A perspective on massive random-access

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***Abstract*—This paper discusses the contemporary problem of providing multiple-access (MAC) to a massive number of uncoordinated users. First, we define a random-access code for K_a-user Gaussian MAC to be a collection of norm-constrained vectors such that the noisy sum of any K_a of them can be decoded with a given (suitably defined) probability of error. An achievability bound for such codes is proposed and compared against popular practical solutions: ALOHA, coded slotted ALOHA, CDMA, and treating interference as noise. It is found out that as the number of users increases existing solutions become vastly energy-inefficient.**

Second, we discuss the asymptotic (in blocklength) problem of coding for a K-user Gaussian MAC when K is proportional to blocklength and each user's payload is fixed. It is discovered that the energy-per-bit vs. spectral efficiency exhibits a rather curious tradeoff in this case.

I. Introduction

An interesting technological challenge for the next generation of wireless standards is to provide coexistence over the same band of a massive number of infrequently communicating devices. This problem has attracted attention in the world of the licensed spectrum (3GPP and 5G-PPP) under the name of *mMTC* (massive machine-type communication), and in the world of unlicensed spectrum under the name of *LP-WANs* (low-power wide-area networks).

One may be inclined to dismiss the novelty of the challenge by refering back to the classical multiple-access channel (MAC) question. There are, however, several interesting and new aspects of this reincarnation of the problem: small size of the payload leads to *finite-blocklength* (FBL) effects [1], only a small fraction of users are active at any given time (*random-access*), but the total number of active users can still be comparable with the overall blocklength (*massive multiple-access*) and users access channel without any prior resource requests to the base station (*grantless* or *grantfree* [2]).

Various subsets of these issues have been observed and discussed in the past. The FBL questions for a K-user MAC have been studied in [3], but their bounds and normal approximations require evaluating probabilities in 2^K-dimensional spaces, and thus are only computable for very modest values of K. Classical literature on the topic of multiple-access may roughly be split into three categories: information theoretic (Ahslwede-Liao [4], [5]), the network-theoretic (starting with ALOHA [6] and going to content-resolution [7], [8]) and the coding-theoretic (CDMA [9], [10] and a closely-related adder-MAC [11], [12]). Already 30 years ago R. Gallager [13] called for "a coding technology that is applicable for a large set of transmitters of which a small, but variable, subset simultaneously use the channel." It appears (to this author) that this call has not been completely answered still. One reason for this could be that the models in each of three categories are different and thus solutions are not directly comparable. Our first goal, thus, is to define a notion of random-access code that would appeal to all three communities. This we do next.

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Fix integer $K_a \geq 1$ – the number of active users – and let $P_{Y|X} = P_{Y|X_1,\ldots,X_{K_a}} : \mathcal{X}^{K_a} \to \mathcal{Y}$ be a memoryless MAC satisfying permutation invariance condition: the distribution $P_{Y|X_1,\ldots,X_{K_a}}(\cdot|x_1,\ldots,x_{K_a})$ coincides with $P_{Y|X_1,\ldots,X_{K_a}}(\cdot|x_{\pi(1)},\ldots,x_{\pi(K_a)})$ for any $x^{K_a} \in \mathcal{X}^{K_a}$ and any permutation π.

Definition 1. An (M, n, ϵ) random-access code for the K_a-user channel $P_{Y|X^{K_a}}$ is a pair of (possibly randomized) maps – the encoder $f : [M] \to \mathcal{X}^n$ and the decoder $g : \mathcal{Y}^n \to \binom{[M]}{K_a}$ – satisfying:

$$\frac{1}{K_a}\sum_{j=1}^{K_a} \mathbb{P}[E_j] \leq \epsilon\,,$$

where $E_j \triangleq \{W_j \notin g(Y^n)\} \cup \{W_j = W_i \text{ for some } i \neq j\}$ is the j-th user error event, $W_1,\ldots,W_{K_a}$ are independent and uniform on $[M]$ and $X_j = f(W_j)$.

In other words, we have K_a users generating codewords from the *same codebook* and the decoder's job is to provide an estimate of the transmitted list. The error measures the average fraction of correctly guessed messages. The key differences from the usual information-theoretic K-user MAC are: a) users are forced to employ the same codebook; b) decoding is done upto permutation of messages; c) the error event is defined per-user, as opposed to global for all users. Before explaining the rationale for our definition we make technical remarks.

In the remainder of this paper we exclusively focus on the Gaussian MAC (GMAC), given by

$$Y = X_1 + \cdots + X_{K_a} + Z\,, \qquad Z \sim \mathcal{N}(0,1)\,. \quad (1)$$

In this case the blocklength n is also called the number of real degrees of freedom (rdof). Naturally, for the GMAC we require in addition that $\|f(j)\|_2^2 \leq nP$ a.s. and in this case we say that the code achieves *energy-per-bit* $\frac{E_b}{N_0} \triangleq \frac{nP}{2\log_2 M}$ and the *system spectral efficiency* $S = \frac{K_a}{n}\log_2 M$ measured in bits/rdof.

We have chosen to postulate that any collision between the chosen messages automatically results in error. Note that

$$\mathbb{P}[\cup_{i\neq j}\{W_j = W_i\}] \leq \frac{\binom{K_a}{2}}{M}\,, \quad (2)$$

which is negligible in every practical situation. Thus, the details of handling colliding messages are not important. (A good alternative is to let the decoder output a multiset and measure error by the multiset difference.) These subtleties are important for the zero-error questions, though: compare B_s-codes and (s,t)-plans in [14].

To place Def. 1 in context, recall that the traditional way of treating random-access in information-theoretic literature is by way of partially active users ("T-out-of-N MAC"). There is classical literature on the topic [14]–[18] and a recent work by D. Guo and colleagues [19], [20] extends the model to "massive" number of users. From our point of view, however, the T-out-of-N model is not completely satisfactory because a) the total number N of users (active plus inactive) affects the results; and b) it conflates the user identification problem ("who was active") with the actual data transmission. For example, in the network-theoretic studies it is common to think of MAC layer's job as that of delivering packets not identifying who sent them. The reasoning is that part of the payload ("headers") contains identifying information. In the regime of small 100-bit payloads particular details of how this identification is done affects performance rather severely and precludes honest evaluation of, e.g., ALOHA.

Our chief aim with Def. 1 was to propose a model in which the total number of users could be taken infinite and for which ALOHA would be a valid achievability. Note that taking $N = \infty$ automatically precludes the possibility of user identification. Naturally, we were lead to the idea of forcing users employ the same codebook (or randomized encoder, as in the case of ALOHA). Note that the idea of non-identifiable users has been discussed before [21], as well as the idea of computing throughput (i.e., $1-\epsilon$) by counting the number of correctly decoded packets [22].

In the special case of GMAC, there is another exciting connection: good random-access codes are closely related to good matrices for noisy compressed sensing (CS). Indeed, if we focus on deterministic codes and ignore message collisions between users then arranging all codewords in a $n \times M$ matrix A, the goal of the decoder is just that of support recovery of a K_a-sparse $U \in \{0,1\}^M$ based on $Y = AU + Z$ with $Z \sim \mathcal{N}(0, I_n)$. This connection between MAC and CS has been observed and explored earlier [23] and more recently [24]. We point out, however, that compared to the standard CS problem we have a rather peculiar setting: the dimension of U is terribly large (at least 2^{100} in practice), the distortion is measured by the Hamming distance (see (11) below) and the reconstruction $\hat{U}$ is required to be strictly K_a-sparse itself.

What makes Def. 1 an attractive way to think about random-access is the fact that many popular schemes become achievability bounds and, consequently, can be compared against each other. This is what we do below. In Section II we prove a random coding existence bound. Then in Section III we evaluate this bound and also compare to several popular schemes: (slotted) ALOHA, coded slotted ALOHA, treating

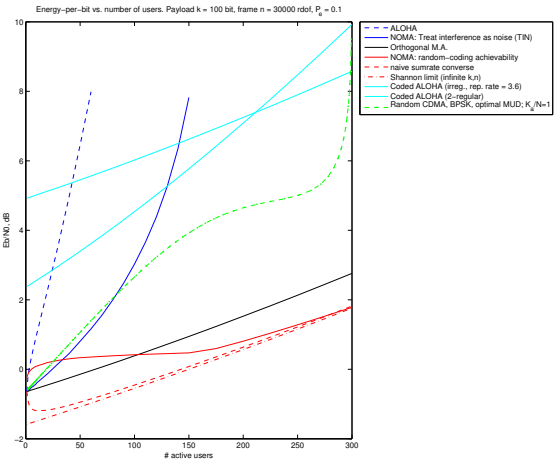

Fig. 1. The tradeoff between the $\frac{E_b}{N_0}$ and number of active users.

interference as noise (TIN), CDMA with optimal multi-user detector (MUD) and the (non-grantless) strategy of orthogonalization (TDMA/FDMA). The conclusions are not favorable to the known solutions.

In Section IV we discuss the standard (non random-access) K-user GMAC in the asymptotic $n \to \infty$ regime, but with $K = \mu n$ and a per-user error criterion. We discover that in this setting the trade-off between energy-per-bit and spectral efficiency is dramatically different from the familiar $\frac{E_b}{N_0} = \frac{2^{2S}-1}{2S}$.

II. Random coding bound

Theorem 1. *Fix $P' < P$. There exists an (M, n, ϵ) random-access code for K_a-user GMAC satisfying power-constraint P and*

$$\epsilon \le \sum_{t=1}^{K_a} \frac{t}{K_a} \min(p_t, q_t) + p_0 \,, \tag{3}$$

where

$$p_0 = \frac{\binom{K_a}{2}}{M} + K_a \mathbb{P}\left[\frac{1}{n}\sum_{j=1}^{n} Z_j^2 > \frac{P}{P'}\right] , \tag{4}$$

$$p_t = e^{-nE(t)}, \tag{5}$$

$$E(t) = \max_{0\le\rho,\rho_1\le 1} -\rho\rho_1 t R_1 - \rho_1 R_2 + E_0(\rho,\rho_1)$$

$$E_0 = \rho_1 a + \frac{1}{2}\log(1-2b\rho_1)$$

$$a = \frac{\rho}{2}\log(1+2P't\lambda) + \frac{1}{2}\log(1+2P't\mu) \tag{6}$$

$$b = \rho\lambda - \frac{\mu}{1+2P't\mu},\ \mu = \frac{\rho\lambda}{1+2P't\lambda} \tag{7}$$

$$\lambda = \frac{P't - 1 + \sqrt{D}}{4(1+\rho_1\rho)P't}, \tag{8}$$

$$D = (P't-1)^2 + 4P't\frac{1+\rho\rho_1}{1+\rho}$$

$$R_1 = \frac{1}{n}\log M - \frac{1}{nt}\log(t!) \tag{9}$$

$$R_2 = \frac{1}{n}\log\binom{K_a}{t} \tag{10}$$

$$q_t = \inf_{\gamma} \mathbb{P}[I_t \le \gamma] + \exp\{n(tR_1 + R_2) - \gamma\}$$

where random variable I_t is defined in (13) *below.*

Remark 1. Similar bound holds for any MAC satisfying the following symmetry constraint: For any two sets $S_1 \subset K_a$ and $S_2 = [K_a] \setminus S_1$ we should have

$$\mathbb{P}\left[\frac{P_{Y|X}(Y|x_{S_1}, x_{S_2})}{P_{Y|X}(Y|x_{S_1}, \bar{x}_{S_2})} > t \middle| X_{S_1} = x_{S_1}, X_{S_2} = x_{S_2}\right]$$

is a function of $(t, x_{S_2}, \bar{x}_{S_2})$ and is independent of x_{S_1}. In particular, all additive channels with additive noise satisfy this constraint.

Remark 2. If we set $\rho_1 = 1$ and $q_t = 1$ we get the identification bound from [20, Section IV.B].

Proof. We generate the M codewords $c_1, \ldots, c_M \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, P')$. Upon transmission if $\|c_{W_j}\|_2^2 > nP$, user j transmits 0 instead. For any $S \subset [M]$ we set $c(S) \triangleq \sum_{j \in S} c_j$. The decoder outputs the set $\hat{S}$ of cardinality K_a minimizing $\|c(\hat{S}) - Y^n\|_2^2$ (ties happen with probability zero, so we ignore them). We analyze probability of error next. Say that W_j is unique if $W_j \neq W_i$ for any $i \neq j$. Let

$$G = \frac{1}{K_a} \sum_{j=1}^{K_a} 1\{W_j \notin \hat{S}, W_j\text{–unique}\}\,.$$

We need to upper bound $\mathbb{E}[G]$. To that end, first note that function G is bounded by 1 and thus we can change the measure over which $\mathbb{E}$ is taken at the expense of adding a total variation distance. We replace measure by the one under which a) $W_1, \ldots, W_{K_a}$ are sampled uniformly without replacement from $[M]$; and b) under which $X_j = c_{W_j}$ (instead of $X_j = c_{W_j} 1\{\|c_{W_j}\|_2^2 \leq nP\}$ under the true measure). The total variation between the true measure and the new one is easily bounded by p_0.

If we let $S = \{W_1, \ldots, W_{K_a}\}$ be a random K_a-subset of $[M]$, then under the new measure, we have

$$G = \frac{1}{K_a}|S \setminus \hat{S}| = \frac{1}{2K_a} d_H(S, \hat{S})\,, \qquad (11)$$

where $d_H(\cdot,\cdot)$ is the Hamming distance. We define events $F_t \triangleq \{|S \setminus \hat{S}| = t\}$ and are only left to show $\mathbb{P}[F_t] \leq \min(p_t, q_t)$. From symmetry, we may assume $S = \{1, \ldots, K_a\}$.

First, consider the Gallager-type bound. Let S_0 and S_0' be generic subsets of size t in $[K_a]$ and in $[M] \setminus [K_a]$, respectively. Then, recalling that $Y = c([K_a] \setminus S_0) + c(S_0) + Z$, we define error events $F(S_0, S_0') \triangleq \{\|c(S_0) - c(S_0') + Z\|_2 < \|Z\|_2\}$ and $F(S_0) \triangleq \cup_{S_0'} F(S_0, S_0')$. Next, fix $\rho, \rho_1 \in [0,1]$ and $\lambda > 0$. Using Chernoff bound and identity

$$\mathbb{E}\left[e^{-\gamma\|\sqrt{a}Z+u\|_2^2}\right] = \frac{e^{-\frac{\gamma\|u\|_2^2}{1+2a\gamma}}}{(1+2a\gamma)^{\frac{n}{2}}}\,, \quad \forall \gamma > -\frac{1}{2a} \qquad (12)$$

we estimate

$$\mathbb{P}[F(S_0, S_0')|c(S_0), Z] \leq e^{E_1(c(S_0), Z)}\,,$$

where $E_1 = \lambda\left(\|Z\|_2^2 - \frac{\|c(S_0)+Z\|_2^2}{1+2tP'\lambda}\right)$. Next, we invoke Gallager's ρ-trick, i.e. $\mathbb{P}[\cup_j A_j] \leq (\sum_j \mathbb{P}[A_j])^\rho$ for any $\rho \in [0,1]$, to get

$$\mathbb{P}[F(S_0)|c(S_0), Z] \leq \binom{M-K_a}{t}^\rho e^{\rho E_1(c(S_0), Z)}\,.$$

Taking here expectation over c_{S_0} and employing (12) again, we get

$$\mathbb{P}[F(S_0)|Z] \leq \left(\frac{M^t}{t!}\right)^\rho e^{+b\|Z\|_2^2 - na}\,,$$

where a, b are given by (6)-(7). Finally, applying Gallager's ρ-trick again, we get

$$\mathbb{P}[\cup_{S_0} F(S_0)] \leq \mathbb{E}\left[\left(\sum_{S_0} \mathbb{P}[F(S_0)|Z]\right)^{\rho_1}\right] = e^{-nE(t)}\,,$$

where we also set λ as in (8), which is optimal under a fixed ρ, ρ_1.

Next we bound $\mathbb{P}[F_t]$ differently. Define information density[1]

$$i_t(a; y|b) = nC_t + \frac{\log e}{2}\left(\frac{\|y-b\|_2^2}{1+P't} - \|y-a-b\|_2^2\right)\,,$$

where $C_t = \frac{1}{2}\log(1+P't)$. Fix γ and consider the event $\tilde{F} = \{I_t \leq \gamma\}$ where

$$I_t = \min_{S_0} i_t(c(S_0); Y|c(S_0^c))\,, \qquad (13)$$

where minimum is over all t-subsets of $[K_a]$. Note that $F(S_0, S_0') = \{i_t(c(S_0'); Y|c(S_0^c)) > i_t(c(S_0); Y|c(S_0^c))\}$ and thus we have

$$\mathbb{P}[F(S_0)|c_1, \ldots, c_{K_a}, Y] \leq \exp\{ntR_1 - \gamma\} 1_{\tilde{F}^c} + 1_{\tilde{F}}$$

according to the usual properties of information density, e.g. [25, Prop. 17.1]. Taking further union over S_0 we get that the first term is to be multiplied by $\exp\{nR_2\}$, which yields q_t. □

III. Numerical evaluation

On Fig. 1 we compare various MAC strategies in the following setting: each active user is sending $k = 100$ bits, the frame length is $n = 30000$ and the target per-user probability of error is 10%. This is a regime of interest for LP-WANs such as LoRaWAN and Weightless.

We describe briefly how each curve was obtained. The TIN[2] achievability is the following: Generate codewords $\{c_i\}$ according to i.i.d. $\mathcal{N}(0, P')$. Decoder's list consists of top K_a codewords closest to the received channel output Y. Assuming that $c_1, \ldots, c_{K_a}$ were transmitted, the (per-user) probability of error can be bounded similarly to the DT bound in [1] to get

$$\mathbb{E}\exp\{-|i(X^n; Y^n) - \log M|^+\} + \mathbb{P}[\sum_{i=1}^n X_i^2 > nP] \qquad (14)$$

where $i(x^n; y^n) = \frac{\log e}{2}\left(\frac{\|y^n\|_2^2}{1+K_aP'} - \frac{\|y^n - x^n\|_2^2}{1+(K_a-1)P'}\right) + nC_{TIN}(P')$ and $C_{TIN}(P') = \frac{1}{2}\log\left(1 + \frac{P'}{1+(K_a-1)P'}\right)$. If we omit the second

[1] To see that this is indeed an information density, suppose that $X_1, \ldots, X_{K_a} \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, P'I_n)$ are the input to the MAC. Then we have for any subset $S_0 \subset [K_a]$ that $\log \frac{dP_{Y|X_{S_0}, X_{S_0^c}}}{dP_{Y|X_{S_0^c}}}$ is a function of $(X(S_0), X(S_0^c), Y)$ only. Thus, we may define this log-likelihood (whose expectation is $I(X_{S_0}; Y|X_{S_0^c}^c)$) in terms of the latter three n-vectors, as opposed to all (K_a+1) n-vectors.

[2] In practical terms TIN curve corresponds to CDMA with matched filter detector (i.e. without MUD)

term in (14) and approximate by the CLT we get that TIN coding achieves about[3]

$$\log M \approx nC_{TIN}(P) - \sqrt{\frac{nP\log^2 e}{1+K_aP}}Q^{-1}(\epsilon)\,,$$

ALOHA was done by partitioning n-long frame into m subframes (we optimized m for each K_a) and letting users randomly select a subframe. Decoding works only if there were no collision and a single-user decoder succeeded. Note that the steep increase in ALOHA is primarily due to a large number of subframes required to avoid collisions. This difficulty is alleviated with Coded Slotted ALOHA (CSA) [27], [28] which requires a lot fewer subframes in order to tolerate the given number of users. For example, for 2-regular CSA we assumed that each packet is retransmitted twice (for 3 dB loss in $\frac{E_b}{N_0}$) but that packets of all users can be recovered if $K_a < \frac{m}{2}$; we also ignored the overhead required for including pointers to other packets. TDMA curve corresponds to a non-random-access strategy of orthogonalizing into blocklength $\frac{n}{K_a}$ for each user. "Random CDMA" corresponds to another non-random-access strategy[4] where each of the users employ random ± 1 signatures of length $N = K_a$ modulated by their binary ± 1 symbols. Optimal MUD results in each user seeing an effective single-user BI-AWGN channel with SNR found from Tanaka's formula [29], [30]. *In all of these computations, we assumed that good single-user codes are used that achieve the two-term capacity-dispersion approximation [1] for the AWGN or BI-AWGN channels.* Consequently, the above curves are only (optimistic) approximations.

If we let $n \to \infty$ while holding K_a fixed, then the Shannon limit $\frac{k}{n} \to \frac{1}{2K_a}\log_2(1+K_aP)$ with $\epsilon \to 0$, which we plotted for reference. The "naive sumrate converse" is a conjectured converse for a (non-random-access) K_a-GMAC from [3, Section V].

The random coding curve is a firm bound, computed by optimizing over P' in Theorem 1 (we only used q_t-bound for $t = 1$). Interestingly, for $K_a \le 50$ the terms with $t = 1, 2, 3$ contribute the most, while for $K_a \gtrsim 170$, most of the contribution occurs at t close to K_a. This should be puzzling, since information-theoretically the symmetric rate for K_a-GMAC equal $\frac{1}{2K_a}\log(1+PK_a)$, corresponds to the term $t = K_a$ in the random-coding bound. This effect is illuminated by Section IV: when K_a (and spectral efficiency) is small, the dominant constraint is the FBL penalty on $\frac{E_b}{N_0}$ due to finite number k of bits [31]; when K_a is large, the multi-access interference dominates.

IV. Asymptotics

Consider a standard K-user GMAC [32, Section 4.7] and suppose users are sending $\log_2 M$ bits each (with *different* codebooks!), but the probability of error is defined as $\epsilon = \frac{1}{K}\sum_{j=1}^{K}\mathbb{P}[W_j \neq \hat{W}_j]$, where $\hat{W}_j \in [M]$ are decoder decisions.

Fix ϵ and $0 < \mu \ll 1$ (the density of users per rdof). What is the fundamental tradeoff $\frac{E_b}{N_0}$ vs. system spectral efficiency tradeoff for $K, n \to \infty$ with $K = \mu n$. More exactly, we define

$$\left(\frac{E_b}{N_0}\right)_{\min} = \mathcal{E}^*(S,\mu,\epsilon) \triangleq \inf\left\{\frac{nP}{2\log_2 M}\right\}\,,$$

where infimum is taken over all (n, M, P) for which there exist K-codebooks each of size M decodable with per-user probability of error $\le \epsilon$ over K-GMAC; the ratio $\frac{K\log M}{n} = S$ is the system spectral efficiency. Note that, although we do allow $n \to \infty$, this question implicitly falls in the domain of the FBL information theory. Indeed, for a fixed spectral efficiency S we must have $\log M = \frac{S}{\mu}$ regardless of the blocklength.

What is a typical value of μ? Consider a metropolitan area with 10^6-10^7 devices. The sub-GHz ISM band is about 20 MHz wide. Therefore, if each of the devices is active a few times per hour, we get the ratio of 10^3-10^4 rdof per active user at any given moment. Note that this number is unlikely to vary by much in the near future.

Results of comparison are shown on Fig. 2. Here we describe how each curve was obtained. For convenience, let us denote $P_{tot} = KP$, so that $\frac{E_b}{N_0} = \frac{P_{tot}}{2S}$.

For the *converse* we have the best of two bounds: The Fano's inequality: $(1-\epsilon)S \le \frac{1}{2}\log(1+P_{tot}) + \mu h(\epsilon)$ and the bound following from the fact that each user only transmits finitely many bits $\frac{S}{\mu}$, namely from [31, Theorem 2]:

$$\frac{S}{\mu} \le -\log Q\left(\sqrt{\frac{P_{tot}}{\mu}} + Q^{-1}(1-\epsilon)\right)\,.$$

For the *TDMA/FDMA achievability* we divide n channel uses evenly among all K users and compute smallest P guaranteeing existence of a single-user AWGN code of rate S, blocklength $\frac{1}{\mu}$ and error probability ϵ. The precise achievability bound that was used is Shannon's bound, e.g. [1, Theorem 13]. For the *TIN achievability* we used (14) with power $P = \frac{P_{tot}}{\mu n}$. As $n \to \infty$ the second term in (14) converges to zero, while first converges to (this is *not* an approximation)

$$\mathbb{P}[Z > u] + M\mathbb{P}\left[Z > \sqrt{\frac{P_{tot}}{(1+P_{tot})\mu}} - u\right]\,, \quad (15)$$

where $Z \sim \mathcal{N}(0,1)$ and $\ln M = \frac{P_{tot}}{2(1+P_{tot})\mu} - \sqrt{\frac{P_{tot}}{(1+P_{tot})\mu}}u$. Finally, to apply *Theorem 1 bound* we drop the second term in R_1, cf. (9) since users now employ different codebooks. We then find the smallest P_{tot} so that $E_{min}(P_{tot}, \epsilon) > 0$, where the latter is given by

$$\min_{\epsilon \le \theta \le 1}\max_{0 \le \rho, \rho_1 \le 1} -\rho\rho_1\theta S - \rho_1\mu h(\theta) + E_0(\rho, \rho_1)$$

and is the worst exponent among all terms p_t, cf. (5), with $t \ge \epsilon K_a$. We note that, for spectral efficiencies above the point marked by A on the plot the dominant term in Theorem 1 corresponds to $t = K$; between A and B the dominant term is $\epsilon K < t < K$; and below B the dominant term is $t = \epsilon K$. This partially explains the effects observed in evaluation on Fig. 1 too.

[3]We could have instead generated spherical codebook, resulting in improved dispersion [26]. Numerically, the difference, however, is imperceptible.

[4]This strategy is not truly random-access because it requires the receiver to know the signatures of active users. It does provide a good benchmark for what would be possible with MUD-based CDMA.

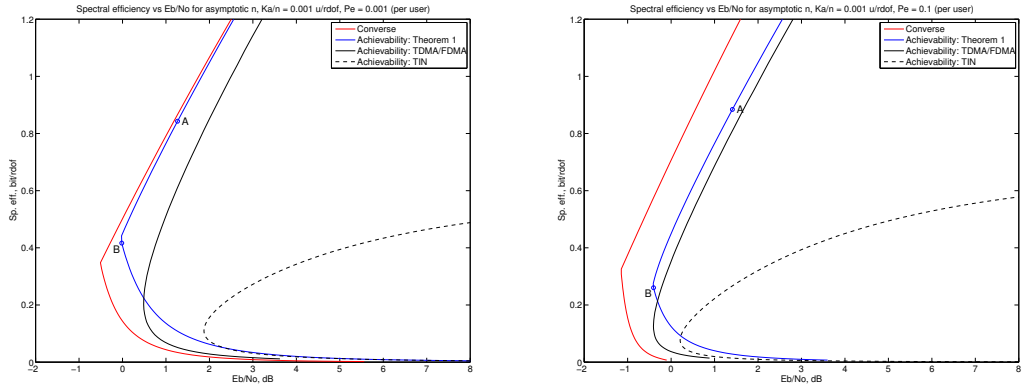

Fig. 2. The tradeoff between the $\frac{E_b}{N_0}$ and system spectral efficiency for the (non random-access) K-user GMAC in the regime $K, n \to \infty$ with $\mu = \frac{K}{n} = 10^{-3}$, per-user probability of error $\epsilon = 10^{-3}$ (left) and $\epsilon = 10^{-1}$ (right). All curves are firm bounds (not approximations).

ACKNOWLEDGMENT

Author is thankful for stimulating discussions with A. Collins, M. Feder, A. Frolov, D. Guo, G. Kabatianskii, S. Kowshik, F. R. Kschischang, B. Li, K. Narayanan, E. Telatar and, especially, Or Ordentlich.

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