Unified models of the QCD axion and supersymmetry breaking

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Abstract

Similarities between the gauge meditation of supersymmetry breaking and the QCD axion model suggest that they originate from the same dynamics. We present a class of models where supersymmetry and the Peccei–Quinn symmetry are simultaneously broken. The messengers that mediate the effects of these symmetry breakings to the Standard Model are identical. Since the axion resides in the supersymmetry breaking sector, the saxion and the axino are heavy. We show constraints on the axion decay constant and the gravitino mass.

1. Introduction

One of the most serious problems of the standard model, the so-called strong CP problem [1–3], is elegantly solved by the Peccei–Quinn (PQ) mechanism [4]. Another problem, the hierarchy problem, is considerably relaxed by low energy supersymmetry (SUSY) [5–8]. The precise gauge coupling unification at a high energy scale also motivates low energy SUSY [9–11].

There are several hints for a potential connection between these two physical ideas. First, models of SUSY breaking often involve spontaneous breaking of global symmetry. In fact, it is one of the sufficient conditions for SUSY breaking [12]. It would be illuminating to identify this global symmetry with the PQ symmetry.

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Second, if the PQ symmetry breaking field resides in the SUSY breaking sector, the super partners of the axion, namely the saxion and the axino, may obtain large masses [13–16]. Such a model is free from the cosmological problems associated with light saxions and axinos (see [17] and references therein).

Finally, one realization of the PQ mechanism, the KSVZ model [18,19], has the following superpotential,

$$W = ZQ\bar{Q}, \tag{1}$$

where Z is a PQ charged field with a non-zero vacuum expectation value (VEV), and Q and $\bar{Q}$ are PQ and standard model gauge (especially $SU(3)_c$) charged fields. If the chiral field Z also obtains a non-zero F term VEV, the SUSY breaking is mediated to super partners of standard model particles via the gauge interaction. This is nothing but the gauge mediation of SUSY breaking [20–24] with messenger fields Q and $\bar{Q}$.

Motivated by these hints, we propose a model where SUSY and the PQ symmetry are simultaneously broken, and the messenger fields that mediate SUSY breaking and the anomaly of the PQ symmetry are in fact the same. The model provides a unification for the physics of SUSY breaking and the PQ mechanism.

2. Unification of SUSY and PQ symmetry breaking

2.1. Simultaneous SUSY and PQ symmetry breaking in a single sector

We introduce chiral fields M_+ and M_-, whose $U(1)_{\mathrm{PQ}}$ charges are $+1$ and -1, respectively. The PQ symmetry is broken by introducing a chiral field X and a superpotential coupling,

$$W \supset \kappa X (M_+ M_- - v^2), \tag{2}$$

where κ and v are constants. SUSY is broken by lifting the flat direction $M_+ M_- = v^2$. To achieve this, we introduce chiral fields Z_+ and Z_-, and couple them to $M_\pm$ via mass terms. The superpotential of this minimal model is then given by

$$W = \kappa X (M_+ M_- - v^2) + \lambda' r v Z_+ M_- + \frac{\lambda'}{r} v Z_- M_+, \tag{3}$$

where λ' and r are constants. By phase rotations of chiral fields, we take all constants in Eq. (3) to be real.

The simultaneous breaking of the PQ symmetry and SUSY via the superpotential in Eq. (3) is discussed in [13,14]. As is shown in section 3, this model is the low energy effective theory of a dynamical SUSY breaking model with a deformed moduli constraint (the IYIT model) [25,26], and is studied by [16,27] in the context of the heavy scalar scenario [28–32]. Direct coupling between the SUSY and the PQ breaking sectors is also analysed in [15] using an effective field theory, while [33–35] connect the two sectors indirectly via quantum corrections involving messengers.

For $\lambda' < \kappa$, the VEVs of the fields are given by

$$\langle M_+ \rangle = r v \sqrt{1 - \frac{\lambda'^2}{\kappa^2}}, \quad \langle M_- \rangle = \frac{v}{r} \sqrt{1 - \frac{\lambda'^2}{\kappa^2}},$$

$$\langle Z_+ \rangle = \langle Z_- \rangle \equiv z, \quad \langle X \rangle = -\frac{\lambda' z}{\kappa \sqrt{1 - \lambda'^2/\kappa^2}}, \tag{4}$$

up to a $U(1)_{PQ}$ rotation. The PQ symmetry is broken by the VEVs of the charged fields $M_\pm$ and $Z_\pm$, where z is undetermined at tree level. If $\lambda' > \kappa$, the VEVs of $M_\pm$ and $Z_\pm$ vanish, and the PQ symmetry is not broken. Thus we will adopt the above hierarchy and also assume that $\lambda' \ll \kappa$ for simplicity. SUSY is predominantly broken by the F terms of $Z_\pm$,

$$F_{Z_\pm} = -\lambda' v^2. \tag{5}$$

2.2. Mass spectrum

The chiral field X and a linear combination of $M_\pm$ obtain a large mass κv. We may integrate them out and parametrize $M_\pm$ as

$$M_+ \to rv \times \exp(-\frac{A}{v\sqrt{r^2 + 1/r^2}}),$$
$$M_- \to \frac{v}{r} \times \exp(\frac{A}{v\sqrt{r^2 + 1/r^2}}), \tag{6}$$

where A is a chiral field. The effective superpotential of $Z_\pm$ and A is then given by

$$W_{\text{eff}} = \lambda f^2 Z_+ \exp(\frac{A}{\sqrt{2}f}) + \lambda f^2 Z_- \exp(-\frac{A}{\sqrt{2}f}), \tag{7}$$

where $f \equiv v\sqrt{(r^2 + 1/r^2)/2}$ and $\lambda \equiv 2\lambda'/(r^2 + 1/r^2)$. We note that most of the following discussion relies only on this effective superpotential, and not on the UV completion in Eq. (3).

Let us first calculate the masses of scalar components of $Z_\pm$ and A. We decompose scalar components as

$$Z_\pm \to \left(z + \frac{\pm\rho_H + \rho_L}{2}\right)\exp\left(i\frac{\pm\theta_H + \theta_L}{2z}\right),$$
$$A \to \frac{s + i\phi}{\sqrt{2}}. \tag{8}$$

Expanding the scalar potential, we obtain the mass terms,

$$V_{\text{mass}} = \frac{1}{2}\lambda^2 f^2 \left(\theta_H + \frac{z}{f}\phi\right)^2$$
$$+ \frac{1}{2}\lambda^2 f^2 \left(\rho_H + \frac{z}{f}s\right)^2 + \lambda^2 f^2 s^2. \tag{9}$$

The mass eigenstates and eigenvalues are given by

$$a = \frac{\phi - \epsilon\theta_H}{\sqrt{1 + \epsilon^2}}, \quad b = \frac{\theta_H + \epsilon\phi}{\sqrt{1 + \epsilon^2}}, \quad \epsilon \equiv \frac{z}{f}$$
$$\begin{pmatrix} \sigma_+ \\ \sigma_- \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} s \\ \rho_H \end{pmatrix},$$
$$\tan\alpha = \frac{2\epsilon}{1 + \epsilon^2 + \sqrt{1 + 6\epsilon^2 + \epsilon^4}},$$
$$m_a = 0, \quad m_b = \lambda f\sqrt{1 + \epsilon^2},$$
$$m_{\sigma_\pm}^2 = \frac{1}{2}\lambda^2 f^2 \left[3 + \epsilon^2 \pm \sqrt{1 + 6\epsilon^2 + \epsilon^4}\right]. \tag{10}$$

Scalar fields ρ_L and θ_L are massless at tree level but obtain masses through quantum corrections, as we will see later. The remaining massless field, a, is the axion.

Next we consider the masses of the fermionic components of $Z_\pm$ and A. The quadratic terms of $\delta Z_\pm \equiv Z_\pm - z$ and A in the superpotential in Eq. (3) are

$$W_{\text{eff,quad}} = \frac{1}{2}\lambda z A^2 + \lambda f A \frac{1}{\sqrt{2}}(\delta Z_+ - \delta Z_-). \tag{11}$$

The mass eigenstates $\psi_\pm$ and eigenvalues are

$$\begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \psi_A \\ \psi_{Z_H} \end{pmatrix}, \quad \tan\beta = \frac{\sqrt{\epsilon^2 + 4} - \epsilon}{2},$$
$$m_{\psi_\pm} = \frac{1}{2}\lambda f \times \left[\sqrt{\epsilon^2 + 4} \pm \epsilon\right], \tag{12}$$

where ψ_A and ψ_{Z_H} are the fermionic components of A and $Z_H \equiv (Z_+ - Z_-)/\sqrt{2}$, respectively. The fermionic component of $Z_L \equiv (Z_+ + Z_-)/\sqrt{2}$ is the goldstino and is eaten by the gravitino via the super Higgs mechanism.

The expressions for the mass eigenstates are simplified in the limit $\epsilon \ll 1$ or $\epsilon \gg 1$. In the limit $\epsilon \ll 1$, where the PQ symmetry is dominantly broken by the VEVs of $M_\pm$, we have

$$a = \phi, \ b = \theta_H, \ \sigma_+ = s, \ \sigma_- = \rho_H, \tag{13}$$

$$m_b = \lambda f, \ m_{\sigma_+} = \sqrt{2}\lambda f, \ m_{\sigma_-} = \lambda f, \tag{14}$$

$$\psi_\pm = \frac{1}{\sqrt{2}}\left(\psi_A \mp \psi_{Z_H}\right), \tag{15}$$

$$m_{\psi_\pm} = \lambda f. \tag{16}$$

In the limit $\epsilon \gg 1$, where the PQ symmetry is dominantly broken by the VEVs $\langle Z_\pm \rangle$, we obtain

$$a = -\theta_H, \ b = \phi, \ \sigma_+ = s, \ \sigma_- = \rho_H, \tag{17}$$

$$m_b = \lambda z, \ m_{\sigma_+} = \lambda z, \ m_{\sigma_-} = \frac{\sqrt{2}\lambda f^2}{z}, \tag{18}$$

$$\psi_+ = \psi_A, \ \psi_- = \psi_{Z_H}, \tag{19}$$

$$m_{\psi_+} = \lambda z, \ m_{\psi_-} = \frac{\lambda f^2}{z}, \tag{20}$$

where the masses of $\sigma_- = \rho_H$ and $\psi_- = \psi_{Z_H}$ are suppressed by the large Majorana masses λz of $\sigma_+ = s$ and $\psi_+ = \psi_A$.

2.3. Sgoldstino potential in the minimal model

As we have seen, the directions ρ_L and θ_L, which correspond to the sgoldstino components, are massless at tree level. Accordingly, z is undetermined at tree level. Here we discuss the stabilization of the sgoldstino in the minimal model given by Eq. (3).

Quantum corrections generate a potential for the scalar component of $Z_L \equiv (Z_+ + Z_-)/\sqrt{2}$,

$$\Delta V_\pm(Z_L) = \frac{\lambda^4 f^4}{512\pi^2}\Big[8(1 + \epsilon^2)^2 \ln(1 + \epsilon^2)$$
$$+ 2(3 + \epsilon^2 + \sqrt{1 + 6\epsilon^2 + \epsilon^4})^2 \ln\frac{3 + \epsilon^2 + \sqrt{1 + 6\epsilon^2 + \epsilon^4}}{2}$$

$$+ 2(3 + \epsilon^2 - \sqrt{1 + 6\epsilon^2 + \epsilon^4})^2 \ln \frac{3 + \epsilon^2 - \sqrt{1 + 6\epsilon^2 + \epsilon^4}}{2}$$

$$- (\epsilon - \sqrt{4 + \epsilon^2})^4 \ln \frac{(\epsilon - \sqrt{4 + \epsilon^2})^2}{4}$$

$$- (\epsilon + \sqrt{4 + \epsilon^2})^4 \ln \frac{(\epsilon + \sqrt{4 + \epsilon^2})^2}{4} \Bigg]$$

$$\simeq \begin{cases} \frac{\lambda^4 f^2}{32\pi^2}(2\ln 2 - 1)|Z_L|^2 & : |Z_L| \lesssim f \\ \frac{\lambda^4 f^4}{16\pi^2} \ln \frac{|Z_L|}{f} & : |Z_L| \gtrsim f, \end{cases} \tag{21}$$

where $\epsilon = |Z_+|/f$.

The supergravity effect induces a tadpole term for Z_L,

$$V(Z_L) = \Delta V_{\pm}(Z_L) + (-2\sqrt{2}\lambda f^2 m_{3/2} Z_L + \text{h.c.}), \tag{22}$$

where $m_{3/2}$ is the gravitino mass. We take $m_{3/2}$ to be real by a $U(1)_R$ rotation. The gravitino mass is related with the magnitude of the SUSY breaking by the (almost) vanishing cosmological constant condition

$$\sqrt{3} m_{3/2} = |F_{Z_L}|/M_{\text{Pl}} = \sqrt{2}\lambda \frac{f^2}{M_{\text{Pl}}}. \tag{23}$$

The tadpole term induces the VEV of Z_L and the messenger scale [36]. Assuming $|\langle Z_L \rangle| \lesssim f$, we obtain

$$\langle Z_L \rangle = \frac{64\sqrt{2}\pi^2}{2\ln 2 - 1} \frac{m_{3/2}}{\lambda^3} = \frac{128\pi^2}{\sqrt{3}(2\ln 2 - 1)\lambda^2} \frac{f^2}{M_{\text{Pl}}}. \tag{24}$$

For small λ, the formula (24) yields $|\langle Z_L \rangle| > f$. In such a parameter region, the potential of Z_L given by the quantum correction is logarithmic, and cannot stabilize Z_L against the tadpole term. Instead Z_L is stabilized around $\langle Z_L \rangle \sim M_{\text{Pl}}$ by the supergravity effect. Later, we couple $Z_{\pm}$ to the messenger field. If $\langle Z_L \rangle$ is as large as M_{Pl}, the gauge mediated soft masses of supersymmetric standard model (SSM) particles are smaller than the gravitino mass. Thus, in the following, we concentrate on the parameter region where $\langle Z_L \rangle \ll M_{\text{Pl}}$. Then in the minimal model, $\langle Z_L \rangle$ is at the most $\mathcal{O}(f)$.

2.4. Simultaneous mediation of SUSY breaking and the anomaly of the PQ symmetry

The simplest possibility of the mediation is to introduce a pair of standard $SU(3)_c$ charged chiral fields Q and $\bar{Q}$ with the coupling,

$$W = yZ_+ Q\bar{Q}. \tag{25}$$

The precise gauge coupling unification is maintained if Q and $\bar{Q}$ are complete multiplets of the $SU(5)$ GUT gauge group. Soft masses generated by loop corrections from the KSVZ axion sector is discussed in [37–39], while the PQ breaking field is not the dominant source of the SUSY breaking. Soft masses from the F term of the axion multiplet are analyzed in [40] using an effective field theory.

The mass terms of the scalar component of the messenger field are given by

$$V_{\text{mass}} = \begin{pmatrix} Q^* & \bar{Q} \end{pmatrix} \begin{pmatrix} y^2 \langle Z_+ \rangle^2 & y F_{Z_+}^* \\ y F_{Z_+} & y^2 \langle Z_+ \rangle^2 \end{pmatrix} \begin{pmatrix} Q \\ \bar{Q}^* \end{pmatrix}. \tag{26}$$
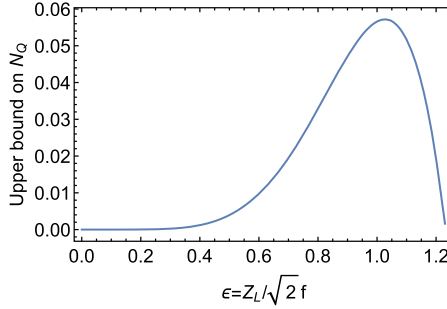
Fig. 1. Upper bound on the multiplicity of the messenger N_Q for the minimal model.

To avoid tachyonic masses for the messenger fields, we require that

$$y > \frac{|F_{Z_+}|}{\langle Z_+ \rangle^2} = \frac{2\lambda f^2}{\langle Z_L \rangle^2}. \tag{27}$$

On the other hand, the quantum correction from the messenger loop generates a potential term for the SUSY breaking field,

$$\Delta V_{\text{mes}} \simeq \frac{N_Q y^2}{32\pi^2} F_{Z_L}^2 \ln \frac{|Z_L|^2}{\mu^2}, \tag{28}$$

where N_Q is the multiplicity of the messenger field. By requiring that this potential does not destabilize the SUSY breaking vacuum, we obtain

$$\frac{N_Q y^2 \lambda^2 f^4}{8\pi^2 |\langle Z_L \rangle|^2} < \frac{\partial^2 \Delta V}{\partial |\langle Z_L \rangle|^2} \equiv 2m_Z^2. \tag{29}$$

The bounds on y in Eqs. (27) and (29) are compatible if

$$N_Q < \frac{4\pi^2 \langle Z_L \rangle^6}{\lambda^4 f^8} m_Z^2. \tag{30}$$

In Fig. 1, we show the upper bound on N_Q as a function of Z_L. Here we have evaluated m_Z using $\Delta V_\pm$ in Eq. (21). It is evident that the upper bound is too severe and is inconsistent with $N_Q \gtrsim 3$, which leads us to extend the model to stabilize the sgoldstino.

2.5. Stabilization of the sgoldstino in extended models: model-independent analysis

By coupling the sgoldstino to other chiral multiplets, quantum corrections from these multiplets give additional contributions to the mass of the sgoldstino. Here we assume that a positive squared mass m_Z^2 is generated from a quantum correction. (For setups which generate a negative squared mass, see [41–46].) Even in this generic situation, we show that there is a lower bound on the axion decay constant and the gravitino mass.

The VEV of Z_L is given by

$$\langle Z_L \rangle = \frac{4}{\sqrt{3}} \frac{\lambda^2 f^4}{M_{\text{Pl}} m_Z^2}, \tag{31}$$
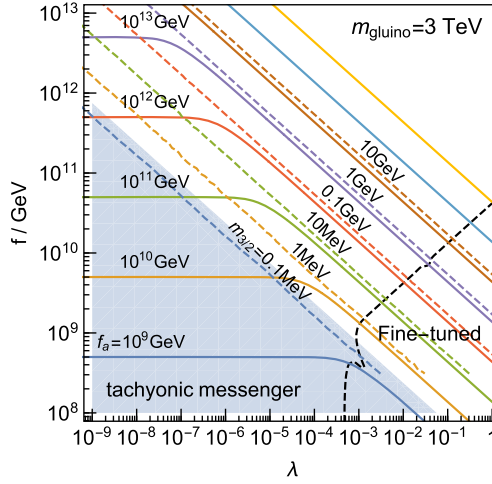
and the gauge mediated gluino mass is given by

Fig. 2. The model-independent bounds on (λ, f) and the contours of the axion decay constant f_a and the gravitino mass $m_{3/2}$. The blue shaded region is excluded as the messenger field is tachyonic. The region below the black dashed line requires fine-tuning. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

$$m_{\tilde{g}} = \frac{\alpha_3}{4\pi} \frac{F_{Z_L}}{Z_L} = \frac{\alpha_3}{4\pi} \frac{\sqrt{6}}{4} \frac{m_Z^2 M_{\mathrm{Pl}}}{\lambda f^2}. \tag{32}$$

For given λ, f, and $m_{\tilde{g}}$, m_Z^2 is fixed,

$$m_Z^2 = \frac{8\pi}{\alpha_3} \sqrt{\frac{2}{3}} \frac{m_{\tilde{g}} \lambda f^2}{M_{\mathrm{Pl}}}. \tag{33}$$

There are two bounds that must be considered. One is Eq. (30),

$$N_Q < \frac{\alpha_3^5}{8\sqrt{6}\pi^3} \frac{\lambda^3 f^6}{m_{\tilde{g}}^5 M_{\mathrm{Pl}}}. \tag{34}$$

Another is

$$m_Z^2 > \frac{1}{2} \left| \frac{\partial^2 \Delta V_\pm}{\partial |\langle Z_L \rangle|^2} \right|. \tag{35}$$

Otherwise we need fine-tuning between $\Delta V_\pm$ and additional contributions to obtain a required value of m_Z^2. In Fig. 2, we show the constraints on (λ, f) as well as the contours of the axion decay constant f_a,

$$f_a = \sqrt{2\left(M_+^2 + M_-^2 + Z_+^2 + Z_-^2\right)}, \tag{36}$$

and the gravitino mass $m_{3/2}$. Here we assume that the messenger is in the **5** representation of the $SU(5)$ GUT group, so $N_Q = 5$. For the most part, the axion decay constant is dominated by the VEVs of $M_\pm$ in the left half of the parameter space and the VEVs of $Z_\pm$ in the right half. The blue shaded region is excluded as the messenger field becomes tachyonic. The region below a black dashed line calls for fine-tuning. We obtain lower bounds from them,

$$f_a \gtrsim 1.7 \times 10^9 \left(\frac{m_{\tilde{g}}}{3\,\mathrm{TeV}}\right)^{2/3} \mathrm{GeV}, \tag{37}$$

$$m_{3/2} \gtrsim 0.2 \times \left(\frac{m_{\tilde{g}}}{3 \text{ TeV}} \right)^{5/3} \text{ MeV}. \tag{38}$$

A final issue to consider in this general approach is the tunneling rate per unit volume between the false and true vacua, $\Gamma/V = Ae^{-B}$ [47]. Using the result from [48,49], we can estimate the bounce action with

$$B = 8\pi \left(\frac{\langle Z_L \rangle}{(\sqrt{2}\lambda f^2)^{1/2}} \right)^4.$$

For the valid parameter space, $B > 10^9$, and so we expect our SUSY breaking vacuum to take much longer than the age of the Universe to decay.

2.6. Cosmology

We now address several cosmological topics that may affect the parameter space of our model.

Our model contains a SUSY preserving vacuum where the messengers obtain nonzero VEVs, so we must ensure that the SUSY breaking vacuum is selected during cosmological evolution. Following the discussion in [50], in the early Universe we assume that the SSM particles are in thermal equilibrium and therefore the sgoldstino field potential obtains finite temperature corrections from the messenger fields. We also take the sgoldstino field to be stabilized at the origin initially due to a positive Hubble-induced mass. The messenger masses become tachyonic about $\langle Z_L \rangle = 0$ as the universe cools, which in turn causes them to develop VEVs. To reach the SUSY-breaking vacuum, the sgoldstino field must obtain a sufficiently large VEV before this occurs. Since this condition references the masses of the messengers only about the origin, the model independent analysis performed in [50] should be applicable, and we obtain

$$\frac{y}{\sqrt{2}} < \left(\frac{3^{3/4}}{15} \frac{2g^2 + g'^2}{2} \right)^{2/5} \left(\frac{m_{3/2}}{M_{PL}} \right)^{1/5}. \tag{39}$$

Combining this with Eq. (35) and Eq. (27), we obtain

$$f_a \gtrsim 2.6 \times 10^{10} \left(\frac{m_{\tilde{g}}}{3 \text{ TeV}} \right)^{2/3} \text{ GeV}, \tag{40}$$

$$m_{3/2} \gtrsim 1.6 \times \left(\frac{m_{\tilde{g}}}{3 \text{ TeV}} \right)^{5/3} \text{ MeV}. \tag{41}$$

Hence vacuum selection raises the lower bounds by a factor $\mathcal{O}(10)$.

Another potential concern is that the sgoldstino, which may be produced in the early universe by thermal or nonthermal processes, might affect Big Bang Nucleosynthesis (BBN). The relevant decay modes of the sgoldstino are $Z_L \to aa$ and $Z_L \to gg$ with decay rates

$$\Gamma_{Z_L \to aa} = \frac{m_Z^3}{128\pi} \left(\frac{\langle Z_L \rangle}{2f^2 + \langle Z_L \rangle^2} \right)^2, \tag{42}$$

$$\Gamma_{Z_L \to gg} = \frac{\alpha_3^2}{128\pi^3} \frac{m_z^3}{z^2}, \tag{43}$$

respectively. Looking to the parameter space in Fig. 2, the former decay dominates in most of the area where $\langle Z_\pm \rangle$ controls the axion decay constant, while the latter decay dominates for a majority of the remaining allowed parameter space. Sgoldstino decay into gravitinos dominates in the upper right portion of the parameter space but the gravitino is heavy in this region and so it

is not favored. In both of the relevant regions, the decay time is short enough that the sgoldstino does not affect BBN.

It should also be noted that the super partners of the axion obtain large masses. This is a merit of the setup described above [13–16]. In general, the super partners of the axion obtain only small masses, typically smaller than the masses of SSM particles. Since they couple to SSM particles very weakly while being light, they cause various cosmological problems (see [17] and references therein). These problems are particularly serious in gauge mediation, where the SUSY breaking scale is small and the super partners of the axion are light. In our setup, since the axion multiplet resides in the SUSY breaking sector, the super partners of the axion can be much heavier than SSM particles and do not cause cosmological problems.

The only light particle that could affect cosmology is the gravitino due to either demanding a low reheating temperature [51–53] or overclosing the Universe [54,55]. The latter issue could be resolved by diluting the gravitino abundance through large entropy production. Possible example mechanisms include using the sgoldstino [50,56,57], saxion [58–61], messenger fields [62] or hidden sector fields [41]. Note that the sgoldstino and saxion masses are determined by the parameters of our model and the condition for successful entropy dilution could pin down the viable parameter space. For example, entropy dilution via sgoldstino is possible for a gravitino mass of $\mathcal{O}(1 \text{ GeV})$ [50]. We leave the discussion about the saxion for future work.

2.7. Alignment of CP phases

An interesting feature of our model is that the phases of the gravitino mass and the gaugino masses are aligned with each other. This is because the phase of the VEV of $\langle Z_{\pm} \rangle$, which generates the messenger scale, is aligned with the gravitino mass in a phase convention where the F term of the SUSY breaking field Z_L is real. Thus, the CP phase of the $B\mu$ term (in a convention where the μ term is real) due to the supergravity effect [63] is absent in our model. This feature would be advantageous if one requires that SUSY particles are light (e.g. to explain the experimental anomaly of the muon anomalous magnetic moment [64–66] by SUSY particles [67–69]) while the gravitino mass is large (e.g. to be consistent with a large reheating temperature).

3. Example of an extended model: low energy theory of the IYIT model

3.1. Effective theory of the IYIT model

Let us consider a vector-like SUSY breaking sector based on $SU(2)$ hidden strong gauge dynamics [25,26]. We introduce four chiral fields which are in the fundamental representation of $SU(2)$, q_i ($i = 1$–4), and six singlet chiral fields, Z_+, Z_-, $Z_{0,a}$ ($a = 1$–4). We assume $U(1)_{PQ}$ charges shown in Table 1, and consider the following superpotential,

$$W = \lambda_+ Z_+ q_1 q_2 + \lambda_- Z_- q_3 q_4 \tag{44}$$
$$+ Z_{0,a} \left(\lambda_a^{13} q_1 q_3 + \lambda_a^{14} q_1 q_4 + \lambda_a^{23} q_2 q_3 + \lambda_a^{24} q_2 q_4 \right),$$

where the λ's are constants, and summation over a is assumed. The genericity of the superpotential can be guaranteed by symmetries. One concrete example of $U(1)_R$ and Z_4 charges is shown in Table 1.

Below the dynamical scale of the hidden $SU(2)$, Λ, the theory is described by meson fields $M_{ij} \simeq q_i q_j / \eta \Lambda$ with the deformed moduli constraints, $\text{Pf} M_{ij} = \Lambda^2 / \eta^2$ [70]. Here and hereafter,

Table 1
Charge assignment of chiral fields.

	q_1	q_2	q_3	q_4	Z_+	Z_-	$Z_{0,a}$	$Q\bar{Q}$
$U(1)_R$	0	0	0	0	2	2	2	0
$U(1)_{PQ}$	$-1/2$	$-1/2$	$+1/2$	$+1/2$	1	-1	0	-1
Z_4	1	1	1	1	2	2	2	2

we assume the naive dimensional analysis to count factors of $\eta \sim 4\pi$ [71,72]. The deformed moduli constraint may be expressed by introducing a Lagrange multiplier field X,

$$W_{\text{eff}} = \kappa X \left(M_{12}M_{34} + M_{13}M_{24} + M_{14}M_{23} - \frac{\Lambda^2}{\eta^2} \right). \tag{45}$$

The tree-level superpotential in Eq. (44) becomes

$$W_{\text{tree}} = \lambda_+ \frac{\Lambda}{\eta} Z_+ M_{12} + \lambda_- \frac{\Lambda}{\eta} Z_- M_{34} \tag{46}$$

$$+ \frac{\Lambda}{\eta} Z_{0,a} \left(\lambda_a^{13} M_{13} + \lambda_a^{14} M_{14} + \lambda_a^{23} M_{23} + \lambda_a^{24} M_{24} \right).$$

We define

$$M_- \equiv M_{12}, \ M_+ \equiv M_{34}, \tag{47}$$

$$M_{0,1} \equiv \frac{1}{\sqrt{2}} \left(M_{13} + i M_{24} \right), M_{0,2} \equiv \frac{1}{\sqrt{2}} \left(M_{13} - i M_{24} \right),$$

$$M_{0,3} \equiv \frac{1}{\sqrt{2}} \left(M_{14} + i M_{23} \right), M_{0,4} \equiv \frac{1}{\sqrt{2}} \left(M_{14} - i M_{23} \right).$$

Then the effective superpotential in Eq. (45) is given by

$$W_{\text{eff}} = \kappa X \left(M_+ M_- + \frac{1}{2} M_{0,a}^2 - \frac{\Lambda^2}{\eta^2} \right). \tag{48}$$

By $SU(4)$ rotations of $M_{0,a}$ and $Z_{0,a}$, the total superpotential can be simplified as

$$W = \kappa X \left(M_+ M_- + \frac{1}{2} c_{ab} M_{0,a} M_{0,b} - \frac{\Lambda^2}{\eta^2} \right) \tag{49}$$

$$+ \lambda_+ \frac{\Lambda}{\eta} Z_+ M_- + \lambda_- \frac{\Lambda}{\eta} Z_- M_+ + \lambda_{0,a} \frac{\Lambda}{\eta} Z_{0,a} M_{0,a}$$

with c_{ab} as a unitary matrix. We will work with only one pair of neutral fields (Z_0, M_0), which corresponds to the generic case that there exists a mild hierarchy in the neutral coupling constants so that the effect of only one neutral field dominates. Therefore, after a redefinition of constants, we have the effective superpotential

$$W = \kappa X (M_+ M_- + \frac{c}{2} M_0^2 - v^2)$$

$$+ \lambda' r v Z_+ M_- + \lambda' \frac{1}{r} Z_- M_+ + \lambda'_0 v Z_0 M_0. \tag{50}$$

The coupling constant κ originates from strong dynamics and is expected to be large. The absolute value of the coupling constant c is at maximum unity. To maximize the quantum correction,

we assume $|c| = 1$ in the following. We also assume that $\lambda'_0 v^2 > \lambda f^2$, since otherwise M_0 obtains a VEV instead of $M_\pm$. The vacuum is then given by

$$\langle M_+ \rangle = rv, \ \ \langle M_- \rangle = \frac{1}{r} v, \ \ \langle Z_+ \rangle = \langle Z_- \rangle = z,$$

$$\langle M_0 \rangle = \langle Z_0 \rangle = 0. \tag{51}$$

3.2. Stabilization of the sgoldstino by neutral fields in the IYIT model

To estimate the quantum correction from Z_0 and M_0, we use the parametrization [73]

$$M_+ \to r\sqrt{v^2 - M_0^2/2}, \ M_- \to \frac{1}{r}\sqrt{v^2 - M_0^2/2}. \tag{52}$$

Here we have neglected the dependence on A, which is irrelevant for the quantum correction from Z_0 and M_0 to Z_L. The effective superpotential of Z_L and Z_0, M_0 is given by

$$W_{\text{eff}} \simeq \lambda f^2 (Z_+ + Z_-)\sqrt{1 - \frac{M_0^2}{2v^2}} + \lambda'_0 v Z_0 M_0$$

$$\simeq \sqrt{2}\lambda f^2 Z_L - \frac{\sqrt{2}}{4} R^2 \lambda Z_L M_0^2 + \lambda_0 f Z_0 M_0,$$

$$R \equiv \frac{f}{v} > 1, \lambda_0 \equiv \frac{1}{R}\lambda'_0. \tag{53}$$

The quantum correction to the potential of Z_L from Z_0 and M_0 is given by

$$\Delta V_0 = \frac{\lambda^4 R^4 f^4}{64\pi^2} f(\frac{\lambda R^2 z}{\lambda_0 f}) \left(1 + O\left((\lambda R/\lambda_0)^4 \right) \right) \tag{54}$$

$$\simeq \begin{cases} \frac{\lambda^4 R^8 f^2}{96\pi^2} \left(\frac{\lambda}{\lambda_0} \right)^2 |Z_L|^2 & : \lambda R^2 |Z_L| \lesssim \lambda_0 f \\ \frac{\lambda^4 R^4 f^4}{16\pi^2} \ln\frac{\lambda R^2 |Z_L|}{\lambda_0 f} & : \lambda R^2 |Z_L| \gtrsim \lambda_0 f, \end{cases}$$

$$f(x) = (4 + x^2)^{-2}\Big[32 + 20x^2 + 3x^4$$

$$+ \left(16 - 4\sqrt{1 + 4/x^2} + 8x^2 + x^4 - 6x\sqrt{4 + x^2} \right.$$

$$- x^3\sqrt{4 + x^2} \Big) \ln\left(1 + \frac{x^2}{2} - x\sqrt{1 + x^2/4} \right)$$

$$+ \left(16 + 4\sqrt{1 + 4/x^2} + 8x^2 + x^4 + 6x\sqrt{4 + x^2} \right.$$

$$+ x^3\sqrt{4 + x^2} \Big) \ln\left(1 + \frac{x^2}{2} + x\sqrt{1 + x^2/4} \right)\Big].$$

In this model, m_Z^2 is given by

$$m_Z^2 = \frac{\lambda^4 R^8 f^2}{96\pi^2} \left(\frac{\lambda}{\lambda_0} \right)^2 + \frac{1}{2}\frac{\partial^2 \Delta V_\pm}{\partial |\langle Z_L \rangle|^2}. \tag{55}$$
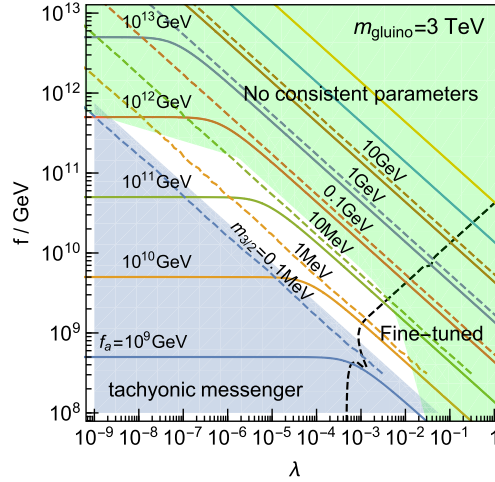
Fig. 3. The bounds on (λ, f) for the IYIT model and the contours of the axion decay constant f_a and the gravitino mass $m_{3/2}$. The blue shaded region is excluded as the messenger field is tachyonic. The region below the black dashed line requires fine-tuning. There is no consistent parameter (λ_0, R) to yield the green shaded region. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

3.3. Parameter window of the IYIT model

Let us now discuss constraints on the parameter space. The constraint from the stability of the vacuum, $\lambda_0' v^2 > \lambda f^2$, is

$$\lambda R < \lambda_0. \tag{56}$$

Constants $\lambda' r$, λ'/r and λ_0' are dimensionless coupling constants in the IYIT model, and are at the most $O(1)$. This gives upper bounds on λ_0 and R,

$$\lambda R^3 < 1, \tag{57}$$

$$\lambda_0 R < 1. \tag{58}$$

Finally, the potential of Z_L becomes logarithmic for $\lambda R^2 Z_L > \lambda_0 f$, and cannot stabilize the sgoldstino against the tadpole term, so

$$\lambda R < \sqrt{\frac{4\pi}{\sqrt{2}\alpha_3} \frac{m_{\tilde{g}}}{f}} \lambda_0^{1/2}. \tag{59}$$

By combining the bounds in Eqs. (56)–(59), we obtain upper bounds on R^4/λ_0,

$$\frac{R^4}{\lambda_0} < \begin{cases} \lambda^{-2} & \text{Eqs. (56), (57)} \\ \lambda^{-8/3} h^2 & \text{Eqs. (57), (59)} \\ \lambda^{-10/3} h^{10/3} & \text{Eqs. (58), (59)} \end{cases}, \quad h \equiv \sqrt{\frac{4\pi}{\sqrt{2}\alpha_3} \frac{m_{\tilde{g}}}{f}} \tag{60}$$

These give upper bounds on m_Z^2.

In Fig. 3, we show the constraints on (λ, f). The meaning of the blue shaded region and the black dashed line are the same as in Fig. 2. In the green shaded region, the bound on m_Z^2 from Eqs. (55) and (60) is inconsistent with the required value of m_Z^2 shown in Eq. (33). We obtain the bounds on the axion decay constant f_a and the gravitino mass $m_{3/2}$

$$10^9 \text{ GeV} \lesssim f_a \lesssim 10^{12} \text{ GeV}, \tag{61}$$

$$0.1 \text{ MeV} \lesssim m_{3/2} \lesssim 10 \text{ MeV}, \tag{62}$$

for a gluino mass $\mathcal{O}(\text{TeV})$. It is interesting that the allowed range of f_a is consistent with the axion dark matter scenario [74–77].

4. Summary

In this letter, we have presented a model that tackles several outstanding issues in the Standard Model and its supersymmetric extension.

We have examined a minimal hidden sector that consists of a superpotential with a $U(1)$ symmetry, which we identify with the PQ symmetry, and messenger quarks that carry $SU(3)_c$ charges. Supersymmetry and this PQ symmetry are spontaneously broken while lowest order supergravity effects create the messenger scale. Quantum effects generate a potential for the sgoldstino and force constraints on model parameters to ensure the stability of the SUSY-breaking vacuum. These constraints proved to be too stringent and required that we supplement the minimal model with extra matter fields. We have shown that classes of models that share features with ours, such as a quantum mechanically induced sgoldstino mass and a minimal messenger sector, automatically obtain lower bounds on the axion decay constant and gravitino mass. This fact encouraged us to supplement our minimal model in the hopes that such attractive features could be preserved and expanded upon in a stable extended model.

An IYIT model with $SU(2)$ gauge dynamics is a natural candidate for such an extended model since the minimal model is easily embedded in the $U(1)$ charged subsector of this larger model. Combining the inequalities from vacuum stability and IYIT coupling constants, upper bounds for the sgoldstino mass were derived. The resulting window in parameter space was found to restrict the gravitino mass to lie between $0.1 \text{ MeV} \lesssim m_{3/2} \lesssim 10 \text{ MeV}$ and the axion decay constant to $10^9 \text{ GeV} \lesssim f_a \lesssim 10^{12} \text{ GeV}$, which is the suitable range for invisible axion dark matter.

Acknowledgements

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