Asynchronous Quasi-Consensus of Heterogeneous Multiagent Systems With Nonuniform Input Delays

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***Abstract*—This paper analyzes the consensus problem in heterogenous nonlinear multiagent systems. The multiagent systems not only have nonidentical nonlinear dynamics for all agents, but also have different network topologies for position and velocity interactions. An asynchronous sampled-data control without any input delays is first proposed, the information of each agent is only sampled at its own sampling instants and need not be sampled at other sampling instants. Then, quasi-consensus in heterogenous multiagent systems is proved by Lyapunov stability theory. When asynchronous sampled-data control has nonuniform input delays, sufficient conditions for quasi-consensus in heterogenous multiagent systems are further obtained. The upper bound of quasi-consensus errors is estimated. Finally, numerical simulations are provided to verify the effectiveness of theoretical results.**

***Index Terms*—Asynchronous sampled-data control, distributed tracking, heterogeneous multiagent systems, nonuniform input delays, quasi-consensus.**

I. Introduction

CONSENSUS of multiagent systems has received considerable attentions in recent years, due to its broad applications in formation control of unmanned air vehicles, coordination of multiple robots, synchronization of distributed harmonic oscillators, and so on. The main merit of consensus is that a group of agents can reach an agreement on certain quantities of interest by distributed interactions between each agent and its neighbors.

Previous studies on consensus have mainly focused on multiagent systems with first-order dynamics [1]–[3], second-order dynamics [4]–[6], high-order systems or linear systems [7]–[9], and nonlinear systems [10]–[12]. Most of these studies have been concerned with homogeneous systems which had the same network topologies and the same dynamics of nodes. However, heterogeneous systems are usually ubiquitous in real applications due to the differences of intrinsic dynamics of systems, uncertainties and disturbances of parameters, perturbations of systems, heterogeneities of network topologies, and so on. Admittedly, heterogeneous multiagent systems are more difficult to achieve consensus than homogeneous systems. Considerable efforts have been devoted to the study on coordination of heterogeneous systems and much progress has been made so far. Both synchronization of heterogenous networks and consensus of heterogeneous multiagent systems have obtained fruitful results. Consensus of heterogeneous multiagent systems, which consisted of single and double integrator agents, was studied in [13] and [14]. Based on an adaptive fuzzy distributed controller, heterogeneous multiagent systems with nonidentical nonlinear dynamics arising from uncertainties, unmeasured states, and external disturbances was studied in [15]. In the absence of complete synchronization manifold, bounded synchronization of heterogeneous complex networks was proved in [16] and [17]. Synchronization of heterogeneous networked systems involving both dynamics and parameters was presented in [18]. Quasi-synchronization or quasi-consensus in heterogeneous nonlinear multiagent systems was investigated by employing control inputs in [19] and [20]. In addition to the heterogeneous networked systems with nonidentical dynamics of nodes, a heterogeneous network with nonidentical network topologies, where the position and velocity interactions were different, was studied in [21]. To our knowledge, however, few works have focused on the networked systems that are heterogeneous in both dynamics of nodes and topologies of network structures. This is the first motivation. In this paper, we consider nonlinear multiagent systems with nonidentical node dynamics and different network topologies.

Depending on the coupling of heterogenous networked systems, quasi-synchronization could be guaranteed for certain networks without any external controllers [22]. In general, however, it is essential to add an external controller to heterogeneous systems to reach synchronization or consensus. Many control strategies have been employed to coordinate heterogeneous networked systems, such as continuous feedback control [23], [24], discontinuous feedback control [25], adaptive control [26]–[29], intermittent control [30], impulsive control [19], [31], and so on. With the development of digital technologies, sampled-data control has been widely used in cooperative control of both homogeneous and heterogeneous multiagent systems [32]–[39]. When each agent has different

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sampling instants with other agents, asynchronous sampled-data control is applicable to the practical cases [32]–[35]. Recently, the consensus tracking problem for heterogeneous nonlinear systems has been studied in [34] based on an asynchronous sampled-data control. The average consensus of second-order systems with communication delays has been investigated in [35] via the asynchronous edge-event triggered control. In contrast to the asynchronous case, synchronous sampled-data control has unified sampling instants for all agents [36]–[39], and even is necessary for stochastic sampling [40], [41]. By proposing synchronous sampling protocols which only used sampled position data, sufficient and necessary criteria for consensus of second-order multiagent systems were derived in [39]. Note that agents in heterogeneous multiagent systems are nonidentical, it is reasonable to adopt an asynchronous sampling control for heterogeneous systems.The delayed-input approach is an useful tool to deal with sampled-data-based system [34], [37], thus the sampled-data-based system is converted to a delayed system which can be viewed as a system with input delay [42]–[44]. This paper adopts asynchronous sampling control. Furthermore, there are two different asynchronous sampling mechanisms in two heterogeneous network topologies.

Consensus can be classified as leader-following consensus (tracking consensus) [6], [8], [11] or leaderless consensus [4], [5], [10] on the basis of whether or not a multiagent system has a leader. Since the heterogeneity harms consensus in multiagent systems, a group of heterogeneous agents usually cannot reach consensus. It is necessary to introduce a common objective (a leader) to coordinate all heterogeneous agents. Therefore, the leader plays an important role in coordinated control of heterogeneous multiagent systems. In other words, leader-following consensus provides an effective way to analyze the consensus problem of heterogeneous multiagent systems [19]–[21], [25], [31], [34].

Motivated by the above mentioned discussion, this paper investigates the consensus problem in heterogeneous leader-following multiagent systems by applying asynchronous sampled-data controls. The main contributions of this paper are summarized as follows. First, heterogeneities of node dynamics and network topologies are considered together. Specifically, individuals of second-order multiagent systems have nonidentical nonlinear dynamics. Moreover, the network topologies of the position and velocity interactions are different. Second, asynchronous sampled-data controls are employed to solve the consensus problem for heterogeneous second-order multiagent systems. Each agent has different sampling instants with other agents. In other words, the information of each agent is only sampled at its own sampling instants and need not be sampled at the sampling instants of its neighbors. In addition, the sampling instants of position information and velocity information are also different. Third, the asynchronous sampling control with nonuniform input delays is considered.

The rest of this paper is organized as follows. In Section II, basic concepts of algebraic graph theory and problem formulation are presented. In Section III, sufficient conditions for heterogeneous multiagent systems with both delay-free input and nonuniform input delays are obtained. In Section IV, we illustrate our theoretical results with numerical simulations. Finally, the conclusion is summarized in Section V.

II. Preliminaries and Problem Formulation

In this section, notations are first introduced. Then, basic concepts of algebraic graph theory and the model formulation of heterogeneous multiagent systems are provided.

Throughout this paper, $\mathbb{R}$ and $\mathbb{N}$ stand for the sets of real numbers and natural numbers, respectively. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ represent the n-dimensional Euclidean space and the set of $n \times m$-dimensional real matrices, respectively. Let I_n and O_n (I and O) be the n-dimensional (appropriate dimension) identity matrix and zero matrix, respectively. $1_n \in \mathbb{R}^n$ ($0_n \in \mathbb{R}^n$) is a vector with each entry being one (zero). For a symmetric matrix, $M > 0$ means that M is positive definite, and $\lambda_{\min}(M)$ denotes the minimum eigenvalue of M. Symbol $\|\cdot\|$ is the Euclidean norm. $\mathrm{diag}\{A_i\}_{i=1}^n$ signifies the $n \times n$ block-diagonal matrix with A_i ($1 \le i \le n$) being its ith diagonal block. Notation $\otimes$ indicates the Kronecker product. Symbol $*$ indicates the symmetric parts of a symmetric matrix.

A. Algebraic Graph Theory

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ be a weighted directed graph with the set of nodes $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$, the set of directed edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and an adjacency matrix $A = [a_{ij}]_{N\times N}$. A directed edge e_{ij} is denoted by an ordered pair of nodes (v_j, v_i), and $e_{ij} \in \mathcal{E}$ if and only if $a_{ij} > 0$. If $e_{ij} \in \mathcal{E}$, then node j is called a neighbor of node i. The set of neighbors of node i is denoted by $\mathcal{N}_i$. A directed path from node v_j to v_i is a sequence of directed edges $e_{i,i_1}, e_{i_1,i_2}, \ldots, e_{i_l,j}$ in the digraph with distinct nodes v_{i_m}, $m = 1, 2, \ldots, l$. A digraph has a directed spanning tree if there exists one node which has a direct path to all other nodes. The Laplacian matrix $L = [l_{ij}]_{N\times N}$ of the graph $\mathcal{G}$ is defined as: $l_{ij} = -a_{ij}$ for $i \neq j$, and $l_{ii} = -\sum_{j=1, j\neq i}^{N} l_{ij}$. A graph is undirected if $e_{ji} \in \mathcal{E}$ is equivalent to $e_{ij} \in \mathcal{E}$.

B. Heterogeneous Nonlinear Multiagent System

Consider a heterogeneous leader-following multiagent system with one leader and N followers. The leader is an isolated agent with second-order dynamics governed by

$$\begin{aligned} \dot{x}_0(t) &= v_0(t) \\ \dot{v}_0(t) &= f_0(x_0(t), v_0(t), t) \end{aligned} \tag{1}$$

where $x_0(t) \in \mathbb{R}^n$ and $v_0(t) \in \mathbb{R}^n$ are the position and velocity states, respectively. The intrinsic dynamics of the leader $f_0 : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}^n$ is a nonlinear continuous vector-valued function.

The followers of heterogeneous multiagent system composed of N nonlinear agents described by

$$\begin{aligned} \dot{x}_i(t) &= v_i(t) \\ \dot{v}_i(t) &= f_i(x_i(t), v_i(t), t) + u_i(t), \quad i = 1, 2, \ldots, N \end{aligned} \tag{2}$$

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where $x_i(t) \in \mathbb{R}^n$, $v_i(t) \in \mathbb{R}^n$, and $u_i(t) \in \mathbb{R}^n$ are the position state, velocity state, and control input of the ith agent, respectively. The intrinsic dynamics of the ith agent $f_i : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}^n$ is a nonlinear continuous vector-valued function.

It is assumed that the leader-following multiagent systems (1) and (2) are heterogeneous. In other words, All the intrinsic dynamics $f_0, f_1, f_2, \ldots, f_N$ are different with each other.

Assumption 1: The leader has a directed path to each follower agent.

Assumption 2: For the nonlinear functions $f_0, f_1, \ldots, f_N$, there exist constants $p_i > 0$ and $q_i > 0$ such that

$$\|f_i(x, v, t) - f_i(y, z, t)\| \leq p_i\|x - y\| + q_i\|v - z\|$$

$i = 0, 1, 2, \ldots, N,$ for all $x, v, y, z \in \mathbb{R}^n$.

Assumption 3: The states of the leader are bounded, that is, for any initial condition $[x_0(0), v_0(0)]$, there exist constants $\mu > 0$ and $T = T(x_0(0), v_0(0))$ such that $\|x_0(t)\| \leq \mu$ and $\|v_0(t)\| \leq \mu$ for all $t > T$.

Remark 1: Assumption 1 is a necessary condition for achieving consensus of multiagent systems and has been widely adopted in [19]–[21] and [45]. Assumption 2 is a mild Lipschitz condition [19], [20], [22] and holds for many nonlinear systems. Assumption 3 is reasonable for several systems, and the similar assumptions are adopted in many literatures, such as [18]–[20], [34], [45], and [46].

Definition 1 [17], [19]: The heterogenous leader-following multiagent system (1) and (2) is said to achieve quasi-consensus to a compact set $\mathcal{M}$ if for any initial conditions

$$\lim_{t\to\infty} \text{dist}(x_i(t) - x_0(t), \mathcal{M}) = 0$$
$$\lim_{t\to\infty} \text{dist}(v_i(t) - v_0(t), \mathcal{M}) = 0, i = 1, 2, \ldots, N$$

where $\text{dist}(x^*, \mathcal{M})$ denotes the distance from a point x^* to the compact set $\mathcal{M}$.

Lemma 1 (Jensen Inequality [47]): For a given $n \times n$-matrix $R > 0$ and for all continuous function ω in $[a, b] \to \mathbb{R}^n$, the following inequality holds:

$$\int_a^b \omega^T(u)R\omega(u)du \geq \frac{1}{b-a}\left(\int_a^b \omega^T(u)du\right)R\left(\int_a^b \omega(u)du\right).$$

III. Main Results

In this section, consensus for heterogeneous leader-following multiagent systems (1) and (2) is analyzed. Two consensus protocols involving asynchronous sampling and heterogeneous topologies are proposed. Sufficient criteria of quasi-consensus for heterogeneous systems with both delay-free input and input of nonuniform delays are derived.

A. Consensus Analysis for Heterogeneous Multiagent System With Delay-Free Input

In view of the advantages of the sample-data control, such as robustness and low cost, this paper first considers the following delay-free asynchronous sampling protocol:

$$\begin{aligned} u_i(t) = {} & \alpha_i \sum_{j=1}^{N} a_{ij}\left[x_j\left(t^j_{k_j(t)}\right) - x_i\left(t^i_k\right)\right] \\ & + \beta_i \sum_{j=1}^{N} b_{ij}\left[v_j\left(s^j_{r_j(t)}\right) - v_i\left(s^i_r\right)\right] \\ & - \alpha_i\gamma_i^a\left[x_i\left(t^i_k\right) - x_0\left(t^0_{k_0(t)}\right)\right] \\ & - \beta_i\gamma_i^b\left[v_i\left(s^i_r\right) - v_0\left(s^0_{r_0(t)}\right)\right] \end{aligned} \quad (3)$$

for $t \in [t^i_k, t^i_{k+1}) \cap [s^i_r, s^i_{r+1}), k, r \in \mathbb{N}, \ i = 1, 2, \ldots, N$, where $\{t^i_k | k \in \mathbb{N}\}$ are the sampling instants of position information satisfying $0 = t^i_0 < t^i_1 < \ldots < t^i_k < \ldots$ and $t^i_{k+1} - t^i_k \leq h_{ai} \leq h_a$ with $h_a > 0$; $\{s^i_r | r \in \mathbb{N}\}$ are the sampling instants of velocity information satisfying $0 = s^i_0 < s^i_1 < \ldots < s^i_r < \ldots$ and $s^i_{r+1} - s^i_r \leq h_{bi} \leq h_b$ with $h_b > 0$. Denote $h = \max\{h_a, h_b\}$. $k_j(t) = \max\{k | t^j_k \leq t\}$ and $r_j(t) = \max\{r | s^j_r \leq t\}$ represent the latest sampling numbers of position information and velocity information of the jth ($j \in \mathcal{N}_i$) agent at time t, respectively. Adjacency matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ indicate the network topologies of position and velocity interactions among all the N follower agents, respectively. α_i and β_i are the coupling strengths of the ith agents. γ_i^a and γ_i^b are the pinning gains of the ith agent in the network topologies A and B, respectively. $\gamma_i^\ell > 0$ if the ith agent is pinned by the leader, otherwise $\gamma_i^\ell = 0$, $\ell = a, b$.

Remark 2: In the protocol (3), the position information and velocity information are assumed to be exchanged over two different interaction topologies. Additionally, the sampling instants in position and velocity interaction topologies are also independent of each other.

Remark 3: Since the interaction topologies of position and velocity are heterogeneous and the dynamics of all agents are nonidentical, it is reasonable to assume that the sampling instants of all agents are nonidentical and asynchronous. In addition, position information and velocity information have different sampling instants. More specifically, the sampling instants $\{t^i_k\}_{k=1}^{+\infty}$ of position information of agent i are independent of the sampling instants $\{t^j_{k'}\}_{k'=1}^{+\infty}$ of position information of agent j. At time t, the protocol (3) uses the kth sampling position information $x_i(t^i_k)$ of agent i and the k'th sampling position information $x_j(t^j_{k'})$ of agent j, where $j \in \mathcal{N}_i$. However, k and k' may be different. The same rule holds for the sampling instants of velocity information. If both position and velocity of all agents are sampled at the same instants, then the sampling is synchronous.

Let $\widetilde{x}_i(t) = x_i(t) - x_0(t)$ and $\widetilde{v}_i(t) = v_i(t) - v_0(t)$, $i = 1, 2, \ldots, N$. Combining systems (1) and (2) and consensus protocol (3), one derives the following error systems:

$$\begin{aligned}\dot{\tilde{x}}_i(t) &= \tilde{v}_i(t)\\ \dot{\tilde{v}}_i(t) &= f_i(x_i(t), v_i(t), t) - f_i(x_0(t), v_0(t), t)\\ &\quad + \alpha_i \sum_{j=1}^{N} a_{ij}\Big[\tilde{x}_j\Big(t^j_{k_j(t)}\Big) - \tilde{x}_i\big(t^i_k\big)\Big] - \alpha_i\gamma_i^a\tilde{x}_i\big(t^i_k\big)\\ &\quad + \beta_i \sum_{j=1}^{N} b_{ij}\Big[\tilde{v}_j\Big(s^j_{r_j(t)}\Big) - \tilde{v}_i\big(s^i_r\big)\Big] - \beta_i\gamma_i^b\tilde{v}_i\big(s^i_r\big)\\ &\quad + \delta_i(t), \quad t \in \big[t_k, t_{k+1}\big) \cap \big[s^i_r, s^i_{r+1}\big) \end{aligned} \tag{4}$$

where $\delta_i(t) = \delta_{i1}(t) + \delta_{i2}(t)$, and

$$\begin{aligned}\delta_{i1}(t) &= f_i(x_0(t), v_0(t), t) - f_0(x_0(t), v_0(t), t)\\ \delta_{i2}(t) &= \alpha_i \sum_{j=1}^{N} a_{ij}\Big[x_0\Big(t^j_{k_j(t)}\Big) - x_0\big(t^i_k\big)\Big]\\ &\quad + \beta_i \sum_{j=1}^{N} b_{ij}\Big[v_0\Big(s^j_{r_j(t)}\Big) - v_0\big(s^i_r\big)\Big]\\ &\quad - \alpha_i\gamma_i^a\Big[x_0\big(t^i_k\big) - x_0\Big(t^0_{k_0(t)}\Big)\Big]\\ &\quad - \beta_i\gamma_i^b\Big[v_0\big(s^i_r\big) - v_0\Big(s^0_{r_0(t)}\Big)\Big] \end{aligned} \tag{5}$$

$i = 1, 2, \ldots, N$.

Together with Assumptions 2 and 3, there exist constants $\nu_i > 0$ and $\overline{T} > 0$ such that, for all $t > \overline{T}$

$$\|f_i(t, x_0(t), v_0(t)) - f_0(t, x_0(t), v_0(t))\| \le \nu_i$$

$i = 1, 2, \ldots, N$.

Remark 4: $\delta_{i1}(t)$ reflects the heterogeneity of dynamics between the leader and the ith follower. If the dynamics of each follower is identical to the dynamics of the leader, then $\delta_{i1}(t)$ is equal to zero. $\delta_{i2}(t)$ implies the asynchrony of sampled-data control, and the nonidentity of position and velocity interaction topologies. If the asynchronous sampling degrades into the synchronous sampling, that is, both position and velocity of the leader and all the followers are sampled at the same instants, then $\delta_{i2}(t)$ will vanish.

Defining $\tau_i(t) = t - t^i_k$ for $t \in [t^i_k, t^i_{k+1})$ and $\sigma_i(t) = t - s^i_r$ for $t \in [s^i_r, s^i_{r+1})$. The error systems (4) can be rewritten as

$$\begin{aligned}\dot{\tilde{x}}_i(t) &= \tilde{v}_i(t)\\ \dot{\tilde{v}}_i(t) &= f_i(x_i(t), v_i(t), t) - f_i(x_0(t), v_0(t), t)\\ &\quad - \alpha_i \sum_{j=1}^{N} l^a_{ij}\tilde{x}_j\big(t - \tau_j(t)\big) - \alpha_i\gamma_i^a\tilde{x}_i(t - \tau_i(t))\\ &\quad - \beta_i \sum_{j=1}^{N} l^b_{ij}\tilde{v}_j\big(t - \sigma_j(t)\big) - \beta_i\gamma_i^b\tilde{v}_i(t - \sigma_i(t))\\ &\quad + \delta_i(t), \quad t \in \big[t^i_k, t^i_{k+1}\big) \cap \big[s^i_r, s^i_{r+1}\big) \end{aligned} \tag{6}$$

where $L^a = [l^a_{ij}]_{N\times N}$ and $L^b = [l^b_{ij}]_{N\times N}$ are the Laplacian matrices of A and B, respectively.

Denoting $\gamma^\ell = \mathrm{diag}\{\gamma_i^\ell\}_{i=1}^N$ and $H^\ell = L^\ell + \gamma^\ell$, $\ell = a, b$. Let H^ℓ_n be a block matrix, where the nth column is the same as the nth column of H^ℓ and other columns are zero blocks.

From (6), one can further get

$$\begin{aligned}\dot{\tilde{x}}(t) &= \tilde{v}(t)\\ \dot{\tilde{v}}(t) &= F(t) - \sum_{n=1}^{N}\big[\big(K^a H^a_n\big)\otimes I_n\big]\tilde{x}_n(t - \tau_n(t))\\ &\quad - \sum_{m=1}^{N}\Big[\Big(K^b H^b_m\Big)\otimes I_n\Big]\tilde{x}_m(t - \sigma_m(t)) + \delta(t) \end{aligned} \tag{7}$$

where $\tilde{x}(t) = [\tilde{x}_1^T(t), \tilde{x}_2^T(t), \ldots, \tilde{x}_N^T(t)]^T$, $\tilde{v}(t) = [\tilde{v}_1^T(t), \tilde{v}_2^T(t), \ldots, \tilde{v}_N^T(t)]^T$, $K^a = \mathrm{diag}\{\alpha_i\}_{i=1}^N$, $K^b = \mathrm{diag}\{\beta_i\}_{i=1}^N$, $\delta(t) = [\delta_1^T(t), \delta_2^T(t), \ldots, \delta_N^T(t)]^T$, and

$$F(t) = \begin{bmatrix} f_1(x_1(t), v_1(t), t) - f_1(x_0(t), v_0(t), t)\\ f_2(x_2(t), v_2(t), t) - f_2(x_0(t), v_0(t), t)\\ \vdots\\ f_N(x_N(t), v_N(t), t) - f_N(x_0(t), v_0(t), t)\end{bmatrix}.$$

Let $\tilde{y}(t) = [\tilde{x}^T(t), \tilde{v}^T(t)]^T$. System (7) can be transformed into the following matrix form:

$$\begin{aligned}\dot{\tilde{y}}(t) &= D\tilde{y}(t) + \tilde{F}(t) - \sum_{n=1}^{N}\tilde{K}^a\tilde{H}^a_n\tilde{y}(t - \tau_n(t))\\ &\quad - \sum_{m=1}^{N}\tilde{K}^b\tilde{H}^b_m\tilde{y}(t - \sigma_m(t)) + \Delta(t) \end{aligned} \tag{8}$$

where

$$D = \begin{bmatrix} O_N & I_N\\ O_N & O_N\end{bmatrix}\otimes I_n,\ \tilde{F}(t) = \begin{bmatrix} 0_{nN}\\ F(t)\end{bmatrix}$$

$$\tilde{H}^a_n = \begin{bmatrix} O_N & O_N\\ H^a_n & O_N\end{bmatrix}\otimes I_n,\ \tilde{H}^b_m = \begin{bmatrix} O_N & O_N\\ O_N & H^b_m\end{bmatrix}\otimes I_n$$

$$\Delta(t) = \begin{bmatrix} 0_{nN}\\ \delta(t)\end{bmatrix}$$

and

$$\tilde{K}^\ell = \begin{bmatrix} O_N & O_N\\ O_N & K^\ell\end{bmatrix}\otimes I_n,\ \ell = a, b.$$

Note that $\tilde{F}^T(t)\tilde{F}(t) = F^T(t)F(t)$. Hence, it follows from Assumption 2 that:

$$\tilde{F}^T(t)\tilde{F}(t) \le \tilde{y}^T(t)\Pi\tilde{y}(t) \tag{9}$$

where

$$\Pi = 2\begin{bmatrix}\mathrm{diag}\{p_i^2\}_{i=1}^N & O_N\\ O_N & \mathrm{diag}\{q_i^2\}_{i=1}^N\end{bmatrix}\otimes I_n.$$

Choose the Lyapunov–Krasovskii functional candidate as follows:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t) \tag{10}$$

where

$$\begin{aligned} V_1(t) &= \tilde{y}^T(t)E\tilde{y}(t)\\ V_2(t) &= \sum_{i=1}^{N}\int_{t-h_{ai}}^{t} e^{\lambda(s-t)}\tilde{y}^T(s)P_i\tilde{y}(s)ds\\ V_3(t) &= \sum_{i=1}^{N}\int_{t-h_{bi}}^{t} e^{\lambda(s-t)}\tilde{y}^T(s)Q_i\tilde{y}(s)ds \end{aligned}$$

$$V_4(t)=\sum_{i=1}^{N}h_{ai}\int_{-h_{ai}}^{0}\int_{t+\theta}^{t}e^{\lambda(s-t)}\dot{\tilde{y}}^T(s)R_i\dot{\tilde{y}}(s)dsd\theta$$

$$V_5(t)=\sum_{i=1}^{N}h_{bi}\int_{-h_{bi}}^{0}\int_{t+\theta}^{t}e^{\lambda(s-t)}\dot{\tilde{y}}^T(s)W_i\dot{\tilde{y}}(s)dsd\theta$$

with $\lambda>0$, $E>0$, $P_i>0$, $Q_i>0$, $R_i>0$, and $W_i>0$, $i=1,2,\ldots,N$.

Theorem 1: Suppose that Assumptions 1 and 2 hold. The trajectory of the error system (8) exponentially converges into a compact set

$$\mathcal{M}=\left\{\varepsilon\in\mathbb{R}^{2nN}:\|\varepsilon\|\le\sqrt{\frac{\rho}{\lambda_{\min}(E)\lambda}\sum_{i=1}^{N}\sup_{t\in[0,\infty)}\|\delta_i(t)\|^2}\right\}$$

if there exist constants $\lambda>0$, $\rho>0$, $h_{ai}>0$ and $h_{bi}>0$, and matrices $E>0$, $P_i>0$, $Q_i>0$, $R_i>0$, $W_i>0$, S_1 and S_2 with appropriate dimensions, $i=1,2,\ldots,N$, such that

$$\Omega=\begin{bmatrix}\Omega_{11}&\Omega_{12}&S_1&O&O&\Omega_{16}&\Omega_{17}&S_1\\ *&\Omega_{22}&S_2&O&O&\Omega_{26}&\Omega_{27}&S_2\\ *&*&-I&O&O&O&O&O\\ *&*&*&\Omega_{44}&O&\Omega_{46}&O&O\\ *&*&*&*&\Omega_{55}&O&\Omega_{57}&O\\ *&*&*&*&*&\Omega_{66}&O&O\\ *&*&*&*&*&*&\Omega_{77}&O\\ *&*&*&*&*&*&*&-\rho I\end{bmatrix}<0 \quad (11)$$

where

$$\Omega_{11}=\lambda E+\sum_{i=1}^{N}(P_i+Q_i)-\sum_{i=1}^{N}\left(e^{-\lambda h_{ai}}R_i+e^{-\lambda h_{bi}}W_i\right)+S_1D+D^TS_1^T+\Pi$$

$$\Omega_{12}=E-S_1+D^TS_2^T$$

$$\Omega_{16}=\left[e^{-\lambda h_{a1}}R_1-S_1\widetilde{K}^a\widetilde{H}_1^a,\,e^{-\lambda h_{a2}}R_2-S_1\widetilde{K}^a\widetilde{H}_2^a\ \ldots,\,e^{-\lambda h_{aN}}R_N-S_1\widetilde{K}^a\widetilde{H}_N^a\right]$$

$$\Omega_{17}=\left[e^{-\lambda h_{b1}}W_1-S_1\widetilde{K}^b\widetilde{H}_1^b,\,e^{-\lambda h_{b2}}W_2-S_1\widetilde{K}^b\widetilde{H}_2^b\ \ldots,\,e^{-\lambda h_{bN}}W_N-S_1\widetilde{K}^b\widetilde{H}_N^b\right]$$

$$\Omega_{22}=\sum_{i=1}^{N}\left(h_{ai}^2R_i+h_{bi}^2W_i\right)-S_2-S_2^T$$

$$\Omega_{26}=\left[-S_2\widetilde{K}^a\widetilde{H}_1^a,-S_2\widetilde{K}^a\widetilde{H}_2^a,\ldots,-S_2\widetilde{K}^a\widetilde{H}_N^a\right]$$

$$\Omega_{27}=\left[-S_2\widetilde{K}^b\widetilde{H}_1^b,-S_2\widetilde{K}^b\widetilde{H}_2^b,\ldots,-S_2\widetilde{K}^b\widetilde{H}_N^b\right]$$

$$\Omega_{44}=-\mathrm{diag}\left\{e^{-\lambda h_{ai}}(P_i+R_i)\right\}_{i=1}^N$$

$$\Omega_{46}=\mathrm{diag}\left\{e^{-\lambda h_{ai}}R_i\right\}_{i=1}^N$$

$$\Omega_{55}=-\mathrm{diag}\left\{e^{-\lambda h_{bi}}(Q_i+W_i)\right\}_{i=1}^N$$

$$\Omega_{57}=\mathrm{diag}\left\{e^{-\lambda h_{bi}}W_i\right\}_{i=1}^N$$

$$\Omega_{66}=-2\mathrm{diag}\left\{e^{-\lambda h_{ai}}R_i\right\}_{i=1}^N$$

$$\Omega_{77}=-2\mathrm{diag}\left\{e^{-\lambda h_{bi}}W_i\right\}_{i=1}^N.$$

Proof: See Appendix A for the details. ■

Remark 5: Due to the difference between the leader and each follower, δ_{i1} may not vanish. Since asynchronous sampling protocol (3) is employed, t_k^i, $t_{k0(t)}^0$, and $t_{kj(t)}^j$, $j\in\mathcal{N}_i$ may be nonidentical. Likewise, s_r^i, $s_{r0(t)}^0$, and $s_{rj(t)}^j$, $j\in\mathcal{N}_i$ are usually different with each other. Hence, $\delta_{i2}(t)$ is nonzero, $i=1,2,\ldots,N$. In other words, $\sum_{i=1}^N\|\delta_i(t)\|^2$ cannot tend to zero. That is, only quasi-consensus (bounded consensus) can be reached if $\sum_{i=1}^N\sup_{t\in[0,\infty)}\|\delta_i(t)\|^2$ is finite. Furthermore, $\sum_{i=1}^N\|\delta_i(t)\|^2$ will not vanish as the heterogeneities of dynamics, even if both position and velocity of the leader and all followers are sampled at the same instants.

Proposition 1: Under Assumptions 2 and 3, there exists a positive constant ϱ_i such that

$$\sup_{t\in[0,\infty)}\|\delta_i(t)\|\le\varrho_i,\quad i=1,2,\ldots,N.$$

Proof: According to Assumptions 2 and 3, there exists a $\widetilde{T}>0$ such that for all $t>\widetilde{T}$ one has

$$\begin{aligned}\|\delta_i(t)\|&\le\|f_i(x_0(t),v_0(t),t)-f_0(x_0(t),v_0(t),t)\|\\&\quad+\alpha_i\sum_{j=1}^{N}a_{ij}\left\|x_0\left(t_{kj(t)}^j\right)-x_0\left(t_k^i\right)\right\|\\&\quad+\beta_i\sum_{j=1}^{N}b_{ij}\left\|v_0\left(s_{rj(t)}^j\right)-v_0\left(s_r^i\right)\right\|\\&\quad+\alpha_i\gamma_i^a\left\|x_0\left(t_k^i\right)-x_0\left(t_{k0(t)}^0\right)\right\|\\&\quad+\beta_i\gamma_i^b\left\|v_0\left(s_r^i\right)-v_0\left(s_{r0(t)}^0\right)\right\|\\&\le\nu_i+2\mu\left[\sum_{j=1}^{N}\left(\alpha_ia_{ij}+\beta_ib_{ij}\right)+\alpha_i\gamma_i^a+\beta_i\gamma_i^b\right].\end{aligned}$$

Let $\varrho_i=\nu_i+2\mu[\sum_{j=1}^N(\alpha_ia_{ij}+\beta_ib_{ij})+\alpha_i\gamma_i^a+\beta_i\gamma_i^b]$. This completes the proof. ■

The following theorem is a straightforward conclusion by combining Theorem 1 and Proposition 1.

Theorem 2: If Assumptions 1–3 and conditions of Theorem 1 hold, then the trajectory of the error system (8) exponentially converges into a compact set

$$\mathcal{M}=\left\{\varepsilon\in\mathbb{R}^{2nN}:\|\varepsilon\|\le\sqrt{\frac{\rho}{\lambda_{\min}(E)\lambda}\sum_{i=1}^{N}\varrho_i^2}\right\}.$$

Remark 6: The result of Theorem 2 shows that quasi-consensus in heterogeneous multiagent systems (1) and (2) can be reached. In addition, an upper bound can be estimated. Some papers apply Barrier Lyapunov function to guarantee the boundedness of error [28], [45], [48]. If $\lim_{t\to\infty}\delta_i(t)=0$, $i=1,2,\ldots,N$, then consensus in heterogeneous multiagent system (1) and (2) can be achieved.

If the position information and velocity information have the same sampling frequencies, that is $\{t_k^i\}_{k=1}^{+\infty}$ is the same with $\{s_k^i\}_{k=1}^{+\infty}$ for $i=1,2,\ldots,N$, then $h_{ai}=h_{bi}$ and system (8) is rewritten as

$$\dot{\tilde{y}}(t)=D\tilde{y}(t)+\widetilde{F}(t)-\sum_{n=1}^{N}\widetilde{H}_n\tilde{y}(t-\tau_n(t))+\Delta(t)\quad(12)$$

where

$$\widetilde{H}_n = \begin{bmatrix} O_N & O_N \\ K^a H_n^a & K^b H_n^b \end{bmatrix} \otimes I_n.$$

Let $h_i = h_{ai} = h_{bi}$. Choose the Lyapunov–Krasovskii functional candidate as follows:

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{13}$$

where

$$\begin{aligned} V_1(t) &= \widetilde{y}^T(t)E\widetilde{y}(t) \\ V_2(t) &= \sum_{i=1}^{N} \int_{t-h_i}^{t} e^{\lambda(s-t)}\widetilde{y}^T(s)Q_i\widetilde{y}(s)ds \\ V_3(t) &= \sum_{i=1}^{N} h_i \int_{-h_i}^{0}\int_{t+\theta}^{t} e^{\lambda(s-t)}\dot{\widetilde{y}}^T(s)R_i\dot{\widetilde{y}}(s)dsd\theta \end{aligned}$$

with $\lambda > 0$, $E > 0$, $Q_i > 0$, and $R_i > 0$, $i = 1, 2, \dots, N$.

By Theorem 1, the following corollary holds directly for system (12).

Corollary 1: Suppose that Assumptions 1–3 hold. The trajectory of the error system (12) exponentially converges into a compact set

$$\mathcal{M} = \left\{ \varepsilon \in \mathbb{R}^{2nN} : \|\varepsilon\| \le \sqrt{\frac{\rho}{\lambda_{\min}(E)\lambda}\sum_{i=1}^{N}\varrho_i^2} \right\}$$

if there exist constants $\lambda > 0$, $\rho > 0$, and $h_i > 0$, and matrices $E > 0$, $Q_i > 0$, $R_i > 0$, S_1, and S_2 with appropriate dimensions, $i = 1, 2, \dots, N$, such that

$$\omega = \begin{bmatrix} \omega_{11} & \omega_{12} & S_1 & O & \omega_{15} & S_1 \\ * & \omega_{22} & S_2 & O & \omega_{25} & S_2 \\ * & * & -I & O & O & O \\ * & * & * & \omega_{44} & \omega_{45} & O \\ * & * & * & * & \omega_{55} & O \\ * & * & * & * & * & -\rho I \end{bmatrix} < 0 \tag{14}$$

where the blocks of ω are given by

$$\begin{aligned} \omega_{11} &= \lambda E + \sum_{i=1}^{N} Q_i - \sum_{i=1}^{N} e^{-\lambda h_i}R_i + S_1 D + D^T S_1^T + \Pi \\ \omega_{12} &= E - S_1 + D^T S_2^T \\ \omega_{15} &= \Big[e^{-\lambda h_1}R_1 - S_1\widetilde{H}_1, e^{-\lambda h_2}R_2 - S_1\widetilde{H}_2 \\ &\qquad , \dots, e^{-\lambda h_N}R_N - S_1\widetilde{H}_N\Big] \\ \omega_{22} &= \sum_{i=1}^{N} h_i^2 R_i - S_2 - S_2^T \\ \omega_{25} &= \big[-S_2\widetilde{H}_1, -S_2\widetilde{H}_2, \dots, -S_2\widetilde{H}_N\big] \\ \omega_{44} &= -\mathrm{diag}\Big\{e^{-\lambda h_i}(Q_i + R_i)\Big\}_{i=1}^{N} \\ \omega_{45} &= \mathrm{diag}\Big\{e^{-\lambda h_i}R_i\Big\}_{i=1}^{N} \\ \omega_{55} &= -2\mathrm{diag}\Big\{e^{-\lambda h_i}R_i\Big\}_{i=1}^{N}. \end{aligned}$$

Remark 7: If the dynamics of the leader and all followers are identical and the topologies of position and velocity are the same, then the multiagent systems are homogeneous. If asynchronous protocol (3) is applied, then only quasi-consensus of homogeneous systems can be reached under Assumptions 1–3 since $\delta_{i2}(t)$ cannot vanish. Furthermore, if sampling protocol is synchronous, then position and velocity of all the agents are sampled at the same instants. Thus, consensus of homogeneous systems will be reached since $\delta_i(t) = 0$ for all $i = 1, 2, \dots, N$.

B. Consensus Analysis With Nonuniform Input Delays

Because of the limited rate of network communication, networked systems usually exist input delays. Since multiagent system (2) is heterogeneous, it is reasonable to consider nonuniform input delays. Adopting the nonuniform delayed protocol $u_i(t - d_i)$ for multiagent system (2), where d_i is a time delay of the ith control input with $0 < d_i < d < h$, d is a positive constant.

Based on the sampled-data protocol (3) with nonuniform input delays, the following error systems can be obtained:

$$\begin{aligned} \dot{\widetilde{x}}_i(t) &= \widetilde{v}_i(t) \\ \dot{\widetilde{v}}_i(t) &= f_i(x_i(t), v_i(t), t) - f_i(x_0(t), v_0(t), t) \\ &\quad + \alpha_i \sum_{j=1}^{N} a_{ij}\Big[\widetilde{x}_j\Big(t_{k_j(t)}^j - d_i\Big) - \widetilde{x}_i\big(t_k^i - d_i\big)\Big] \\ &\quad + \beta_i \sum_{j=1}^{N} b_{ij}\Big[\widetilde{v}_j\Big(s_{r_j(t)}^j - d_i\Big) - \widetilde{v}_i\big(s_r^i - d_i\big)\Big] \\ &\quad - \alpha_i\gamma_i^a\widetilde{x}_i\big(t_k^i - d_i\big) - \beta_i\gamma_i^b\widetilde{v}_i\big(s_r^i - d_i\big) \\ &\quad + \widetilde{\delta}_i(t), \quad t \in \big[t_k^i, t_{k+1}^i\big) \cap \big[s_r^i, s_{r+1}^i\big) \end{aligned} \tag{15}$$

where $i = 1, 2, \dots, N$, and

$$\begin{aligned} \widetilde{\delta}_i(t) &= f_i(x_0(t), v_0(t), t) - f_0(x_0(t), v_0(t), t) \\ &\quad + \alpha_i \sum_{j=1}^{N} a_{ij}\Big[x_0\Big(t_{k_j(t)}^j - d_i\Big) - x_0\big(t_k^i - d_i\big)\Big] \\ &\quad + \beta_i \sum_{j=1}^{N} b_{ij}\Big[v_0\Big(s_{r_j(t)}^j - d_i\Big) - v_0\big(s_r^i - d_i\big)\Big] \\ &\quad - \alpha_i\gamma_i^a\Big[x_0\big(t_k^i - d_i\big) - x_0\Big(t_{k_0(t)}^0 - d_i\Big)\Big] \\ &\quad - \alpha_i\gamma_i^b\Big[v_0\big(s_r^i - d_i\big) - v_0\Big(s_{r_0(t)}^0 - d_i\Big)\Big]. \end{aligned} \tag{16}$$

Similar to Proposition 1, based on Assumptions 2 and 3, there exists a positive constant $\widetilde{\varrho}_i$ such that $\sup_{t\in[0,\infty)} \|\widetilde{\delta}_i(t)\| \le \widetilde{\varrho}_i$, $i = 1, 2, \dots, N$.

Then, systems (15) can be rewritten as the following compact form:

$$\begin{aligned} \dot{\widetilde{y}}(t) &= D\widetilde{y}(t) + \widetilde{F}(t) - \sum_{n,j=1}^{N} \widetilde{K}^a\widetilde{H}_{jn}^a\widetilde{y}\big(t - \tau_n(t) - d_j\big) \\ &\quad - \sum_{m,j=1}^{N} \widetilde{K}^b\widetilde{H}_{jm}^b\widetilde{y}\big(t - \sigma_m(t) - d_j\big) + \widetilde{\Delta}(t) \end{aligned} \tag{17}$$

where

$$\widetilde{\delta}(t) = \left[\widetilde{\delta}_1^T(t), \widetilde{\delta}_2^T(t), \dots, \widetilde{\delta}_N^T(t)\right]^T$$
$$\widetilde{\Delta}(t) = \left[0_{nN}^T, \widetilde{\delta}^T(t)\right]^T, \ \widetilde{H}_{jn}^a = \begin{bmatrix} O_N & O_N \\ H_{jn}^a & O_N \end{bmatrix} \otimes I_n$$
$$\widetilde{H}_{jm}^b = \begin{bmatrix} O_N & O_N \\ O_N & H_{jm}^b \end{bmatrix} \otimes I_n$$

the elements of matrix H_{ij}^ℓ are all zero except the (i,j)th entry is the same as the (i,j)th entry of H^ℓ, $\ell = a, b$, $i, j = 1, 2, \dots, N$.

Theorem 3: Suppose that Assumptions 1–3 hold. The trajectory of the error system (17) exponentially converges into a compact set

$$\mathcal{M} = \left\{ \varepsilon \in \mathbb{R}^{2nN} : \|\varepsilon\| \le \sqrt{\frac{\rho}{\lambda_{\min}(E)\lambda} \sum_{i=1}^N \widetilde{\varrho}_i^2} \right\}$$

if there exist constants $\lambda > 0$, $\rho > 0$, $h_{ai} > 0$, and $h_{bi} > 0$, and matrices $E > 0$, $P_i > 0$, $Q_i > 0$, $R_i > 0$, $W_i > 0$, S_1, and S_2 with appropriate dimensions, $i = 1, 2, \dots, N$, such that

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & S_1 & O & O & \Phi_{16} & \Phi_{17} & S_1 \\ * & \Phi_{22} & S_2 & O & O & \Phi_{26} & \Phi_{27} & S_2 \\ * & * & -I & O & O & O & O & O \\ * & * & * & \Phi_{44} & O & \Phi_{46} & O & O \\ * & * & * & * & \Phi_{55} & O & \Phi_{57} & O \\ * & * & * & * & * & \Phi_{66} & O & O \\ * & * & * & * & * & * & \Phi_{77} & O \\ * & * & * & * & * & * & * & -\rho I \end{bmatrix} < 0 \quad (18)$$

where

$$\Phi_{11} = \lambda E + \sum_{i=1}^N (P_i + Q_i) - N \sum_{i=1}^N e^{-\lambda(h_{ai}+d)} R_i - N \sum_{i=1}^N e^{-\lambda(h_{bi}+d)} W_i + S_1 D + D^T S_1^T + \Pi$$
$$\Phi_{12} = E - S_1 + D^T S_2^T$$
$$\Phi_{16} = \left[1_N^T \otimes e^{-\lambda(h_{a1}+d)} R_1, 1_N^T \otimes e^{-\lambda(h_{a2}+d)} R_2, \dots, 1_N^T \otimes e^{-\lambda(h_{aN}+d)} R_N\right] - S_1 \widetilde{K}^a \left[\hbar_1^a, \hbar_2^a, \dots, \hbar_N^a\right]$$
$$\Phi_{17} = \left[1_N^T \otimes e^{-\lambda(h_{b1}+d)} W_1, 1_N^T \otimes e^{-\lambda(h_{b2}+d)} W_2, \dots, 1_N^T \otimes e^{-\lambda(h_{bN}+d)} W_N\right] - S_1 \widetilde{K}^b \left[\hbar_1^b, \hbar_2^b, \dots, \hbar_N^b\right]$$
$$\hbar_i^\ell = \left[\widetilde{H}_{1i}^\ell, \widetilde{H}_{2i}^\ell, \dots, \widetilde{H}_{Ni}^\ell\right], \ i = 1, 2, \dots, N, \ \ell = a, b$$
$$\Phi_{22} = N \sum_{i=1}^N \left[(d + h_{ai})^2 R_i + (d + h_{bi})^2 W_i\right] - S_2 - S_2^T$$
$$\Phi_{26} = -S_2 \widetilde{K}^a \left[\hbar_1^a, \hbar_2^a, \dots, \hbar_N^a\right]$$
$$\Phi_{27} = -S_2 \widetilde{K}^b \left[\hbar_1^b, \hbar_2^b, \dots, \hbar_N^b\right]$$
$$\Phi_{44} = -\text{diag}\left\{e^{-\lambda(d+h_{ai})}(P_i + N R_i)\right\}_{i=1}^N$$
$$\Phi_{46} = \text{diag}\left\{1_N^T \otimes e^{-\lambda(d+h_{ai})} R_i\right\}_{i=1}^N$$
$$\Phi_{55} = -\text{diag}\left\{e^{-\lambda(d+h_{bi})}(Q_i + N W_i)\right\}_{i=1}^N$$
$$\Phi_{57} = \text{diag}\left\{1_N^T \otimes e^{-\lambda(d+h_{bi})} W_i\right\}_{i=1}^N$$
$$\Phi_{66} = -2\text{diag}\left\{I_N^T \otimes e^{-\lambda(d+h_{ai})} R_i\right\}_{i=1}^N$$
$$\Phi_{77} = -2\text{diag}\left\{I_N^T \otimes e^{-\lambda(d+h_{bi})} W_i\right\}_{i=1}^N.$$

Proof: See Appendix B for the details. ■

If the networked leader-following systems (1) and (2) have the uniform input delays $u_i(t-d)$ for all $i = 1, 2, \dots, N$, then system (17) should be changed as follows:

$$\dot{\widetilde{y}}(t) = D\widetilde{y}(t) + \widetilde{F}(t) - \sum_{n=1}^N \widetilde{K}^a \widetilde{H}_n^a \widetilde{y}(t - \tau_n(t) - d) - \sum_{m=1}^N \widetilde{K}^b \widetilde{H}_m^b \widetilde{y}(t - \sigma_m(t) - d) + \widetilde{\Delta}_d(t) \quad (19)$$

where $\widetilde{\Delta}_d(t) = [0_{nN}^T, \widetilde{\delta}_d^T(t)]^T$, $\widetilde{\delta}_d(t) = [\widetilde{\delta}_{d1}^T(t), \widetilde{\delta}_{d2}^T(t), \dots, \widetilde{\delta}_{dN}^T(t)]^T$ and

$$\begin{aligned}\widetilde{\delta}_{di}(t) = {} & f_i(x_0(t), v_0(t), t) - f_0(x_0(t), v_0(t), t) \\ & + \alpha_i \sum_{j=1}^N a_{ij}\left[x_0\left(t_{k_j(t)}^j - d\right) - x_0\left(t_k^i - d\right)\right] \\ & + \beta_i \sum_{j=1}^N b_{ij}\left[v_0\left(s_{r_j(t)}^j - d\right) - v_0\left(s_r^i - d\right)\right] \\ & - \alpha_i \gamma_i^a\left[x_0\left(t_k^i - d\right) - x_0\left(t_{k_0(t)}^0 - d\right)\right] \\ & - \alpha_i \gamma_i^b\left[v_0\left(s_r^i - d\right) - v_0\left(s_{r_0(t)}^0 - d\right)\right]. \end{aligned} \quad (20)$$

Combining Theorems 1 and 3 derives the following result directly.

Corollary 2: Suppose that Assumptions 1–3 hold. The trajectory of the error system (19) exponentially converges into a compact set

$$\mathcal{M} = \left\{ \varepsilon \in \mathbb{R}^{2nN} : \|\varepsilon\| \le \sqrt{\frac{\rho}{\lambda_{\min}(E)\lambda} \sum_{i=1}^N \widetilde{\varrho}_i^2} \right\}$$

if there exist constants $\lambda > 0$, $\rho > 0$, $h_{ai} > 0$, $h_{bi} > 0$, and $d > 0$, and matrices $E > 0$, $P_i > 0$, $Q_i > 0$, $R_i > 0$, $W_i > 0$, S_1, and S_2 with appropriate dimensions, $i = 1, 2, \dots, N$, such that

$$\phi = \begin{bmatrix} \phi_{11} & \phi_{12} & S_1 & O & O & \phi_{16} & \phi_{17} & S_1 \\ * & \phi_{22} & S_2 & O & O & \Omega_{26} & \Omega_{27} & S_2 \\ * & * & -I & O & O & O & O & O \\ * & * & * & \phi_{44} & O & \phi_{46} & O & O \\ * & * & * & * & \phi_{55} & O & \phi_{57} & O \\ * & * & * & * & * & \phi_{66} & O & O \\ * & * & * & * & * & * & \phi_{77} & O \\ * & * & * & * & * & * & * & -\rho I \end{bmatrix} < 0 \quad (21)$$

where Ω_{26} and Ω_{27} are defined in Theorem 1, and the other blocks of ϕ are defined as follows:

$$\phi_{11} = \lambda E + \sum_{i=1}^{N}(P_i + Q_i) + S_1 D + D^T S_1^T + \Pi - \sum_{i=1}^{N}\left[e^{-\lambda(h_{ai}+d)}R_i + e^{-\lambda(h_{bi}+d)}W_i\right]$$

$$\phi_{12} = E - S_1 + D^T S_2^T$$

$$\phi_{16} = \left[e^{-\lambda(h_{a1}+d)}R_1, e^{-\lambda(h_{a2}+d)}R_2, \ldots, e^{-\lambda(h_{aN}+d)}R_N\right] - S_1\widetilde{K}^a\left[\widetilde{H}_1^a, \widetilde{H}_2^a, \ldots, \widetilde{H}_N^a\right]$$

$$\phi_{17} = \left[e^{-\lambda(h_{b1}+d)}W_1, e^{-\lambda(h_{b2}+d)}W_2, \ldots, e^{-\lambda(h_{bN}+d)}W_N\right] - S_1\widetilde{K}^b\left[\widetilde{H}_1^b, \widetilde{H}_2^b, \ldots, \widetilde{H}_N^b\right]$$

$$\phi_{22} = \sum_{i=1}^{N}\left[(h_{ai}+d)^2 R_i + (h_{bi}+d)^2 W_i\right] - S_2 - S_2^T$$

$$\phi_{44} = -\text{diag}\left\{e^{-\lambda(h_{ai}+d)}(P_i + R_i)\right\}_{i=1}^{N}$$

$$\phi_{46} = \text{diag}\left\{e^{-\lambda(h_{ai}+d)}R_i\right\}_{i=1}^{N}$$

$$\phi_{55} = -\text{diag}\left\{e^{-\lambda(h_{bi}+d)}(Q_i + W_i)\right\}_{i=1}^{N}$$

$$\phi_{57} = \text{diag}\left\{e^{-\lambda(h_{bi}+d)}W_i\right\}_{i=1}^{N}$$

$$\phi_{66} = -2\text{diag}\left\{e^{-\lambda(h_{ai}+d)}R_i\right\}_{i=1}^{N}$$

$$\phi_{77} = -2\text{diag}\left\{e^{-\lambda(h_{bi}+d)}W_i\right\}_{i=1}^{N}.$$

Remark 8: Compared with the evident delays d_i in delayed input $u_i(t - d_i)$, protocol (3) has hidden delays induced by asynchronous sampling. Since each agent and its neighbors are sampled asynchronously, delays are induced in transmissions of information.

IV. Numerical Simulations

In this section, numerical simulations are provided to illustrate the theoretical results.

Example 1: Consensus of heterogeneous multiagent systems with delay-free input.

Consider a heterogeneous multiagent system (2) composed of three agents. The intrinsic nonlinear dynamics f_i of the ith agent is described by a Chua's circuit

$$f_i(x_i(t), v_i(t), t) = A_i v_i(t) + B_i g(v_i(t)) \tag{22}$$

where $x_i(t), v_i(t) \in \mathbb{R}^3$, $g(v_i(t)) = [0.5(|v_{i1} + 1| - |v_{i1} - 1|), 0, 0]^T$, and

$$A_1 = \begin{bmatrix} -0.0750 & 0.3000 & 0 \\ 0.0300 & -0.0300 & 0.0300 \\ 0 & -0.5400 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0.1750 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -0.0675 & 0.2700 & 0 \\ 0.0300 & -0.0300 & 0.0300 \\ 0 & -0.5400 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0.1000 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

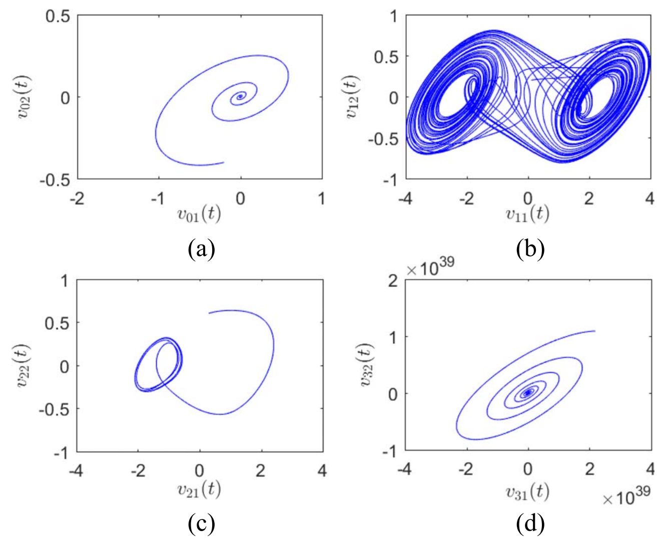

Fig. 1. Trajectories of the leader and three followers.

$$A_3 = \begin{bmatrix} -0.0975 & 0.3900 & 0 \\ 0.0300 & -0.0300 & 0.0300 \\ 0 & -0.5400 & -0.0030 \end{bmatrix}, B_3 = \begin{bmatrix} 0.0500 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The nonlinear dynamics of the leader f_0 is also described by a Chua's circuit

$$f_0(x_0(t), v_0(t), t) = A_0 v_0(t) + B_0 g(v_0(t)) \tag{23}$$

where

$$A_0 = \begin{bmatrix} -0.0750 & 0.3000 & 0 \\ 0.0300 & -0.0300 & 0.0300 \\ 0 & -0.5400 & -0.0300 \end{bmatrix}, B_0 = \begin{bmatrix} 0.0100 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The trajectories of the leader and three followers are depicted in Fig. 1.

It is shown from Fig. 1 that four agents have different dynamics with each other. Four agents are stable, chaotic, oscillating, and unstable, respectively. Furthermore, f_i satisfies

$$\begin{aligned} &\|f_i(x_i(t), v_i(t), t) - f_i(x_0(t), v_0(t), t)\| \\ &\quad \leq \|A_i(v_i(t) - v_0(t))\| + \|B_i(g(v_i(t)) - g(v_0(t)))\| \\ &\quad \leq (\|A_i\| + \|B_i\|)\|v_i(t) - v_0(t)\|. \end{aligned} \tag{24}$$

Assume that the Laplacian matrices of the position topology and velocity topology are chosen by

$$L^a = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}, \quad L^b = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}.$$

Chosen $\gamma^a = \{0, 3.5, 0\}$, $\gamma^b = \{2.9, 0, 0\}$, $K^a = \{7, 7.7, 8.4\}$, and $K^b = \{6.5, 7.15, 7.8\}$ for consensus protocol (3). Let $\lambda = 0.1$ and $\rho = 4$. By solving LMI (11), an allowance bound of sampling intervals is obtained as $h = 0.045$. For simplicity, asynchronous period sampling is considered. Chosen $h_{0x} = h_{0v} = 0.04, h_{a1} = 0.04, h_{a2} = 0.035, h_{a3} = 0.02, h_{b1} = 0.02, h_{b2} = 0.015$, for $h_{b3} = 0.01$ for asynchronous sampling protocol (3).

The trajectories of the position and velocity states of four agents are sketched in Figs. 2 and 3, respectively.

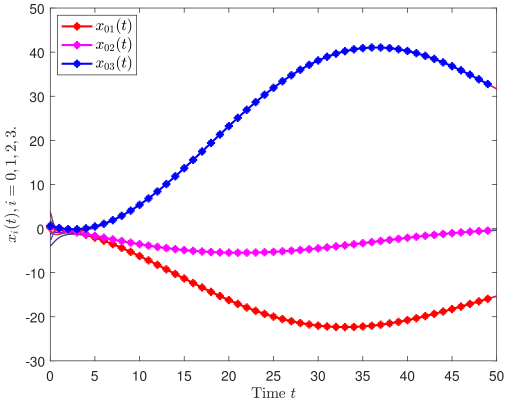

Fig. 2. Trajectories of the position states of four agents.

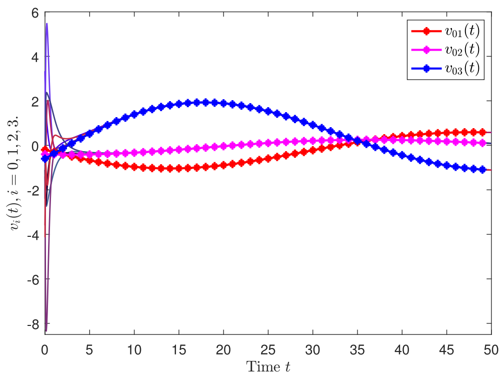

Fig. 3. Trajectories of the velocity states of four agents.

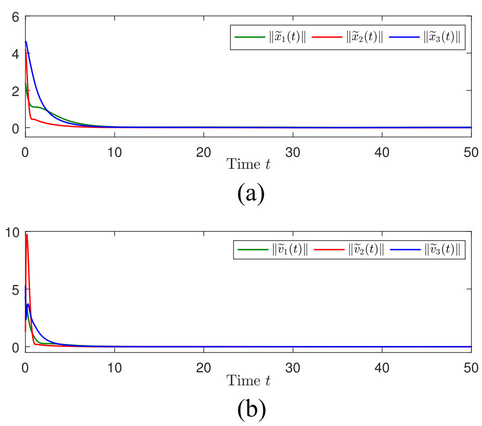

Fig. 4. Error trajectories of (a) the position states and (b) the velocity states.

The marked dotted lines are three states of the position and velocity of the leader in Figs. 2 and 3. The other lines represent three states of all the followers. Figs. 2 and 3 show that the followers can track the leader with asynchronous sampling information and heterogeneous topologies of position and velocity interactions.

The error trajectories of the position and velocity states between each follower and the leader are sketched in Fig. 4(a) and (b), respectively.

It is shown from Fig. 1(a) and (23) that three states of the position and velocity of the leader tend to constant vector and zero vector, respectively. Combining (5) and (22) gives

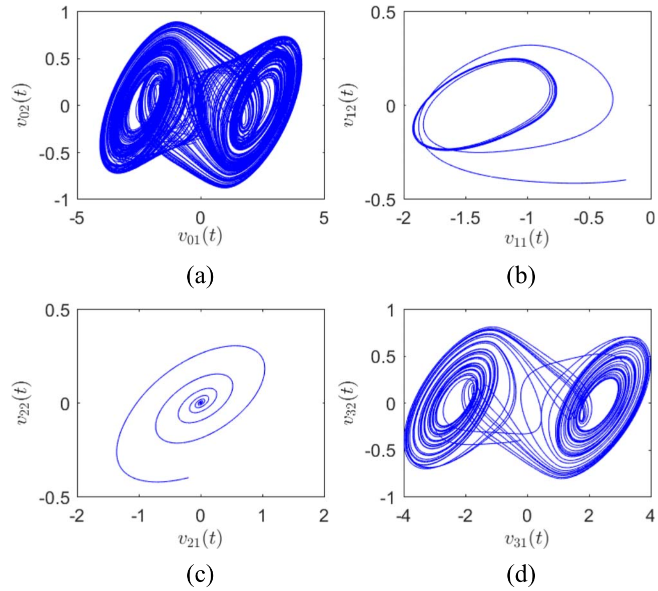

Fig. 5. Trajectories of the leader and three followers.

$\lim_{t\to\infty} \|\delta_i(t)\| = 0$, $i = 1, 2, \ldots, N$. Therefore, consensus in heterogeneous (1) and (2) is reached based on Theorem 1 and Remark 6. Consensus is also verified in Figs. 2–4.

Example 2: Consensus of heterogeneous multiagent systems with nonuniform input delays.

Suppose that a heterogeneous leader-following multiagent systems composed of four agents have the same forms as Example 1, but the parameters of the nonlinear dynamics are different, where

$$A_0 = \begin{bmatrix} -2.5000 & 10.0000 & 0 \\ 1.0000 & -1.0000 & 1.0000 \\ 0 & -18.0000 & 0 \end{bmatrix}, B_0 = \begin{bmatrix} 5.8333 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} -0.0660 & 0.2700 & 0 \\ 0.0300 & -0.0330 & 0.0300 \\ 0 & -0.5400 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0.0930 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -0.0975 & 0.3900 & 0 \\ 0.0300 & -0.0300 & 0.0300 \\ 0 & -0.5400 & -0.0300 \end{bmatrix}, B_2 = \begin{bmatrix} 0.0300 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -0.0625 & 0.2500 & 0 \\ 0.0250 & -0.0250 & 0.0250 \\ 0 & -0.4500 & 0 \end{bmatrix}, B_3 = \begin{bmatrix} 0.1458 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The trajectories of the leader and three followers are drawn in Fig. 5.

Chosen the same values of $\gamma^a, \gamma^b, K^a, K^b, \lambda$, and ρ as Example 1. By solving LMI (18), an allowance bound of sampling intervals is obtained as $h = 0.029$. Set $h_{0x} = h_{0v} = 0.02, h_{a1} = 0.025, h_{a2} = 0.015, h_{a3} = 0.008, h_{b1} = 0.02, h_{b2} = 0.028$, and $h_{b3} = 0.016$. Given the nonuniform input delays as $d_1 = 0.01, d_2 = 0.02$, and $d_3 = 0.015$. The trajectories of the position and velocity states of four agents are sketched in Figs. 6 and 7, respectively.

Similar to Example 1, the marked dotted lines are three states of the position and velocity of the leader in Figs. 6 and 7. The other lines represent three states of all the followers. Figs. 6 and 7 show that the followers can track the leader

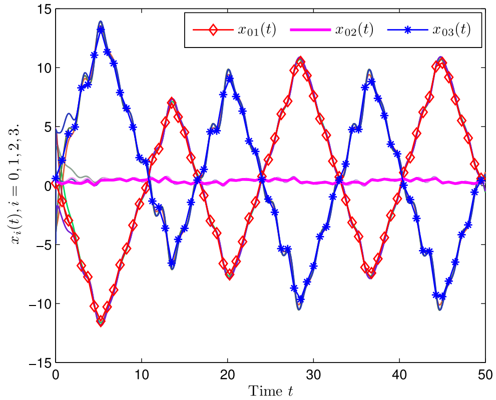

Fig. 6. Trajectories of the position states of four agents.

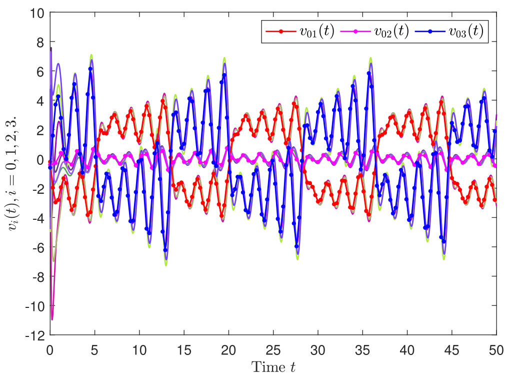

Fig. 7. Trajectories of the velocity states of four agents.

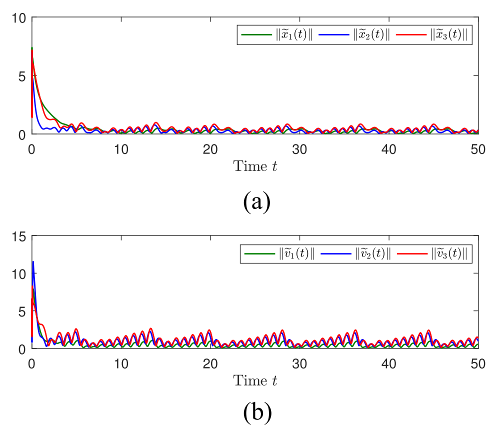

Fig. 8. Error trajectories of (a) the position states and (b) the velocity states.

within a bounded range. In other words, quasi-consensus is achieved in heterogeneous multiagent systems with asynchronous sampling and nonuniform input delays.

The error trajectories of the position and velocity states between each follower and the leader are sketched in Fig. 8(a) and (b), respectively.

Since the leader is bounded in Fig. 5(a), we have from (16) and (22) that $\sup_{t\in[0,\infty)} \|\tilde{\delta}_i(t)\|$ is bounded, $i = 1, 2, \ldots, N$. That is, quasi-consensus in heterogeneous multiagent systems is reached. Fig. 8(a) and (b) shows that the errors of the position and velocity between the leader and each follower cannot vanish.

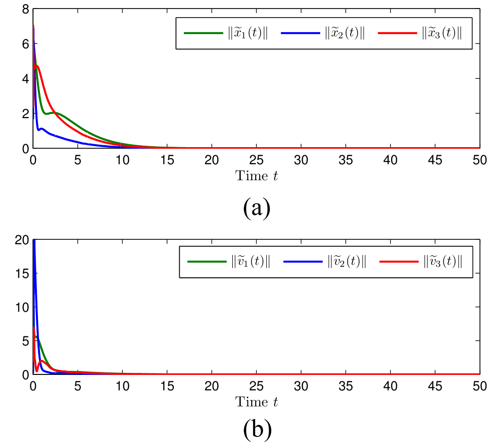

Fig. 9. Error trajectories for identical dynamics and synchronous sampling.

If the dynamics of three followers are chosen as $f_1 = f_2 = f_3 = f_0$ and synchronous sampling is adopted by choosing sampling interval $h = 0.02$, then $\tilde{\delta}_i(t) = 0$. Thus, consensus in leader-following systems is reached. Fig. 9(a) and (b) shows that the errors of the position and velocity between the leader and each follower tend to zero.

V. Conclusion

This paper has analyzed the consensus problem of heterogeneous second-order leader-following nonlinear multiagent systems. The position and velocity interactions have different topologies. Asynchronous sampled-data protocols are proposed. Each agent has independent sampling instants and is only sampled at its own sampling instants. In addition, sampling mechanisms over position and velocity interaction topologies are also independent of each other. Sufficient conditions for quasi-consensus of heterogeneous multiagent systems are first obtained by employing a delay-free asynchronous sampled-data control. Then, the results are extended to the control input with nonuniform input delays. Although the leader-following multiagent systems have nonidentical nonlinear dynamics, heterogeneous interaction topologies, asynchronous sampled data and even nonuniform input delays, quasi-consensus can still be reached. That is, all the followers can track the leader within a bound range. The upper bound of quasi-consensus errors is estimated. Future works include the studies on the consensus problem for heterogeneous multiagent systems with switching topologies and stochastic sampling.

Appendix A
Proof of Theorem 1

Let $U(t) = \dot{V}(t) + \lambda V(t) - \rho\Delta^T(t)\Delta(t)$. Calculating the derivation of $V(t)$ along the trajectory of (8) and substituting the derivation of $V(t)$ into $U(t)$ gives

$$U(t) \leq 2\tilde{y}^T(t)E\dot{\tilde{y}}(t) + \tilde{y}^T(t)\left[\lambda E + \sum_{i=1}^{N}(P_i + Q_i)\right]\tilde{y}(t)$$

$$\begin{aligned}
&+ \dot{\tilde{y}}^T(t)\left[\sum_{i=1}^{N}\left(h_{ai}^2 R_i + h_{bi}^2 W_i\right)\right]\dot{\tilde{y}}(t) - \rho\Delta^T(t)\Delta(t)\\
&-\sum_{i=1}^{N} e^{-\lambda h_{ai}}\tilde{y}^T(t-h_{ai})P_i\tilde{y}(t-h_{ai})\\
&-\sum_{i=1}^{N} e^{-\lambda h_{bi}}\tilde{y}^T(t-h_{bi})Q_i\tilde{y}(t-h_{bi})\\
&-\sum_{i=1}^{N} e^{-\lambda h_{ai}} h_{ai}\int_{t-h_{ai}}^{t}\dot{\tilde{y}}^T(s)R_i\dot{\tilde{y}}(s)ds\\
&-\sum_{i=1}^{N} e^{-\lambda h_{bi}} h_{bi}\int_{t-h_{bi}}^{t}\dot{\tilde{y}}^T(s)W_i\dot{\tilde{y}}(s)ds. \qquad (25)
\end{aligned}$$

Based on the Jensen inequality of Lemma 1, we have

$$\begin{aligned}
&-h_{ai}\int_{t-h_{ai}}^{t}\dot{\tilde{y}}^T(s)R_i\dot{\tilde{y}}(s)ds\\
&= -h_{ai}\int_{t-h_{ai}}^{t-\tau_i}\dot{\tilde{y}}^T(s)R_i\dot{\tilde{y}}(s)ds - h_{ai}\int_{t-\tau_i}^{t}\dot{\tilde{y}}^T(s)R_i\dot{\tilde{y}}(s)ds\\
&\le -(h_{ai}-\tau_i)\int_{t-h_{ai}}^{t-\tau_i}\dot{\tilde{y}}^T(s)R_i\dot{\tilde{y}}(s)ds - \tau_i\int_{t-\tau_i}^{t}\dot{\tilde{y}}^T(s)R_i\dot{\tilde{y}}(s)ds\\
&\le -\left[\tilde{y}(t-\tau_i)-\tilde{y}(t-h_{ai})\right]^T R_i\left[\tilde{y}(t-\tau_i)-\tilde{y}(t-h_{ai})\right]\\
&\quad - \left[\tilde{y}(t)-\tilde{y}(t-\tau_i)\right]^T R_i\left[\tilde{y}(t)-\tilde{y}(t-\tau_i)\right]\\
&= -\left[\tilde{y}^T(t-\tau_i), \tilde{y}^T(t-h_{ai})\right]\begin{bmatrix} R_i & -R_i\\ * & R_i\end{bmatrix}\begin{bmatrix}\tilde{y}(t-\tau_i)\\ \tilde{y}(t-h_{ai})\end{bmatrix}\\
&\quad - \left[\tilde{y}^T(t), \tilde{y}^T(t-\tau_i)\right]\begin{bmatrix} R_i & -R_i\\ * & R_i\end{bmatrix}\begin{bmatrix}\tilde{y}(t)\\ \tilde{y}(t-\tau_i)\end{bmatrix}. \qquad (26)
\end{aligned}$$

Similarly,

$$\begin{aligned}
&-h_{bi}\int_{t-h_{bi}}^{t}\dot{\tilde{y}}^T(s)W_i\dot{\tilde{y}}(s)ds\\
&\le -\left[\tilde{y}(t-\sigma_i)-\tilde{y}(t-h_{bi})\right]^T W_i\left[\tilde{y}(t-\sigma_i)-\tilde{y}(t-h_{bi})\right]\\
&\quad - \left[\tilde{y}(t)-\tilde{y}(t-\sigma_i)\right]^T W_i\left[\tilde{y}(t)-\tilde{y}(t-\sigma_i)\right]\\
&= -\left[\tilde{y}^T(t-\sigma_i), \tilde{y}^T(t-h_{bi})\right]\begin{bmatrix} W_i & -W_i\\ * & W_i\end{bmatrix}\begin{bmatrix}\tilde{y}(t-\sigma_i)\\ \tilde{y}(t-h_{bi})\end{bmatrix}\\
&\quad - \left[\tilde{y}^T(t), \tilde{y}^T(t-\sigma_i)\right]\begin{bmatrix} W_i & -W_i\\ * & W_i\end{bmatrix}\begin{bmatrix}\tilde{y}(t)\\ \tilde{y}(t-\sigma_i)\end{bmatrix}. \qquad (27)
\end{aligned}$$

Let $e(t) = [\tilde{y}^T(t), \dot{\tilde{y}}^T(t), \tilde{F}^T(t), \xi_1^T(t), \xi_2^T(t), \xi_3^T(t), \xi_4^T(t), \Delta^T(t)]^T$, where

$$\begin{aligned}
\xi_1(t) &= \left[\tilde{y}^T(t-h_{a1}), \tilde{y}^T(t-h_{a2}), \ldots, \tilde{y}^T(t-h_{aN})\right]^T\\
\xi_2(t) &= \left[\tilde{y}^T(t-h_{b1}), \tilde{y}^T(t-h_{b2}), \ldots, \tilde{y}^T(t-h_{bN})\right]^T\\
\xi_3(t) &= \left[\tilde{y}^T(t-\tau_1(t)), \tilde{y}^T(t-\tau_2(t)), \ldots, \tilde{y}^T(t-\tau_N(t))\right]^T\\
\xi_4(t) &= \left[\tilde{y}^T(t-\sigma_1(t)), \tilde{y}^T(t-\sigma_2(t)), \ldots, \tilde{y}^T(t-\sigma_N(t))\right]^T.
\end{aligned}$$

Substituting (9), (26), and (27) into (25) derives

$$U(t) \le e^T(t)\Omega e(t).$$

It follows from the condition $\Omega < 0$ that:

$$\dot{V}(t) + \lambda V(t) - \rho\Delta^T(t)\Delta(t) < 0. \qquad (28)$$

From (28), one can get the following inequality directly:

$$V(t) < \rho\int_0^t e^{\lambda(s-t)}\Delta^T(s)\Delta(s)ds + e^{-\lambda t}V(0).$$

Consequently, $\lim_{t\to\infty} y^T(t)Ey(t) \le (\rho/\lambda)\sum_{i=1}^{N}\sup_{t\in[0,\infty)}\|\delta_i(t)\|^2$.

According to the conditions of Theorem 1, consensus errors $\tilde{x}(t), \tilde{v}(t)$ exponentially converge to set $\mathcal{M}$ under protocol (3). This completes the proof.

Appendix B
Proof of Theorem 2

Construct the Lyapunov–Krasovskii functional

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t)$$

where

$$\begin{aligned}
V_1(t) &= \tilde{y}^T(t)E\tilde{y}(t)\\
V_2(t) &= \sum_{i=1}^{N}\int_{t-d-h_{ai}}^{t} e^{\lambda(s-t)}\tilde{y}^T(s)P_i\tilde{y}(s)ds\\
V_3(t) &= \sum_{i=1}^{N}\int_{t-d-h_{bi}}^{t} e^{\lambda(s-t)}\tilde{y}^T(s)Q_i\tilde{y}(s)ds\\
V_4(t) &= N\sum_{i=1}^{N}(h_{ai}+d)\int_{-h_{ai}-d}^{0}\int_{t+\theta}^{t}\\
&\qquad e^{\lambda(s-t)}\dot{\tilde{y}}^T(s)R_i\dot{\tilde{y}}(s)dsd\theta\\
V_5(t) &= N\sum_{i=1}^{N}(h_{bi}+d)\int_{-h_{bi}-d}^{0}\int_{t+\theta}^{t}\\
&\qquad e^{\lambda(s-t)}\dot{\tilde{y}}^T(s)W_i\dot{\tilde{y}}(s)dsd\theta
\end{aligned}$$

with the constant $\lambda > 0$, and matrices $E > 0, P_i > 0, Q_i > 0, R_i > 0$, and $W_i > 0$.

Let $e(t) = [\tilde{y}^T(t), \dot{\tilde{y}}^T(t), \tilde{F}^T(t), \xi_a^T(t), \xi_b^T(t), \xi_\tau^T(t), \xi_\sigma^T(t), \tilde{\Delta}^T(t)]^T$, where

$$\begin{aligned}
\xi_a(t) &= \left[\tilde{y}^T(t-h_{a1}-d), \tilde{y}^T(t-h_{a2}-d)\right.\\
&\qquad \left. , \ldots, \tilde{y}^T(t-h_{aN}-d)\right]^T\\
\xi_b(t) &= \left[\tilde{y}^T(t-h_{b1}-d), \tilde{y}^T(t-h_{b2}-d)\right.\\
&\qquad \left. , \ldots, \tilde{y}^T(t-h_{bN}-d)\right]^T\\
\xi_\tau(t) &= \left[\xi_{\tau 1}^T(t), \xi_{\tau 2}^T(t), \ldots, \xi_{\tau N}^T(t)\right]^T\\
\xi_{\tau i}(t) &= \left[\tilde{y}^T(t-\tau_i(t)-d_1), \tilde{y}^T(t-\tau_i(t)-d_2)\right.\\
&\qquad \left. , \ldots, \tilde{y}^T(t-\tau_i(t)-d_N)\right]^T\\
\xi_\sigma(t) &= \left[\xi_{\sigma 1}^T(t), \xi_{\sigma 2}^T(t), \ldots, \xi_{\sigma N}^T(t)\right]^T\\
\xi_{\sigma i}(t) &= \left[\tilde{y}^T(t-\sigma_i(t)-d_1), \tilde{y}^T(t-\sigma_i(t)-d_2)\right.\\
&\qquad \left. , \ldots, \tilde{y}^T(t-\sigma_i(t)-d_N)\right]^T, \quad i = 1, 2, \ldots, N.
\end{aligned}$$

The remainder proof is similar to Theorem 1, therefore, it is omitted.

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