Factorization for Substructures of Boosted Higgs Jets[☆]

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Abstract

We present a perturbative QCD factorization formula for substructures of an energetic Higgs jet, taking the energy profile resulting from the $H \to b\bar{b}$ decay as an example. The formula is written as a convolution of a hard Higgs decay kernel with two b-quark jet functions and a soft function that links the colors of the two b quarks. We derive an analytical expression to approximate the energy profile within a boosted Higgs jet, which significantly differs from those of ordinary QCD jets. This formalism also extends to boosted W and Z bosons in their hadronic decay modes, allowing an easy and efficient discrimination of fat jets produced from different processes.

Keywords: QCD, Jet Substructure

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The Higgs boson, which is responsible for the electroweak symmetry breaking mechanism in the Standard Model (SM), has been discovered at the Large Hadron Collider (LHC) with its mass around 125 GeV. Though its couplings to other particles seem to be consistent with the SM, the ultimate test as to whether this observed particle is the SM Higgs boson relies on the measurement of the trilinear Higgs coupling that appears in Higgs pair production. A Higgs boson is predominantly produced at rest via gluon fusion processes at the LHC. It has been shown [1] that the cross section of the Higgs pair production increases rapidly with center-of-mass energy of hadron colliders. With much higher collision energy in the partonic process, preferred for exploring the trilinear Higgs coupling, the Higgs boson and its decay products will be boosted. An energetic Higgs boson can also be associately produced with other SM particles, such as W, Z bosons, top quarks and jets [2].

The SM Higgs boson decays into a pair of bottom quark and antiquark dominantly. When the Higgs boson is highly boosted, this pair of bottom quarks may appear as a single jet and cannot be unambiguously discriminated from an ordinary QCD jet. A similar challenge applies to the identification of other boosted heavy particles, e.g., W bosons, Z bosons, and top quarks, when decaying via hadronic modes. Hence, additional information on internal structures of these boosted jets (such as their masses, energy profiles, and configurations of subjets) is required for the experimental identification. Many theoretical efforts were

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devoted to the exploration of heavy particle jet properties based on event generators [3–5]. Recently, the perturbative QCD (pQCD) formalism, including fixed-order calculations [6] and the resummation technique [7], was employed to investigate jet substructures. The alternative approach based on the soft-collinear effective theory and its application to jet production at an electron-positron collider were presented in Refs. [8, 9].

In this Letter we develop a pQCD factorization formula to describe the internal jet energy profile (JEP) of the boosted jet resulting from the $H \to b\bar{b}$ decay, with energy E_{J_H} and invariant mass m_{J_H}. The basic idea of our theoretical approach is as follows. A Higgs boson is a colorless particle, while its decay products, the bottom quark and antiquark, are colored objects and dressed by multiple gluon radiations to form a system with mass of $\mathcal{O}(m_{J_H})$ and energy of $\mathcal{O}(E_{J_H})$. The invariant mass m_J of the bottom quark and its collimated gluons, with energy of $\mathcal{O}(E_{J_H})$, typically satisfies the hierarchy $E_{J_H} \gg m_{J_H} \gg m_J$. Based on the factorization theorem, QCD dynamics characterized by different scales must factorize into soft, collinear, and hard pieces, separately. First, the Higgs jet function J_H is factorized from a Higgs boson production process at the leading power of m_{J_H}/E_{J_H}. Then the b-quark jet function is defined at the leading power of m_J/m_{J_H} [7], soft gluons with energy of $\mathcal{O}(m_{J_H})$ are absorbed into a soft function S, and the remaining energetic gluons with energy $\mathcal{O}(E_{J_H})$ and invariant mass of $\mathcal{O}(m_{J_H})$ go into the hard Higgs decay kernel H.

The Higgs JEP is then factorized at leading power of m_J/m_{J_H} into a convolution of the hard kernel with two b-quark jet functions and a soft function that links colors of the two b quarks. We will demonstrate a simple scheme, in which the soft gluons are absorbed into one of the b-quark jets, forming a fat jet, and the soft function reduces to unity. The other b-quark jet is a thin jet to avoid double counting of soft radiation. Evaluating the decay $H \to b\bar{b}$ up to leading order (LO) in the coupling constant α_s and substituting the light-quark jet function in [7] for the b-quark jet functions, we predict the Higgs JEP. Since a Higgs boson is massive and a color singlet, its JEP dramatically differs from that of ordinary QCD jets. Below, we present the derivation of the JEP of a Higgs boson decaying into a bottom-quark pair.

The four-momentum of the Higgs jet can be written as $P_{J_H} = E_{J_H}(1, \beta_{J_H}, 0, 0)$, with $\beta_{J_H} = \sqrt{1-(m_{J_H}/E_{J_H})^2}$. We define the Higgs jet function at the scale μ as

$$J_H(m_{J_H}^2, E_{J_H}, R, \mu^2) = \frac{2(2\pi)^3}{E_{J_H}} \sum_{N_J} \langle 0|\phi(0)|N_J\rangle\langle N_J|\phi^\dagger(0)|0\rangle \tag{1}$$
$$\times\delta(m_{J_H}^2 - \hat{m}_{J_H}^2(N_J, R))\delta(E_{J_H} - E(N_J))\delta^{(2)}(\hat{n}_{J_H} - \hat{n}(N_J)),$$

where the coefficient has been chosen to satisfy $J_H^{(0)} = \delta(m_{J_H}^2 - m_H^2)$ at the zeroth order in the Yukawa coupling. m_H represents the Higgs boson mass, and R the Higgs jet cone radius. The three δ-functions in the above definition specify the Higgs jet invariant mass, energy, and unit momentum direction of the set N_J of final-state particles, respectively. After applying the aforementioned factorization procedure, J_H is written as

$$J_H(m_{J_H}^2, E_{J_H}, R, \mu^2) = \frac{1}{E_{J_H}} \Pi_{i=1,2} \int dm_{J_i}^2 dE_{J_i} d^2\hat{n}_{J_i} \int d\omega S(\omega, R, \mu_f^2) J_i(m_{J_i}^2, E_{J_i}, R_i, \mu_f^2)$$
$$\times H(P_{J_1}, P_{J_2}, R, \mu^2, \mu_f^2)\delta(m_{J_H}^2 - P_{J_1}\cdot P_{J_H} - P_{J_2}\cdot P_{J_H} - \omega) \times \delta(E_{J_H} - E_{J_1} - E_{J_2})\delta^{(2)}(\hat{n}_{J_H} - \hat{n}_{J_1+J_2}), \tag{2}$$

where the factorization scale μ_f is introduced by the b-quark jet functions J_i. m_{J_i} (E_{J_i}, P_{J_i}, R_i) is the invariant mass (energy, momentum, radius) of the b-quark jets, and the soft function takes the form $S^{(0)} = \delta(\omega)$ at LO with the variable $\omega \equiv P_S \cdot P_{J_H}$, where P_S is the soft gluon momentum.

To describe the Higgs JEP, we define the jet energy function $J_H^E(m_{J_H}^2, E_{J_H}, R, r, \mu^2)$ by including in Eq. (2) a step function $\Theta(r - \theta_j)$ for every final-state particle j. The final-state particles with non-vanishing step functions (i.e., emitted within the test cone of radius r) and associated with the b-quark jet J_i are grouped into the b-quark jet energy function $J_i^E(m_{J_i}^2, E_{J_i}, R_i, r_i, \mu_f^2)$. The energetic final-state particles outside the b-quark jets and within the test cone are absorbed into the hard kernel H^E. The other final-state particles outside the test cone are absorbed back into their original functions. In this work, we will consider only the LO hard kernel, for which $H^E = H^{(0)}$. We then arrive at

$$J_H^E(m_{J_H}^2, E_{J_H}, R, r, \mu^2) = \frac{1}{E_{J_H}}\Pi_{i=1,2}\int dm_{J_i}^2 dE_{J_i} d^2\hat{n}_{J_i}\int d\omega S(\omega, R, \mu_f^2)$$
$$\times\sum_{i\neq j} J_i^E(m_{J_i}^2, E_{J_i}, R_i, r_i, \mu_f^2) J_j(m_{J_j}^2, E_{J_j}, R_j, \mu_f^2) H^{(0)}(P_{J_1}, P_{J_2}, R, \mu^2, \mu_f^2)$$
$$\times\delta(m_{J_H}^2 - P_{J_1}\cdot P_{J_H} - P_{J_2}\cdot P_{J_H} - \omega)\delta(E_{J_H} - E_{J_1} - E_{J_2})\delta^{(2)}(\hat{n}_{J_H} - \hat{n}_{J_1+J_2}), \quad (3)$$

where the LO hard kernel is

$$H^{(0)} = \frac{N_c}{2\pi^3}\left(\frac{m_b}{v}\right)^2 \frac{(E_{J_1}E_{J_2})^2[1-\cos(\theta_{J_1}+\theta_{J_2})]}{(P_{J_H}^2 - m_H^2)^2 + \Gamma_H^2 m_H^2}, \quad (4)$$

with the number of colors N_c, the b-quark mass m_b, the vacuum expectation value v, the Higgs decay width Γ_H, and the polar angle θ_{J_i} of the b-quark jet J_i relative to the Higgs jet axis.

We have the freedom to choose the jet parameters R_i and r_i, whose values depend on the scheme adopted to factorize the soft radiation in the Higgs jet into different convolution pieces in Eq. (3). A simple scheme is to take J_1 as a thin jet, such that its entire energy is counted when a sufficient amount of the thin jet is within the test cone as specified below. For that, we set $R_1 = r_1 = r$, which increases from the minimal value 0.1 in our numerical analysis. This choice leads to the simplification of J_1^E, $J_1^E(m_{J_1}^2, E_{J_1}, r, r, \mu_f^2) = E_{J_1} J_1(m_{J_1}^2, E_{J_1}, r, \mu_f^2)\Theta(ar - \theta_{J_1})$, with $a \sim \mathcal{O}(1)$ being a geometric factor. The scheme also includes a fat jet J_2 with a large cone radius $R_2 = R$, which then absorbs all soft radiation in the Higgs jet. The energy function of the fat jet $J_2^E(m_{J_2}^2, E_{J_2}, R, r_2 = r, \mu_f^2)$, which contributes to the Higgs JEP as $\theta_{J_2} \leq ar$, will take the result derived in the resummation technique [7, 10].

Next, we integrate out the dependence on the Higgs jet invariant mass by taking the first moment in the Mellin transformation of J_H^E, defined as $\bar{J}_H^E(1, E_{J_H}, R, r, \mu^2) \equiv \int J_H^E(m_{J_H}^2, E_{J_H}, R, r, \mu^2) dm_{J_H}^2/(RE_{J_H})^2$. To perform the integration over $\hat{n}_{J_2}$, we write the corresponding δ-function as

$$\begin{aligned}\delta^{(2)}(\hat{n}_{J_H} - \hat{n}_{J_1+J_2}) &= \delta\left(\frac{\mathbf{P}_{J_H}}{|\mathbf{P}_{J_H}|} - \frac{\mathbf{P}_{J_1}+\mathbf{P}_{J_2}}{|\mathbf{P}_{J_1}+\mathbf{P}_{J_2}|}\right),\\ &= \frac{|\mathbf{P}_{J_1}+\mathbf{P}_{J_2}|}{|\mathbf{P}_{J_2}|}\delta\left(\frac{|\mathbf{P}_{J_1}+\mathbf{P}_{J_2}|}{|\mathbf{P}_{J_H}||\mathbf{P}_{J_2}|}\mathbf{P}_{J_H} - \frac{\mathbf{P}_{J_1}}{|\mathbf{P}_{J_2}|} - \hat{n}_{J_2}\right),\end{aligned} \quad (5)$$

where the ratio is given by $|\mathbf{P}_{J_1}+\mathbf{P}_{J_2}|/|\mathbf{P}_{J_2}| = (E_{J_1}\cos\theta_{J_1} + E_{J_2}\cos\theta_{J_2})/E_{J_2}$. The angular relation $E_{J_1}\sin\theta_{J_1} = E_{J_2}\sin\theta_{J_2}$ is then demanded. The integration over E_{J_2} is trivial. The b-quark jet masses m_{J_i}, whose typical values are much lower than m_H, are negligible in the hard kernel. The integrations over $m_{J_H}^2$ and $m_{J_i}^2$ can then be done trivially, with $\int dm_{J_i}^2 J_i(m_{J_i}^2, E_{J_i}, R_i, \mu_f^2) = 1 + O(\alpha_s) \approx 1$.

The soft function is defined as a vacuum expectation value of two Wilson links in the directions $\bar{\xi}_{J_i} = (1, \hat{n}_{J_i})/\sqrt{2}$ with $\hat{n}_{J_i} = \mathbf{P}_{J_i}/|\mathbf{P}_{J_i}|$, $i = 1, 2$. An explicit next-to-leading-order (NLO) calculation in the Mellin (N) space gives

$$S^{(1)} = \frac{\alpha_s C_F}{\pi(RE_{J_H})^2}\ln\frac{\bar{\xi}_{J_1}^2\bar{\xi}_{J_2}^2}{4(\bar{\xi}_{J_1}\cdot\bar{\xi}_{J_2})^2}\left(\frac{1}{\epsilon} + \ln\frac{4\pi\mu_f^2\bar{N}^2}{R^4E_{J_H}^2e^{\gamma_E}}\right), \quad (6)$$

where the color factor $C_F = 4/3$ and the moment $\bar{N} \equiv N\exp(\gamma_E)$, with γ_E being the Euler constant. The off-shellness $\bar{\xi}_{J_i}^2$ associated with the b-quark jets implies that $S^{(1)}$ contains the collinear dynamics which has been absorbed into the jet functions. Hence, the subtraction of the collinear divergences from the soft function is necessary to avoid double counting. The collinear divergences from loop momenta collimated to the b quark ($\bar{b}$ quark) can be collected with the $\bar{b}$ quark (b quark) line being replaced by the eikonal line in the direction n_{J_1} (n_{J_2}) that appear in the b-quark jet definitions [6]. The NLO subtraction term for the latter with the same cone radius R is obtained by substituting the vector n_{J_2} for $\bar{\xi}_{J_1}$ in $S^{(1)}$. After this subtraction, we have

$$S^{(1)} - S^{(1)}_{n_{J_2}} = \frac{\alpha_s C_F}{\pi (R E_{J_H})^2} \ln \frac{\bar{\xi}^2_{J_1} (\bar{\xi}_{J_2} \cdot n_{J_2})^2}{(\bar{\xi}_{J_1} \cdot \bar{\xi}_{J_2})^2 n^2_{J_2}} \left(\frac{1}{\epsilon} + \ln \frac{4\pi \mu_f^2 \bar{N}^2}{R^4 E^2_{J_H} e^{\gamma_E}} \right), \tag{7}$$

to which we can further impose the condition $4(\bar{\xi}_{J_2} \cdot n_{J_2})^2 / n^2_{J_2} = R^2$ for defining a quark (or gluon) jet [7]. Because the thin jet J_1 contributes only the overall normalization in $\bar{J}^E_H$, the choice of n_{J_1} is arbitrary. We then utilize this freedom, and choose n_{J_1} such that $S^{(1)}_{n_{J_1}}$ has the logarithmic coefficient the same as of $S^{(1)} - S^{(1)}_{n_{J_2}}$. This choice is possible, because of $\bar{\xi}_{J_1} \cdot \bar{\xi}_{J_2} \sim (m_H / E_{J_H})^2 \sim O(r)$ in the considered kinematic region. The further collinear subtraction leads to

$$S^{(1)} - S^{(1)}_{n_{J_1}} - S^{(1)}_{n_{J_2}} \approx 0, \tag{8}$$

so the soft function in this special scheme is given by $S(\omega, R, \mu_f^2) \approx \delta(\omega)$.

Equation (3) then reduces to

$$\begin{aligned} \bar{J}^E_H(1, E_{J_H}, R, r) &= \frac{1}{R^2 (E_{J_H})^3} \frac{1}{\pi^2} \left(\frac{m_b}{v} \right)^2 \int dE_{J_1} \int d\cos\theta_{J_1} (E_{J_1} \cos\theta_{J_1} + E_{J_2} \cos\theta_{J_2}) \\ &\quad \times \left[E^2_{J_1} \Theta(ar - \theta_{J_1}) + R^2 E^3_{J_2} \bar{J}^E_2(1, E_{J_2}, R, r) \Theta(ar - \theta_{J_2}) \right] \\ &\quad \times \frac{E_{J_1} E_{J_2} [1 - \cos(\theta_{J_1} + \theta_{J_2})]}{\{2 E_{J_1} E_{J_2} [1 - \cos(\theta_{J_1} + \theta_{J_2})] - m_H^2\}^2 + \Gamma_H^2 m_H^2}, \end{aligned} \tag{9}$$

where the light-quark jet functions are set at the factorization scale $\mu_f^2 = (E_J R)^2 / \bar{N}$, the renormalization scale for $\bar{J}^E_H$ is chosen to be $\mu = E_{J_H} r / R$ [7], and the Mellin transformation $\bar{J}^E_2(1, E_{J_H}, R, r) \equiv \int J^E_2(m^2_{J_2}, E_{J_H}, R, r) dm^2_{J_2} / (R E_{J_2})^2$ has been inserted.

The choice of the merging parameter a is a matter of factorization schemes, and the difference arising from distinct a's will be compensated by the corresponding distinct hard kernels H^E. That is, a larger a means more contribution to the Higgs JEP from the b-quark jets, and less contribution from H^E. Since we neglect H^E and consider only the LO hard kernel $H^{(0)}$ here, our analysis will be more consistent, as a larger a is chosen. Below, we set a in Eq. (9) to its maximal allowed value, $a = 2$, according to the prescription of the cone algorithm, and predict the Higgs JEP with $E_{J_H} = 500$ GeV and $E_{J_H} = 1000$ GeV, both with a cone radius of $R = 0.7$. A JEP is defined as

$$\Psi(E_J, R, r) = \frac{\bar{J}^E(1, E_J, R, r)}{\bar{J}^E(1, E_J, R, R)}. \tag{10}$$

It is interesting to note that a simple expression can be derived for the JEP of a boosted Higgs jet after applying the narrow width approximation for the Higgs boson propagator. It yields

$$\Psi(E_{J_H}, R, r) = \frac{\int_{z_m(r)}^1 dz z(1-z)[1 + \Psi_q(z E_{J_H}, R, r)]}{\int_{z_m(R)}^1 dz z(1-z)[1 + \Psi_q(z E_{J_H}, R, R)]}, \tag{11}$$

where the integration variable $z = E_{J_1} / E_{J_H}$, the lower limit $z_m(r) = \hat{m}_H^2 / (\hat{m}_H^2 + a^2 r^2)$, the small parameter $\hat{m}_H \equiv m_H / E_{J_H}$, and Ψ_q denotes the light-quark JEP [7, 10]. Compared to the energy profiles of QCD jets [7], the Higgs JEP is lower at small r due to the dead-cone effect, and increases faster with r once the energetic b-quark jets start to contribute.

Our formalism can be readily extended for studying boosted W and Z bosons in their hadronic decay modes, by inserting their masses and widths, since Eq. (11) is coupling- and spin-independent. As shown in Fig. 1, the predicted W, Z and Higgs JEPs are well consistent with those from Pythia8 [11] for the heavy-boson jet energy $E_J = 500$ GeV. For the Pythia8 comparison, we have used the 4C tune which was shown to agree well with the ATLAS data for the JEPs of QCD jets with energies ranging from 30 GeV to

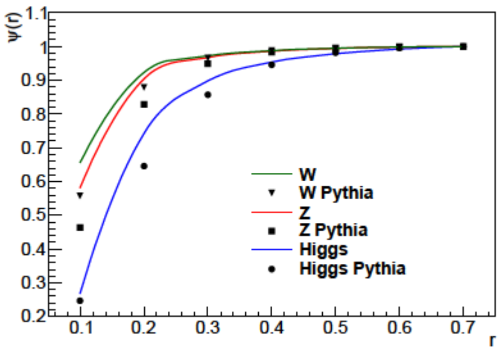

Figure 1: Comparison of W, Z and Higgs JEPs to Pythia8 predictions, for $E_J = 500$ GeV and $R = 0.7$.

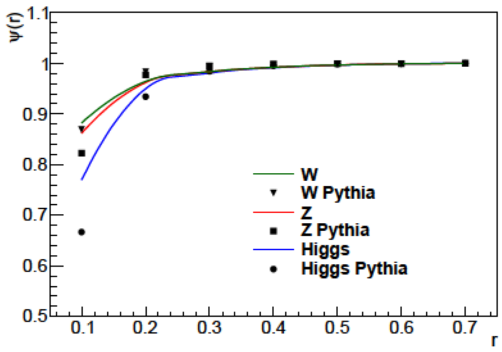

Figure 2: Comparison of W, Z and Higgs JEPs to Pythia8 predictions, for $E_J = 1000$ GeV and $R = 0.7$.

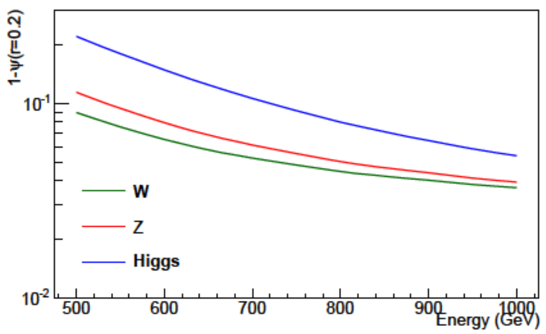

Figure 3: Energy dependence of the JEPs for Higgs, W, and Z for fixed $r = 0.2$ and $R = 0.7$.

600 GeV [12]. In the analysis we include the effects of initial-state and final-state radiations, hadronization, and beam remnants, but turn off the effects from multiple parton interactions. Similar agreement is also observed, cf. Fig. 2, for these boosted electroweak bosons at 1 TeV, though the deviation at $r = 0.1$ is larger. From these figures, we can get a rough estimate of the effective radii needed to capture all of the radiation for these boosted electroweak bosons. We find that the effective radius needed is $R \approx 0.7$ and $R \approx 0.3$ for the Higgs boson at 500 GeV and 1 TeV, respectively, and 0.4 and 0.25 for either W or Z bosons. Moreover, the W and Z JEPs are thinner than the Higgs JEP due to their smaller masses. This is further demonstrated via the energy dependence of the JEP for a fixed r value ($r = 0.2$) in Fig. 3: the separation of the W, Z, and Higgs jets becomes more difficult as the jet energy increases, because the ratio of the mass to the jet energy becomes smaller. Note that the simple expression in Eq. (11) is derived under the aforementioned approximations, at the LO accuracy for the hard kernels, and with the neglect of soft links among the heavy-boson jet and other subprocesses, such as beams and other final-state particles. The associated theoretical uncertainties can be reduced by taking into account relevant corrections, and will be addressed in a future work. Furthermore, the questions of how techniques like trimming [13], pruning [14, 15], soft-drop [16, 17], and other similar techniques affect the JEP, how subleading corrections affect the JEP, and what happens to the profile if we loosen some of the aforementioned approximations will be investigated in a future work.

In conclusion, we have applied the pQCD factorization to formulate the JEP of a boosted colorless heavy-particle (such as W, Z, Higgs, W' and Z' boson) jet, which is found to differ dramatically from the ordinary QCD JEPs with similar energy and jet radius. The formalism is greatly simplified by considering inside the boosted jet a thin jet and a fat jet, which absorbs all the soft-gluon effect, when the heavy-particle decays into a quark and antiquark pair. More interestingly, the analytical expression for the JEPs of the electroweak bosons allows for an easy and efficient discrimination of different production processes for these boosted jets. The implementation of this discrimination method can further help suppress QCD background to signals of boosted W and Z jets, after applying conventional kinematic selections.

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