A Joint Detection and Decoding Receiver Design for Polar Coded MIMO Wireless Transmissions

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***Abstract*—This work develops a novel design of joint detection and decoding receiver for multiple-input multiple output (MIMO) wireless transmissions that utilizes polar codes in forward error correction (FEC). To optimize the overall receiver performance, we integrate the polar code constraints during signal detection by relaxing and transforming FEC code constraints from the original Galois field to the real field. We propose a novel joint linear programming (LP) optimization formulation that takes into consideration the transformed polar code constraints when designing a novel receiver robust against practical obstacles including channel state information (CSI) errors, additive noises, co-channel interferences, and pilot contamination. Our newly proposed joint LP formulation can also be integrated with reduced complexity polar decoders such as successive cancellation (SC) and successive cancellation list (SCL) decoders to deliver superior receiver performance at low cost.**

I. Introduction

Multiple-input multiple-output (MIMO) transceivers have been widely adopted in modern wireless communication networks owing to their ability to achieve high spectral efficiency. As a result, designing robust and efficient receivers for MIMO communication systems has also been extensively investigated in the literature. At the same time, forward error correction (FEC) codes play a critical role to combat against possible detection errors caused by noise, interferences, and channel distortions in MIMO wireless systems. Well-known FEC channel codes that have been studied and utilized effectively in advanced communication systems include convolutional codes, turbo codes and low-density parity-check (LDPC) codes. Ideally, optimum receiver that minimizes the probability of error in detection of transmitted symbols can be obtained by the maximum likelihood detector (MLD) under the FEC codeword constraints. However, such ideal joint maximum likelihood detection and decoder is computationally expensive and can be costly to implement for many practical FEC codes of sufficient code length.

In practice, many MIMO receivers apply symbol detection in independently from the FEC decoders. The common receiver architecture of disjoint MIMO detection followed by FEC decoding is attributable to the NP-hard high complexity of incorporating Galois field FEC constraints in the ML detection that operates strictly in real (or complex) field. To overcome the difficulty posed by the conflicting fields of operation for detection and decoding, existing joint MIMO detection and FEC decoding receivers typically utilize the concept of soft information exchange between soft decoder and detector to form a turbo receiver [1], [2].

One recent development that may change this traditional turbo processing receiver is the work by Feldman et al. [3] that successfully transformed FEC code constraints from Galois field to a set of linear inequalities called fundamental polytope in the real field. Integrating the linear FEC constraints with the symbol detection formulation, the authors of [4] developed an l_1-norm based joint detector for flat-fading MIMO channels whereas Flangan [5] also proposed a unified framework for linear programming (LP) receivers. In [6]–[10], Wang and coworkers proposed various joint detection-decoding schemes for LDPC-based systems and demonstrated substantial performance improvement over traditional disjoint receivers when channel state information (CSI) is not accurately known. In addition to strengthening performance in point-to-point communication links, code constraints can also be used in multi-user scenarios to suppress co-channel interferences [8], [11]. It is important to note that the number of transformed linear constraints grow exponentially with the weight of the FEC code's binary parity check matrix. Thus, the sparse parity check matrix of LDPC codes make such codes particularly attractive for joint detection and decoding based on the aforementioned transformation of FEC constraints.

In recent years, another class of interesting FEC codes known as polar codes, discovered by Arıkan [12] [13] in 2008, has also shown strong potential in wireless communications. In particular, polar codes have been adopted as FEC codes for physical downlink control channels (PDCCH) in 5G cellular standard. Polar codes have been shown to approach channel capacity over binary discrete memoryless channels (B-DMCs) without high encoding and decoding complexity. Practical use of polar codes has been investigated in AWGN channels [13]–[15] and fading channels [16]. Popular decoder schemes include successive cancellation (SC) decoding introduced by Arıkan [12] and successive cancellation list decoding introduced in [17].

In order to develop an optimized joint detection-decoding receiver that incorporates polar code constraints, we must overcome a new challenge that the number of linear constraints in fundamental polytope [3] is often large since the polar code parity check matrix is far from sparse. In fact for polar codes, not only the number of constraints makes the LP receivers impractical, but a decoder based on such polytope will also fail in polar coded systems, as illustrated in [18]. Instead, another relaxed polytope has been proposed in [18] according to the factor graph representation of polar codes that obtains the ML-certificate property. This new polytope is in a space dimension of $O(N \log N)$ where N is the block length.

In this work, we first present a generic MLD formulation for a FEC coded MIMO detector in Section II before relaxing into LP receiver formulation. In Section III, we incorporate polar code constraints by incorporating the approach of [18] to develop a novel joint LP receiver that effectively exploits the rich and important diversity of FEC code constraints. This new receiver is robust against practical non-idealities including CSI errors, channel noises, co-channel interferences, and pilot contamination. We present simulation results in Section IV to illustrate the efficacy of the proposed receiver design before concluding the paper in Section V.

II. System Model

Consider a polar-coded MIMO transmission system in which information bits are first encoded with polar encoder as FEC codewords before being mapped into QAM data symbols of constellation $\tilde{Q}$. At the receiver side, the objective is to recover the source bits despite channel distortions, interferences, and channel noises. Our goal is to design a joint detector that incorporates the polar code information for improved performance.

A polar code of rate $R = K/N$ is specified by $(N, K, \mathcal{I}^c)$, where $N = 2^n$ is the codeword length, K is the number of information bits in a codeword. Let $\mathcal{I} \subseteq \{1, \ldots, N\}$ denote the set of indices that are the information bits and $\mathcal{I}^c$ be its compliment which is known as frozen (non-information bearing) bits. Let $\mathbf{u} = [u_1, u_2, \cdots, u_N]$ denote the binary information vector and $\mathbf{b} = [b_1, b_2, \cdots, b_N]$ denote the binary codeword vector. There is an invertible mapping of $\mathbf{b} = \mathbf{u}\mathbf{G_N}$ between $\mathbf{u}$ and $\mathbf{b}$ where $\mathbf{G_N}$ is the generator matrix of the polar code. $\mathbf{G_N}$ is defined through $\mathbf{G_N} = \mathbf{B_N}\mathbf{F}^{\otimes n}$, where $\mathbf{B_N}$ is a bit reversal operator defined in [12], and $\mathbf{F}^{\otimes n}$ denotes n-fold Kronecker power of polarization kernel

$$\mathbf{F} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}.$$

Using the concept of channel polarization, N identical realization of the channel can be transformed into N virtual bit-channels, which become polarized as either extremely noisy or completely error-free as N grows asymptotically to approach infinity. Consequently, the crucial step in construction of polar codes is to sort the virtual bit-channels based on their capacity and to select the K most reliable ones out of N bit-channels to carry information bits. The remaining $N-K$ bit channels will carry the frozen bits (set to 0). As shown in [12], the ratio of reliable virtual channels to the whole set of virtual channels K/N converges to the channel capacity for asymptotically large N. As a result, polar codes can achieve channel capacity as the codeword size becomes asymptotically large in B-DMCs.

A. Channel Model

For length N polar codeword and Q-ary QAM constellation, the symbol length is $M = N/Q$. We consider a spatial multiplexing MIMO system with n transmit antennas and m receive antennas. In such a system, we can transmit one codeword in $T = M/n$ time instants. For each time instant t we can define complex QAM symbol $\tilde{x}_{i,t} \in \tilde{Q}$ and QAM vector $\tilde{\mathbf{x}}[t] = [\tilde{x}_{1,t} \cdots, \tilde{x}_{n,t}]^T$ as the transmission signal vector. Let each received symbol be $\tilde{y}_{i,t} \in \mathbb{C}$ and the received signal vector be $\tilde{\mathbf{y}}[t] = [\tilde{y}_{1,t}, \cdots, \tilde{y}_{m,t}]^T$. By defining $\tilde{\mathbf{H}} = [\tilde{h}_{ij}] \in \mathbb{C}^{m \times n}$ as the linear MIMO channel matrix for a flat fading wireless channel, we can write the received signal vector as

$$\tilde{\mathbf{y}}[t] = \tilde{\mathbf{H}}\tilde{\mathbf{x}}[t] + \tilde{\mathbf{n}}[t] \tag{1}$$

in which $\tilde{\mathbf{n}}[t] = [\tilde{n}_{1,t}, \cdots, \tilde{n}_{m,t}]^T \in \mathbb{C}^m$ is the additive white Gaussian noise (AWGN) vector whose elements are i.i.d. complex random variables and $\tilde{n}_{i,t} \sim \mathcal{CN}(0, \sigma_n^2)$. Note that the elements of $\tilde{\mathbf{H}}$ are typically unknown and are estimated by utilizing the pilot symbols.

B. Maximum Likelihood Receiver

If the channel is known or estimated at the receiver, the optimal maximum likelihood detector (MLD) that minimizes the probability of error for each transmission at time instant t would aim to solve the problem

$$\min_{\tilde{\mathbf{x}} \in \tilde{Q}^n} \parallel \tilde{\mathbf{y}}[t] - \tilde{\mathbf{H}}\tilde{\mathbf{x}} \parallel_2^2 \tag{2}$$

It should be noted that this MLD receiver does not yet take into consideration the fact that a data symbol vector $\tilde{\mathbf{x}}$ must be output bits of an FEC codeword. In other words, only valid FEC codewords should be considered in the MLD receiver. Eliminating invalid codewords in MLD would have taken into consideration of the Galois field code constraints within the detection stage to minimize the probability of producing wrong symbol sequence by the detector to achieve better performance. Codeword constraints are even more critical when our estimate of the channel matrix $\tilde{\mathbf{H}}$ is itself inaccurate.

We let each FEC codeword of length N, $\mathbf{b} = [b_1\ b_2\ \cdots b_N]$ span T data vectors possibly with the help of padding bits, i.e.,

$$\tilde{\mathcal{M}}(\mathbf{b}) = \{\tilde{\mathbf{x}}[1],\ \tilde{\mathbf{x}}[2],\ \cdots,\ \tilde{\mathbf{x}}[T]\}$$

where $\tilde{\mathcal{M}}(\cdot)$ is the mapping of FEC bits to data symbols for transmission. Consequently, the optimum receiver can be written as

$$\min_{\mathbf{b}} \sum_{k=1}^{T} \parallel \tilde{\mathbf{y}}[k] - \tilde{\mathbf{H}}\tilde{\mathbf{x}}[k] \parallel_2^2 \tag{3a}$$

$$\tilde{\mathcal{M}}(\mathbf{b}) = \{\tilde{\mathbf{x}}[1],\ \tilde{\mathbf{x}}[2],\ \cdots,\ \tilde{\mathbf{x}}[T]\} \tag{3b}$$

$$\mathbf{b} \in \mathcal{F} \tag{3c}$$

where $\mathcal{F}$ denotes the set of all valid FEC codewords of length N. The optimization problem above is a non-convex integer optimization problem and is extremely difficult to solve because it requires exhaustive search over all valid set of symbols $\tilde{\mathcal{M}}(\mathbf{b}) = \{\tilde{\mathbf{x}}[1], \ \tilde{\mathbf{x}}[2], \ \cdots, \ \tilde{\mathbf{x}}[T]\}$ that satisfy the coding constraints. This leads to an NP-hard problem whose complexity grows exponentially with N. Furthermore, the constraint that requires $\mathbf{b} \in \mathcal{F}$, is defined in Galois field and obviously is a non-convex constraint when considering the real-field optimization in Eqs. (3). In the next section we show how we can modify the cost function and the constraints in (3) into an LP optimization problem that serves as a unified joint receiver.

III. LP Receiver with Polar Coding Constraints

A. Reformulation of Objective Function

To formulate the LP receiver, our first step is to modify the objective function by changing the l_2 norm in (2) to l_1 norm metric. The use of l_1 norm as an optimization metric has been used in data analysis and parameter estimation because it is robust to the impulsive noises and other man-made radio interferences [4]. To simplify the l_1 notation, we shall reformulate the problem into real field only. Consequently, we can transform our system from complex to real by defining

$$\mathbf{y}[t] = \begin{bmatrix} \mathrm{Re}\{\tilde{\mathbf{y}}[t]\} \\ \mathrm{Im}\{\tilde{\mathbf{y}}[t]\} \end{bmatrix}, \mathbf{x}[t] = \begin{bmatrix} \mathrm{Re}\{\tilde{\mathbf{x}}[t]\} \\ \mathrm{Im}\{\tilde{\mathbf{x}}[t]\} \end{bmatrix}, \mathbf{n}[t] = \begin{bmatrix} \mathrm{Re}\{\tilde{\mathbf{n}}[t]\} \\ \mathrm{Im}\{\tilde{\mathbf{n}}[t]\} \end{bmatrix} \tag{4}$$

and

$$\mathbf{H} = \begin{bmatrix} \mathrm{Re}\{\tilde{\mathbf{H}}\} & -\mathrm{Im}\{\tilde{\mathbf{H}}\} \\ \mathrm{Im}\{\tilde{\mathbf{H}}\} & \mathrm{Re}\{\tilde{\mathbf{H}}\} \end{bmatrix} \tag{5}$$

Given the new notations in the real field, we can write our system equation that characterizes the relationship between the channel input $\mathbf{x}[t]$ and the channel output $\mathbf{y}[t]$ into

$$\mathbf{y}[t] = \mathbf{H}\mathbf{x}[t] + \mathbf{n}[t]. \tag{6}$$

To transform (3b) also from complex to real, we define $\mathcal{M}(\cdot)$ to be the mapping from bit vector $\mathbf{b}$ of length N to $\mathbf{x} = \{\mathbf{x}[1], \mathbf{x}[2], \ldots, \mathbf{x}[T]\}$ where $\mathbf{x}[t], 1 \le t \le T$ is the set of real transmission symbols defined in (4).

We can classically reformulate our problem into a linear programming problem with two sets of generalized vector inequalities by introducing a set of slack variables $e_{i,t} \ge 0, 1 \le i \le m, 1 \le t \le T$. We also define $\mathbf{e}[t]$ to be a vector of these slack variables at each time instant t, i.e. $\mathbf{e}[t] = [e_{t,1}, \cdots, e_{t,m}]$. Consequently we can reformulate the optimization problem of (3) into

$$\begin{aligned} \min \quad & \sum_{t=1}^{T} \sum_{i=1}^{m} e_{i,t} \\ \text{s.t.} \quad & \mathbf{H}\mathbf{x}[t] - \mathbf{e}[t] \preceq \mathbf{y}[t] \\ & -\mathbf{H}\mathbf{x}[t] - \mathbf{e}[t] \preceq -\mathbf{y}[t] \\ & \mathcal{M}(\mathbf{b}) = \{\mathbf{x}[1], \mathbf{x}[2], \ \cdots, \ \mathbf{x}[T]\} \\ & \mathbf{b} \in \mathcal{F} \end{aligned} \tag{7}$$

Note that $x \preceq y$ denotes $x_i \le y_i$ for every coordinate i.

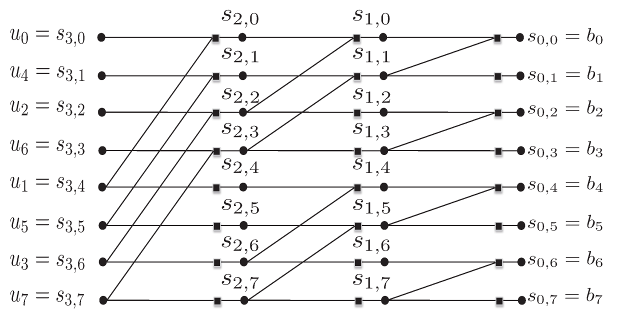

Fig. 1. factor graph representation of a polar code with block length $N = 2^3$

B. Receiver Integration of Polar Coding Constraints

Now we describe how to integrate information from the $\mathbf{b} \in \mathcal{F}$ codeword constraint into the linear programming optimization of (7) when polar codes are adopted as FEC. The authors of [3] described how to transform coding constraints in Galois field into certain linear constraints in real field that are known as the fundamental polytope $\bar{\mathcal{Q}}$ based on the parity check matrix originally defined in Galois field. This polytope conversion scales well for LDPC codes because of the sparsity of the parity check matrix. However, the dense parity check matrix of polar codes not only leads to overwhelming large number of constraints, but also leads to frequent failure of a decoder based on such polytope $\bar{\mathcal{Q}}$ [18].

Recursive structure of polar codes leads to a sparse graph representation with $O(N \log N)$ auxiliary variables which is shown for block length $N = 2^3$ in Figure 1. Taking advantage of the sparse factor graph representation, a new polytope can be defined in a space of dimension $O(N \log N)$ [18]. For this reason, we shall exploit this polytope to generate linear coding constraints that can be incorporated into the LP receiver of (7). We name the corresponding polytope $\mathcal{P}$. The graph of Figure 1 shows how a polar codeword $\mathbf{b}$ can be constructed from binary vector $\mathbf{u}$ by a one to one mapping through the generator matrix $\mathbf{G_N}$, $\mathbf{b} = \mathbf{u}\mathbf{G_N}$. The circle nodes on the graph represent a total of $N(1 + \log N)$ binary variables whereas the squares represent the check nodes. If all the check nodes are satisfied, then $\mathbf{b}$ is a valid codeword.

An example of a check node constraint in Figure 1 is $u_0 \oplus u_1 \oplus s_{2,0} = 0$, where $\oplus$ denotes modulo-2 summation. To define the relaxed polytope $\mathcal{P}$, we let the variables in the graph be real variables instead of binary. Note that each constraint involves only either 3 or 2 variables. Therefore, for each check node $j \in \mathcal{J}$ with 3 neighbors $\mathcal{N}(j) = \{a_1, a_2, a_3\}$, the local minimal convex polytope of check node j is $\mathcal{P}_j$ and can be very simply defined by below linear inequalities

$$\begin{aligned} & 0 \le a_1 \le a_2 + a_3, \\ & 0 \le a_2 \le a_3 + a_1, \\ & 0 \le a_3 \le a_1 + a_2, \\ & a_1 + a_2 + a_3 \le 2 \end{aligned} \tag{8}$$

For each check node $j \in \mathcal{J}$ with only two neighbors $\mathcal{N}(j) = \{a_1, a_2\}$, the local polytope $\mathcal{P}_j$ is defined by

$$0 \leq a_1 = a_2 \leq 1 \tag{9}$$

Moreover, let the cutting plane T defined by setting all frozen variables whose indices belong to $\mathcal{I}^c$ to zero. Therefore, the polytope $\mathcal{P}$ is the intersection of all local minimal polytopes, and the cutting planes $\mathcal{T}$

$$\mathcal{P} = \left(\bigcap_j \mathcal{P}_j \right) \cap \mathcal{T} \tag{10}$$

Therefore, we can write down the linear coding constraints by enforcing all the variables of the factor graph to be inside the polytope $\mathcal{P}$, i.e. $\mathbf{s} \in \mathcal{P}$ where $\mathbf{s}$ denotes all the variables of the factor graph. These constraints can be added to (7) as relaxed version of (3c). Therefore, the final formulation for the LP-based joint detection and decoding receiver can be simply written as

$$\begin{aligned} \min \quad & \sum_{t=1}^{T} \sum_{i=1}^{m} e_{i,t} \\ \text{s.t.} \quad & \mathbf{H}\mathbf{x}[t] - \mathbf{e}[t] \preceq \mathbf{y}[t] \\ & -\mathbf{H}\mathbf{x}[t] - \mathbf{e}[t] \preceq -\mathbf{y}[t] \\ & \mathcal{M}(\mathbf{b}) = \{\mathbf{x}[1], \mathbf{x}[2], \cdots, \mathbf{x}[T]\} \\ & \mathbf{s} \in \mathcal{P} \subseteq [0,1]^{N(1+\log N)} \end{aligned} \tag{11}$$

Note that all $N(1 + \log N)$ variables of $\mathbf{s}$ are optimization variables of (11). As a result, the detector generates an estimate for each of these variables including $\mathbf{b}$. We denote the estimated bit vector as $\hat{\mathbf{b}} = [\hat{b}_1 \; \hat{b}_2 \; \cdots \hat{b}_N]$. Each element $0 \leq \hat{b}_i \leq 1$ can be used to calculate the likelihood of b_i to be 0 or 1. Once the log-likelihood ratio was generated by our joint LP receiver, a soft-input decoder such as SC or SCL can further decode $\hat{\mathbf{b}}$ to produce a final output $\hat{\mathbf{u}}$ which denotes an estimation of uncoded source bits.

IV. Simulation Results

We now present a set of simulation tests and results to demonstrate the performance of the proposed joint LP receiver in terms of bit error rate (BER) and block error rate (BLER). In particular, we will compare the joint LP receiver with a conventional decoupled MMSE detector in which detection and decoding are performed sequentially.

In our simulation test, we adopt a 4×4 MIMO wireless communication system model with QPSK modulation over a flat Rayleigh fading channel. For FEC, we adopt a polar code of rate = 1/2 with length $N = 128$. At the receiver, the flat fading MIMO channel matrix is estimated and is subjected to estimation errors. More specifically, our estimate of the channel matrix $\hat{\mathbf{H}}$ is assumed to be $\hat{\mathbf{H}} = \mathbf{H} + \mathbf{E}$, where $\mathbf{H} = [h_{ij}] \in \mathbb{R}^{2m \times 2n}$ is the real transformation of the complex channel matrix $\tilde{\mathbf{H}}$, based on (5). Therefore, $\mathbf{H} = [h_{ij}] \in \mathbb{R}^{2m \times 2n}$ consists of i.i.d. elements such that $h_{i,j} \sim \mathcal{N}(0, 1)$. The estimation error matrix $\mathbf{E} = [e_{ij}] \in \mathbb{R}^{2m \times 2n}$ also consists of i.i.d. random elements such that $e_{i,j} \sim \mathcal{N}(0, \alpha \frac{\sigma_n^2}{2})$, where σ_n^2 is the noise variance and $\alpha \geq 1$ depends on whether there are enough pilot symbols to estimate the channel accurately or not. In our simulations, we investigate two cases of $\alpha = 1$ (enough pilot symbols) and $\alpha = 2$ (short pilot length leading to estimation error variance to be twice the noise variance). After the detection stage, SCL decoder with list size $L = 1$ and $L = 4$ has been used. BER and BLER has been calculated for both the decoder output and the detection output by hard slicing the soft information that it generated.

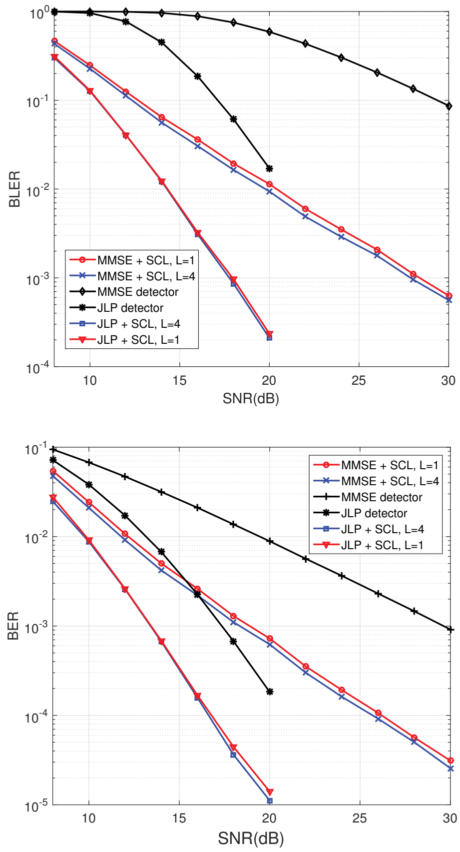

Fig. 2. Performance comparison of joint LP detector using code constraints versus decoupled MMSE, $\alpha = 1$

Figure 2 shows the BER and BLER of the proposed joint linear programming receiver (JLP) with the benchmark MMSE receiver. Three different parameter settings are considered, respectively. The first set of results compares the JLP detector without SCL decoding with the MMSE detector without SCL decoding. It can be clearly seen from Figure 2 that our proposed JLP receiver substantially outperforms the MMSE detector by integrating the FEC codeword constraint information during detection. The more consistent detector output can improve the BER by as much as 12dB in terms of signal-to-noise ratio (SNR) at BER rate of 10^{-3}. We further compare the effect of SCL decoding after detection output based on JLP or MMSE. It is clear from Figure 2 that both BLER and BER are substantially reduced when ensuing polar decoding is adopted. In terms of BLER, the performance improvement by the JLP over MMSE is as high as 10dB for both SCL based

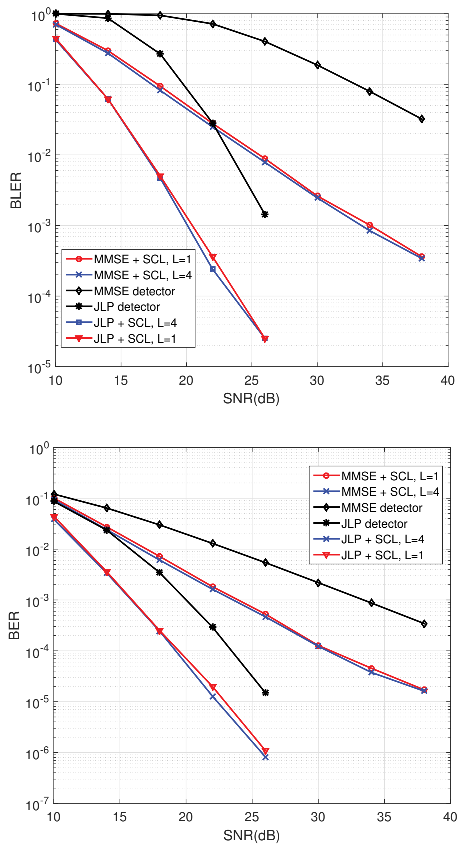

Fig. 3. Performance comparison of joint LP detector using code constraints versus decoupled MMSE, $\alpha = 2$

on $L = 1$ and $L = 4$.

Next, Figure 3 illustrates the performance comparison when channel estimation error variance is quite large, at twice the channel white noise variance, i.e. $\alpha = 2$. In this case the performance gain of JLP over MMSE is even more evident. JLP together with SCL decoder outperforms MMSE with SCL decoder by nearly 16 dB at BER rate of 1.5×10^{-5}. The performance improvement in JLP is well expected since MMSE only relies the received signals without integrating the code constraints. As a result, MMSE signal detection is significantly more vulnerable to channel estimation error. On the other hand, JLP takes advantage of FEC coding constraints and can be more robust against channel estimation error.

V. Conclusion

We proposed a robust receiver design for MIMO systems by adopting the capacity-approaching polar codes for forward error correction (FEC). We incorporate relaxed polar code constraints to formulate a novel joint linear programming (JLP) optimization problem. The proposed receiver is more robust against channel estimation error and other non-idealities at the wireless receivers. Our results demonstrate superior receiver performance particularly when the knowledge of the wireless channel is inaccurate due to practical obstacles such as short pilot length or pilot contamination. Our proposed JLP detector can also directly interface with well-known low complexity polar decoders such as SC and SCL decoders for effective error correction. Future works may consider theoretical analysis of the proposed receiver as well as further complexity reduction of the joint linear programming receivers.

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References

[1] B. Lu, X. Wang, and K. R. Narayanan, "LDPC-based space-time coded OFDM systems over correlated fading channels: Performance analysis and receiver design," *IEEE Transactions on Communications*, vol. 50, no. 1, pp. 74–88, 2002.

[2] B. M. Hochwald and S. Ten Brink, "Achieving near-capacity on a multiple-antenna channel," *IEEE transactions on communications*, vol. 51, no. 3, pp. 389–399, 2003.

[3] J. Feldman, M. J. Wainwright, and D. R. Karger, "Using linear programming to decode binary linear codes," *IEEE Transactions on Information Theory*, vol. 51, no. 3, pp. 954–972, 2005.

[4] T. Cui, T. Ho, and C. Tellambura, "Linear programming detection and decoding for MIMO systems," in *Information Theory, 2006 IEEE International Symposium on*. IEEE, 2006, pp. 1783–1787.

[5] M. F. Flanagan, "A unified framework for linear-programming based communication receivers," *IEEE Transactions on Communications*, vol. 59, no. 12, pp. 3375–3387, 2011.

[6] K. Wang and Z. Ding, "Joint turbo receiver for LDPC-coded MIMO systems based on semi-definite relaxation," *arXiv preprint arXiv:1803.05844*, 2018.

[7] K. Wang, W. Wu, and Z. Ding, "Joint detection and decoding of LDPC coded distributed space-time signaling in wireless relay networks via linear programming," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP), Florence, Italy*, 2014, pp. 1925–1929.

[8] K. Wang and Z. Ding, "Robust receiver design based on FEC code diversity in pilot-contaminated multi-user massive MIMO systems," in *IEEE Intl. Conf. on Acoust., Speech and Signal Process. (ICASSP), Shanghai, China*, 2016.

[9] K. Wang, H. Shen, W. Wu, and Z. Ding, "Joint detection and decoding in LDPC-based space-time coded MIMO-OFDM systems via linear programming," *IEEE Trans. Signal Process.*, vol. 63, no. 13, pp. 3411–3424, 2015.

[10] K. Wang, W. Wu, and Z. Ding, "Diversity combining in wireless relay networks with partial channel state information," in *IEEE Intl. Conf. on Acoust., Speech and Signal Process. (ICASSP), South Brisbane, Queensland*, 2015, pp. 3138–3142.

[11] K. Wang and Z. Ding, "FEC code anchored robust design of massive MIMO receivers," *IEEE Trans. Wireless Commun.*, vol. 15, no. 12, pp. 8223–8235, 2016.

[12] E. Arıkan, "Channel polarization: A method for constructing capacity-achieving codes for symmetric binary-input memoryless channels," *IEEE Transactions on Information Theory*, vol. 55, no. 7, pp. 3051–3073, 2009.

[13] E. Arikan, "A performance comparison of polar codes and Reed-Muller codes," *IEEE Communications Letters*, vol. 12, no. 6, 2008.

[14] I. Tal and A. Vardy, "How to construct polar codes," *IEEE Transactions on Information Theory*, vol. 59, no. 10, pp. 6562–6582, 2013.

[15] D. Wu, Y. Li, and Y. Sun, "Construction and block error rate analysis of polar codes over awgn channel based on gaussian approximation," *IEEE Communications Letters*, vol. 18, no. 7, pp. 1099–1102, 2014.

[16] A. Bravo-Santos, "Polar codes for the Rayleigh fading channel," *IEEE Communications Letters*, vol. 17, no. 12, pp. 2352–2355, 2013.

[17] I. Tal and A. Vardy, "List decoding of polar codes," in *Information Theory Proceedings (ISIT), 2011 IEEE International Symposium on*. IEEE, 2011, pp. 1–5.

[18] N. Goela, S. B. Korada, and M. Gastpar, "On LP decoding of polar codes," in *Information Theory Workshop (ITW), 2010 IEEE*. IEEE, 2010, pp. 1–5.