Robustness of Interdependent Geometric Networks Under Inhomogeneous Failures

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***Abstract*—Complex systems such as smart cities and smart power grids rely heavily on their interdependent components. The failure of a component in one network may lead to the failure of the supported component in another network. Components which support a large number of interdependent components may be more vulnerable to attacks and failures. In this paper, we study the robustness of two interdependent networks under node failures. By modeling each network using a random geometric graph (RGG), we study conditions for the percolation of two interdependent RGGs after inhomogeneous node failures. We derive analytical bounds on the interdependent degree thresholds (k_1, k_2), such that the interdependent RGGs percolate after removing nodes in G_i that support more than k_j nodes in G_j $(\forall i, j \in \{1, 2\}, i \neq j)$. We verify the bounds using numerical simulation, and show that there is a trade-off between k_1 and k_2 for maintaining percolation after the failures.**

I. Introduction

Large-scale networks are integral parts of complex cyber-physical systems and smart cities. Various entities from different networks rely on each other and boost each other's performance. For example, smart power grids depend on the designated communication network for effective control. At the same time, control centers and communication networks receive electrical power from the power grid to compute and transmit control signals. Failures in one network could have an impact on the other network, and could eventually lead to the failure of the whole system.

Cascading failures of interdependent networks under uniformly random node removals were first studied in [1]. Each network is modeled as a random graph. Nodes in one random graph depend on nodes in the other random graph, and a node is functional if both itself and its interdependent node are in the giant components of the their own graphs. After initial node failures in the first graph, their interdependent nodes in the second graph fail. Because of these failures, some nodes may be disconnected from the giant component of the second graph and then fail, and the failures of the disconnected nodes propagate back to nodes in the first graph.

Physical networks are spatially embedded. Nearby nodes are more likely to be connected, whereas distant nodes are less likely to be connected. Geographical proximity also applies to interdependence. In the Italy blackout in September 2003, where a cascading failure between power and communication networks took place, it is reported that every server in the communication network is connected to the geographically closest power station [1]. In [2], [3], an interdependent lattices model was proposed and the effect of removing nodes randomly or in a geographical region was analyzed. In [4], an interdependent Random Geometric Graphs (RGG) model was proposed. This model allows the two networks to have different sizes and average link lengths and also captures one-to-multiple dependencies with spatial constraints. The authors in [4] studied bounds on the percolation thresholds of interdependent RGGs. They studied the effect of random failures where every node fails independently with the same probability, and geographical failures where nodes are removed in a geographical region.

In real-world networks, failures may not be equally likely for every node. In a virus epidemic that infects smart grids, a power station with a large number of controllable communication nodes is more prone to infection and failure. Moreover, a power station that support a larger number of communication nodes is more likely to suffer physical attacks. Cyber attack is recognized as one of the major threats to the power grid [5], and the attack on a control center that controls a large number of power plants could have a more severe impact. In this paper, we study the robustness of interdependent RGGs under a failure model where the failure of each node depends on the number of its interdependent nodes. Our paper extends the analysis of degree-dependent failures in a single RGG in [6]. To the best of our knowledge, our paper is the first to address robustness of interdependent RGGs to non-uniform node failures in an analytical manner.

The main contributions of this paper are as follows.

1) We derive upper bounds on the interdependent degree thresholds (k_1, k_2), such that the interdependent RGGs percolate after the removals of nodes in G_i that are interdependent with more than k_j nodes in G_j $(\forall i, j \in \{1, 2\}, i \neq j)$.
2) We obtain a new closed-form upper bound on

the degree threshold k, such that a single RGG remains percolated after the removals of nodes with more than k neighbors. We then apply the result to obtain upper bounds on the interdependent degree thresholds (k_1, k_2) for interdependent RGGs.

3) We verify the bounds using numerical simulation, and show that there is a trade-off between k_1 and k_2 for maintaining percolation after the failures.

II. MODEL

We study two interdependent Poisson boolean models $G_{\text{IntDep}} = (G_1(\lambda_1, d_1), G_2(\lambda_2, d_2), d_{\text{dep}})$. In each graph $G_i(\lambda_i, d_i)$, nodes are generated by a Poisson point process of density λ_i in an infinite two-dimensional plane. Two nodes are connected if they are within distance d_i ($\forall i \in \{1, 2\}$). Every node in G_i is interdependent with all nodes in G_j within distance d_{dep} ($\forall i, j \in \{1, 2\}, i \neq j$). Every node must have at least one interdependent node to be functional.

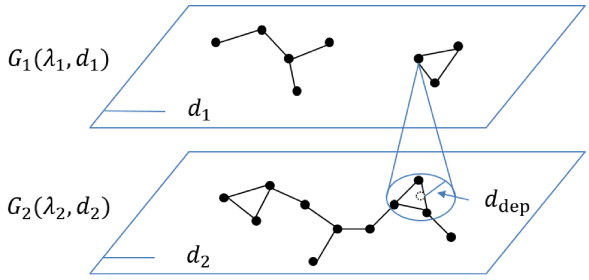


Fig. 1: Illustration of interdependent networks $G_{\text{IntDep}} = (G_1(\lambda_1, d_1), G_2(\lambda_2, d_2), d_{\text{dep}})$.

Recall the definition of a mutual component and an infinite mutual component in interdependent graphs, as given in [4].

Definition 1. Let V_i^0 denote nodes in a connected component in $G_i(\lambda_i, d_i)$, $\forall i \in \{1, 2\}$. If each node in $V_i \subseteq V_i^0$ has at least one supply node in $V_j \subseteq V_j^0$ within d_{dep}, $\forall i, j \in \{1, 2\}, i \neq j$, then nodes V_1 and V_2 form a *mutual component* of G_{IntDep}.

If, in addition, V_i contains an infinite number of nodes, $\forall i \in \{1, 2\}$, then V_1 and V_2 form an *infinite mutual component*.

The graph G_{IntDep} *percolates* if there exists an infinite mutual component. The conditions for percolation have been studied in [4], where nodes in both graphs are generated by homogeneous Poisson point processes. In this paper, we study the percolation of G_{IntDep} after removing nodes that have a large number of interdependent nodes. The removed nodes potentially have a significant impact on the percolation of G_{IntDep}, since they support a large number of nodes in the other graph. A power station that provides electrical power for a large number of routers, or a control center responsible for control of smart power stations in a large region, may both be vital in maintaining the robustness of the system.

The effect of removing such nodes cannot be viewed as a "thinning" of the original point process, since the remaining nodes do not follow Poisson point processes. This is therefore the main technical challenge in studying the percolation of G_{IntDep}, after the removal of all nodes with a large number of interdependent nodes. To address this issue, we develop mappings from Poisson point processes with inhomogeneous node removals to discrete models, and utilize the discrete models to derive conditions for the percolation of G_{IntDep}.

III. PERCOLATION AFTER INHOMOGENEOUS NODE FAILURES

In this section, we provide a sufficient condition for G_{IntDep} to percolate, after removing all the nodes that are interdependent with a large number of nodes.

We first introduce a structure which is useful in our analysis. Consider a $(2D - 2d) \times (D - 2d)$ rectangle $R(D)$, where $d = \max(d_1, d_2, d_{\text{dep}})$. If the mutual component in G_{IntDep} forms a horizontal crossing[1] over $R(D)$, and a vertical crossing over the $(D - 2d) \times (D - 2d)$ square within the left half of $R(D)$, then the rectangle is *complete*. See Fig. 2 for an illustration.

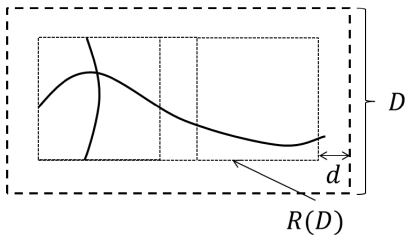


Fig. 2: Complete rectangle $R(D)$.

Consider two complete rectangles that have a sufficiently large overlapping area. The mutual component across one rectangle is connected to the mutual component across the other rectangle. Intuitively, if the probability that the rectangle is complete, denoted by $p_c[R(D)]$, is sufficiently large, then the mutual components across infinitely many overlapping rectangles could form an infinite mutual component. This method has been used to study the percolation of $G(\lambda, d)$ and G_{IntDep} in [7], [4].

In this section, we study G_{IntDep} which satisfies the condition that $p_c[R(D)]$ *is greater than 0.8639 for*

[1]A mutual component forms a horizontal crossing over a rectangle $[x_1, x_2] \times [y_1, y_2]$, if among the nodes in the mutual component, there exist a sequence of connected nodes in G_i, $(x_i^1, y_i^1), \ldots, (x_i^m, y_i^m)$, such that $x_i^2, \ldots, x_i^{m-1} \in [x_1, x_2]$, $x_i^1 < x_1$, $x_i^m > x_2$, $y_i^1, \ldots, y_i^m \in [y_1, y_2]$, $\forall i \in \{1, 2\}$. A vertical crossing is defined analogously.

some value D. For a single Poisson boolean model $G(\lambda, d)$, the probability $\tilde{p}_c[R(D)]$ that a rectangle is complete[2] approaches 1 as the size of the rectangle approaches infinity, as long as $G(\lambda, d)$ percolates [8]. Moreover, $\tilde{p}_c[R(D)] > 0.8639$ for some D is a sufficient condition for the percolation of $G(\lambda, d)$ [9]. Thus, the condition that $\tilde{p}_c[R(D)] > 0.8639$ for some D is equivalent to the condition that $G(\lambda, d)$ percolates.

The next theorem proves, under the assumption $p_c[R(D)] > 0.8639$ for some value D, that G_{IntDep} still contains an infinite mutual component after the removal of nodes that have a large number of interdependent nodes.

Theorem 1. *If* $p_c[R(D)] > 0.8639$ *for some* D*, then there exist* k_1, k_2 *such that* G_{IntDep} *contains an infinite mutual component, after removing all the nodes in* G_i *with more than* k_j *interdependent nodes in* G_j*,* $\forall i, j \in \{1, 2\}, i \neq j$.

Proof. We map the percolation of G_{IntDep} to the percolation of a square lattice L. See Fig. 3 for an example. The state of a bond in L is determined by the points in G_{IntDep} in the rectangle containing the bond. A bond is open if the following conditions are satisfied.

1) Rectangle $R(3^t D)$ is complete.
2) There are no more than k_i nodes from G_i in a $(2 \times 3^t D) \times (3^t D)$ rectangle, $\forall i \in \{1, 2\}$.

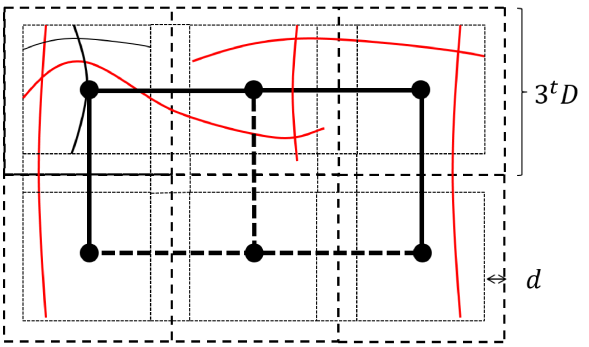


Fig. 3: 1-dependent bond percolation on square lattice L. Solid line segments of length D represent open bonds, and dashed line segments represent closed bonds.

The event that a bond in L is open is dependent on the event that an adjacent bond is open, since they both depend on the point processes of G_{IntDep} in an overlapping area. However, the events that two non-adjacent bonds in L are open are independent, since their associated rectangles do not overlap and the point processes in non-overlapping areas are independent. Thus, L is a *1-dependent bond percolation model* on a square lattice. If the probability that a bond in L is open is larger than 0.8639, then L percolates [9]. This is in contrast with the independent bond percolation model where the percolation threshold is 0.5 [7].

The percolation of L implies the percolation of G_{IntDep}. Consider two adjacent open bonds in L. The rectangles associated with the two bonds are both complete and the mutual components in both rectangles are joined by the nodes in the overlapping area of the two rectangles. For example, red curves across rectangles in Fig. 3 represent the mutual components in the rectangles, and they are joined if the open bonds are connected. If there is an infinite component in L, then there is an infinite mutual component in G_{IntDep}.

Moreover, none of the nodes adjacent to the open bonds are interdependent with more than k_i nodes in G_i, $\forall i \in \{1, 2\}$. For each open bond in L, the $(2 \times 3^t D) \times (3^t D)$ rectangle contains at most k_i nodes in G_i. Therefore, each node from G_i in the $(2 \times 3^t D - 2d) \times (3^t D - 2d)$ rectangle R, where $d \geq d_{\text{dep}}$, is interdependent with no more than k_j nodes from G_j. To conclude, nodes in G_i that have no more than k_j interdependent nodes $\forall i, j \in \{1, 2\}, i \neq j$ form an infinite mutual component.

Next, we compute the probability that a bond in L is open. Recall that $p_c[R(3^t D)]$ is the probability that rectangle $R(3^t D)$ is complete. A result from [9] (proof of Theorem 2 therein) indicates that if $p_c[R(D)] > 0.8639$, then $p_c[R(3D)] > p_c[R(D)]$, where $R(3D)$ is a $(6D-2d) \times (3D-2d)$ rectangle. Thus, $p_c[R(3^t D)]$ is an increasing function in t, provided that $p_c[R(D)] > 0.8639$.

Since both $p_c[R(3^t D)]$ and $-1/(2 \times 3^{2t} D^2 \lambda_i)$ are increasing functions in t, their sum is an increasing function in t. Let t be the smallest integer such that

$$p_c[R(3^t D)] - \frac{1}{2 \times 3^{2t} D^2 \lambda_1} - \frac{1}{2 \times 3^{2t} D^2 \lambda_2} > 0.8639.$$

Let N_i^t denote the number of nodes from G_i in a $(2 \times 3^t D) \times (3^t D)$ rectangle. The random variable N_i^t follows a Poisson distribution with mean $\mathbb{E}[N_i^t] = 2 \times 3^{2t} D^2 \lambda_i$ and variance $\text{Var}[N_i^t] = 2 \times 3^{2t} D^2 \lambda_i$. Let $k_i = 4 \times 3^{2t} D^2 \lambda_i$. The probability that there are no more than k_i nodes from G_i in $R(3^t D)$ can be bounded using the Chebyshev's inequality.

$$\begin{aligned} \Pr(N_i^t \geq k_i) &\leq \Pr(|N_i^t - \mathbb{E}[N_i^t]| \geq \mathbb{E}[N_i^t]) \\ &\leq \frac{\text{Var}[N_i^t]}{(\mathbb{E}[N_i^t])^2} = \frac{1}{2 \times 3^{2t} D^2 \lambda_i}. \end{aligned}$$

The probability that either $N_1^t \geq k_1$ or $N_2^t \geq k_2$ can be bounded by the union bound

$$\Pr(N_1^t \geq k_1 \cup N_2^t \geq k_2) \leq \frac{1}{2 \times 3^{2t} D^2 \lambda_1} + \frac{1}{2 \times 3^{2t} D^2 \lambda_2}.$$

The probability that both $N_1^t < k_1$ and $N_2^t < k_2$ is at least $1 - \Pr(N_1^t \geq k_1 \cup N_2^t \geq k_2)$.

[2]That is, a connected component in $G(\lambda, d)$ in the rectangle forms a horizontal and a vertical crossing.

For any two events A and B, $\Pr(A\cap B) \geq \Pr(A)+\Pr(B)-1$. Therefore, the probability that $R(3^tD)$ is complete and that there are no more than k_i nodes from G_i in $R(3^tD)$ is at least

$$p_c[R(3^tD)] + \left(1 - \frac{1}{2\times 3^{2t}D^2\lambda_1} - \frac{1}{2\times 3^{2t}D^2\lambda_2}\right) - 1 > 0.8639.$$

Since the probability that a bond is open is at least 0.8639, L percolates. To conclude, G_{IntDep} contains an infinite mutual component, in which every node in G_i has no more than k_j interdependent nodes in G_j, $\forall i,j \in \{1,2\}, i \neq j$. □

IV. A NEW APPROACH FOR COMPUTING THE DEGREE THRESHOLD

For the approach discussed in Section III, it is difficult to obtain an analytical expression for the probability $p_c[R(3^tD)]$ that a rectangle is complete. In this section, we develop a new approach that computes a closed-form solution to obtain degree thresholds. We first present the result for a single graph $G(\lambda,d)$ in Theorem 2, and then extend the result to G_{IntDep} in Theorem 3.

Theorem 2. *If there exist λ, d, k_{th} satisfying*

$$\sum_{k=8}^{k_{th}} \frac{\left(\frac{98d^2\lambda}{9}\right)^k}{k!} e^{-\frac{98d^2\lambda}{9}} \max\left[1 - 8\left(\frac{97}{98}\right)^k, 0\right] > 0.8639,$$

then there exists an infinite component in $G(\lambda,d)$, after removing all the nodes with degree more than k_{th}.

Proof. We map the percolation of $G(\lambda,d)$ to a 1-dependent bond percolation model on a square lattice L. The state of a bond in L is determined by the point process in a $2D\times D$ rectangle $\hat{R}(D)$, where $D = 7d/3$. See Fig. 4 for an illustration. A bond is open if the following conditions are satisfied.

1) There are no more than k_{th} nodes in $\hat{R}(D)$.
2) Each of the eight small squares contains at least one node.

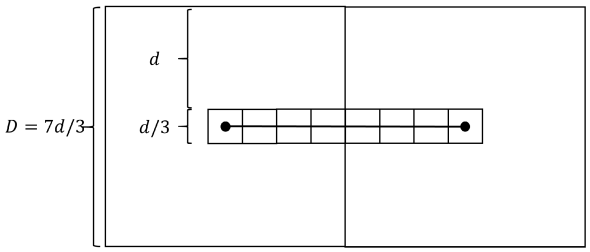


Fig. 4: A single bond of length D and its associated $2D \times D$ rectangle.

Notice that the rectangles $\hat{R}(D)$ associated with two non-adjacent bonds do not overlap, while $\hat{R}(D)$ associated with two adjacent bonds overlap. Therefore, the event that a bond is open is independent of the event that a non-adjacent bond is open, but is dependent on the event that an adjacent bond is open. Therefore, we study 1-dependent bond percolation in L.

We prove that the percolation of L implies the percolation of $G(\lambda,d)$ after removing all nodes with degree larger than k_{th}. If a bond is open in L, each small $d/3\times d/3$ square contains at least one node in $G(\lambda,d)$. Nodes in two adjacent small squares are connected, because they are within distance $\sqrt{5}d/3 < d$. Moreover, all the nodes in the small squares have degrees no more than k_{th}, because the total number of nodes in $\hat{R}(D)$ is no more than k_{th} and the distance between a node in the small squares and a node outside the rectangle $\hat{R}(D)$ is more than d. Next, consider two adjacent open bonds in L. The small squares that contain the intersection of the two bonds overlap. Thus, the nodes in the small squares adjacent to the two open bonds are connected. If the open bonds form an infinite component in L, then there is an infinite component in $G(\lambda,d)$ that contains only nodes with degrees no more than k_{th}.

Next, we compute the probability that a bond is open. Let $S = 2D^2 = 98d^2/9$. The number of nodes in $\hat{R}(D)$ follows a Poisson distribution, i.e.,

$$\Pr(N(S)=k) = \frac{(\lambda S)^k}{k!} e^{-\lambda S}.$$

Given that a realization of the Poisson point process generates k nodes in $\hat{R}(D)$, the k nodes are located uniformly in $\hat{R}(D)$. The probability that there is no node in a small square of size $S' = d/3\times d/3$ within $\hat{R}(D)$ is

$$\Pr(N(S')=0|N(S)=k) = \left(\frac{S-S'}{S}\right)^k.$$

The probability that there exists at least one small square which contains no node, among a total of h small squares, is upper bounded by the union bound

$$h\Pr(N(S')=0|N(S)=k) = h\left(\frac{S-S'}{S}\right)^k.$$

The probability that each of the h small squares contains at least one node, given that the total number of nodes in $\hat{R}(D)$ is k, is at least

$$\Pr(E|N(S)=k) \geq \max\left[1 - h\left(\frac{S-S'}{S}\right)^k, 0\right].$$

The probability that a bond is open is therefore

$$\begin{aligned} p &= \sum_{k=0}^{k_{th}} \Pr(N(S)=k)\Pr(E|N(S)=k) \\ &\geq \sum_{k=8}^{k_{th}} \frac{(98d^2\lambda/9)^k}{k!} e^{-98d^2\lambda/9} \max\left[1 - 8\left(\frac{97}{98}\right)^k, 0\right]. \end{aligned}$$

From [9], $p > 0.8639$ is a sufficient condition for the percolation of L. Therefore, if λ, d, k_{th} satisfy $p > 0.8639$, then $G(\lambda, d)$ percolates after removing all nodes with degree more than k_{th}. □

This result can be applied to interdependent networks to obtain a closed-form expression for the interdependent degrees k_1 and k_2 such that G_{IntDep} percolates after removing all nodes which are interdependent with more than k_i nodes in G_i ($\forall i \in \{1, 2\}$). Theorem 3 provides a sufficient condition under which G_{IntDep} percolates after removing nodes that are interdependent with a large number of nodes, under the condition $d_1 = d_2 = d_{\text{dep}}$.

Theorem 3. *If there exist λ_i, d, k_i satisfying*

$$\prod_{i=1}^{2} \sum_{k=8}^{k_i} \frac{(\frac{98d^2\lambda_i}{9})^k}{k!} e^{-\frac{98d^2\lambda_i}{9}} \max\left[1 - 8\left(\frac{97}{98}\right)^k, 0\right] > 0.8639$$

then there exists an infinite mutual component in $G_{IntDep} = (G_1(\lambda_1, d), G_2(\lambda_2, d), d_{dep} = d)$, after removing all the nodes in G_i that have more than k_j interdependent nodes in G_j ($\forall i, j \in \{1, 2\}, i \neq j$).

Proof. The proof is similar to that for Theorem 2. In the construction of the 1-dependent bond percolation model L, a bond is open if

1) There are no more than k_i nodes from G_i in $\hat{R}(D)$ ($\forall i \in \{1, 2\}$).
2) Each of the eight small squares contains at least one node from G_i ($\forall i \in \{1, 2\}$).

Given that the point processes in G_1 and G_2 are independent, the probability that a bond is open is the product of the probabilities that both conditions are satisfied for nodes in G_1 and G_2. From Theorem 2, the probability that nodes in G_i satisfy both conditions is at least

$$p_i = \sum_{k=8}^{k_i} \frac{(98d^2\lambda_i/9)^k}{k!} e^{-98d^2\lambda_i/9} \max\left[1 - 8\left(\frac{97}{98}\right)^k, 0\right].$$

If $p_1 p_2 > 0.8639$, open bonds in L form an infinite component.

Finally, we prove that the percolation of L implies the percolation of G_{IntDep} after removing nodes with a large number of interdependent nodes. By the same analysis as the proof of Theorem 2, and noting that nodes of G_1 and G_2 in the same small square are within distance $\sqrt{2}d/3 < d_{\text{dep}}$, nodes in G_{IntDep} adjacent to the open bonds of L form an infinite mutual component, and none of them has more than k_i interdependent nodes from G_i ($\forall i \in \{1, 2\}$). □

V. Simulation Results

In this section, we first provide numerical results based on two finite-size interdependent RGGs in a $[0, 20] \times [0, 20]$ area. Nodes of the two RGGs are generated by Poisson point processes with densities $\lambda_1 = \lambda_2 = 2.5$. The connection distances and the interdependent distance are identical: $d_1 = d_2 = d_{\text{dep}} = 1$. Before failures, the initial interdependent RGGs are percolated, as shown in Fig. 5.

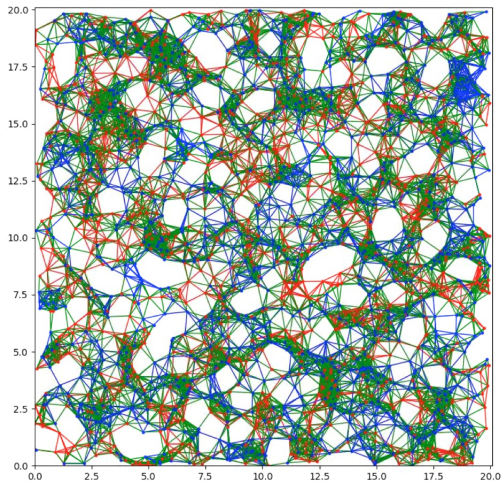
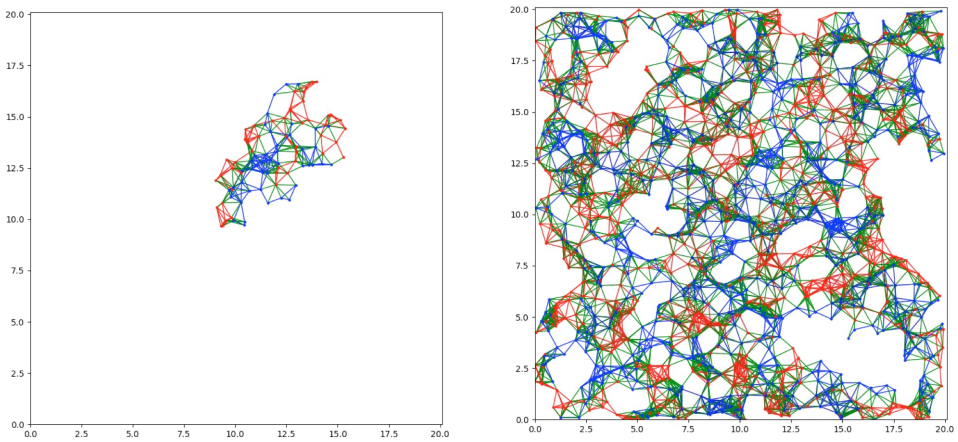


Fig. 5: Initial network with $\lambda_1 = \lambda_2 = 2.5$ and $d_1 = d_2 = d_{dep}$ before any failures. Red links and blue links belong to G_1 and G_2, respectively. Interdependent links are shown in green.

After the removal of all nodes in G_1 with more than $k_2 = 8$ interdependent nodes in G_2 and all nodes in G_2 with more than $k_1 = 8$ interdependent nodes in G_1, the remaining graph is illustrated in Fig. 6a, where only 94 nodes remain out of the initial 2000 nodes and most of the network has lost its functionality. We repeat the same experiment with $k_1 = k_2 = 10$ and as shown in Fig. 6b, 1475 nodes out of 2000 nodes have stayed functional. Based on the results of Theorems 1 and 3, one can conclude the existence of a critical "threshold" for k_1 and k_2 for which removing nodes with more interdependent neighbors does not affect the robustness of network. To observe the transition, we set $k_1 = k_2 = k$ and plot the ratio of the size of the largest mutual component after failures to the size of the initial network as k changes. The result is shown in Fig. 7.

Small values of k_1 and k_2 lead to more severe failures in the network, since a larger number of nodes are removed. There is a trade-off between the parameters k_1 and k_2 in the sense that if we increase one of them, small values of the other may not have the same devastating effect. To observe this trade-off, consider two finite interdependent RGG in a $[0, 20] \times [0, 20]$ area where nodes are generated according to Poison point processes with densities $\lambda_1 = \lambda_2 = 5$ and $d_1 = d_2 = d_{\text{dep}} = 1$. We have simulated the failures for a wide range of values for k_1 and k_2 and obtained the ratios of the size of the largest mutual component



(a) $k = 8$ (b) $k = 10$

Fig. 6: Resulting networks after removing nodes with interdependent degree more than k. Red links and blue links belong to G_1 and G_2, respectively. Interdependent links are shown in green.

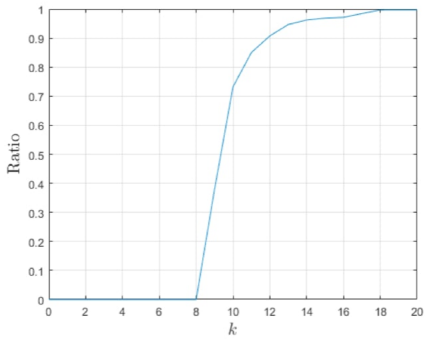


Fig. 7: Transition to critical region: $\lambda_1 = \lambda_2 = 2.5$, $d_1 = d_2 = d_{dep} = 1$.

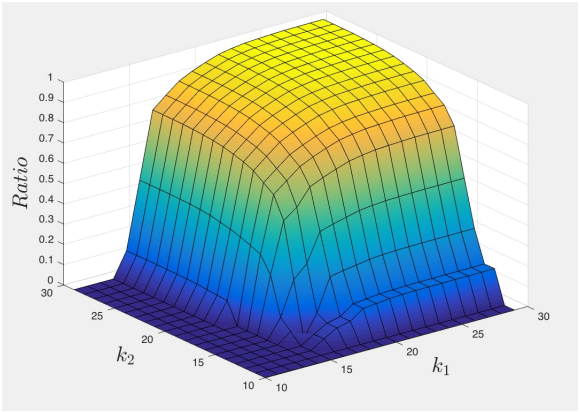


Fig. 8: Ratio of the size of the largest mutual component to the size of original network for different values of k_1, k_2: $\lambda_1 = \lambda_2 = 5$, $d_1 = d_2 = d_{dep} = 1$.

to the size of the network. The result is depicted in Fig. 8.

VI. Conclusion

In this paper we analyzed the percolation of interdependent random geometric networks in an inhomogeneous failure scenario. We proved that even after removing all the nodes with more interdependent neighbors than a certain threshold, the network stays percolated. Using simulations, we demonstrated a trade-off between the interdependent degree thresholds: by increasing one, the network stays functional for smaller values of the other. The analysis of such failures can be used to consider more realistic cascading failure models where a node fails if a certain ratio of its interdependent neighbors fail.

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