Joint Turbo Receiver for LDPC-Coded MIMO Systems Based on Semi-definite Relaxation

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***Abstract*—Semi-definite relaxation (SDR) has demonstrated the capability of approaching maximum-likelihood (ML) performance. In this work, we first develop a new SDR-based detector that exploits forward error correction (FEC) code information in the detection stage. The joint SDR detector substantially improves overall receiver performance by generating highly reliable information to downstream decoder. For further performance improvement, we integrate the joint SDR detector with decoder using a feedback link to form an iterative turbo receiver. Meanwhile, we propose a simplified SDR receiver that solves only one SDR problem per codeword instead of solving multiple SDR problems in the iterative turbo processing. This simplification significantly reduces the complexity of SDR turbo receiver, while maintaining a similarly superior error performance.**

I. Introduction

Multiple-input multiple-output (MIMO) technology offers the potential for high data rates and/or reliable transmissions by utilizing advanced wireless transceiver design. In the receiver end, joint ML detection and decoding is known to be optimum in terms of minimum error probabilities. However, ML detection and decoding is NP-hard. Alternatively, a number of linear receivers, such as matched filter, zero-forcing and minimum mean squared error receivers, have been extensively investigated. Other more sophisticated receivers, such as successive/parallel interference cancellation, have also been studied. Furthermore, turbo processing is capable of approaching MIMO capacity by exchanging extrinsic information between detector and decoder [1]. Nonetheless, the soft detector in the turbo receiver incurs exponential complexity in the computation of exact log-likelihood ratio (LLR). Thus, of strong importance is the investigation of new ways to reduce the complexity of turbo receiver.

For uncoded MIMO systems, SDR has recently become a popular technique to approximate the ML solutions because of its upper-bounded polynomial complexity and its guaranteed approximation error [2]. Several earlier works [3], [4] developed SDR detection in proposing multiuser detection for CDMA transmissions. Among them, the authors of [5] proposed an SDR-based multiuser detector for M-ary PSK signaling. Another work in [6] presented an efficient SDR implementation of blind ML detection of signals that utilize orthogonal space-time block codes. Furthermore, multiple SDR detectors of 16-QAM signaling were compared and shown to be equivalent in [7]. SDR has also been adopted in the design of lower-complexity turbo receiver. Instead of enumerating through the large candidate list that grows exponentially with the number of simultaneously transmitted MIMO data symbols, the authors of [8] solve one SDR problem for each coded bit and this approach results in negligible performance loss. The authors of [9] further developed two soft-output SDR detectors that are significantly less complex while exhibiting only slight degradation than full-list turbo receivers in performance. More recently, as a follow-up paper of [9], the authors of [10] extended the efficient SDR receivers from 4-QAM (QPSK) to higher-order QAM signaling by presenting two customized algorithms for solving the SDR demodulators.

FEC codes have long been integrated into data communications to effectively combat noises and interferences. However, because FEC takes place in the finite field whereas modulated symbol is in the Euclidean space of real/complex field, detection and decoding are typically performed disjointly. In this work, we present a novel receiver based on SDR for joint detection and decoding. In our design, FEC codes not only are used for decoding, but also are integrated as constraints within the detection optimization formulation [11], [12], [13]. To further boost performance, the joint SDR detector is incorporated into iterative turbo processing receiver. The proposed joint SDR turbo receiver has lower complexity compared with the original full-list turbo receiver, while achieving similar bit error rate (BER) in overall performance. Furthermore, we present a simplified algorithm, in which only one SDR is solved in the initial iteration for each codeword and a simple approximation can generate the requisite LLR in subsequent iterations. Compared with existing SDR-based turbo receivers in [9] which use randomizations or Bernoulli trials to generate a preliminary candidate list which is further enriched by bit flipping, we can directly generate the candidate list without many additional steps and sophisticated data structures.

II. System Model

We consider an N_t-input N_r-output spatial multiplexing MIMO system. The channel is assumed to be flat-fading. The baseband equivalent model at time k can be expressed as

$$\mathbf{y}_k^c = \mathbf{H}_k^c \mathbf{s}_k^c + \mathbf{n}_k^c, \quad k = 1, \ldots, K, \tag{1}$$

where $\mathbf{y}_k^c \in \mathbb{C}^{N_r \times 1}$ is the received signal, $\mathbf{H}_k^c \in \mathbb{C}^{N_r \times N_t}$ denotes the MIMO channel matrix, $\mathbf{s}_k^c \in \mathbb{C}^{N_t \times 1}$ is the transmitted signal, and $\mathbf{n}_k^c \in \mathbb{C}^{N_r \times 1}$ is an additive Gaussian noise vector, each element of which is independent and follows $\mathcal{CN}(0, 2\sigma_n^2)$.

To simplify subsequent problem formulation, the complex-valued signal model can be transformed into the real field by letting

$$\mathbf{y}_k = \begin{bmatrix} \text{Re}\{\mathbf{y}_k^c\} \\ \text{Im}\{\mathbf{y}_k^c\} \end{bmatrix}, \mathbf{s}_k = \begin{bmatrix} \text{Re}\{\mathbf{s}_k^c\} \\ \text{Im}\{\mathbf{s}_k^c\} \end{bmatrix}, \mathbf{n}_k = \begin{bmatrix} \text{Re}\{\mathbf{n}_k^c\} \\ \text{Im}\{\mathbf{n}_k^c\} \end{bmatrix},$$

and

$$\mathbf{H}_k = \begin{bmatrix} \text{Re}\{\mathbf{H}_k^c\} & -\text{Im}\{\mathbf{H}_k^c\} \\ \text{Im}\{\mathbf{H}_k^c\} & \text{Re}\{\mathbf{H}_k^c\} \end{bmatrix}.$$

Consequently, the received signal vector is given by

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{s}_k + \mathbf{n}_k, \quad k = 1, \ldots, K. \tag{2}$$

In this study, we choose capacity-approaching LDPC code for the purpose of forward error correction. Further, we assume the transmitted symbols are generated from QPSK constellation, i.e., $s_{k,i}^c \in \{\pm 1 \pm j\}$ for $k = 1, \ldots, K$ and $i = 1, \ldots, N_t$. The spatial multiplexing first places the codeword first along the spatial dimension and then along the temporal dimension.

III. Joint ML-SDR Detection

A. MIMO SDR Detection

Based on the assumption of Gaussian noise, it can be easily shown that the optimal ML detection is equivalent to the following discrete least squares problem

$$\min_{\mathbf{x}_k \in \{\pm 1\}^{2N_t}} \sum_{k=1}^{K} \|\mathbf{y}_k - \mathbf{H}_k \mathbf{x}_k\|^2. \tag{3}$$

However, this problem is NP-hard. Instead, SDR can generate an *approximate* solution to the ML problem in polynomial time. To apply SDR, define the rank-1 semi-definite matrix with the help of auxiliary variable $t_k \in \{-1, +1\}$

$$\mathbf{X}_k = \begin{bmatrix} \mathbf{x}_k \\ t_k \end{bmatrix} \begin{bmatrix} \mathbf{x}_k^T & t_k \end{bmatrix} = \begin{bmatrix} \mathbf{x}_k \mathbf{x}_k^T & t_k \mathbf{x}_k \\ t_k \mathbf{x}_k^T & t_k^2 \end{bmatrix}, \tag{4}$$

and denote the cost matrix by

$$\mathbf{C}_k = \begin{bmatrix} \mathbf{H}_k^T \mathbf{H}_k & \mathbf{H}_k^T \mathbf{y}_k \\ -\mathbf{y}_k^T \mathbf{H}_k & ||\mathbf{y}_k||^2 \end{bmatrix}. \tag{5}$$

Using the property of trace $\mathbf{v}^T \mathbf{Q} \mathbf{v} = \text{tr}(\mathbf{v}^T \mathbf{Q} \mathbf{v}) = \text{tr}(\mathbf{Q} \mathbf{v} \mathbf{v}^T)$, ML detection in Eq. (3) can be relaxed to SDR by removing the rank-1 constraint on $\mathbf{X}_k$. Thus, the SDR formulation is

$$\begin{aligned} \min_{\{\mathbf{X}_k\}} \quad & \sum_{k=1}^{K} \text{tr}(\mathbf{C}_k \mathbf{X}_k) \\ \text{s.t.} \quad & \mathbf{X}_k(i,i) = 1, \ k = 1, \ldots, K, i = 1, \ldots, 2N_t + 1, \\ & \mathbf{X}_k \succeq 0, \ k = 1, \ldots, K. \end{aligned} \tag{6}$$

Remark: The SDR problems formulated in most papers target a single time snapshot, since their system of interest is uncoded. Here, for subsequent integration of code information, we consider a total of K snapshots that form an FEC codeword.

B. FEC Code Integration

Consider an (N_c, K_c) LDPC code. Let $\mathcal{M}$ and $\mathcal{N}$ be the index set of check nodes and variable nodes of the parity check matrix, respectively, i.e., $\mathcal{M} = \{1, \ldots, N_c - K_c\}$ and $\mathcal{N} = \{1, \ldots, N_c\}$. Denote the neighbor set of the m-th check node as $\mathcal{N}_m$ and let $\mathcal{S} \triangleq \{\mathcal{F} \,|\, \mathcal{F} \subseteq \mathcal{N}_m \text{ with} \, |\mathcal{F}| \, \text{odd}\}$. Then one characterization of fundamental polytope is captured by the following forbidden set (FS) constraints [14]

$$\sum_{n \in \mathcal{F}} f_n - \sum_{n \in \mathcal{N}_m \setminus \mathcal{F}} f_n \le |\mathcal{F}| - 1, \ \forall m \in \mathcal{M}, \forall \mathcal{F} \in \mathcal{S} \tag{7}$$

plus the box constraints for bit variables

$$0 \le f_n \le 1, \quad \forall n \in \mathcal{N}. \tag{8}$$

Recall that the bit variables $\{f_n\}$ are mapped by modulators into transmitted data symbols in $\mathbf{x}_k$. It is important to note that the parity check inequalities (7) can help tighten our detection solution of $\mathbf{x}_k$ by explicitly forbidding the bad configurations of $\mathbf{x}_k$ that are inconsistent with FEC codewords. Thus, a joint detection and decoding algorithm can take advantage of these linear constraints by integrating them within the SDR problem formulation.

C. Symbol-to-Bit Mapping

To anchor the FS constraints into the SDR formulation in Eq. (6), we need to connect the bit variables f_n's with the data vectors $\mathbf{x}_k$'s or the matrix variables $\mathbf{X}_k$'s. As stated in [2], if $(\mathbf{x}_k^*, t_k^*)$ is an optimal solution to (6), then the final solution should be $t_k^* \mathbf{x}_k^*$, where t_k^* controls the sign of the symbol. In fact, Eq. (4) shows that the first $2N_t$ elements of last column or last row are exactly $t_k \mathbf{x}_k$. Hence, for QPSK modulation, the mapping constraints for time instant $k = 1, \ldots, K$ can be simply written as follows

$$\begin{aligned} \mathbf{X}_k(i, 2N_t + 1) &= 1 - 2f_{2N_t(k-1)+2i-1}, \\ \mathbf{X}_k(i + N_t, 2N_t + 1) &= 1 - 2f_{2N_t(k-1)+2i}. \end{aligned} \tag{9}$$

IV. Iterative Turbo SDR Receiver

A. Turbo Receiver Structure

The structure of a typical turbo receiver for MIMO systems is shown in Fig. 1. The MIMO detector takes in received signals and *a priori* information (often in the format of LLR), and outputs soft information of each bit, denoted by L_{D1} in the figure. After subtracting the prior information L_{A1} from L_{D1}, we have the extrinsic information $L_{E1} = L_{D1} - L_{A1}$. L_{E1} is de-interleaved to become L_{A2} as the input to channel decoder. The path from decoder to detector follows similar processing.

Before diving into the detector design, we review the classical approach of list-based LLR generation. Let $\mathbf{s}_k = \mathcal{M}(\mathbf{b}_k)$ denote the QPSK modulator applied to a vector of polarized bits (± 1), and $\mathbf{L}_{A1,k}$ is the prior LLR vector corresponding to $\mathbf{b}_k$. Here, we note that the polarized bit $b_{i,k} = 1 - 2c_{i,k}$ for coded bit $c_{i,k} \in \{0, 1\}$, where subscript (i, k) denotes the i-th bit at time k. Further, let the vector with superscript

$$
\begin{aligned}
\min_{\{\mathbf{X}_k, f_n\}} \quad & \sum_{k=1}^{K} \operatorname{tr}(\mathbf{C}_k \mathbf{X}_k) + 2\sigma_n^2 \mathbf{L}_{A1}^T \mathbf{f} \\
\text{s.t.} \quad & \mathbf{X}_k(i,i) = 1,\, \mathbf{X}_k \succeq 0, \quad k = 1, \ldots, K, i = 1, \ldots, 2N_t + 1, \\
& \mathbf{X}_k(i, 2N_t + 1) = 1 - 2f_{2N_t(k-1)+2i-1}, \quad k = 1, \ldots, K, i = 1, \ldots, N_t, \\
& \mathbf{X}_k(i + N_t, 2N_t + 1) = 1 - 2f_{2N_t(k-1)+2i}, \quad k = 1, \ldots, K, i = 1, \ldots, N_t, \\
& \sum_{n \in \mathcal{F}} f_n - \sum_{n \in \mathcal{N}_m \setminus \mathcal{F}} f_n \le |\mathcal{F}| - 1, \quad \forall m \in \mathcal{M}, \forall \mathcal{F} \in \mathcal{S}; \\
& 0 \le f_n \le 1, \quad \forall n \in \mathcal{N}.
\end{aligned}
\tag{12}
$$

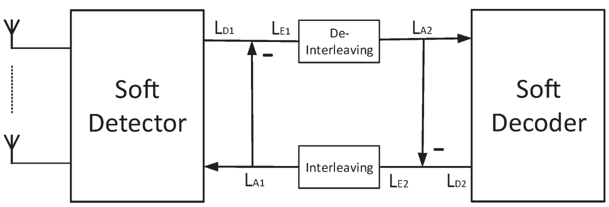

Fig. 1: Structure of Turbo Receiver.

$[i]$ denote a vector excluding the i-th element. Also, denote $\mathcal{L} = \{-1, +1\}^{2N_t}$ and $\mathcal{L}_{i,\pm 1} = \{\mathbf{b} \in \mathcal{L} \,|\, b_i = \pm 1\}$. Following the derivations in [1], the extrinsic LLR of bit $b_{i,k}$ with max-log approximation is given by

$$
\begin{aligned}
L_{E1}(b_{i,k}) \approx & \max_{\mathbf{b}_k \in \mathcal{L}_{i,+1}} \left\{ -\frac{||\mathbf{y}_k - \mathbf{H}_k \mathbf{s}_k||^2}{2\sigma_n^2} + \frac{(\mathbf{L}_{A1,k}^{[i]})^T \mathbf{b}_k^{[i]}}{2} \right\} \\
& - \max_{\mathbf{b}_k \in \mathcal{L}_{i,-1}} \left\{ -\frac{||\mathbf{y}_k - \mathbf{H}_k \mathbf{s}_k||^2}{2\sigma_n^2} + \frac{(\mathbf{L}_{A1,k}^{[i]})^T \mathbf{b}_k^{[i]}}{2} \right\}
\end{aligned}
\tag{10}
$$

It is noted that the cardinality of $\mathcal{L}$ is exponential in N_t. More specifically, in the case of QPSK, $|\mathcal{L}| = 4^{N_t}$. Thus, it is imperative to reduce the list size for practical use, especially in the coming era of massive MIMO. On the other hand, to avoid severe LLR quality degradation, the reduced list should contain the true maximizer or at least the candidates that are close to the true maximizer.

B. Joint MAP-SDR Turbo Receiver

When *a priori* information of each bit is available, *maximum a posterior* (MAP) criterion can be employed instead of ML. According to [15], the likelihood probability $p(\mathbf{y}_k|\mathbf{s}_k) \propto \exp(-||\mathbf{y}_k - \mathbf{H}_k\mathbf{s}_k||^2/(2\sigma_n^2))$ and *a priori* probability $p(\mathbf{s}_k = \mathcal{M}(\mathbf{b}_k)) \propto \exp(\mathbf{L}_{A1,k}^T \mathbf{b}_k/2)$. Therefore, the *a posterior* probability can be given as

$$
\begin{aligned}
p(\mathbf{s}_k|\mathbf{y}_k) & \propto p(\mathbf{y}_k|\mathbf{s}_k) p(\mathbf{s}_k) \\
& \propto \exp\left(-\frac{||\mathbf{y}_k - \mathbf{H}_k \mathbf{s}_k||^2}{2\sigma_n^2} + \frac{\mathbf{L}_{A1,k}^T \mathbf{b}_k}{2} \right).
\end{aligned}
\tag{11}
$$

After taking logarithm and summing over the K time instants, MAP is equivalent to minimizing the new cost function

$$
\sum_{k=1}^{K} \operatorname{tr}(\mathbf{C}_k \mathbf{X}_k) - \sigma_n^2 \mathbf{L}_{A1}^T (\mathbf{1} - 2\mathbf{f}) = \sum_{k=1}^{K} \operatorname{tr}(\mathbf{C}_k \mathbf{X}_k) + 2\sigma_n^2 \mathbf{L}_{A1}^T \mathbf{f}.
$$

By integrating the constraints from Eqs. (6), (7), (8) and (9), the optimization problem in Eq. (12) describes the new joint MAP-SDR detector for QPSK modulation. Notice that our MAP cost function in Eq. (12) is generally applicable to any QAM constellations, whereas the approach in [10] has to approximate the cost function for higher order QAM. For higher order QAM beyond QPSK, the necessary changes for our joint SDR receiver include box relaxation of diagonal elements of $\mathbf{X}_k$ [7] and the modification of symbol-to-bit mapping constraints. We refer interested readers to the works [16], [17], [18], [19] for details of higher order QAM mapping constraints.

With the solution from joint MAP-SDR detector, it is unnecessary to enumerate over the full list $\mathcal{L}$ to generate LLRs as shown in Eq. (10). Instead, we can construct a subset $\overline{\mathcal{L}}_k \subseteq \mathcal{L}$, containing the probable candidates that are within a certain Hamming distance P from the SDR optimal solution $\mathbf{b}_k^*$ [20]. More specifically, $\overline{\mathcal{L}}_k = \{\mathbf{b}'_k \in \mathcal{L} \,|\, d(\mathbf{b}'_k, \mathbf{b}^*_k) \le P\}$, where the Hamming distance $d(\mathbf{b}', \mathbf{b}'') = \operatorname{card}(\{i \,|\, b'_i \ne b''_i\})$. Correspondingly, we have $\overline{\mathcal{L}}_{i,k,\pm 1} = \left\{\mathbf{b}_k \in \overline{\mathcal{L}}_k \,|\, b_{i,k} = \pm 1\right\}$. The Hamming radius P determines the cardinality of $\overline{\mathcal{L}}_k$, that is, $|\overline{\mathcal{L}}_k| = \sum_{j=0}^{P} \binom{2N_t}{j}$. Compared to the full list's size 4^{N_t}, this could significantly reduce the list size with the selection of a small P. We now briefly summarize the steps of this novel turbo receiver:

- S0 To initialize, let the first iteration $\mathbf{L}_{A1} = \mathbf{0}$, and select a value P.
- S1 Solve the joint MAP-SDR given in Eq. (12).
- S2 Generate a list $\overline{\mathcal{L}}_k$ with a given P, and generate extrinsic LLRs $\mathbf{L}_{E1}$ via Eq. (10) with $\mathcal{L}_{i,\pm 1}$ being replaced by $\overline{\mathcal{L}}_{i,k,\pm 1}$.
- S3 Send de-interleaved $\mathbf{L}_{A2}$ to SPA decoder. If maximum iterations are reached or if all FEC parity checks are satisfied after decoding, stop the turbo process; Otherwise, return to S1.

C. Simplified Turbo SDR Receiver

One can clearly see that it is costly for our proposed turbo SDR algorithm to solve one joint MAP-SDR in each iteration (in step S1). To reduce receiver complexity, we can solve one joint MAP-SDR in the first iteration and generate the candidate list by other means in subsequent iterations without repeatedly solving the joint MAP-SDR. In fact, the authors [9] proposed a Bernoulli randomization method to generate such a candidate list based only on the first iteration SDR output and subsequent decoder feedback. We now propose another list generation method for our receiver that is more efficient.

The underlying principle of turbo receiver is that soft detector should use information from both received signals and decoder feedback to improve receiver performance from one iteration to another. During the initial iteration, we solve the joint MAP-SDR with $\mathbf{L}_{A1} = \mathbf{0}$. The extrinsic LLR from this first iteration is denoted as $\mathbf{L}_{E1}^{init}$, which corresponds to the information that can be extracted from received signals. When *a priori* LLR value $\mathbf{L}_{A1}$ becomes available after the first iteration, we combine them directly as $\mathbf{L}_{E1}^{comb} = \mathbf{L}_{E1}^{init} + \mathbf{L}_{A1}$, and perform hard decision on $\mathbf{L}_{E1}^{comb}$ to obtain the bit vector $\mathbf{b}_k^*$ for each snapshot k, i.e., $\mathbf{b}_k^* = \text{sign}(\mathbf{L}_{E1}^{comb})$. We then can generate list $\hat{\mathcal{L}}_k$ as before according to a pre-specified P.

We note that $\mathbf{L}_{A1}$ varies from iteration to iteration, as does $\mathbf{L}_{E1}^{comb}$. If $\mathbf{L}_{A1}$ converges towards a "good solution", it would enhance $\mathbf{L}_{E1}^{comb}$. If $\mathbf{L}_{A1}$ is moving towards a "poor solution", then the initial LLR $\mathbf{L}_{E1}^{init}$ should help readjust $\mathbf{L}_{E1}^{comb}$ to certain extent. In particular, the joint MAP-SDR detector (in the first iteration) can provide a reliably good starting point $\mathbf{L}_{E1}^{init}$ for the turbo receiver, with which additional information that can be extracted from resolving MAP-SDR in subsequent iterations is quite limited. As will be shown in our simulations, this simple receiver scheme can generate output performance that is close to the original algorithm that requires solving joint MAP-SDR in each iteration.

V. Simulation Results

In the simulation tests, a MIMO system with $N_t = 4$ and $N_r = 4$ is assumed. The MIMO channel coefficients are assumed to be ergodic Rayleigh fading. QPSK modulation is used and a regular (256,128) LDPC code with column weight 3 is employed. For LDPC decoding, log-domain sum-product algorithm (SPA) is used.

A. Joint ML-SDR Receiver Performance

In this subsection, we demonstrate the power of code integration by showing the performances of non-iterative receivers. We name the formulation in Eq. (6) as *disjoint ML-SDR*, while we call the formulation in Eq. (12) *joint ML-SDR* by setting $\mathbf{L}_{A1} = \mathbf{0}$. With the optimal SDR solution $\{\mathbf{X}_k^*\}$, there are 2 typical approaches to retrieve the final solution $\hat{\mathbf{s}}_k$, namely, rank-1 approximation and Gaussian randomization. We caution that randomization is, however, not suitable for soft decoding, because the magnitudes of the randomized symbols do not reflect the actual reliability level. Moreover, we found that rank-1 approximation produces almost same performance by directly using the last column of $\mathbf{X}_k^*$. Therefore, here we use the last column as the demodulated symbols, the BER curves of which are shown in Fig. 2. In the performance evaluation, we consider 1) hard slicing on demodulated symbols, 2) weighted bit flipping (WBF) decoding and 3) SPA decoding. In some sense, hard slicing shows the "pure" gain

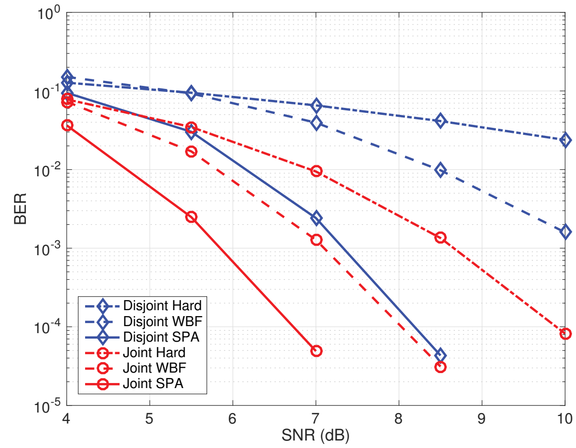

Fig. 2: BER comparisons of disjoint and joint SDR receivers: Direct approach using the final column of $\mathbf{X}_k$.

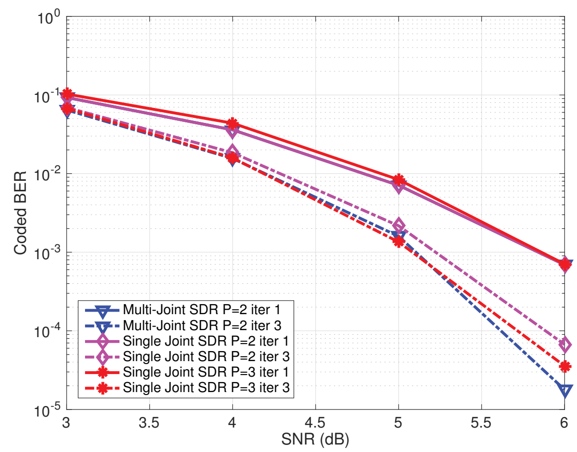

Fig. 3: BER comparisons of multi SDR and single SDR turbo receivers at 1st and 3rd iteration.

by incorporating code constraints. WBF is a hard decoding algorithm that performs moderately well and SPA using LLR is the best. If we compare the SPA curves, the SNR gain is 1.5 dB at BER = 10^{-4}. For other curves, the gains are even larger.

B. Joint MAP-SDR Turbo Receiver Performance

We investigate the performance of joint MAP-SDR versus simplified turbo receiver. The BER curve of *single joint SDR* turbo receiver, which only runs joint MAP-SDR receiver in the initial iteration, is shown in Fig. 3 in comparison with the *multi-joint SDR* that runs joint MAP-SDR in each iteration. We choose two Hamming radii $P = 2$ and 3 for single joint SDR, while that for multi-joint SDR is fixed at 2. It is clear that they all perform equally well in the first iteration since the same joint MAP-SDR is invoked in that iteration. At the 3rd iteration, single joint SDR slightly degrades, especially for $P = 2$, but the performance degradation is acceptable in trade for such low complexity.

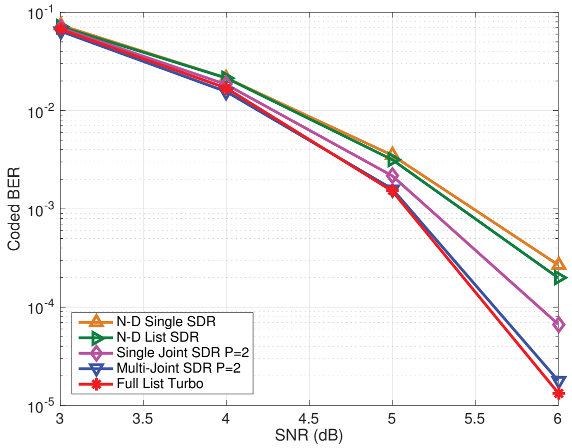

Fig. 4: BER comparisons of different turbo receivers.

C. Comparison with Other Turbo Receivers

Now we compare our proposed joint SDR turbo receivers with the *full list* turbo receiver, which applies Eq. (10) directly using the exponential-sized list, as well as those SDR turbo receivers from [9], which we name as "N-D List SDR" and "N-D Single SDR", respectively. The "N-D List SDR" solves SDRs in each iteration while "N-D Single SDR" runs one SDR in the first iteration only. For the N-D methods, we employ same setting as in the paper [9]: 25 randomizations, (at most) 25 preliminary elements in the list, of which 5 elements are used for enrichment. All BER curves plotted in Fig. 4 are after the 3rd iteration of turbo processing. For our joint MAP-SDR and its simplified SDR turbo receivers, Hamming radius $P = 2$ for list generation. The performance advantage of our receivers is clear around BER = 10^{-4} compared with the N-D methods. Both our multi SDR and single SDR receiver outperform its counterpart, and our single SDR receiver even outperforms "N-D List SDR" that solves SDRs in each iteration. Moreover, our multi-joint SDR performs very close to the full list turbo receiver, while SDR complexity is only polynomial.

VI. Conclusion

This work presents a novel MIMO receiver design by integrating code constraints to improve detector performance. The proposed joint MAP-SDR turbo receiver performs similarly as the full list turbo receiver, while computation cost is reduced from exponential to polynomial. Moreover, the joint SDR receivers outperform existing SDR-based turbo receivers with a noticeable gain. To strengthen this work, we will further investigate complexity-performance tradeoffs in the future. In addition, we would like to extend the current work to higher order QAM constellations. Last but not least, we remark that the concept of joint receiver design can be very effective when there exist RF imperfections, such as phase noise [21].

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