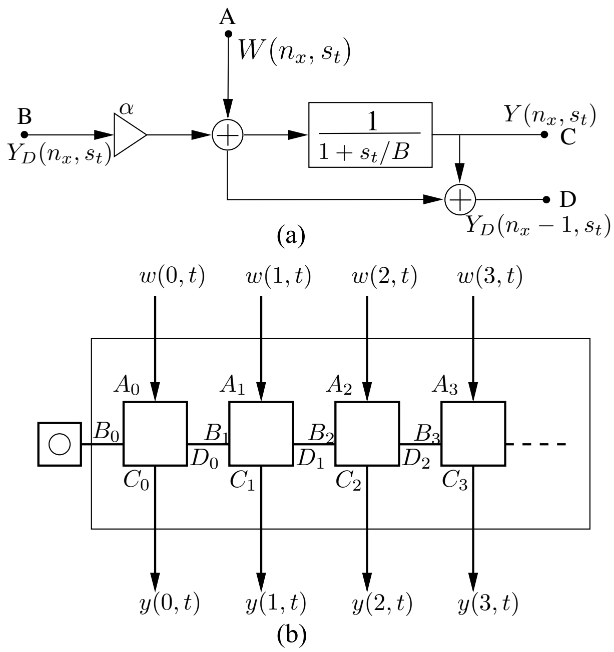

Fig. 6. (a) SFG of a single 2-D analog module (2DAM) used to realize the proposed IIR beam filter; (b) spatial interconnection of 2-input-2-output analog modules used to realize the proposed IIR beam filter.

VI. CMOS/BiCMOS Realization of MD Δ-Σ ADCs for Full Array Receivers

We propose the conceptual implementation of a full antenna array aperture that applies the proposed MD spatio-temporal Δ-Σ algorithm to the ADCs. Typical applications incclude microwave and mm-wave imaging, wireless communications and emerging 5G wireless systems, phased-array radar, and radio astronomy instrumentation. The required operations can be realized over large bandwidths (multiple GHz) using combinations of active and passive RLC elements in a CMOS or BiCMOS RF-IC technology. In the proposed system, the previously described 2-D IIR analog beam filter is implemented in RF-IC form using CMOS circuits and embedded in a MD feedback loop such that the quantization noise generated by the 1-bit ADCs is shaped in both spatial and temporal frequency domains. The overall goal is for the ROSs of the noise and distortion to lie outside the ROS of the signals of interest (far-field plane waves).

The receiver front-ends consist of LNAs having sufficient bandwidth and low enough NF to meet application specifications. The amplified and low-pass filtered signals of each element in the antenna array must be digitized using a dedicated ADC for direct-conversion digital aperture array applications such as phased-array radars, radio-astronomy instrumentation, and mobile base stations for wireless communications. The digitization of the antenna array signals requires fast temporal sampling at rates greater than the Nyquist frequency, and the ADC resolution should be high enough to achieve sufficient linearity and low quantization noise. However, the relatively high bandwidth of RF applications necessitate ADC sampling rates in the range of hundreds of MHz to several GHz. Such high sampling rate requirements necessitate the use of highly parallel ADC architectures, such as flash converters. As mentioned earlier, a N-bit flash ADC requires 2^N parallel comparators, which indicates that chip area and power consumption grows exponentially with the resolution of each ADC. The proposed MD noise-shaping method allows the resolution of such converters to be greatly reduced (ideally, to 1-bit) without sacrificing ENOB and SNDR, thus reducing both chip area and power consumption.

VII. Conclusion

A 2-D feedback topology is proposed based on a MD spatio-temporal extension of the $\Delta-\Sigma$ algorithm used in conventional high-resolution ADCs. By expanding the $\Delta-\Sigma$ concept to both space as well as time dimensions while discretizing space and keeping time continuous, it was shown that analog mixed-signal 2-D IIR filters and low-resolution (ideally 1-bit) ADC/DAC pairs can be used in antenna array processing to effectively remove quantization noise from plane wave signals of interest. The proposed technique has been analyzed theoretically, and is believed to be suitable for experimental verification. Future high-bandwidth implementations of the concept will employ CMOS RF-IC technology.

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