2017 IEEE 56th Annual Conference on Decision and Control (CDC)
December 12-15, 2017, Melbourne, Australia

# Understanding the Inefficiency of Security-Constrained Economic Dispatch

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***Abstract*— The security-constrained economic dispatch (SCED) problem tries to maintain the reliability of a power network by ensuring that a single failure does not lead to a global outage. The previous research has mainly investigated SCED by formulating the problem in different modalities, e.g. preventive or corrective, and devising efficient solutions for SCED. In this paper, we tackle a novel and important direction, and analyze the economic cost of incorporating security constraints in economic dispatch. Inspired by existing inefficiency metrics in game theory and computer science, we introduce notion of *price of security* as a metric that formally characterizes the economic inefficiency of SCED as compared to the original problem without security constraints. Then, we focus on the preventive approach in a simple topology comprising two buses and two lines, and investigate the impact of generation availability and demand distribution on the price of security. Moreover, we explicitly derive the worst-case input instance that leads to the maximum price of security. By experimental study on two test-cases, we verify the analytical results and provide insights for characterizing the price of security in general networks.**

## I. INTRODUCTION

The primary goals in power system operation are to minimize operating costs and maintain system reliability [1]. The economic dispatch (ED) problem minimizes generation costs subject to operating constraints [2]. To ensure that failures do not cascade after major disturbances, such as line or generator outages, system operators add security constraints to the economic dispatch problem [3]. The resulting problem is known as security-constrained economic dispatch (SCED) [4], [5]. The typical criteria is that the system must be robust to the failure of any single element, i.e. the solution must satisfy the $N-1$ condition [6].

There are currently two major approaches to SCED. Preventive approaches impose additional operating limits for the post-disturbance configurations, resulting from contingencies, without taking into account the corrective capabilities of the system [4]. In contrast, corrective approaches leverage the system's real-time corrective capabilities after an outage, such as generation rescheduling, switching, congestion management, etc [1]. While preventive approaches are simpler to implement than corrective approaches, the former are overly conservative and more expensive. Nevertheless, majority of SCED implementations today are preventive. Historically, this may be due, in part, to more complex control, sensing, and communication requirements of real-time corrective dispatch. However, recent research has demonstrated that it is possible to efficiently dispatch generators in real-time and distributed manners to rapidly correct for grid disturbances [2], [7], [8].

With the growth of renewables and distributed generation, existing approaches for ensuring system security may not be appropriate for the future grid. The inefficiency of preventive approaches could become more significant due to increased operating uncertainty and greater number of generation sources. The future grid is also more likely to have correlated failures, which necessitates additional contingency considerations beyond $N-1$ [9], leading to more conservative scheduling and higher costs. Hence, it is increasingly important to understand the tradeoffs between different approaches for ensuring security to understand the impact of security constraints on operating costs and their tradeoffs against the benefits of system reliability.

To date, we are not aware of any analysis of the operating costs attributable to security constraints. While there is a large body of literature on SCED, majority of the research have focused on developing efficient and cost-effective algorithms [10], [11]. Understanding the additional costs incurred due to security constraints, as well as how the costs depend on system structure (e.g. network topology, demand profiles, generation availability, etc.), may also provide insights into the most critical components in the system, and in turn guide resource allocation, maintenance decisions, and infrastructure investments.

In this paper, we study the impact of security constraints on operating costs. We focus on preventive approaches as it is the most prevalent approach for ensuring security in current power systems. In particular, we study the cost of ensuring $N-1$ security by investigating the ratio of dispatch costs at the solution of SCED to that at the solution of ED (i.e. removing security constraints from SCED). We refer to this ratio as the price of security. Our metric has an intuitive interpretation and is inspired by inefficiency metrics in game theory and computer science (e.g. price of anarchy, price of stability) [12]. In network routing, the simple topology comprising 2 nodes and 2 edges is known to provide useful insights into the price of anarchy in more general topologies. Motivated by those findings, we focus in this paper on a simple topology comprising 2 buses and 2 lines and completely characterize its price of security.

Our analyses illustrates a few phenomena in the case where there is a generator at each bus – one cheap and the other expensive (see Fig. 1). First, the price of secu-

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This work was supported by NSF grants CNS 1544771, EPCN 1711188, and AMPS 1736448, and the Agency for Science, Technology, and Research (A*STAR).

rity always increases when there is more cheap generation capacity in the system. Second, the price of security is maximized when the lines between the two buses are saturated. This could be expected since the most cheap generation is substituted for expensive generation (to ensure security) when the lines are most heavily utilized. Our analyses also reveals a counter-intuitive phenomenon. Given fixed total demand, having more demand distributed on the cheaper node may increase the price of security. This occurs when the transmission lines are fully utilized, and so additional demand does not change the cost of ensuring security; but when more demand is distributed on the cheaper node, the ED cost is smaller, and therefore the additional cost of ensuring security has a relatively bigger impact on the dispatch cost.

Finally, we investigate numerically the price of security for the PJM (Pennsylvania-New Jersey-Maryland) 5-bus system [13] and illustrate that some of our theoretical results in 2-bus case manifest in general settings. In particular, the numerical results on the PJM 5-bus system show that the impact of generation capacity and demand distribution at the cheap region of the network is similar to that of the 2-bus system. However, finding the worst-case demand that maximizes the price of security is a formidable task that depends on properties of the lines between two regions, aggregate demand, and demand distribution.

## II. System Model

In this section, we introduce the system model and define the ED and SCED problems. Then, we define the proposed metric for measuring the inefficiency of SCED.

### A. Power System

We model the topology of the power network by a directed graph[1] $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the set of nodes (or buses), indexed by $v$, and $\mathcal{E}$ is the set of edges (or lines or branches), indexed by $e$. Let $n := |\mathcal{V}|$ and $m := |\mathcal{E}|$ denote the number of nodes and edges respectively.

Assume each node has exactly one generator and one load.[2] Assume that the generator at node $v$ has a maximum generation capacity $\overline{q}_v \in \mathbf{R}_+$, and it incurs a cost $\alpha_v q_v$ when generating $q_v$, where the coefficient $\alpha_v \in \mathbf{R}_+$. Let $d_v$ denote the demand at node $v$. Define the vectors $\overline{\mathbf{q}} := (\overline{q}_v, v \in \mathcal{V})$, $\mathbf{q} := (q_v, v \in \mathcal{V})$, and $\boldsymbol{\alpha} := (\alpha_v, v \in \mathcal{V})$.

Let $f_e$ denote the power flow on edge $e$ and assume that the edge has a thermal line limit (capacity) $\overline{f}_e$. Define vectors $\overline{\mathbf{f}} := (\overline{f}_e, e \in \mathcal{E})$ and $\mathbf{f} := (f_e, e \in \mathcal{E})$. We assume a DC power flow model and let $\mathbf{H}$ be the $m \times n$ matrix of shift factors that map power injections to line flows. Then the latter are given by

$$\mathbf{f} = \mathbf{H}(\mathbf{q} - \mathbf{d}). \tag{1}$$

[1]Note that in reality the power flow on the links are bidirectional, however, it is common to model the network topology as a directed graph with arbitrary directions on the edges.

[2]Considering linear cost model for generators, this assumption is not restrictive. Multiple generators (resp. loads) at a node can be equivalently represented by a single generator (resp. load) via an appropriate transformation of costs (resp. demands).

### B. Problem Formulation

The ED problem minimizes generation costs subject to operating constraints and is given by:

$$\begin{aligned}
\text{ED} : \min_{\mathbf{q}} \quad & \boldsymbol{\alpha}^{\mathsf{T}}\mathbf{q} && \text{(2a)}\\
\text{s.t.} \quad & \mathbf{0} \leq \mathbf{q} \leq \overline{\mathbf{q}}, && \text{(2b)}\\
& \mathbf{1}^{\mathsf{T}}(\mathbf{q} - \mathbf{d}) = 0, && \text{(2c)}\\
& -\overline{\mathbf{f}} \leq \mathbf{H}(\mathbf{q} - \mathbf{d}) \leq \overline{\mathbf{f}}. && \text{(2d)}
\end{aligned}$$

Constraint (2b) restricts generations to capacities, constraint (2c) enforces supply-demand balance, and constraint (2d) restricts line flows to line limits.

By focusing on robustness to the outage of any single line, we formulate the SCED problem. Associate with the outage of an edge $e \in \mathcal{E}$, an $m-1$ vector $\overline{\mathbf{f}}_{-e} = (f_{e'} : e' \in \mathcal{E}, e' \neq e)$ of line capacities and $(m-1) \times n$ matrix $\mathbf{H}_{-e}$ of shift factors. We are interested in the following SCED problem:

$$\begin{aligned}
\text{SCED} : \min_{\mathbf{q}} \quad & \boldsymbol{\alpha}^{\mathsf{T}}\mathbf{q} && \text{(3a)}\\
\text{s.t.} \quad & \mathbf{0} \leq \mathbf{q} \leq \overline{\mathbf{q}}, && \text{(3b)}\\
& \mathbf{1}^{\mathsf{T}}(\mathbf{q} - \mathbf{d}) = 0, && \text{(3c)}\\
& -\overline{\mathbf{f}} \leq \mathbf{H}(\mathbf{q} - \mathbf{d}) \leq \overline{\mathbf{f}}, && \text{(3d)}\\
& -\overline{\mathbf{f}}_{-e} \leq \mathbf{H}_{-e}(\mathbf{q} - \mathbf{d}) \leq \overline{\mathbf{f}}_{-e}, \forall e. && \text{(3e)}
\end{aligned}$$

Note that SCED contains $2m(m-1)$ more constraints than ED, which are represented by (3e), each of which is associated with a unique line outage.

### C. Price of Security

Our goal is to understand the cost of ensuring security to outage of any single line. Toward this, we define a metric to compare the costs of the solutions to ED and SCED.

Given a network $\mathcal{G}$, cost coefficients $(\alpha_v, v \in \mathcal{V})$, and transmission line limits $(f_e, e \in \mathcal{E})$, let $\boldsymbol{\omega} = (\overline{\mathbf{q}}, \mathbf{d}) \in \Omega$ be an input instance to ED and SCED, where $\Omega$ is the set of all possible different instances of generation capacities and demands that are feasible to both problems. Define $c^{\star}_{\text{ED}}(\boldsymbol{\omega})$ and $c^{\star}_{\text{SC}}(\boldsymbol{\omega})$ as the optimal values of ED and SCED under input instance $\boldsymbol{\omega}$, respectively.

***Definition 1:*** The price of security for instance $\boldsymbol{\omega}$ is

$$\text{PoS}(\boldsymbol{\omega}) := \frac{c^{\star}_{\text{SC}}(\boldsymbol{\omega})}{c^{\star}_{\text{ED}}(\boldsymbol{\omega})}. \tag{4}$$

Note that all feasible solutions of SCED are also feasible for ED, hence, it follows that $c^{\star}_{\text{ED}}(\boldsymbol{\omega}) \leq c^{\star}_{\text{SC}}(\boldsymbol{\omega})$, and hence, $\text{PoS}(\boldsymbol{\omega}) \geq 1$. We are interested in characterizing the instance that lead to the largest value for $\text{PoS}(\boldsymbol{\omega})$, that is, the maximum extra cost of ensuring security.

Since it is difficult to obtain closed form expressions for the solutions to ED and SCED (as a function of $\omega$), obtaining a closed form expression for $\text{PoS}(\boldsymbol{\omega})$ is a challenging task in general. Moreover, system operators typically do not have direct control on demand, and generation availability varies over time. Thus, it is of interest to characterize the worst-case generation availability and demand profile that maximizes the price of security.

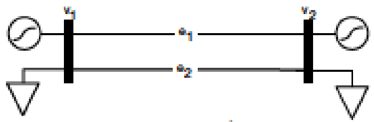



Fig. 1. A simple 2-bus topology

***Definition** 2:* Define the worst-case price of security over all instances in $\Omega$ by:

$$\text{PoS} := \sup_{\omega \in \Omega} \text{PoS}(\omega). \tag{5}$$

We define the worst-case price of security over different generation capacities and demands only, assuming fixed network topology, cost functions, and line limits. This is motivated by the fact that the latter are typically constant over longer time-scales (i.e. days or months) while generation availabilities and demands vary greatly over shorter time-scales (i.e. hours). Moreover, as we will demonstrate, generation capacities and demands alone have complicated and surprising impacts on the price of security. Therefore, we focus on analyzing the worst-case price of security over generation capacities and demands, and leave the analyses with respect to other factors to future work.

To obtain insights into the problem, we begin by analyzing a simple 2-bus topology in the next section. Then, in the subsequent section, we investigate these insights numerically on the PJM 5-bus system [13]

## III. Analysis of 2-Bus Topology

In this section, we analyze the price of security of the simple 2-bus topology shown in Fig. 1, where there are 2 nodes connected by 2 edges. Therefore, $\mathcal{V} = \{v_1, v_2\}$ and $\mathcal{E} = \{e_1, e_2\}$. All proofs are given in [14].

First, by specializing ED to the 2-bus topology, we obtain

$$\begin{aligned}
&\text{ED-2B}: \\
\min_{q_1,q_2} \quad & \alpha_1 q_1 + \alpha_2 q_2 \\
\text{s.t.} \quad & 0 \le q_1 \le \overline{q}_1, && \text{(6a)}\\
& 0 \le q_2 \le \overline{q}_2, && \text{(6b)}\\
& (q_1 - d_1) + (q_2 - d_2) = 0, && \text{(6c)}\\
& -\overline{f}_1 \le \frac{B_1(q_1 - d_1 - q_2 + d_2)}{2(B_1+B_2)} \le \overline{f}_1, && \text{(6d)}\\
& -\overline{f}_2 \le \frac{B_2(q_1 - d_1 - q_2 + d_2)}{2(B_1+B_2)} \le \overline{f}_2, && \text{(6e)}
\end{aligned}$$

where constraints (6a) and (6b) are equivalent to the generation capacity constraint (2b) in ED. Constraint (6c) is the supply-demand balance constraint, and constraints (6d) and (6e) are the line constraints associated with lines $e_1$ and $e_2$, respectively. In addition, $B_1$ and $B_2$ are the sensitivity of the flows at edges 1 and 2 with respect to changes in the phase difference between head and tail nodes. By substituting equation (6c) into inequalities (6d) and (6e), the latter two inequalities are equivalent to the following single constraint:

$$-f^{\text{ED}} \le q_1 - d_1 \le f^{\text{ED}}, \tag{7}$$

where

$$f^{\text{ED}} := (B_1 + B_2) \min\left\{\frac{\overline{f}_1}{B_1}, \frac{\overline{f}_2}{B_2}\right\}. \tag{8}$$

Note that $f^{\text{ED}}$ can be interpreted as the maximum flow from node 1 to node 2. Next, we simplify SCED. By specializing SCED to the 2-bus topology, and making use of the simplification in (7), we obtain the following problem:

$$\begin{aligned}
&\text{SCED-2B}: \\
\min_{q_1,q_2} \quad & \alpha_1 q_1 + \alpha_2 q_2 \\
\text{s.t.} \quad & 0 \le q_1 \le \overline{q}_1, && \text{(9a)}\\
& 0 \le q_2 \le \overline{q}_2, && \text{(9b)}\\
& (q_1 - d_1) + (q_2 - d_2) = 0, && \text{(9c)}\\
& -f^{\text{ED}} \le q_1 - d_1 \le f^{\text{ED}}, && \text{(9d)}\\
& -\overline{f}_1 \le 1/2(q_1 - d_1 - q_2 + d_2) \le \overline{f}_1, && \text{(9e)}\\
& -\overline{f}_2 \le 1/2(q_1 - d_1 - q_2 + d_2) \le \overline{f}_2. && \text{(9f)}
\end{aligned}$$

Note that SCED-2B contains four more constraints than ED-2B – (9e) and (9f) – that reflect the outage of lines $e_2$ and $e_1$. By using a similar procedure to derive (7), we can rewrite (9e) and (9f) into the following compact form:

$$-f^{\text{SC}} \le q_1 - d_1 \le f^{\text{SC}}, \tag{10}$$

where

$$f^{\text{SC}} := \min\{\overline{f}_1, \overline{f}_2\}, \tag{11}$$

is the maximum flow from node 1 to node 2.

We now proceed to analyze the price of security. Recall that this is defined as the largest ratio between the optimal values of SCED-2B and ED-2B. Without loss of generality, we assume for the rest of this section that $\alpha_1 \le \alpha_2$, i.e. the generation cost at node 1 is cheaper than that at node 2. We also refer to the generator at node 1 as the cheap generator and the generator at node 2 as the expensive generator.

### *A. Impact of Generation Capacities*

The following lemma highlights the impact of cheap generation availability on the price of security.

***Lemma** 1:* Let $\boldsymbol{\omega} = (\overline{\mathbf{q}}, \mathbf{d})$ and $\boldsymbol{\omega}' = (\overline{\mathbf{q}}', \mathbf{d})$ be two input instances with identical demand profiles $\mathbf{d}$. If $\overline{q}_1' \le \overline{q}_1$, then $\text{PoS}(\boldsymbol{\omega}') \le \text{PoS}(\boldsymbol{\omega})$.

Let us consider the case where $\overline{q}_1' \le d_1 + d_2 \le \overline{q}_1$, which implies that total demand $d_1 + d_2$ can be fully served by the cheap generator in instance $\boldsymbol{\omega}$, but cannot be fully served by the cheap generator in instance $\boldsymbol{\omega}'$. Lemma 1 implies that, keeping all other factors constant, the price of security is greater when the cheap generation is not limited (i.e. $\overline{q}_1 \ge d_1 + d_2$) versus when cheap generation is limited (i.e. $\overline{q}_1' \le d_1 + d_2$). Therefore, the price of security is higher when there is greater availability of cheap generation. This is, perhaps, expected since more cheap generation is substituted for expensive generation in order to ensure security.

Since we are interested in identifying the instances with the worst-case price of security, for the rest of our analyses, we focus on cases in which the capacity of cheap generation is greater or equal to total demand.

### B. Impact of Demand

Next, we focus on the impact of the demand profile. Let $\boldsymbol{\omega} = (\overline{\mathbf{q}}, \mathbf{d})$ be an instance such that $\overline{q}_1 \geq d_1 + d_2$, i.e. all demand can be served by cheap generation. We proceed to calculate closed-form expressions for $c^\star_{\text{ED}}(\boldsymbol{\omega})$ and $c^\star_{\text{SC}}(\boldsymbol{\omega})$.

First, we compute $c^\star_{\text{ED}}(\boldsymbol{\omega})$. Recall that $f^{\text{ED}}$ defined in (8) can be interpreted as the maximum flow from node 1 (with cheap generation) to node 2 (with expensive generation). Note that there is sufficient cheap generation to serve all demand. Therefore, the optimal solution of ED-2B is to serve the demand $d_1$ at node 1 locally using cheap generation, use as much of the cheap generation as possible to serve the demand $d_2$ at node 2, i.e. $\min\{d_2, f^{\text{ED}}\}$, and serve the remaining demand at node 2 locally using expensive generation, i.e. $[d_2 - f^{\text{ED}}]^+$, where $[\cdot]^+$ denotes the projection onto the nonnegative orthant. It follows that the optimal cost of the economic dispatch problem is given by:

$$c^\star_{\text{ED}}(\boldsymbol{\omega}) = \alpha_1(d_1 + \min\{f^{\text{ED}}, d_2\}) + \alpha_2[d_2 - f^{\text{ED}}]^+. \quad (12)$$

Next, we compute $c^\star_{\text{SC}}(\boldsymbol{\omega})$. Similarly, recall that $f^{\text{SC}}$ defined in (11) can be interpreted as the maximum flow from node 1 to node 2. Therefore, the optimal cost of the security-constrained economic dispatch problem is given by:

$$c^\star_{\text{SC}}(\boldsymbol{\omega}) = \alpha_1(d_1 + \min\{f^{\text{SC}}, d_2\}) + \alpha_2[d_2 - f^{\text{SC}}]^+. \quad (13)$$

It follows that the price of security for $\boldsymbol{\omega}$ is given by:

$$\text{PoS}(\boldsymbol{\omega}) = \frac{\alpha_1(d_1 + \min\{f^{\text{SC}}, d_2\}) + \alpha_2[d_2 - f^{\text{SC}}]^+}{\alpha_1(d_1 + \min\{f^{\text{ED}}, d_2\}) + \alpha_2[d_2 - f^{\text{ED}}]^+}. \quad (14)$$

Observe that $\text{PoS}(\boldsymbol{\omega})$ is small in both low and high load regimes. This is intuitive. In the low load regime, i.e. when $d_1 + d_2 \ll f^{\text{ED}}$, the line limits are not saturated. Therefore, security to outages of any single line is unlikely to increase the dispatch cost significantly. From the definitions in (8) and (11), note that $f^{\text{ED}} \leq 2f^{\text{SC}}$. In the high load regime, i.e. $d_1 + d_2 \gg f^{\text{ED}}$, the expensive generator contributes substantially towards satisfying demand even in ED. Hence, security to outages of any single line has a small impact on the dispatch cost, since in both (12) and (13), the second terms are dominant. The next lemma highlights the impact of cheap demand on the price of security.

***Lemma** 2:* Let $\boldsymbol{\omega} = (\overline{\mathbf{q}}, \mathbf{d})$ and $\boldsymbol{\omega}' = (\overline{\mathbf{q}}, \mathbf{d}')$ be two input instances such that $\overline{q}_1 \geq d_1 + d_2$ and $d_2 = d_2'$. If $d_1' \geq d_1$, then $\text{PoS}(\boldsymbol{\omega}') \leq \text{PoS}(\boldsymbol{\omega})$.

Lemma 2 implies that, given a fixed demand at the expensive node, the price of security is greatest when there is no demand at the cheap node. This is, perhaps, expected since there is no additional cost to ensure security when demand is being served locally (which is the case with demand located at the cheap node). However, Lemma 2 does not specify which distributions of demand (over the two nodes) lead to the greatest price of security. We characterize the latter in the following lemma.

***Lemma** 3:* Fix the total demand $d$ and assume that $\overline{q}_1 \geq d_1 + d_2 = d$. Then, the demand distribution $\mathbf{d} = (d_1 = d - d_2, d_2 = \min\{d, f^{\text{ED}}\})$ yields the maximum price of security, whose value is given by:

$$\text{PoS}(\boldsymbol{\omega}) = \frac{\alpha_1(d_1 + \min\{f^{\text{SC}}, d_2\}) + \alpha_2[d_2 - f^{\text{SC}}]^+}{\alpha_1 d}. \quad (15)$$

Lemma 3 states that, given fixed total demand, the price of security is largest when demand is distributed to the expensive node, but only until the total demand is up to $f^{\text{ED}}$. When total demand increases beyond $f^{\text{ED}}$, having more demand distributed on the cheap node can, in fact, increase the price of security. This is intuitive because, when the transmission lines in economic dispatch are fully utilized, additional demand on either cheap or expensive sides must be fulfilled locally, and therefore does not change the additional cost of ensuring security. When the demand is distributed to the cheap node, the optimum economic dispatch cost is smaller than in the opposite case. Hence, the additional cost of ensuring security has a relatively bigger impact on the dispatch cost when the demand is added to the cheap side.

Finally, the following theorem is a direct consequence of the results in lemmas 1, 2, and 3, and characterizes the worst-case price of security as defined in (5).

***Theorem** 1:* For the 2-bus topology, $\text{PoS}(\boldsymbol{\omega})$ achieves its maximum value when $d_1 = 0$, $d_2 = f^{\text{ED}}$, and $\overline{q}_1 \geq f^{\text{ED}}$. Moreover, the maximum value is given by:

$$\text{PoS} = \frac{\alpha_2}{\alpha_1} - \frac{(\alpha_2 - \alpha_1)f^{\text{SC}}}{\alpha_1 f^{\text{ED}}}. \quad (16)$$

Theorem 1 states that the instance with the greatest price of security is such that all demand is at the expensive node and that demand is equal to the maximum flow from the cheap node to the expensive node in the economic dispatch problem. From the definitions in (8) and (11), it follows that $f^{\text{SC}}/f^{\text{ED}} \leq 1$. Observe that, as $f^{\text{SC}}/f^{\text{ED}} \uparrow 1$, the PoS $\downarrow 1$.

## IV. Numerical Results

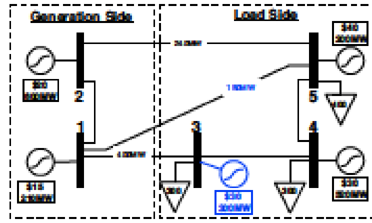
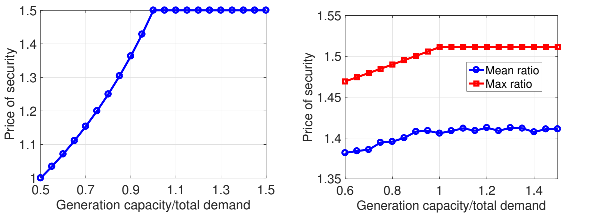


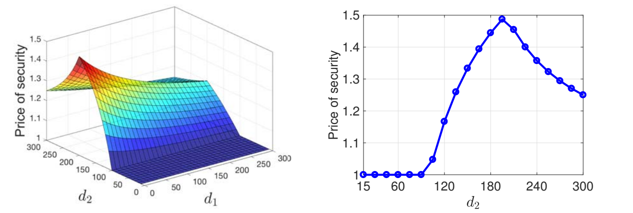

Fig. 2. PJM 5-bus topology; modifications as compared to the original version are highlighted in blue.

We verify the analytical results for the 2-bus topology and investigate the validity of results for the PJM 5-bus topology [13]. For 2-bus case, we set $\alpha_1 = 1, \alpha_2 = 2, \overline{f}_1 = \overline{f}_2 = 100, B_1 = B_2 = 1$. In this way, we get $f^{\text{ED}} = 200$ and $f^{\text{SC}} = 100$. The PJM 5-bus system, as depicted in Fig. 2, has two regions which can be interpreted as generation and load. The generation side has two generators with linear costs $\alpha_1 = 15$ and $\alpha_2 = 20$ which are cheaper than the generators in the load side. We modify the test case in two ways: (i) the line limit of line $(1, 5)$ is set to 150MW to impose a line limit on each line; and (ii) a generator is added at bus 3 to ensure that the SCED problem is always feasible.



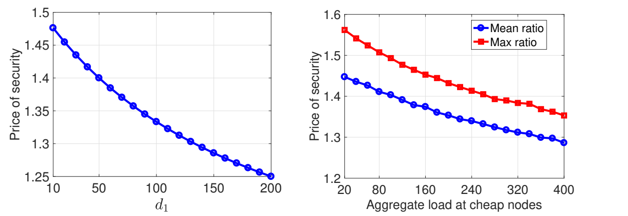

(a) 2-bus topology (b) PJM 5-bus topology

Fig. 3. PoS with fixed demand and different generation capacities



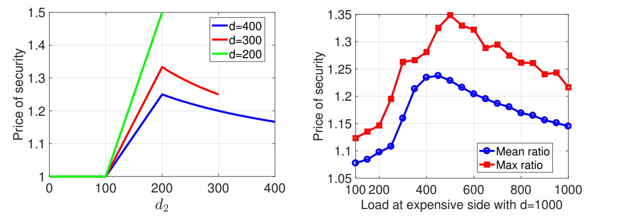

(a) $d_1 \in [0, 300)$, $d_2 \in [0, 300)$ (b) $d_1 = 0$, $d_2 \in [0, 300)$

Fig. 4. PoS for 2-bus network in entire state space of demand distribution



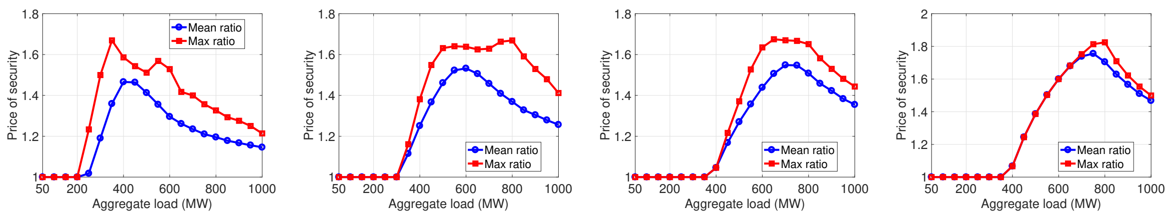

(a) 2-bus topology (b) PJM 5-bus topology

Fig. 5. PoS with fixed expensive demand and different cheap demands




(a) 2-bus topology (b) PJM 5-bus topology

Fig. 6. PoS with fixed aggregate demand and different demand distribution




(a) With 150MW link (b) Without 150MW link (c) Without 150MW link and normalized line limits (d) Without 150MW link, normalized lines, and homogeneous costs

Fig. 7. Results of PJM 5-bus network with different values of demand

## A. Impact of Generation Capacity

In this experiment, we verify the claim in Lemma 1. In Fig. 3(a), while fixing demand ($d_1 = 0, d_2 = 200$), we change the capacity of generator 1 from 50% of demand ($\overline{q}_1 = 100$) to 150% of demand ($\overline{q}_1 = 300$). As stated in Lemma 1, the PoS increases with capacity until it exceeds total demand at 200MW, where it stays constant. In Fig. 3(b), the same result for PJM 5-bus topology is reported and we change the generation capacity of cheap generators (at buses 1 and 2) within $[0.6, 1.5]$ of total demand. Note that even with fixed aggregate demand, the PoS changes with different demand distributions at nodes. Hence, we report the maximum and the average values of 500 random runs each of which with different randomly generated demand profiles. The result exhibits the same behavior as in 2-bus topology and as the generation capacity increases the price of security increases. In summary, the results verify the analysis in Lemma 1, which states intuitively that when the generation capacity is not the bottleneck, higher price of security is expected. Hence, we relax the generation capacity of all generators, in the rest of the experiments.

## B. Impact of Demand Profile

In this set of experiments, we verify the analytical results in lemmas 2, 3 and Theorem 1.

*1) Price of Security in the Entire State Space:* First, we report the PoS for the entire state space of demand distribution in cheap and expensive nodes of 2-bus case in Fig. 4. The most important observation is that the PoS is globally maximized when $d_1 = 0$ and $d_2 = f^{\text{ED}} = 200$, which is consistent with the result in Theorem 1. Another observation in Fig. 4(a) is that given fixed demand at expensive side, the PoS achieves is maximized when $d_1 = 0$, which is consequence of Lemma 2. In Fig. 4(b), the price of security as a function of $d_2$ and for $d_1 = 0$ is reported.

*2) Investigating the Result in Lemma 2:* In Fig. 5, we investigate the result in Lemma 2, which says that given a fixed demand at the expensive node, the PoS decreases as the demand at the cheap node increases. For both topologies, we fix the (aggregate) demand at the expensive side and change the demand at cheap side. As shown in Fig. 5(a), as the demand at cheap node 1 increases, the PoS decreases. In Fig. 5(b), the aggregate demand in expensive nodes is fixed and equal to 400MW and the aggregate load at two nodes 1 and 2 is changed from 20 to 400MW and at each point the average and maximum values are reported. The result demonstrates that as the load at cheap nodes increases, the PoS decreases and when there is no demand at cheap nodes the maximum PoS is attained.

*3) Investigating the Result in Lemma 3:* We fix the aggregate demand and change the distribution of demand at cheap and expensive nodes. In Fig. 6(a), we report the PoS, for 3 different values of aggregate demand in 2-bus topology. The result demonstrates that in all cases the PoS is maximized when the demand $d_2$ at the expensive node is equal to the maximum line capacity and the rest is at the cheap node, which is the result shown in Lemma 3. In Fig. 6(b), the result of the same experiment for PJM 5-bus topology is reported. In this experiment, we fix the total demand at $d = 1000$MW and change the distribution of load at two regions. The result shows the same general behavior in the sense that the PoS reaches its maximum when roughly the demand at expensive node reaches the effective transmission capacity. However, different from explicit characterization of the maximum flow capacity in 2-bus topology in (8) and (11), the effective transmission capacity in PJM 5-bus topology is not straightforward to recognize. Furthermore, different peak values for the maximum and the average PoS imply that even with fixed aggregate demand at expensive side, PoS changes with different demand distribution.

*4) Characterizing the Price of Security in General Networks:* We investigate the PoS for PJM 5-bus topology in more details in the worst-case scenario, where the demand at cheap side is zero. The first result is shown in Fig. 7(a) where the overall behavior is similar to the 2-bus topology, since the PoS is 1 at low load regimes when $d \leq 200$. Then, there is an increasing region (in $[200, 400]$) where the PoS increases, and finally (when $d \geq 400$) the PoS is decreasing.

The result, however, is different from 2-bus topology in a way that the *critical points* (the point at which the price begins to increase, and the one at which the price takes it maximum) are not straightforward function of line properties. Recall that these points are characterized explicitly in (8) and (11) for 2-bus topology. Fig. 7(a) shows that in worst-case, the aggregate demand is less than aggregate line limit from the cheap side to expensive side that is 790MW. Thus, this result shows that characterizing the worst case demand profile is more challenging in PJM 5-bus topology.

To investigate how network topology and line characteristics can impact the two critical points, in three consecutive steps, we simplify PJM 5-bus topology to be similar to 2-bus case. Toward this, we first remove line $(1, 5)$ with capacity 150MW (reported in Fig. 7(b)), second, we normalized the link capacities such that $f_{(1,3)}/B_{(1,3)} = f_{(2,5)}/B_{(2,5)}$ (reported in Fig. 7(c)); recall that $f^{\text{ED}}$ in 2-bus topology is maximized when $\overline{f}_1/B_1 = \overline{f}_2/B_2$; third, we set the homogeneous generator costs at \$15 at generation side and \$40 at demand side (reported in Fig. 7(d)). The observations are as follows: (i) the maximum PoS, located at the second critical point, increases as the network topology simplifies ($1.47 \rightarrow 1.53 \rightarrow 1.55 \rightarrow 1.75$); (ii) the aggregate demand at which the first critical point occurs, i.e, the point where the PoS starts to increase, increases as the networks simplifies (200MW $\rightarrow$ $300 \rightarrow 350 \rightarrow 350$); and finally, the aggregate demand at which the PoS maximizes also increases (400MW $\rightarrow 600 \rightarrow 700 \rightarrow 750$). These observations demonstrate that the worst-case aggregate demand that leads to maximum PoS depends on several characteristics of topology, transmission lines, and cost functions.

## V. Conclusions and Future Directions

This paper studies the economic cost of incorporating security constraints in economic dispatch. We introduce the notion of price of security as a metric that formally characterizes the economic inefficiency of ensuring $N-1$ security. Focusing on security to line outages in a 2-bus topology, we investigate the impact of generation availability and demand distribution on the price of security. Experimental results on the PJM 5-bus system show that some of our theoretical observations manifest in more general settings. This work proposes a new direction on studying the cost of ensuring security in power systems which differs from existing literature that is only concerned with developing approaches to ensure security.

We plan to extend our theoretical results to more general networks. A natural extension is to networks that can be divided into cheap and expensive generation regions. Our theoretical results indicate that the price of security may depend critically on the maximum flow from the cheap to the expensive region.

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