A smart-phone based parking guidance system with predictive parking availability information

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ABSTRACT

Searching for parking has been a problem faced by many drivers, especially in urban areas. With an increasing public demand for parking information and services, as well as the proliferation of advanced smartphones, a range of smartphone-based parking management services began to emerge. Funded by the National Science Foundation, our research aims to explore the potential of smartphone-based parking management services as a solution to parking problems, to deepen our understandings of travelers' parking behaviors, and to further advance the analytical foundations and methodologies for modeling and assessing parking solutions. This paper summarizes progress and results from our research projects on smartphone-based parking management, including parking availability information prediction, parking searching strategy, the development of a mobile parking application, and our next steps to learn and discover new knowledge from its deployment. To predict future parking occupancy, we proposed a practical framework that integrates machine-learning techniques with a model-based core approach that explicitly models the stochastic parking process. The framework is able to predict future parking occupancy from historical occupancy data alone, and can handle complex arrival and departure patterns in real-world case studies, including special event. With the predicted probabilistic availability information, a cost-minimizing parking searching strategy is developed. The parking searching problem for an individual user is a stochastic Markov decision process and is formalized as a dynamic programming problem. The cost-minimizing parking searching strategy is solved by value iteration. Our simulated experiments showed that cost-minimizing strategy has the lowest expected cost but tends to direct a user to visit more parking facilities compared with two greedy strategies. Currently, we are working on implementing the predictive framework and the searching algorithm in a mobile phone application. We are working closely with Arizona State University (ASU) Parking and Transit Services to implement a three-stage pilot deployment of the prototype application around the ASU main campus. In the first stage, our application will provide real-time information and we will incorporate availability prediction and searching guidance in the second and third stages. Once the mobile application is deployed, it will provide unique opportunities to collect data on parking search behaviors, discover emerging scenarios of smartphone-based parking management services, and assess the impacts of such systems.

1. INTRODUCTION

Parking has been a major issue faced by many drivers, especially in downtown areas around the world. Shoup (*1*) conducted empirical studies in the US and suggested that the average time spent searching for a curbside parking space ranged between 3.5 and 14 minutes, which would potentially cause further traffic congestion. Moreover, congestion caused by cruising for parking is a waste of resources and aggravates environmental issues. For example, for a city like Chicago with over 35,000 curbside parking spaces (*2*), cruising to find parking translates into approximately 64 million vehicle-miles travelled, 3 million gallons of gasoline consumed and 30 thousand tons of CO2 emitted every year (*3*).

Advanced parking management services, including parking information provision, parking reservation, and navigation, have emerged to help drivers find parking spaces quickly. The information service provides real-time availability and prices of parking spaces; the reservation service allows drivers to reserve a parking space before departure or on the go, while the navigation service guides them to an open space (*4*). The proliferation of advanced smartphones provides tremendous opportunities for advanced parking management. In fact, the International Parking Institute identified the prevalence of mobile applications the #2 emerging trend in parking in 2015 (*5*). Available services currently on the market include mobile payment (*6*, *7*) and parking information provision and reservation (*8–12*). In addition to parking rates and facility properties provided by almost every parking application, ParkMe also offers real-time parking availability information as a percentage in selected markets. In addition to improving parking experiences for travelers, the systems can be designed to better manage parking demand and reduce traffic congestion and emissions in downtown areas.

Parking availability information is a key input for smart parking guidance algorithms. Existing parking guidance algorithms proposed in the literature mostly use real-time parking occupancy information as an input. For example, Shin & Jun (*13*) proposed a smart parking guidance algorithm with reservation. To recommend the most appropriate parking facility, they considered the real-time status of parking facilities in a city. Caliskan et al. (*14*) proposed a scalable information dissemination algorithm for parking availability information to help drivers reduce searching time.

We argue, however, by further providing time-dependent and predictive future parking probability information as well as smart parking searching guidance, smart-phone based parking management services could be more powerful. This is because that real-time occupancy information itself may not be sufficient as drivers often expect to spend time driving to the desired parking facility, and the probability of finding a parking spot after a reasonable searching time has more impact on their parking behavior than the real-time availability of parking (*15*).

Therefore, our work aims to develop and test a mobile-phone based parking management system that provides predictive parking availability information and guides users to search for parking based on the probabilistic availability information. We proposed a practical framework that integrates machine-learning techniques with a model-based core approach that explicitly accounts for the parking process. The framework is able to predict future parking occupancy from historical occupancy data alone, and can handle complex arrival and departure patterns in real-world case studies, including special event. With the predicted probabilistic availability information, a cost-minimizing parking search strategy is developed. The parking searching problem for an individual user is a stochastic Markov decision process and is formalized as a dynamic programming problem. The cost-minimizing parking searching strategy is solved by value iteration. Our simulated experiments showed that cost-minimizing strategy has the lowest expected cost but tends to direct a user to visit more parking facilities compared with two greedy strategies. Currently, we are working on implementing the predictive framework and the searching algorithm in a mobile phone application. We are working closely with Arizona State University (ASU) Parking and Transit Services to implement a three-stage pilot deployment of the prototype application around the ASU main campus. In the first stage, our application will provide real-time information and we will incorporate availability prediction and searching guidance in the second and last stages. Once the mobile application is deployed, it will provide unique opportunities to collect data on parking search behaviors, discover emerging scenarios of smartphone-based parking management services, and assess the impacts of such systems.

The rest of this paper is organized as follows. Section 2 describes our model-based predictive method and the practical parking availability prediction framework. Case studies with real data from Civic Center garage, San Francisco are presented. Section 3 summarizes our on-going work of parking searching guidance algorithm and the development of the mobile application. Our next steps and future work are discussed in Section 4.

2. PARKING OCCUPANCY PREDICTION

This section describes a practical predictive framework developed by our early research effort. The framework integrates machine-learning techniques with a model-based core approach that explicitly accounts for the parking process. The framework is able to predict future parking occupancy from historical occupancy data alone, and can handle complex arrival and departure patterns in real-world case studies, including special event.

2.1 Prediction Framework

The prediction framework consists of a parameter estimation module and an occupancy prediction module. At the core of both modules is a Markov model describing the stochastic parking process. Additional machine-learning techniques are employed to address some practical considerations pertinent to applying both modules to real data.

2.1.1 Core model-based estimation and prediction methods

Existing parking availability prediction methods in the literature fall into two approaches. One approach starts with modeling the parking arrival and departure process explicitly; and parking availability prediction relies on the estimation of arrival and departure parameters. Previous studies in this category are mainly theoretical, and have at best validated their models using simulation (see (*16*) for a more comprehensive literature review). Another approach, instead of modeling the parking process explicitly, applies statistical and machine learning methods to predict future occupancy directly from the observed occupancy data. These methods include simple regression (*17*, *18*), database system (*19*), neural network (*20*, *21*), and clustering (*22*, *23*). One drawback of these models is the extensive tuning of the model structure. While these methods are able to provide accurate prediction and can be very useful in implementations of parking information systems, their value in exploring innovative parking policies and operational strategies is limited. This is because such statistical and machine learning algorithms are isolated from other aspects of the transportation system; and they are not set up to account for changes in human behaviors and traffic flow in the entire traffic network, which are ultimately reflected in the stochastic arrival and departure processes.

The core predictive methods developed in our work fall into the first approach, where the underlying stochastic parking process is explicit modeled by a discrete-time Markov model. The discrete model focuses on the parking occupancy at a series of discrete time points with time interval Δt. The number of arrival during Δt is assumed to be Poisson with average λ. The number of departure during Δt is assumed to follow a binomial distribution with leaving probability p instead of Poisson. Note that arrival and departure as events can occur at any time and not necessarily at discrete time points. Rather, the proposed model assumes that the arrival rate and leaving probability can only change at discrete time points.

For the proposed discrete-time model, let N_t denote the occupancy at time t. A(t) and D(t) denote the number of arrival and departure during time interval $[t, t+\Delta t)$. Then:

$$N_{t+\Delta t} = \begin{cases} N_t + A(t) - D(t) & if\ N_t + A(t) - D(t) < C \\ C & if\ N_t + A(t) - D(t) \geq C \end{cases}$$

Assume that in a period where $N_t + A(t) - D(t) < C$ always holds; i.e., every incoming vehicle can find a parking spot in the facility. Let E_m denote the expectation of $N_{m\Delta t}$. We have proved in (*16*) that:

$$E_m \coloneqq E(N_{m\Delta t}) = (1-p)^m \left(E_0 - \frac{\lambda}{p}\right) + \frac{\lambda}{p} \tag{2a}$$

Note that if $p = 0$ during a time interval, i.e. no vehicle left the facility, Equation (2a) would degenerate to a linear form (2b) (See (*16*) for detailed derivation).

$$E_m \coloneqq E(N_{m\Delta t}) = \lambda t + E_0 \tag{2b}$$

When a facility is under-saturated, the arrival and departure parameters can be estimated using curve fitting techniques from historical occupancy observations alone based on Equations (2a) and (2b). When a facility is of probability to be recurrently over-saturated, more sophisticated estimation methods may need to be employed. The readers are referred to (*16*) for more details.

The prediction module again employs Equations (2a) or (2b) for under-saturated facilities. It can be engaged both offline or online without requiring additional data other than observed occupancy at the beginning of the day or in real time. Recurrently over-saturated facilities call for the full Markov model instead of the expectation curve alone. However, if a facility is temporarily saturated possibly due to special events, a modified prediction method based on Equations (2a) or (2b) could be adopted. This is further discussed in Section 2.1.3.

2.1.2 Practical considerations

To apply any of the methods described in Section 2.1.1 in practice, additional practical considerations need to be incorporated to handle more complexity in arrival and departure patterns, including special event.

We define the time of day when λ and p change as breaking points. Apparently, (λ, p) must change at local maximum or local minimum of the historical mean occupancy curve. Between the breaking points, the total analysis period is divided into multiple periods within which the historical expectation curve is monotone.

During a period where the historical mean occupancy decreases, it is obvious that $p > 0$ (there are vehicles leaving), which indicates Equation (2a) should be selected for parameter estimation. When historical mean occupancy monotonically increases, p may or may not be zero (there may or may not be vehicles leaving), which indicates that both Equations (2a) and (2b) should be considered and the one with better performance in fitting with the data will be selected. Furthermore, considering the parameters normally do not remain constant for an extended period in real world, we let m represent the largest count of time intervals that (λ, p) could remain constant, and start our regression with a time window of at most m data points. If the resulting R^2 is sufficiently larger than a pre-defined value, move to the next period. Otherwise, reduce one time interval each time form the period and repeat the regression until R^2 meets the criteria. This logic is shown in Figure 1.

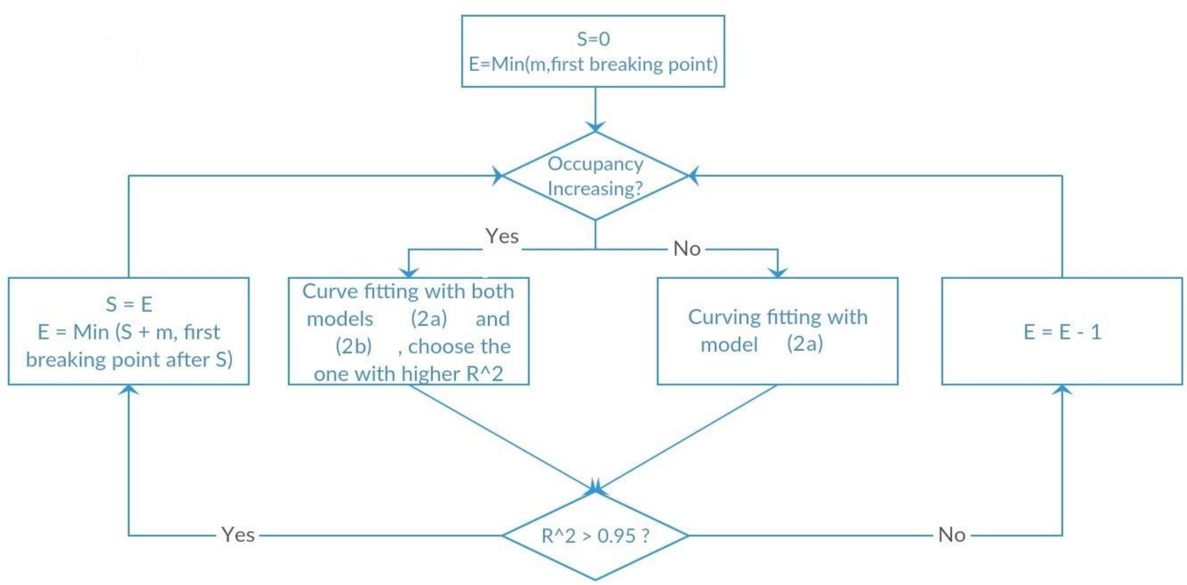

FIGURE 1 ITERATIVE ESTIMATION FRAMEWORK

2.1.3 Handling special event

Even when information on historical special event is insufficient to estimate event-specific arrival and departure parameters, the online prediction can still be refined base on the real-time prediction error using parameters estimated for normal conditions. When the difference between the actual and predicted occupancy is larger than a pre-defined threshold, it is obvious that the historical arrival rates are not able to capture the parking demand pattern at this moment, and a special event might be in place. At this time, the real-time prediction errors could be taken as a simple piece-wise linear approximation of the additional special event arrivals. The online prediction method can then be slightly modified by adding the prediction error from the last time interval to the current prediction value from the original online prediction method. When the difference between actual occupancy and predicted occupancy is smaller than the pre-defined threshold, the original prediction result will be used directly. Note that in this process, no detailed information of the special event (start time, duration, type, etc.) is necessary.

2.2 Real-World Application

Real data from San Francisco are collected to validate the methods and the framework. It is worth mentioning that for all the 14 parking facilities we collected data for, none was ever filled during our three-month data collection period. However, there were days where some parking facilities were very close to saturation due to a special event. Our case studies prove that the parameters estimated offline can lead to accurate predictions of parking facility occupancy both offline and online. Extensive numerical experiments are conducted to compare our methods and framework with several pure machine learning methods found in recent literature. Our model and framework is shown to deliver equal or better performance but requires the computation time that is orders of magnitude less to tune and train the model.

2.2.1 Case Study 1

Case Study 1 is based on historical occupancy data collected from July 6 to July 15, 2015 in the Civic Center garage. Figure 2**Error! Reference source not found.** plots the observed occupancy every 2 minutes for the two weeks. The missing points in the figures are due to closure of the garage and / or lost connection to the database. These points are excluded from the occupancy expectation calculation. Due to the low occupancy between midnight and 7 am, we concentrate our focus on parameter estimation and occupancy prediction starting from 7 am. The time interval Δt is 2 min, and the maximum time window during which (λ, p) remain constant is set to 40 minutes. From hierarchical cluster analysis, the 10 days are first divided into two groups: “workday” and “holiday”. Therefore, we used the days in the “workday” group as our analysis data set (a total of 7 days).

From estimated time-dependent arrival rate the leaving probability, both offline and online prediction for July 16, 2015 are performed. The offline prediction is based on observed occupancy at 7:00 am on July 16, 2015. The absolute relative error (ARE) of offline prediction result is less than 6% throughout the day and increases to less than 30% around midnight (Figure 3). A closer look at the event calendars of venues at Civic Center revealed that June 16th, 2015 had more events after 7 pm[1] compared with the days used for testing. This explains the over-estimation in the evening period for June 16th, 2015, and highlights the limitation of offline prediction. When online prediction is performed every 12 minutes, the ARE is less than 6% during the day and no more than 20% around midnight. When online prediction is performed every 2 minutes, the ARE is reduced to less than 2% during the day and no more than 6% around midnight.

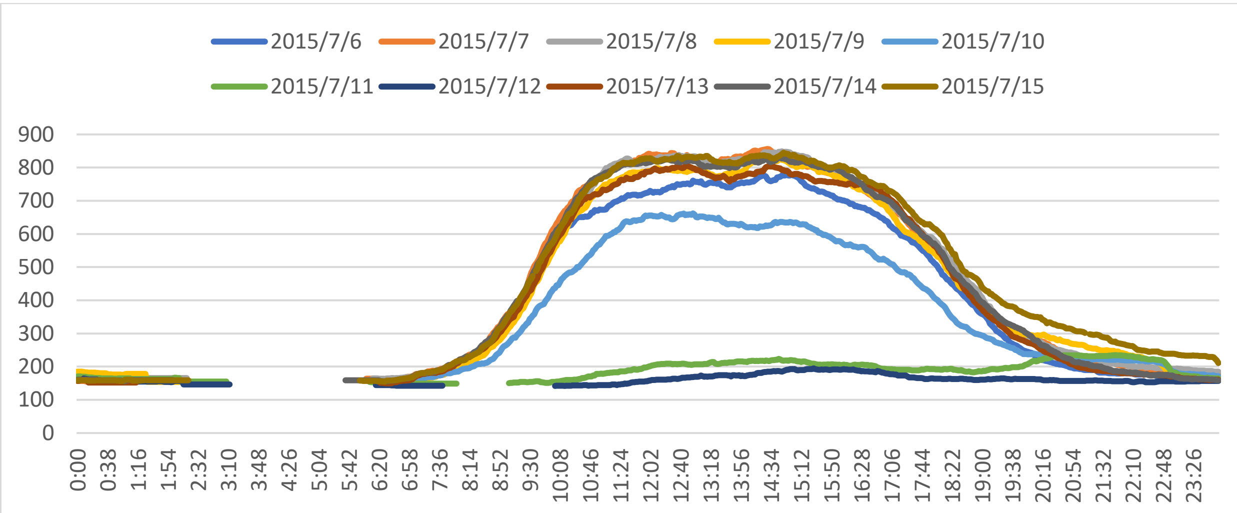

FIGURE 2 OCCUPANCY DATA
(CIVIC CENTER GARAGE, SAN FRANCISCO, JUL 6– 15, 2015)

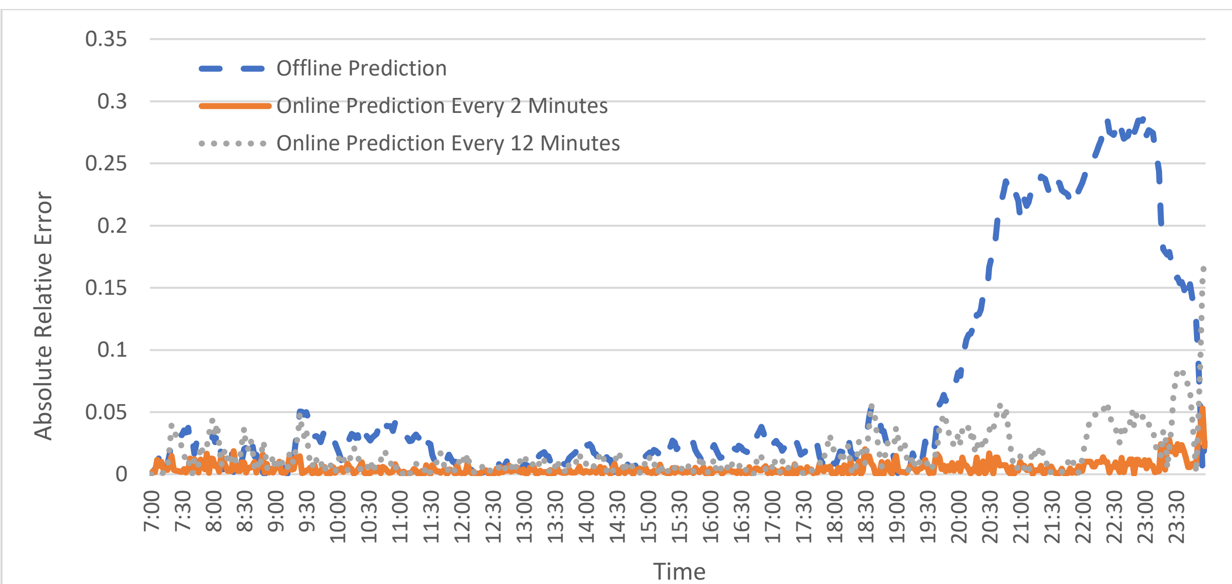

FIGURE 3 RELATIVE ERROR FOR OFFLINE AND ONLINE PREDICTION
(CIVIC CENTER GARAGE, SAN FRANCISCO, JUL 16, 2015)

[1] http://www.sfstation.com/calendar/san-francisco/civic-center/06-16-2015

2.2.2 Case Study 2

This case study aims to examine the performance of our estimation and prediction framework during a highly congested special event. June 19th, 2015 stands out for the Civic Center parking garage with abnormally high parking occupancy. The parking occupancy reached its peak (781 out of 843) at 10:12 pm. Considering some stalls may not be usable due to reservation, double-parking, or other reasons, the facility could be nearly- or even over-saturated with an occupancy of over 90%. A cross-check with San Francisco's event calendar suggests that this congestion was a result of the 100th celebration of San Francisco's City Hall on that day[2]. Since data on historical events of this scale is insufficient to estimate event-specific arrival and departure parameters, the modified online prediction method as described in Section 2.1.3 is implemented.

We focus on a portion of the ingress window where the occupancy kept increasing from 6:48 pm to 10:12 pm. Since June 19th, 2015 is a Friday, data of the following three Fridays was taken as training dataset. Estimated arrival rate and departure probability for a "normal" Friday are generated using the same estimation procedure as in the previous case study. The maximum time window during which (λ, p) remain constant is also set to 12 minutes. The threshold of the difference between actual and predicted occupancy is set to be 10. Both the original and the modified online prediction are implemented.

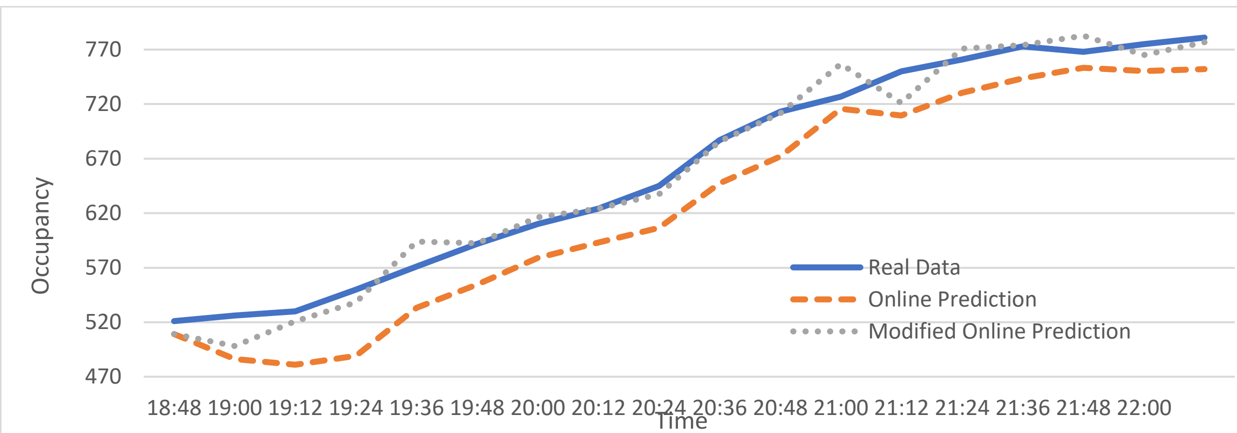

FIGURE 4 OCCUPANCY OF MODIFIED VS. ORIGINAL ONLINE PREDICTION METHODS (CIVIC CENTER GARAGE, SAN FRANCISCO, JUN 19, 2015)

Error! Reference source not found. shows the predicted occupancy for the two online prediction methods. Even without any training data for the special event, the linear approximation in the modified online prediction method is able to handle the extra arrival well. During the study window, while the original online prediction has a mean absolute relative error (MARE) of 5.23%, the modified method successfully reduced the MARE to only 1.74%.

2.2.3 Comparison with pure machine learning methods

To gain further insights on applicability and suitability of the proposed framework, we compared the performance of our approach with the performance of three pure machine learning methods

[2] https://www.sfstation.com/calendar/san-francisco/06-19-2015

found in recent literature since they require the same historical data as in our methods. These methods include artificial neural network (ANN, *20*), wavelet neural network (WNN, *21*), and feature-weighted average method (*23*). ANN and WNN are applicable to online prediction while the feature-weighted average method is for offline prediction only.

<u>Online prediction</u>

Following the same process in Vlahogianni et al (*20*), the ANN structure is genetically optimized, including the number of input nodes (number of previous intervals), the number of hidden nodes, learning rate and momentum. Each neural network is evaluated by a 5-fold cross-validation (*24*)) during model tuning. The implementation is based on Java with Weka API (*25*).

Ji et al. (*21*) used Morlet function as the mother wavelet function for their WNN and the parameters relevant to model structure are selected by experiments. Since it is not clear how exactly the WNN model was tuned in Ji et al. (*21*) while at the same time no well-established package for WNN exists, we instead programmed a brute-force procedure ourselves to determine the WNN structure in Matlab 2017a.

The MARE of the three online prediction methods are displayed in Table 1. It can be observed that our method performs considerably better than WNN and is comparable to ANN. In fact, WNN leads to a MARE of 0.726%; ANN enjoys the lowest MARE of 0.486%; and our method is at 0.489%. A two sample T-test reveals that our model is significantly (at 95% level) better than WNN. On the other hand, ANN is not significantly better than our approach even though it has a slightly lower MARE. In terms of computation time, while our approach took less than 30 seconds to estimate the arrival rates and departure probabilities, it is expected that it would take days to tune the ANN using the same genetic algorithm setting in Vlahogianni et al (*20*). Instead, we performed the genetic algorithm with 10 chromosomes and 10 generations, and the corresponding tuning time is 1 hour 37 minutes.

TABLE 1 MARE FOR ONLINE PREDICTION METHODS

Approach		Case 1	Case 2
Proposed in Our Work	(w/ Original Online Prediction)	0.489%	5.23%
	(w/ Modified Online Prediction)	--	1.74%
ANN		0.486%	2.81%
WNN		0.726%	--

Error! Reference source not found.Error! Reference source not found. plots the ARE from our approach (with both original and modified online prediction) and ANN for case study 2. In this case study, the training data is not sufficient to account for the extra incoming vehicles during the special event. Because of this, both the original online prediction and ANN suffered from relatively large MARE compared to that in case study 1, where the time interval is also 12 minutes. However, with the modified online prediction method, our approach can predict the occupancy more accurately with a MARE of 1.74%, much lower than ANN (MARE of 2.81%) and the original online prediction (MARE of 5.23%).

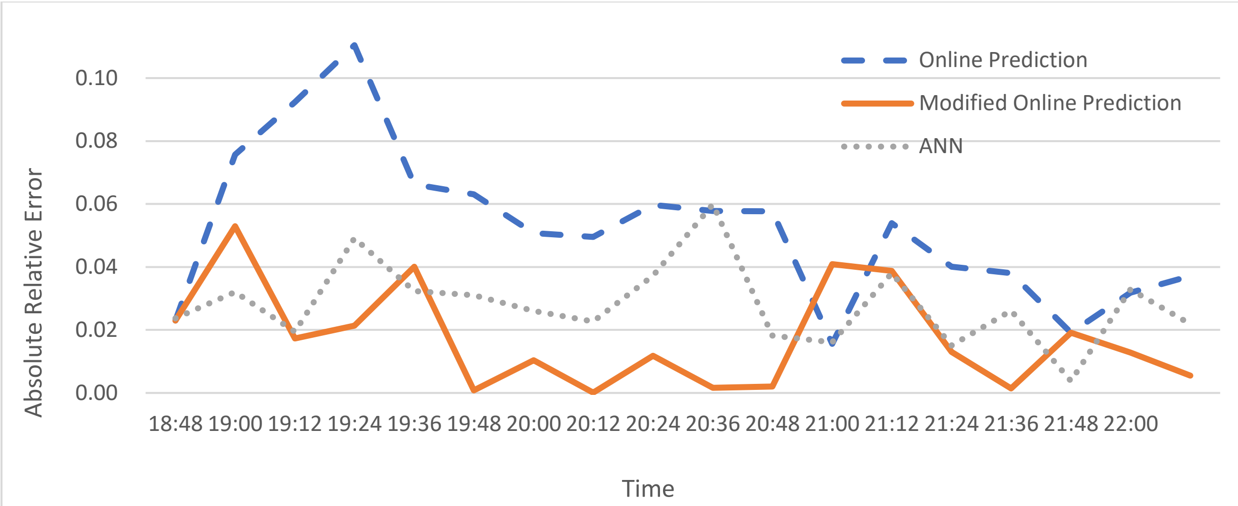

FIGURE 5 ARE OF MODIFIED AND ORIGINAL ONLINE PREDICTION METHODS VS. ANN (CIVIC CENTER GARAGE, SAN FRANCISCO, JUN 19, 2015)

Offline prediction

For offline prediction, we compared our method with a feature-weighted average method in Tamrazian et al. (*23*). His method clusters the time series of daily occupancy into different groups and the prediction is performed referring to the weighted average of the means of each group. The weights in this method are proportional to the number of days in different groups where the days share the same feature as the target day (day of week, weather etc.). The proposed offline prediction method has a 6.363% MARE while the feature-weighted average method has a 7.680% MARE. Our approach leads to significant lower MARE which can be confirmed by a two-sample T-test.

3. PARKING SEARCH AND MOBILE APPLICATION DEVELOPMENT

While Section 2 focuses on the occupancy prediction based on the expectation curve, probabilistic prediction can also be obtained with the full Markov model. More specifically, the probability that a parking facility is not saturated when the user arrives is a valuable input to a parking searching guidance algorithm. Our on-going work proposes an algorithm that considers this probabilistic information as well as other attributes of the parking facility. We are also investigating other heuristic searching guidance algorithms and their performances as compared to the proposed algorithm.

Consider a situation where an individual driver is searching for parking in an area with several parking facilities. A parking searching guidance algorithm will provide a sequence of actions to a driver as to which parking facilities to visit and the order of visit. A driver will keep going down the list until she finds a parking spot. Some greedy searching algorithms include always visiting the closest parking facility and always visiting the facility with the lowest occupancy. We propose an algorithm for parking searching guidance that takes into consideration the probability of a parking facility being saturated when a driver arrives as well as other attributes of the facility such as price and walking distance to his destination. The parking searching problem for an individual user is thus a stochastic Markov decision process and is formalized as a dynamic programming problem. The parking search strategy that minimizes the total expected cost can be obtained through value iteration. We performed numerical experiments on a small network with six nodes and nine links. Various possibilities of how parking availability information is presented and accounted for are investigated. They include predictive probabilistic availability information provided at the beginning of the search and kept unchanged, predictive probability provided in real-time, and real-time updates of perceived probability based on the searching outcome. We solved for the cost-minimizing searching strategy for each of the problem settings, and performed Monte Carlo simulation to compare their performances to the greedy searching strategies. The results showed that the cost-minimizing strategy has the lowest cost as expected, but tends to direct a user to visit more parking facilities compared with the two greedy strategies.

The parking searching strategy is designed from individual drivers' perspective. However, when multiple users adopt the same strategy based on the same inputs and considerations, their actions could change the network condition and the inputs no longer valid. It is, therefore, crucial to investigate the network effect of various parking searching strategies. We are interested in drivers' strategic interactions in the parking competition and the outcomes of such non-cooperative games. We are currently developing methodologies to find the equilibrium of the parking games if it exists. An agent-based simulation model is built using the NetLogo platform. Based on literature in parking search behaviors, we assumed traveler agents make decisions based on a utility function that considers travel distance/time and walking distance, parking lot availability, etc. We are working on implementing various searching strategies in the agent-based simulation and examining the resulting network traffic pattern. We have also performed theoretical analysis on the equilibrium resulting from the parking competition. A strategy-based traffic flow model is proposed to deal with the potential cycling in the network which does not exist in traditional equilibrium analysis. Our preliminary work on a small network with two parking facilities showed that equilibrium exists and can be solved analytically.

To deepen the understanding of travelers' parking behavior and further explore the impact of providing future parking availability and searching strategy, we are currently working closely with ASU Parking and Transit Services to implement a deployment of the prototype application around the ASU main campus. Parking searching trajectory data will be collected through the application. The trajectory data could help discover emerging scenarios of smartphone-based parking management services, and assess the impacts of such systems in a real environment. Additionally, it fills a critical gap in validating and calibrating the theoretical and simulation models using real data. By engaging end users, local transportation agencies, industry, and technology developers, it will generate new knowledge regarding how various stakeholders are interrelated and interact with each other.

4. CONCLUSION AND DISCUSSION

This paper summarizes progress and results from our research projects on smartphone-based parking management, including parking availability information prediction, parking searching strategy, the development of a mobile parking application. A practical framework to predict future parking occupancy from historical occupancy data alone. At the core of this framework is a discrete-time Markov model that describes the stochastic parking process. Several practical considerations for implementing the proposed framework in real world are discussed and machine-learning and other methods proposed to handle more complex arrival and departure patterns, including special event. Our approach delivers equal or better performance comparing to several pure machine learning methods from recent literature, but requires the computation time that is orders of magnitude less to tune and train the model. The cost-minimizing parking searching strategy is solved by formulating the parking searching problem as a stochastic Markov decision process. Our simulated experiments showed that cost-minimizing strategy has the lowest cost as expected but tends to direct a user to visit more parking facilities compared with two greedy strategies. We are working closely with ASU Parking and Transit Services to implement a prototype parking application around the ASU main campus. The parking app provides unique opportunities to collect data on parking search behaviors, discover emerging scenarios of smartphone-based parking management services, and assess the impacts of such systems in a real environment.

For our future work, we plan to collect additional data on recurrently congested facilities to validate the applicability of our predictive framework under saturated situation. Generalizing the existence of parking search equilibrium from two parking facilities to multiple ones is another direction to pursue. Furthermore, cooperative searching strategies will be developed and tested through the deployment of the mobile parking application.

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