

Gaze-based Interaction: A 30 Year Retrospective*

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ABSTRACT

Gaze-based interaction is reviewed, categorized within a taxonomy that splits interaction into four forms, namely diagnostic (off-line measurement), active (selection, look to shoot), passive (foveated rendering, a.k.a. gaze-contingent displays), and expressive (gaze synthesis). Diagnostic interaction is the mainstay of eye-tracked applications, including training or assessment of expertise, and is possibly the longest standing use of gaze due to its mainly offline requirements. Diagnostic analysis of gaze is still very much in demand, especially in training situations such as flight or surgery training. Active interaction is rooted in the desire to use the eyes to point and click, with gaze gestures growing in popularity. Passive interaction is the manipulation of scene elements in response to gaze direction, e.g., to improve frame rate. Expressive eye movement is drawn from its synthesis, which can make use of a procedural (stochastic) model of eye motion driven by goal-oriented tasks such as reading. In discussing each form of interaction, seminal results and recent advancements are reviewed, highlighting outstanding research problems. The survey paper extends an invited proceedings contribution to VS-Games 2017.

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1. Introduction

Motivation for this paper was found in the recent inclusion of eye tracking technology in virtual reality headsets. Acquisitions of eye tracking companies Eye Tribe, Eyefluence and SMI by Facebook (Oculus), Google, and Apple, respectively, were notable events. Other eye tracking developments in helmet-mounted displays (HMDs) include the FOVE, and SMI or Pupil Labs add-ons to the HTC Vive. Interestingly, these HMDs are affordable (~\$600) compared to what was available some 15 years ago (~\$60,000) [1]. Most of these systems, including the one used by the author in 2002, feature binocular eye track-

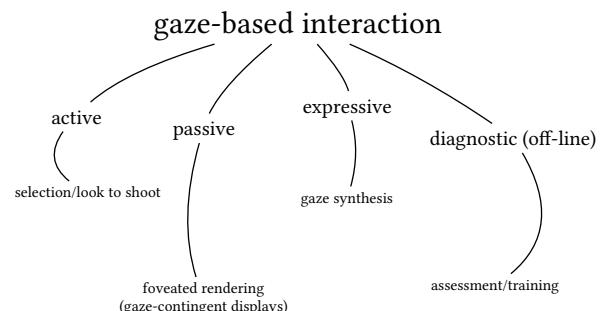


Fig. 1. Gaze interaction taxonomy.

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ing sampling at 60 Hz or better. New systems sport a larger number of infra-red LEDs, e.g., surrounding each eye, and are more comfortable than the author's 2002 HMD custom-built by Virtual Research and ISCAN.

Head-Mounted Displays only constitute one type of eye-tracked display, typically suggestive of immersive interaction in virtual reality. Currently most of these displays make use of 60-120 Hz eye trackers. While being worn on the head but with the immersive display removed, so-called mobile eye trackers can be used for various augmented reality applications such as examination of navigation in public spaces such as evaluating the utility of signage as aids to wayfinding, e.g., in an airport. These are also typically 60-120 Hz devices. More traditional devices, so-called remote or table-mounted, can offer very fast sampling rates, currently up to 2,000 Hz when combined with a chin rest. Generally speaking, eye trackers are usually evaluated in terms of their sampling speed and accuracy, measured in terms of degrees visual angle. Current eye-tracking devices typically boast about 1° visual angle accuracy.

Why has eye tracking suddenly become so popular, or, perhaps more importantly, how is tracked gaze being exploited in virtual reality and other applications? A useful taxonomy for reviewing these applications is shown in Fig. 1, which splits gaze-based interaction into four forms, namely diagnostic (off-line measurement), active (selection, look to shoot), passive (foveated rendering, a.k.a. gaze-contingent displays), and expressive (gaze synthesis).

Diagnostic analysis of gaze, e.g., for assessment of proficiency or training, is mainly performed offline following its recording during performance of some task, often under controlled conditions. Active use of gaze makes use of the real-time (x, y, t) data that eye trackers provide as a streaming signal, similar to the mouse although the eye movement signal is continuous and more noisy than the mouse, which can often show no movement, e.g., when “parked”. Active gaze is often meant to effect selection or some kind of command. Passive gaze usually does not imply any specific user action, however, it implies a change to the display in response to gaze movement. Finally, expressive eye movement implies movement of the eyes that is in turn observed by the user, e.g., movement of the eyes of an avatar or virtual character. This type of eye movement can be produced from processed recorded gaze, i.e., data-driven, or it can be synthesized by procedural (e.g., stochastic) algorithms of eye motion. Such models can be driven by goal-oriented tasks such as reading.

Before reviewing the four forms of gaze-based interaction, a short review of eye movement basics offers some nomenclature and characteristics of gaze.

2. Eye Movement Basics

Detailed human vision is limited to the central 2° visual angle, about the dimension of one’s thumbnail at arm’s length. Outside of this range, visual acuity drops sharply, e.g., about 50% during photopic (daytime) conditions. High visual acuity within the central 2° is due to the tight packing of cone photoreceptors in the central *foveal* region of the retina. Outside foveal vision, the visual field can be delineated further into *parafoveal* vision

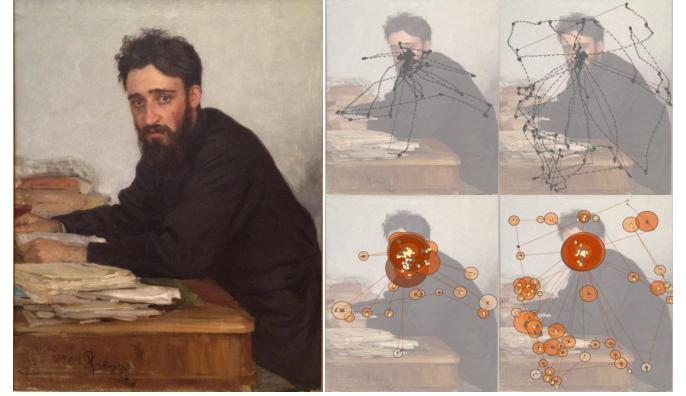


Fig. 2. An update on Yarbus [2], replicating his classic demonstration of task dependency. The painting at left, photographed by the author, is Ilya Efimovich Repin’s *Vsevolod Mikhailovich Garshin (1855-1888)*, 1884, Oil on canvas, Gift of the Humanities Fund, Inc., 1972, The Metropolitan Museum of Art, New York, NY. At upper right is raw (unprocessed) eye movement data recorded at 500 Hz by Nina Gehrer, when performing two visual tasks: gauging the emotion of the subject or free viewing. At lower right is the author’s visualization of microsaccades depicted in bright yellow within fixations shown as orange discs.

(out to about 5°), then *perifoveal* vision (10°), and then *peripheral* vision (all the way out to about 80° on either the temporal or nasal side of each eye). Sundstedt showed a nice depiction of the human visual field in her SIGGRAPH 2010 course notes [3] and subsequent book [4].

Because of the fovea’s limited spatial extent (2°), in order to visually inspect the entire 160°–180° (horizontal) field of view, one needs to reposition the fovea along successive points of *fixation*. Most of viewing time (about 90%) is spent in fixations, which is why detection of these eye movements is of particular importance.

Fixations are characterized by tremor, drift, and *microsaccades* which are used to stabilize gaze on the point of interest on the one hand, but keep the eyes in constant motion on the other, so as to prevent adaptation [5]. This is a consequence of the directional selectivity of retinal and cortical neurons implicated in visual perception [6, 7]. If the eyes were perfectly still, the visual image would fade from view.¹ Pritchard [9] illustrates the three eye movements carrying an image across the retinal photoreceptor mosaic by curved lines away from the center of vision (slow drift), high-frequency (150 Hz) tremor (superimposed on drift), and straight lines representing microsaccades, the fast *flick* movements back toward the center. The magnitude of all these movements is very small; the diameter of the foveal patch shown is 0.05 mm. Microsaccades have received a great deal of attention, as they have been identified as potential indicators of task difficulty (i.e., cognitive load) [10], mental fatigue [11], emotional attention [12],

¹An impressive simulation of this phenomenon was demonstrated by Mahowald and Mead [8] in the design of a silicon retina based on physiological principles—when held still the image faded.

and perceived threat and anxiety [13], among others. For reviews, see Martinez-Conde et al. [14, 15] and Kowler [16].

Note that from an analytical perspective of fixation (or in general event) detection, microsaccades are often seen as signal noise that may be undetectable within the measurement noise introduced by the eye tracker itself [17]. Indeed to detect microsaccades themselves, not only are fast sampling rates required (≥ 300 Hz), but also specialized detection algorithms, with Engbert and Kliegl's [18] being one of the more popular approaches that relies on examination of the median of the eye movement velocity to protect the analysis from noise [19]. An example visualization of detected microsaccades is shown in Fig. 2.

The fovea is repositioned by large jumps of the eyes known as *saccades*. Saccade amplitudes generally range between 1° – 45° visual angle (but can be larger; at about 30° , the head starts to rotate [20]). Saccades and microsaccades show comparable spatiotemporal characteristics, suggesting a dynamic continuum, supporting the hypothesis of a common oculomotor generator [21].

When tracking an object, *smooth pursuits* are used to match the motion of the moving target. When fixating an object, the semi-circular canals of the inner ear provide signals to counter-rotate the eyes when the head turns—this is known as *Vestibulo-Ocular Reflex*, or VOR. The eyes may also rotate in opposite directions during *vergence* movements; when looking close, the eyes converge, when looking far, they diverge. Vergence eye movements are used for depth perception and are tightly coupled to *accommodation*, the focusing of the eye's lens. Further details can be found in the author's monograph on eye tracking methodology [22].

3. Diagnostic Applications

Diagnostic analysis of eye movements generally relies on detection of fixations in an effort to discern what elements of the visual scene attracted the viewer's attention. Note that fixations may themselves be detected by first finding saccades. There are generally two approaches to eye movement event detection: a position-variance approach meant to locate fixations vs. a velocity-based approach generally designed to identify saccades [22]. The sequential pattern of fixations is referred to as the *scanpath* [23]. What is perhaps most relevant is the observation made classically by Yarbus [2]: the pattern of fixations is task-dependent (see also Fig. 2). That is, vision is largely *top-down*, directed by viewing strategy and task demands. However, vision is also *bottom-up*, drawn often involuntarily by eye-catching elements in the scene [24]. Being able to visualize and analyze an expert's strategy, e.g., during inspection or monitoring, is of prime importance to the understanding of expertise. A cogent example lending insight into expertise was given by Law et al. [25] in a virtual laparoscopic training environment: eye movements clearly showed novices fixated on the laparoscope tip while experts, practiced in the tool's manipulation, focused on the target.



Fig. 3. Scanpath visualization during pre-flight checklist using Peysakhovich's [26] edge bundling.

3.1. Assessing and Training Expertise

Ericsson et al. [27] surveyed experts' gaze and noted that experts tend to make shorter fixations, make better use of extrafoveal/peripheral information, and make use of a larger visual span (area around the fixation). Because experts' visual search strategies develop with training, it makes sense to not only analyze visual patterns, e.g., to assess expertise, but also as a means of its development via training. A compelling example of assessment concerns programmers, which shows that novices put in more effort and have more difficulty reading source code during the progression of an introductory programming course [28].

Following a review of literature related to the use of scanpaths in training, we showed that *Gaze-Augmented Think-Aloud* can be particularly effective [29]. This protocol records the eye movements of an expert as they verbalize whatever task they are expert in, and then the video playback is shown to novices as a means of training of the same task. This is a fairly straightforward application of eye tracking, yet it holds a number of important advantages over alternatives where pointing (e.g., with a laser pointer) is involved. Eye movements are faster than hand/limb movements, and perhaps for this reason seem more effortless than pointing. The expert is therefore free to look and make verbal *deictic references* (e.g., "look at this" [30]) without having to consciously think about pointing at something.

3.2. Tracking Dynamic Areas Of Interest

Recorded eye movements of both expert and novice can be used to assess the effectiveness of training. A particularly good example of an eye tracking application in spatial research [31] is flight simulation and training, where the study of visual monitoring is especially important [32]. Fig. 3 illustrates two critical issues in this domain: tracking of dynamic Areas Of Interest (AOIs) and visualization of recorded eye movements. Analysis of AOIs is a popular method of segmenting the scene into individual semantically different zones [33]. When AOIs are dynamic, i.e., they can change their position in time, data analysis becomes problematic [34, 35]. The problem is exacerbated in real environments, where constantly moving objects make the manual coding of a recorded video the most effective technique of eye movement analysis [36]. Fig. 3a shows the use of ArUco fiducial markers [37] as a means of augmenting reality to allow labeling of physical objects, such

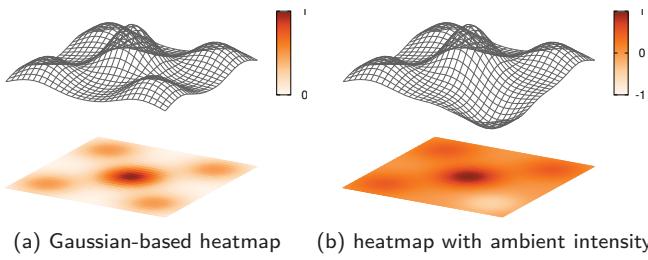


Fig. 4. Mixing Gaussian point spread functions to produce an ambient/focal heatmap. Ambient intensity is modeled by negating the Gaussian, yielding a valley instead of peak.

as the cockpit of an aircraft in this instance. Given camera space coordinates of the corners of the markers and their known real-world dimensions, a Jacobi iterative method can be used to solve for the homography between camera and real-world reference frames [38].

3.3. Visualizing Eye Movements

A common form of eye movement visualization is the scanpath, used to represent saccadic transitions between AOIs. Fig. 3b shows attribute-driven edge bundling visualization of aggregate scanpaths [26]. Color indicates direction of gaze, as identified by the colorwheel at bottom right. Besides scanpaths, the *heatmap*, or attentional landscape, as introduced by Pomplun et al. [39] and popularized by Wooding [40], is used to represent aggregate fixations (both visualizations were predicated by Nodine et al.’s [41] “hotspots” rendered as bar-graphs). Other similar approaches involve gaze represented as height maps [42, 43] or Gaussian Mixture Models [44]. Heatmaps are generated by accumulating exponentially decaying intensity $I(i, j)$ at pixel coordinates (i, j) relative to a fixation at coordinates (x, y) ,

$$I(i, j) = \exp \left(-((x - i)^2 + (y - j)^2) / (2\sigma^2) \right)$$

where the exponential decay is modeled by the Gaussian point spread function (PSF), see Fig. 4. A GPU-based implementation [45] is available for real-time visualization.

Heatmap visualization can also be extended to depict visualization of dynamic ambient/focal visual attention, as expressed by the \mathcal{K} coefficient [46]. Coefficient \mathcal{K}_i is calculated for each fixation as the difference between standardized values (z -scores) of the successive saccade amplitude (a_{i+1}) and the current i^{th} fixation duration (d_i) [47],

$$\mathcal{K}_i = \frac{d_i - \mu_d}{\sigma_d} - \frac{a_{i+1} - \mu_a}{\sigma_a}, \text{ such that } \mathcal{K} = \frac{1}{n} \sum_n \mathcal{K}_i,$$

where μ_d, μ_a are the mean fixation duration and saccade amplitude, respectively, and σ_d, σ_a are the fixation duration and saccade amplitude standard deviations, respectively, computed over all n fixations and hence n \mathcal{K}_i coefficients (i.e., over the entire duration of the scanpath). Locations corresponding to ambient fixations are made to subtract from the mean surface level, i.e., each Gaussian

kernel’s polarity (up or down) is determined by \mathcal{K}_i , using the sign of \mathcal{K}_i to affect the kernel’s direction,

$$I(i, j) = \text{sgn}(\mathcal{K}_i) \exp \left(-((x - i)^2 + (y - j)^2) / (2\sigma^2) \right).$$

3.4. Summary and Further Reading

Eye movement analysis depends to a large extent on detection of fixations in the recorded (or real-time) (x, y, t) eye movement signal. Outstanding problems include better algorithms for scanpath comparison, and better visualizations. In general, visualization is becoming an increasingly significant eye tracking component. In their EuroVis state-of-the-art (STAR) report, Blascheck et al. [48] review and classify visualization techniques for eye movement data. Recent contributions include analysis and visualization of dynamic ambient/focal visual attention [47, 46], and transition matrix analysis of eye movement [49], but more advanced developments are sure to come.

4. Active Applications

Once eye trackers matured sufficiently to produce a real-time signal of the viewer’s (x, y, t) gaze point, they were investigated for their interactive potential. Two seminal contributions from this time are those of Jacob [50] and Starker and Bolt [51]. Both contributions focused on some means of using *dwell time* to effect some kind of system response. Jacob used dwell time as a means of disambiguating gaze-based selection, while Starker and Bolt used it as an interest metric, prompting the system to provide more detail when something was fixated for a prolonged period of time. What is especially notable about Jacob’s contribution was his observation of the *Midas Touch* problem—anything looked at can trigger a response unless some mechanism can be used to prevent it, e.g., dwell time.

4.1. Gaze Gestures

Although dwell time is still heavily relied upon for gaze-based selection, gaze-based gestures have also become popular. For gaze-based eye typing [52], Isokoski’s gaze gestures to off-screen targets showed some potential early on

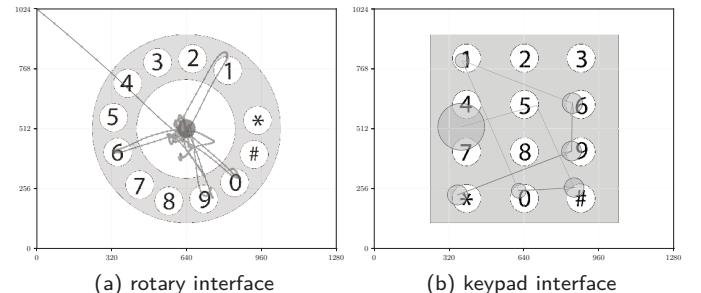


Fig. 5. Entering 9610: a rotary design for gaze-based Personal Identification Number (PIN) entry affords faster input than a traditional keypad grid design.

[53]. Our implementation of *EyeWriter* [54] showed utility of gaze gestures for eye typing, but its performance was slower than dwell-time based eye typing because of its average stroke per character requirement. In the case of eye typing, dwell time outpaced gestures because it was faster, on average, to select one character key with dwell time than say 4 gaze gestures per character.

In the specific case of PIN entry, where the alphabet is small, dwell time is a hindrance to speed. We compared two leading approaches, one based on *PassFaces* [55], and showed that boundary-crossing gestures were faster than dwell time on the given task [56], see Fig. 5. The speed benefits for our rotary interface were similar to those of *Dasher*'s boundary-crossing approach [57]. *Dasher* uses boundary-crossings to select characters and words outpacing traditional dwell-time based character entry during eye-typing, but only after some user training.

The rotary interface shown in Fig. 5a is similar to another well-known gaze-gesture based interface, the *pEYE* menu designed by Huckauf et al. [58, 59]. These circular menus were meant for eye typing, although it was acknowledged that the design could be used for other applications. *pEYE* menus lacked a central fixation point which we included in the rotary interface to exploit center bias [60, 61].

pEYE menus were designed to be hierarchical in nature. Urbina et al. [62] tested various combinations of pie segmentation (e.g., 4, 6, 8, 12 slices) and menu depths (e.g., 2, 3, 4) along with selection via either dwell time or border crossing. They suggested that up to six slices can be effectively and efficiently selected with gaze. Other pie-shaped interfaces include Patidar et al.'s [63] *Quickpie*, a *pEYE* menu with border crossing activation instead of dwell time, similar to our rotary interface.

4.2. Smooth Pursuits

Beyond fixations and gaze gestures, smooth pursuit eye movements have received relatively little attention, especially their automatic detection [64]. Smooth pursuits may feature in the eye tracking signal anytime there are moving objects present, and further, head movements (where they are allowed) may be disguised as eye-tracking events. Although there has been some exploration for pursuit detection with the Kalman filter [65], an interactive method for their detection, mixed with simultaneous detection of fixations and saccades, has been elusive. As an example of their interactive potential, Vidal et al. [66] used Pearson's product-moment correlation to detect synchronization between a user's visual pursuit of a moving object and its time series (trajectory). Using circular motion, Esteves et al. [67] introduced *Orbits*, disambiguating selection of moving targets based on direction and speed, e.g., clockwise vs. anti-clockwise. Providing another way to enter PIN codes, Cymek et al. [68] and then later Freytag et al. [69] both used smooth pursuits to track moving targets. Cymek et al. argued that smooth pursuits might promote user acceptance of pursuit-based, *non-command* style gaze-based interaction, a notion put forth previously by Jacob [70] and Nielsen [71].

4.3. Gameplay and Interaction in 3D Environments

In gameplay, a tempting form of interaction is to use the eyes to point at something to aim or shoot at, as in a first-person shooter [72]. This is particularly effective for arcade-style games (e.g., missile command), as it reduces the amount of mouse movement (although perhaps spinning that large trackball was part of the fun of the old arcade game). Gaze in this context can also be used to orient the viewpoint, as in Tobii's (an eye tracking company) version of *Rise of the Tomb Raider*.

Besides gameplay, gaze can also be used in 3D environments for various interactive tasks, e.g., interaction with multimedia [73]. Instead of gestures or dwell time, in these environments, gaze-based selection can be aided by a mechanical switch used as a selector, e.g., a foot [74]. In 3D applications, gaze can be used to ray-cast a virtual light ray to select an object. Note that if implemented properly, performance with hand pointing can be comparable to gaze-based pointing, if the user does not have to extend their arm to point, e.g., "shoots from the hip" [75]. Recently, Mott et al. suggested a dynamic, cascading (adaptive) form of dwell time for eye-typing applications [76], and Istance and Hyrskykari emphasized the importance of visible targets for efficacy of gaze gestures [77].

4.4. Summary and Further Reading

Sundstedt [4] reviewed various issues of gaze-based gameplay. The Midas Touch is an ever-present consideration. Multi-modality as well as gaze gestures are also interesting emerging alternatives.

5. Passive Applications

Passive use of gaze suggests that the eyes are not used to actively select something, rather, the system responds to gaze in a continuous manner. Passive interaction can be considered more natural than active, since, as Zhai et al. [78] put it, the eyes are a perceptual organ, and are not well suited as interactive motor devices (like the hands). Possibly the best example of passive use of gaze is the foveated, gaze-contingent display, or GCD [79]. GCDs have been used to simulate arbitrary visual fields (including scotoma)



Fig. 6. Real-time visualization of scotoma and spatiochromatic peripheral degradation.

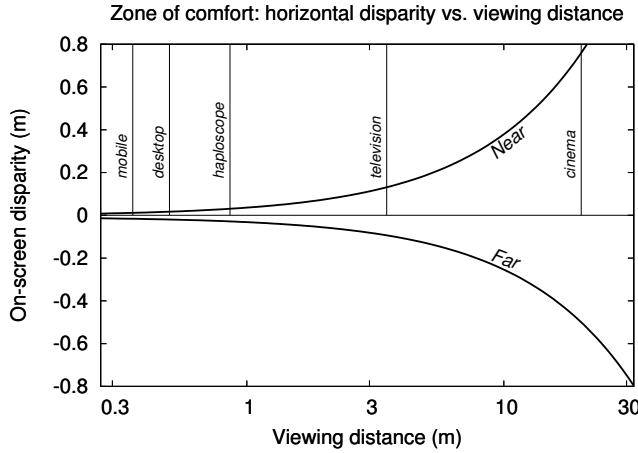


Fig. 7. The zone of comfortable stereo display viewing (based on Shibata et al. [94]) augmented to include short-view distances such as the desktop and haploscope.

[80, 81, 82] and to visualize human spatiochromatic peripheral degradation [83], see Fig 6.

One objective of the GCD is to match human retinal or visual acuity resolutions in an attempt to increase frame rates without the user noticing. Matching foveal resolution only at the point of gaze can free up computational resources in the periphery. In computer graphics, this was classically referred to as view-dependent simplification [84], or Level-Of-Detail (LOD) management [85, 86].

5.1. Foveated Rendering for Speed

There are two main approaches to foveated displays: model- and pixel-based. The model-based approach manipulates graphics geometry prior to rendering, e.g., by reducing the number of triangles to render outside the foveal region. A classic example of this was demonstrated by Luebke et al. [87] although earlier proposals also exist [88]. The pixel-based approach deals with reducing spatiotemporal complexity of pixel data just prior to rendering, e.g., via MIP-mapping [89] or Laplace filtering [90].

Recent examples of foveated rendering include those of Guenter et al. [91] and Patney et al. [92]. Guenter predicted a 100-fold increase in rendering speed at a 70° field of view using three delineations for resolution degradation: foveal, middle, and outer. The effect of these as well as most other gaze-contingent displays is a region of high resolution, with resolution degrading progressively outwards. How the resolution degrades varies—it can be discretized into three levels (as per Guenter et al.), or it can follow a more smoother function resembling that of visual acuity or contrast sensitivity [93].

5.2. Foveated Rendering for Comfort

Apart from rendering speedup, a perhaps more important application of the gaze-contingent display is to promote viewing comfort of 3D displays (e.g., stereo or virtual reality). 3D displays break the natural coupling between

vergence and accommodation (focal distance) by rendering images with non-zero disparity (stimulating vergence) at a fixed display distance [95, 96, 97]. This dissociation—referred to as the *accommodation-vergence conflict*—has been considered to be the primary reason for discomfort (asthenopia) felt by viewers of 3D (stereoscopic) displays, with its source tied to eye strain and fatigue [98, 99].

Fig. 7, adapted from Shibata et al. [94], shows results from their experiment using a dual-lens haploscope monitor arrangement to demarcate a visual comfort zone for various stereoscopic display types, including mobile, desktop, television, and cinema displays. A key insight from their study is that comfortable perception of on-screen disparity is dependent on viewing distance. In cinema, the range extends from 1.6 m to the full screen width, producing a relatively wide range of disparities. A mobile device's range, 0.28–0.44 m, narrows the comfortable on-screen disparity range considerably.

We examined vergence response via gaze disparity, measured at the screen depth, at two mid-range viewing distances: a typical desktop display at a distance of 0.5 m and a haploscope at a distance of 0.86 m, see Fig. 8. We found that vergence error increases away from the $z = 0$ screen plane [100], which we conjectured as objective evidence of the accommodation-vergence conflict.

To reduce visual discomfort, local disparity of the 3D display can be adjusted at the 3D gaze location [101, 102], or alternatively, peripheral blur can be simulated via gaze-contingent depth-of-field [103]. We implemented a real-time depth-of-field display based on the work of Riguer et al. [104]. Peripheral blur is simulated through estimation of the Circle of Confusion (CoC) radius

$$\text{CoC} = a \cdot \left| \frac{f}{d_0 - f} \right| \cdot \left| 1 - \frac{d_0}{d_p} \right|$$

where $a = 1.0$ is modeled lens aperture diameter, $f = 2.2$ is the lens focal length, d_0 is the distance between the focal plane and the lens (objects in this plane at this distance are in sharp focus), and d_p is the distance from the rendered object to the lens. Unlike Mantiuk et al. [105], we did not estimate d_0 as the depth value of the current pixel at the viewer's gaze point, rather, we used gaze depth z directly, and set the depth-of-field focal plane to this distance.

Gaze depth estimation is derived from mapping 2D coordinates to 3D gaze depth, requiring 3D calibration. A binocular eye tracker delivers two eye gaze points, (x_l, y_l)

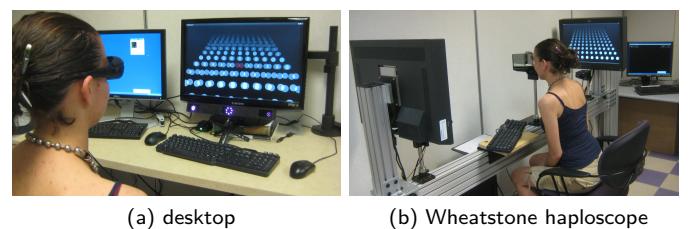


Fig. 8. Desktop and Wheatstone haploscope stereo displays.

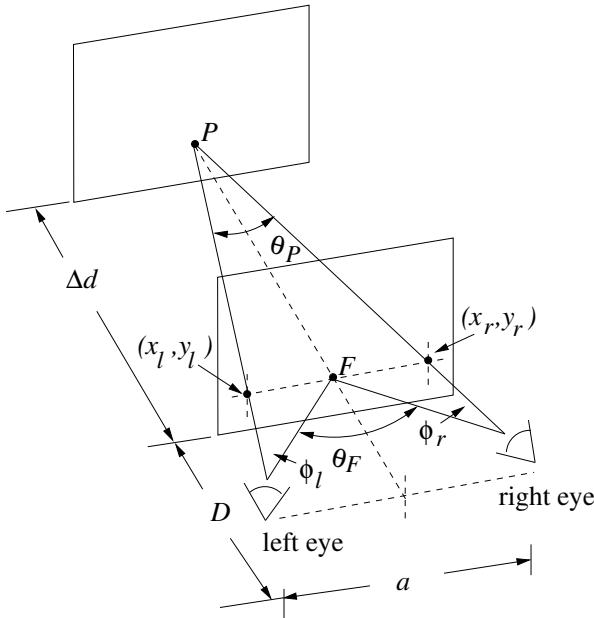


Fig. 9. Binocular disparity of point P w.r.t. fixation point F at viewing distance D with (assumed) interocular distance a [98]. Given gaze coordinates on the image plane, (x_l, y_l) and (x_r, y_r) , gaze depth (Δd) is found via triangle similarity.

for the left eye and (x_r, y_r) for the right, measured in screen coordinates. The horizontal disparity $\Delta x = x_r - x_l$ is sufficient to estimate the gaze depth coordinate $z = (\Delta x D) / (\Delta x - a)$ where D is the viewing distance and a is the inter-ocular distance (e.g., 6.3 cm) [106], see Fig. 9.

5.3. Latency & Saccade Endpoint Prediction

The greatest obstacle to practical utility of foveated rendering is eye tracking latency leading to a delay in the appearance of the central, high-resolution inset. To be indistinguishable from a full-resolution display, the inset should appear within 7 ms of fixation onset [107]. Greater delays (e.g., 15 ms following fixation onset), while detectable, have minimal impact on performance of visual tasks when the radius of the foveal inset is 4° or greater. Due to saccadic suppression, which raises perceptual thresholds for low spatial frequencies and motion signals just before, during, and for about 20–80 ms after each saccade, delays as long as 60 ms do not significantly increase blur detection [108]. Use caution when interpreting these results: the latter pertains to the time following saccade termination (60 ms), the former to the time following fixation onset (7 ms). Either way, there is precious little time within which the foveal region must be updated before the update is noticed. Regarding visual performance, however, Loschky and McConkie's [107] point was that in certain cases the user will tolerate the delay in order to complete whatever task they were trying to accomplish.

One of the most promising recent approaches to beating the gaze-contingent lag was demonstrated via saccade endpoint prediction [111]. The basic premise is straightforward, dating back to Anliker [112] who suggested predict-

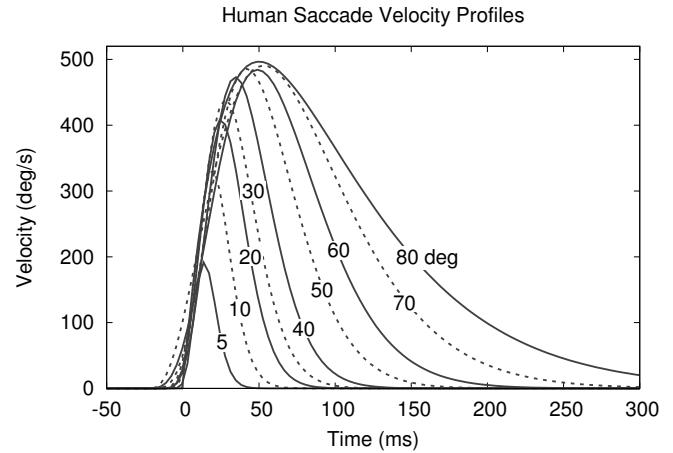


Fig. 10. Models of horizontal saccade velocity profiles ranging from 5°-80°, using the Gamma shaping function provided by Van Opstal and Van Gisbergen [109] to illustrate data depicted by Collewijn et al. [110].

ing saccade termination by mirroring the saccade velocity profile once peak velocity was detected. The assumed symmetry of the velocity profile is an oversimplification, however, as saccades of different amplitudes have differently shaped velocity profiles. The velocity profile of small saccades is symmetrical but is skewed for large saccades, and can be modeled by the expression

$$V(t) = \alpha \left(\frac{t}{\beta} \right)^{\gamma-1} e^{-t/\beta}$$

where time $t \geq 0$, $\alpha, \beta > 0$ are scaling constants for velocity and duration, respectively, and $2 < \gamma < 15$ is the shape parameter that determines the degree of asymmetry [109]. Small values of γ yield asymmetrical velocity profiles and as γ tends to infinity, the function assumes a symmetrical (Gaussian) shape, see Fig. 10 for an illustration. To handle asymmetric velocities, Arabadzhyska et al. [111] built a kind of saccade velocity lookup table for each viewer. The scheme appears effective at predicting the landing position of saccades in mid-flight and thereby offsetting any lag due to latency incurred by the eye tracker.

Other approaches to gaze prediction can be based on scene content instead of on the (real-time) eye movement signal. Examples include utilization of saliency maps [113, 114], often relying on so-called bottom-up models of visual attention [24, 115], or classification of important environment or game objects, e.g., via machine learning [116, 102]. The latter approach can be seen as an example of a growing trend toward modeling of visual attention: combining bottom-up saliency models with contextual top-down information [117].

5.4. Summary and Further Reading

Latency and visual comfort issues are at the forefront of gaze-contingent display research, but several notable approaches have already been proposed to alleviate both. Gaze-contingent depth-of-field is software-based. Recent

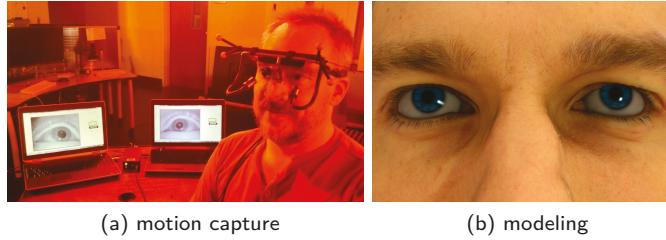


Fig. 11. Eye tracking and motion capture for gaze synthesis.

advancements in hardware e.g., focal surface displays [118], are starting to produce similar effects.

6. Expressive Applications

To bridge Mori’s [119] *Uncanny Valley*, avatars, whether acting in film or games for entertainment or more serious applications, should be modeled with as realistic eye motion as possible. Indeed, Garau et al. [120] found a strong subjective interaction effect between the realism of a character and its gaze: for a more realistic character, more elaborate gaze behavior was preferred. Gaze behavior, e.g., of a game character, influences perceived trust [121]. Gaze behavior of conversational agents can also be used to convey emotion and expression [122, 123]. In virtual reality, eye gaze is critical for correct identification of deictic reference [124]. In film, extreme close shots of the eyes are important for conveying the character’s emotional or perhaps cognitive state, e.g., as in Neil Burger’s 2011 feature film *Limitless*, or Jeff Wadlow’s 2014 *Kick-Ass 2*.

Thus far, eye movements have been modeled at a fairly coarse grain of motion, largely based on Lee et al.’s [125] seminal *Eyes Alive* model, which focused on saccades, implementing what is known as the saccadic *main sequence* [126, 127, 128] (see below). According to Ruhland et al.’s [129] state-of-the-art report on eye modeling, beyond the rapid saccadic shifts of gaze, previous work on eye motion has also included smooth pursuits, vergence, and the coupling of eye and head rotations.

How are avatar fixations animated? Recall that fixations are characterized by tremor, drift, and microsaccades, and that the eyes are in constant motion to prevent adaptation [9]. The eyes are thus never perfectly still. Meanwhile, the perceptual system is sensitive to and amplifies small fluctuations [130], hence when viewing synthetic eye motion it makes sense to consider the jitter and drift that characterize gaze fixation [9].

Rapid advancement of eye tracking technology has revitalized interest in recording eye movements for inclusion in computer graphics [131, 132]. Why not simply use eye trackers to record eye motion and map that motion onto the eyes of an avatar? This is indeed possible (see Fig. 11), however, an eye tracker and an actor emoting expressive eye motions are not always available. Moreover, eye trackers typically inject noise into the recorded signal

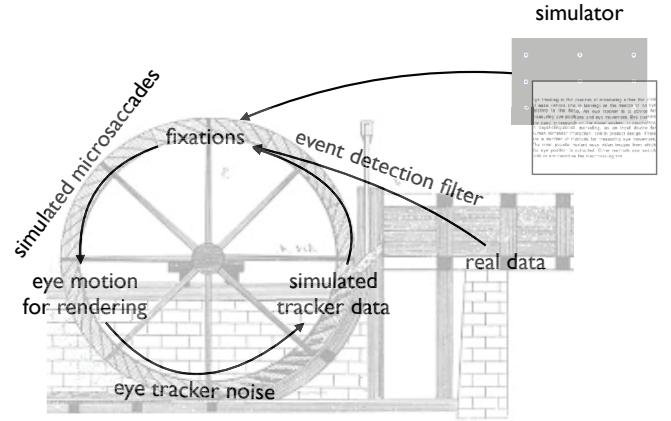


Fig. 12. Procedural eye movement simulation “gristmill”.

[17], which is difficult to separate from the underlying gaze jitter that may be of interest [133].

We have developed a straightforward stochastic model of gaze jitter using $1/f^\alpha$ pink noise as an effective means of simulating microsaccadic jitter [133]. We have shown that adding some, but not too much, jitter is perceived as more natural than omitting it altogether [134]. Why pink noise and not white noise? Three possible signals could trigger microsaccades: fixation error, neural noise, and insufficient retinal motion [135]. Evidence suggests that the three possibilities might not be mutually exclusive, i.e., fixation error and neural noise combine to trigger microsaccades. Recorded neural spikes are superimposed with noise that exhibits non-Gaussian characteristics and can be approximated as $1/f^\alpha$ noise [136].

Pink noise is also suitable for describing physical and biological distributions, e.g., plants [137] and galaxies [138], as well as the behavior of biosystems in general [139].² Aks et al. [141] suggest that memory across eye movements may serve to facilitate selection of information from the visual environment, leading to a complex and self-organizing (saccadic) search pattern produced by the oculomotor system reflecting $1/f^\alpha$ pink noise.

Microsaccadic (fixation) jitter can thus be modeled by pink noise perturbation around the fixation point $\mathbf{p}_{t+h} = \mathbf{p}_t + \mathcal{P}(\alpha = 0.6, \omega_0 = 0.85)$, where $\mathcal{P}(\alpha, \omega_0)$ defines a pink noise filter as a function of two parameters with $1/f^\alpha$ describing the filter’s power spectral density and ω_0 the filter’s unity gain frequency [142]. The pink noise filter takes as input a white noise signal, e.g., modeled by Gaussian noise, $\mathcal{N}(\mu = 0, \sigma = 12/60)$ arcmin visual angle. Setting $\alpha = 1$ produces $1/f$ noise, which has been observed as characteristic of pulse trains of nerve cells belonging to various brain structures [130].

Saccades are modeled by advancing the fixation point \mathbf{p}_t at simulation time t from one look point \mathbf{P}_{i-1} to the next \mathbf{P}_i , i.e., $\mathbf{p}_t = \mathbf{P}_{i-1} + H(t)\mathbf{P}_i$, following fixation dura-

²See Zhou et al. [140] for a discussion of different colors of noise and their point sampling implementations.

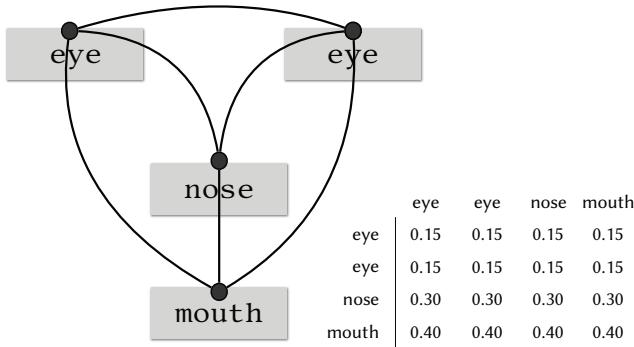


Fig. 13. Face scanning model with implicit self-loops and transition matrix used by Normoyle et al. [121].

tion at point \mathbf{p}_t with $H(t) = \frac{1}{10}t^5 - \frac{1}{4}t^4 + \frac{1}{6}t^3$, a Hermite blending function on the normalized interval $t \in [0, 1]$ used to smoothly advance position \mathbf{p}_t . Saccade durations follow the main sequence $\Delta t = 2.2\theta + 21$ ms which relates saccade duration to amplitude.³ Fixation durations are modeled by normal distribution which can be adjusted to a given task, e.g., $\mathcal{N}(\mu=250, \sigma=50)$ ms for reading.

The procedural eye movement simulation is illustrated in Fig. 12 by a *gristmill* which accepts as input a sequence of fixations with fixation durations. Such a sequence can be produced by simulation, or obtained from eye movement data recorded by an eye tracker. If the latter, then fixations and their durations are extracted via event detection, i.e., filtering. Given a sequence of fixations, jitter can be modeled as pink noise perturbation, as described above, or perhaps as a self-avoiding random walk [144].

6.1. Modeling Gaze Guidance/Allocation

A good approach to simulating a sequence of fixations is by a probabilistic model of gaze guidance, especially given fairly well-known viewing patterns or behavioral contexts. Examples of such models are especially useful for designing virtual characters and human-like robots [145, 146, 147]. A recent example combined a stochastic model of gaze allocation with a heuristic state of behavior to model bidirectional gaze when collaborating over a shared visual space [38]. The stochastic model of gaze allocation, or guidance, can be expressed by a transition matrix used to simulate the probability of making a saccade from one region of interest to the next. Below, two well-studied viewing patterns exemplify the approach: face scanning and reading.

6.1.1. Modeling Face Scanning

When looking at a human face, Yarbus [2] noted early on that “an observer usually pays most attention to the eyes, the lips, and the nose.” This typical triangular scanning pattern has been replicated many times emerging from the

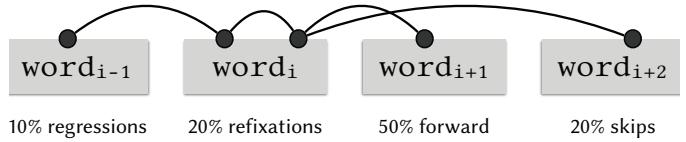


Fig. 14. Reading model (from Rayner [150]).

aggregation of individual scanpaths. However, the scanning pattern varies greatly across individuals [148].

Normoyle et al. [121], see Fig. 13, modeled gaze aversion to instill trust (or lack thereof) by controlling the proportion of time in which the avatar looked toward or away from the viewer. When looking at the viewer’s face, they defined probability distributions aimed at the eye, nose, and mouth, yielding a 4x4 transition matrix, modeled after Buchan et al.’s [149] mean fixation duration distributions when looking at faces under varying task conditions.

Other similar models of face scanning include Lance and Marsella’s [122] Gaze Warping Transformation model, designed to be expressive of basic emotional categories (including anger, disgust, fear, sadness, shame, happiness, and admiration, based on the the Pleasure, Arousal, and Dominance, or PAD, model). Their emotional expressivity was based on modeling saccadic gaze aversion, e.g., looking away depending on emotional makeup. Their model addressed saccades, vestibulo-ocular reflex (VOR), and eye-head coupled gaze movement.

Queiroz et al.’s [123] Gaze Description Language (GDL) used a stochastic model of gaze distribution, wherein weights were tempered by a model of internal state, e.g., concentration, discomfort, distress, or irony. Peterson and Eckstein [61] modeled gaze by an ideal Bayesian observer defined by four free parameters, σ_{nose} , σ_{eyes} , σ_{chin} , and σ_{mouth} , used to set non-uniform prior probabilities of fixations, which are similar to transition matrix probabilities expressed by a transition matrix.

6.1.2. Modeling Reading

Reading consists of *refixations* of the current word, *skips* forward, or *regressions* backwards (e.g., see Fig. 14 and Campbell and Maglio [151] for a model used to detect reading behavior). About 10-15% of saccades are regressions to previously fixated words (or lines, when reading multi-line text). Some of these are within-word regressions, considered re-fixations of the word [150].

Most models of reading are expressed in terms of reading span distribution, including fixation durations and saccade distributions, e.g., refixations, regressions, skips, etc. [152]. Most models also consider lexical content [153], with the two most prominent being E-Z Reader [154] and SWIFT [155]. E-Z Reader is a serial attention shift model while SWIFT is a gradient by attention guidance model [156]. Both are lexically-driven and both are mainly concerned with matching and/or predicting actual reading behavior, e.g., as captured by an eye tracker.

Except for Suppes’ [157, 158] stochastic model of read-

³Other main sequence variants include $\Delta t = 2.7\theta + 37$ ms [127], $\Delta t = 2.7\theta + 23$ ms [110], and $\Delta t = [2 : 2.7]\theta + [20 : 30]$ ms [125, 143].

ing, we are not aware of any model of reading being used for eye movement animation.

6.2. Summary and Further Reading

Realistic eye motion is clearly important for promoting the believability of virtual characters, be they human(oid), robotic, or something else. There is still much left to be done to render lifelike eyes and eye movements. Care should be taken to animate pupil diameter and blinks [134]. Pupil diameter can also be modeled as a pink noise procedural simulation of *pupil unrest, or hippocampus* [159], augmented with a model of light response [160] or based on small random variations to light intensity [161]. Blinks, following Trutoiu's [162] work, can also be modeled procedurally with fast down and slow up phases.

7. Conclusion

Recent research trends suggest that each of the four forms of gaze-based interaction presented here is very vibrant and each offers fertile ground for future work. Gaze-based diagnostics of programmers is a compelling research area developing with potential to impact evidence-based programming language design [163, 164]. Of particular relevance to this work may be estimation of cognitive load, e.g., via pupillometry [165, 166]. Cognitive load measurement is also impacting active applications, e.g., when interacting with smooth pursuit eye movements [167]. Passive applications generally involve manipulation of screen content in response to eye movement. The main applications are improvement of rendering speed or reduction of visual discomfort. Saccade endpoint prediction was noted as a recent notable advancement [111]. Finally, expressive eye movements are generally important for applications that feature animated characters, e.g., in film, games, or virtual reality where conversational agents are employed. A recent example of where such characters are likely to play an important role is in virtual reality simulations that make use of virtual humans. Volonte et al. recently used diagnostic eye tracking to evaluate the effect of virtual humans on visual attention in inter-personal simulations [168]. Adding expressive eye movements to such characters is likely to enhance the experience of such simulations further still.

Analysis and synthesis of eye movements presents interesting diagnostic and interactive possibilities with exciting challenges. Gaze opens an additional bidirectional channel of information to the user. Because gaze is associated with cognitive processing (e.g., Just and Carpenter's [169, 170] *eye-mind assumption*) and emotional expression [171], it is a particularly rich source of information.

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