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Simple arithmetic: not so simple for highly math anxious individuals

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Abstract

Fluency with simple arithmetic, typically achieved in early elementary school, is thought to be one of the building blocks of mathematical competence. Behavioral studies with adults indicate that math anxiety (feelings of tension or apprehension about math) is associated with poor performance on cognitively demanding math problems. However, it remains unclear whether there are fundamental differences in how high and low math anxious individuals approach overlearned simple arithmetic problems that are less reliant on cognitive control. The current study used functional magnetic resonance imaging to examine the neural correlates of simple arithmetic performance across high and low math anxious individuals. We implemented a partial least squares analysis, a data-driven, multivariate analysis method to measure distributed patterns of whole-brain activity associated with performance. Despite overall high simple arithmetic performance across high and low math anxious individuals, performance was differentially dependent on the fronto-parietal attentional network as a function of math anxiety. Specifically, low—compared to high—math anxious individuals perform better when they activate this network less—a potential indication of more automatic problem-solving. These findings suggest that low and high math anxious individuals approach even the most fundamental math problems differently.

Key words: math anxiety; simple arithmetic; attention; fMRI; PLS

Math anxiety, characterized by feelings of tension and apprehension about math (Richardson and Suinn, 1971), is highly prevalent and is a strong predictor of math performance in the USA and world-wide (OECD, 2013): Across OECD countries, an estimated 31% of 15-year-old students report getting very nervous when solving math problems, and 14% of variation in math anxiety explains the variation in math performance (OECD, 2013). Given the significance of this phenomenon, understanding how and why math

anxiety relates to poor math performance may provide insights into identifying the factors that lead to higher math achievement.

Behavioral studies have consistently indicated that high math anxious individuals perform poorly, compared to low math anxious individuals, on complex, working-memory-intensive arithmetic problems (e.g. Ashcraft and Kirk, 2001), in line with the reasoning that their heightened anxiety elicits situation-related worries and negative thoughts (e.g.

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McLaughlin et al., 2007) that consume working memory resources needed for optimal math performance (Ashcraft and Kirk, 2001; Beilock, 2008). However, it remains unclear whether and how math anxiety is associated with poor performance on numerical tasks that do not rely heavily on working memory resources. Some studies find no differences between high and low math anxious individuals in simple arithmetic performance, for example, in a standardized math achievement test (Ashcraft et al., 1998) or when problems are presented in a paper-and-pencil format (Faust et al., 1996). Other work shows that high math anxious individuals perform worse than their low math anxious counterparts on the most basic numerical tasks. For instance, high (compared to low) math anxious individuals exhibit increased numerical distance effects (i.e. reduced efficiency in determining which of the two digits is larger when they are in closer in numerical distance) and slower and less accurate counting of objects (Maloney et al., 2011; Maloney et al., 2010; Núñez-Peña and Suárez-Pellicioni, 2014).

Why does the performance of high and low math anxious individuals on the most basic math tasks sometimes look different and sometimes not? One possibility is that there are subtle differences in the way that high and low math anxious individuals perform on numerical tasks, characterized by differential recruitment of neural resources. These differences may not be easily detected by commonly used behavioral measures, such as reaction time or accuracy, particularly if high math anxious individuals are using a compensatory strategy or exerting more effort to solve the math problems. Supporting this idea, recent studies reveal that—even when controlling for arithmetic ability-math anxiety is associated with altered event-related potential amplitude when verifying simple arithmetic problems (Suárez-Pellicioni et al., 2013) and during the early stages of simple arithmetic (Klados et al., 2015). High math anxious individuals display reduced deactivation in the default mode network during number comparison and bisection tasks (Pletzer et al., 2015), reflecting an increased demand for inhibition of negative thoughts. Further, high math anxious individuals perform better (while low math anxious individuals perform worse) on simple arithmetic when brain stimulation is applied to regions involved in cognitive control (e.g. dorsolateral prefrontal cortex [dlPFC]; Sarkar et al., 2014). Together, these findings suggest that high and low math anxious individuals may approach the math problems in fundamentally different ways, and their performance may depend on differential patterns of neural activity.

Here, we examine the possibility that math anxiety is associated with differences in how individuals solve simple arithmetic problems for the following reasons. First, it has been suggested that working memory is involved in all kinds of mental arithmetic, including single-digit arithmetic (DeStefano and LeFevre, 2004). Given that high levels of anxiety may impact one's available working memory resources, it is possible that high math anxious individuals' capacity to mentally solve simple arithmetic problems may be compromised. Second, there is a variation in how frequently adults remember arithmetic facts directly from memory to solve simple arithmetic, and this further varies by operation type: for example 66-76% for addition; 78–97% for multiplication; 58% for subtraction; 57% for division (Campbell and Xue, 2001; Hecht, 1999; LeFevre et al., 1996). When adults fail to remember arithmetic facts, they may solve arithmetic problems by using a back-up (procedural) strategy (Groen and Parkman, 1972). Given that changes in working memory load are associated with variations in strategy use (e.g. Imbo et al., 2007), it is possible that math anxiety may relate to differential use of strategies. Third, while efficiency in both procedural and retrieval strategy is important for adults to successfully solve simple arithmetic problems (Campbell and Xue, 2001), increased use of retrieval strategy on simple arithmetic is known to be associated with positive affect in math and better performance on more complex arithmetic problems (LeFevre et al., 1996). Given these reasons, it is of particular interest to examine whether low and high math anxious individuals solve simple arithmetic problems in different ways.

Thus, in the current work, we sought to better characterize the differences in how high and low math anxious individuals approach simple arithmetic, a building block for more complex arithmetic problems. Participants indicated whether simple addition or subtraction problems presented with various types of solutions (true, false close-split, false far-split: e.g. 3+5=8, '7 -5=3', '2 +4=14') were correct while undergoing functional MRI scans. We analyzed fMRI data from a relatively large sample (16 high and 32 low math anxious individuals, approximately matched in gender¹) to delineate the neural networks associated with performance on simple arithmetic across high and low math anxious individuals. Similar to past neuroimaging work (e.g. Pletzer et al., 2015), we used a group analysis approach² to increase our detectability of math-anxiety-related differences in the patterns of neural networks during problem solving. To account for group differences associated with basic arithmetic ability, we administered a standardized basic arithmetic task in a paper-and-pencil format outside the scanner.

The goal of the current study was to examine how the relation between brain activities and simple arithmetic performance may vary as a function of math anxiety. We implemented a behavioral partial least squares (PLS) analysis (e.g. McIntosh and Lobaugh, 2004), which is a multivariate, data-driven approach tailored to capture distributed patterns of whole-brain activities that relate to behavioral measures. This technique differs from traditional GLM analyses that detect differences in signal intensity in specific brain regions and individual voxels between two groups or experimental conditions. PLS analyses examine distributed patterns of activity that relate to behavior and experimental conditions. Using singular value decomposition (SVD), the PLS analysis method uncovers latent variables (linear combinations of brain voxel activities and behavioral measures) that maximally co-vary with each other. The PLS analysis method is a particularly useful approach for our study, because the cognitive and affective processes related to math performance and anxiety are likely to be spatially distributed across the whole brain, involving multiple neural networks, rather than localized in specific brain regions/ voxels. By using a data-driven approach, we are able to characterize the profile of whole-brain activity associated with simple arithmetic performance across high and low math anxious groups.

- 1 Hopko et al. (2003) report that females report higher levels of math anxiety than males. We approximately matched gender in each math anxious group to reduce gender-related biases. In our sample, there was no significant gender difference in AMAS score, t(45) = -1.59, P = 0.12.
- While there are disadvantages, such as assuming linearity and assigning individuals to arbitrary groups, group analysis approaches are considered appropriate for detecting the presence of an effect, particularly when researchers aim to initially establish a relationship between the variables (Preacher et al., 2005).
- The PLS analysis avoids Type II errors by using a data-driven approach that minimizes top-down biases, which could limit the ability to find strong and reproducible effects. In addition, unlike standard univariate GLM analyses that require correction for multiple comparisons due to the shear number of statistical tests performed, in the PLS analysis only one statistical test is performed and tested against a null distribution, which is formulated by permuting the data and performing SVD on the permuted data.

Despite overall high performance across high and low math anxious individuals, we provide evidence that simple arithmetic performance depends on the fronto-parietal attentional network differently as a function of math anxiety. Specifically, the low—compared to high—math anxious individuals perform better when they activate this attention network less—a potential indication of more automatic problem-solving in less math anxious individuals. To our knowledge, this is the first study to utilize a multivariate network-based analysis to differentiate how high and low math anxious people solve simple arithmetic problems. Our results show that lower levels of math anxiety are tied to solving simple arithmetic problems in a more automatic way, possibly freeing up individuals' attentional capacity for more attention demanding mathematical endeavors.

Methods

Participants

A large pool of participants from the Chicago area (N = 1034) completed a prescreening survey that includes Abbreviated Math Anxiety Scale (AMAS; Hopko et al., 2003).4 High and low math anxious groups were identified based on previously established quartiles (first quartile ranges for low math anxious group and third or fourth quartile ranges for high math anxious group) of AMAS scores from a large, urban college population (N = 14 678; Maloney, 2011). Sixty individuals participated in the fMRI scanning session.⁵ A total of 12 participants were excluded from the analysis: One

- 4 There was an error in administering the rating scale for AMAS. A different rating scale (1 = Some anxiety; 2 = Moderate anxiety; 3 = Quite a bit of anxiety; 4 = High anxiety; 5 = Extreme anxiety; 6 = Prefer not to answer) from the original rating scale used in Hopko et al.'s (2003) AMAS (1 = Low anxiety; 2 = Some anxiety; 3 = Moderate anxiety; 4 = Quite a bit of anxiety; 5 = High anxiety) was used. Considering that the descriptive labels were one scale higher for the current rating scale than the original scale, it may be possible that the scores derived from the current scale represent higher math anxiety scores than if one completed the original rating scale. To address this, we performed a separate online study on Amazon Mechanical Turk, where a total of 201 participants who previously completed the original AMAS, approximately matched in gender, age, and previous AMAS scores, were randomly assigned to complete the original or current AMAS at a later time point. In this online study, we failed to find a difference between the original and current scales other than the current scale lowering the mean scale value by 0.3. Critically, the correlation between the original and revised scales was observed to be high (r = 0.76), comparable to that of repeated administration of the original scale (r = 0.81). As such, we believe that two math anxiety scales can be treated equivalently. Additional analyses from the current study including 18 participants who scored the lowest on AMAS (excluding 14 participants who scored higher) among low math anxious group that approximately matches the number of 16 high math anxious participants were conducted (Supplementary Results). The behavioral and fMRI results after excluding 14 low math anxious individuals (those who scored higher on math anxiety among the low math anxious group) remain similar to the results including all low math anxious individuals in the current study. In other words, the current results are reproducible when excluding the individuals that one might suspect to have been misidentified as low math anxious individuals due to administering a rating scale with each response category labeled one scale higher than that of the original rating scale. As there is no evidence that these individuals are different from other low math anxious individuals, and given that the current and original scales can be considered equivalent, we include all the low math anxious individuals in the current study.
- The scanning session took approximately 75 min and post-scanning session took approximately 45 min.

high math anxious female's head motion spiked more than 1 mm and five participants (four low math anxious males, one high math anxious female) discontinued the scan due to discomfort or claustrophobia. One participant was excluded due to a recruitment error. Five participants were excluded due to unidentifiable math anxiety scores as a different rating scale for AMAS was used in the current study (see footnote 4).6 The remaining participants included 32 low math anxious individuals (16 females) aged from 19 to 35 years (M = 25.47, s.d. = 4.83) with an average AMAS⁷ score of 1.36 (s.d. = 0.19), and 16 high math anxious individuals (11 females) aged from 21 to 39 years (M = 27.13, s.d. = 5.43) with an average AMAS score of 3.13 (s.d. = 0.57). All participants were righthanded as assessed by the Edinburgh Handedness Inventory (Oldfield, 1971), and self-reported no history of diagnosed learning disability or ADHD.8 All participants gave informed consent in accordance with the Institutional Review Board, University of Chicago, and were compensated with \$40 for participation.

Simple arithmetic verification. Participants verified whether simple arithmetic problems were correctly solved in the scanner, indicating if a problem was correct with their index finger or if there was a mistake with their middle finger. Half of the trials had the correct solution (true). Incorrect solutions were 'split' either close (1 or 2 digits away) or far⁹ (7 or 8 digits away) from the correct answer. Problems were in the form of a+b=c for addition and a - b = c for subtraction, where $1 \le a \le 17$, $1 \le b \le 17$ $\otimes \otimes \otimes = > C$ were included to control for button pressing responses; participants indicated if they saw a 'C' with index finger and 'M' with middle finger. Participants completed 1056 trials divided equally among 8 scans. Each scan contained four sets of problems (16 arithmetic and 17 jitter) that alternated

- 6 These participants were removed due to responding to 'Prefer not to answer' to one or more items in the AMAS used in the current study. Considering that these participants may have responded to this category either due to high or low levels of anxiety, simply removing those items may not accurately reflect their math anxiety scores.
- 7 Average of nine AMAS items (alpha = 0.94) ranges from 1 to 5.
- While we have not collected information about other mental illnesses, alcohol consumption, smoking, or medication use, we are not aware of any existing evidence that suggests that high and low math anxious individuals exhibit differences in these dimensions. In the current study, high and low math anxious individuals were approximately matched in age, gender and basic math and reading abilities. In another laboratory study, where information about alcohol consumption was collected, there was no significant difference between 100 high and 98 low math anxious participants' alcohol consumption [$\chi^2(1)$] = 0.23, P = 0.63; Chang, 2017].
- Previous work (Faust et al., 1996; Suárez-Pellicioni et al., 2013) showed that high math anxious individuals demonstrate difficulty in processing extremely incorrect solutions (14 or 23 units away from the correct answer). One possible reason that these researchers have observed math anxiety-related group differences for these false-far split problems is that high math anxious individuals are not taking the advantage of plausibility strategy (which is quicker than exhaustive verification strategy) for these types of problems and are demonstrating difficulties in inhibiting irrelevant information.

In our study, we did not find behavioral differences between math anxious groups in verifying false-far split problems that are 7 or 8 units away from the correct answer, nor differences in how brain activity and performance were related between these types of problems. It is possible that math-anxiety-related group differences may emerge when the presented false problems are more dramatically incorrect from the correct answer (e.g. 14 units in Suárez-Pellicioni et al., 2013, or 23 units in Faust et al., 1996).

between addition and subtraction. Each set of arithmetic/jitter trials were cued by a word stimulus 'Add' or 'Subtract.' The order of problems was counterbalanced across participants: Lists were generated through m-sequencing (using an in-house MATLAB [version 7.9 R2009a: The MathWorks, Inc., Natick, MA] script) and sequence randomization was verified with autocorrelations to the 16th lag. Half of participants started with addition problems and the other half started with subtraction problems. Each arithmetic trial was displayed for 2000 ms, jitter trial for 1500 ms, and cue display for 500 ms. Trials were separated by a 500 ms-intertrial interval. Figure 1 shows sample trials.

Covariates. Woodcock Johnson-III (WJ-III; Woodcock et al., 2001) math fluency subtest was administered by paper and pencil after scanning. Participants were given a 3-min time limit to solve simple addition, subtraction, and multiplication problems (a total of 160 problems). Raw WJ-III math fluency scores (percent correct) and scanner variability 10 were used as covariates for relevant analyses.

fMRI data acquisition and analysis. Stimulus presentation and paradigm timing were achieved using E-Prime 2.0 Professional (Psychology Software Tools). All imaging data were acquired on a 3.0 Tesla whole body scanner (Philips Achieva) with an 8channel head coil. The first two volumes of each functional run were discarded to allow for equilibrium effects, and odd slices were acquired followed by even slices. Functional volumes (151 per run, 30 slices each, with 0.5 mm gap) covered the entire brain (T2*-weighted echoplanar imaging sequence using a zshimming algorithm to reduce susceptibility artifact [Gu et al., 2002]; $3 \times 3 \times 4$ mm voxels; repetition time (TR), 2000 ms; echo time (TE), 25 ms; flip angle, 77°; field of view (FOV), 192 mm; matrix, 64×64 ; transverse plane). A high-resolution anatomical (magnetization-prepared, rapid-acquisition gradient echo) image was also obtained (1 mm isotropic voxels; TR, 8 ms TE, 3.5 ms; FOV, 240 mm; matrix, 240 \times 228; transverse plane; 181 slices). In the scanner, a projector back-projected stimuli viewed through a mirror by participants, and each functional run was synchronized with the onset of the first stimulus to ensure accuracy of event timing. Fiber optic button press boxes measured response times and accuracy.fMRI data were processed and analyzed using SPM8 (Statistical Parametric Mapping; Wellcome Department of Cognitive Neurology, UCL, London, UK), following standard procedure. Functional images were rigid-body motion-corrected, slice-time corrected, spatially normalized into standard MNI space, and smoothed using a Gaussian kernel of 5-mm FWHM. Each individual's data were modeled using a 2 (arithmetic operation: addition, subtraction) × 3 (presented solution: true, close, far) random effects factorial ANOVA. The estimated motion parameters were entered into the model as repressors of no interest. A temporal high pass filter with a cutoff of 128s was applied to remove slow signal drifts. For each participant, six t-contrast maps were computed, one for each presented solution for each arithmetic operation.

Second-level analyses. A behavioral PLS analysis (https://www.rot man-baycrest.on.ca/index.php? section=84), a multivariate and data-driven approach (Berman et al., 2014; Krishnan et al., 2011;

10 A later group of participants (n = 27; 17 low and 10 high math anxious individuals) were scanned by an upgraded Philips Achieva 3.0T scanner. Identical scanner parameters were used across all participants and similar results were obtained after controlling for scanner variability.

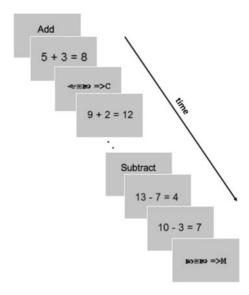


Fig. 1. Simple arithmetic task. Participants pressed a left button with their index finger when the presented solution to the problem is correct (or when they saw the letter 'C') and pressed a right button with their middle finger when the presented solution to the problem is incorrect (or when they saw the letter 'M').

McIntosh and Lobaugh, 2004), was performed to identify wholebrain patterns of activity that distinguish differences between math anxious groups for addition and subtraction separately. Two thousand permutation tests were performed to obtain pvalues for latent variables (LV; groupings of patterns of activity) and 2000 bootstrap samples with replacement were used to obtain 95% confidence intervals for mean correlation between brain and behavior scores. Statistical significance was determined using a threshold of 3.0 bootstrap ratio (salience [weights]/SE [reliability]) and clusters larger than 10 voxels.

Among the regions showing group differences from PLS analysis, we ran post-hoc region of interest (ROI) analyses to confirm whether these regions are significantly related to performance for high and low math anxious groups separately for addition and subtraction. Spherical ROIs with a radius of 5 mm were centered on the peak coordinate for each region.

Results

Behavioral results

Simple arithmetic (fMRI task). We confirmed that high and low math anxious individuals performed the same behaviorally on simple arithmetic verification problems, regardless of arithmetic operation or presented solution (that is, correct solution or incorrect solution either close or far from the correct solution). We controlled for variations in speed-accuracy tradeoffs by measuring inverse efficiency scores (IES; Townsend and Ashby, 1978), dividing response time by accuracy (higher scores indicate worse performance). 11 In a 2 (arithmetic operation: addition or subtraction) × 3 (presented solution: true, closesplit, far-split) repeated measures ANOVA with math anxious group as a between-subjects factor, we found that high and low math anxious individuals performed similarly across different

11 Similar results were obtained when response time and accuracy were analyzed separately (Supplementary Results). By reducing the number of statistical tests required, a composite performance measure such as IES lowers the probability of type I errors.

Table 1. Means and standard errors of inverse efficiency score for arithmetic verification task

Inverse efficiency score (ms)	Low math anxious group ($N = 32$)		High math anxious group (N $=$ 16)	
	Mean	SE	Mean	SE
Addition true	1076	40.2	1121	63.3
Addition close	1329	63.7	1384	92.4
Addition far	1134	43.1	1182	63.9
Subtraction true	1291	58.3	1310	74.8
Subtraction close	1517	91.1	1578	136.8
Subtraction far	1295	57.7	1315	72.2

types of problems. Critically, there were no significant main effects of math anxiety and no interactions between math anxiety and arithmetic operation or presented solution on behavioral performance for these simple problems (Fs < 0.21; Ps > 0.67).

Participants were faster and committed less errors with addition compared to subtraction problems [F (1, 92) = 67.22, P < 0.001; Table 1]. For the effects of presented solution, due to violation of the assumption of sphericity (P < 0.001), degrees of freedom were corrected using Huynh-Feldt estimates of sphericity ($\epsilon > 0.79$). Participants performed differently depending on the type of solution presented, F (1.6, 72.6) = 45.96, P < 0.001. Post hoc comparisons using the Bonferroni correction indicated that participants performed more efficiently (faster and more accurate) when the presented solution was true or far from the correct solution, compared to when the presented solution was close to the correct solution (Ps < 0.001). True solution performance did not significantly differ from far-split solution performance (P = 0.35). Since there were differences in performance between types of arithmetic operation and presented solution, we examined the neural networks associated with performance on each operation and presented solution separately.

Basic arithmetic (paper-and-pencil). High and low math anxious groups performed similarly on WJ-III fluency, t(46) = -0.97, P = 0.34.

fMRI results: brain activity ~ performance relation

A behavioral PLS analysis was implemented to examine the patterns of relationship between brain activity and behavioral performance (as measured by IES) in high and low math anxious individuals.12 Two statistically significant LVs emerged from the PLS analysis.

First latent variable. Similarities in the relation between brain activity and performance across high and low math anxious groups were observed in the first latent variable, accounting for 54% of the covariance (P = 0.002) among addition problems, and 61% covariance (P = 0.002) among subtraction problems (Figure 2). Better performance was associated with greater brain activity mainly in the default mode network (DMN) across addition and subtraction (Supplementary Table S3). Given that the DMN is typically more active when tasks are easy and require less attention (Buckner et al., 2008), and increased task demands are associated with 'task-induced-deactivation,' i.e. increased deactivation in the DMN (McKiernan et al., 2003), the positive relation between the DMN activity and task performance supports our intuition that the addition and subtraction problems were

relatively easy to complete (required less cognitive effort) for individuals who performed better.

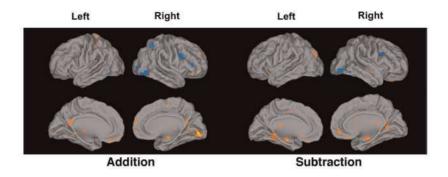
While DMN activity was positively associated with performance, we observed that better performance was negatively associated with activity in a network of regions implicated in working memory such as the inferior/middle/superior frontal gyrus, and the cingulate gyrus/supplementary motor area (BA 6, 32; Supplementary Table S4). Activation in these regions is often associated with completing tasks requiring working memory (Bressler and Menon, 2010), typically decreasing when DMN activity increases (Greicius et al., 2003). Thus, the negative association between performance and activity in this task-related network complemented the positive association between the DMN and performance.

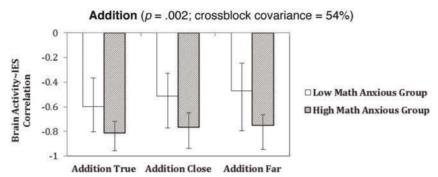
Since these results were not differentiated by arithmetic operation or math anxious group, it seems that both high and low math anxious individuals performed better on these tasks when they were engaging in less effortful/attention processing and instead were more automatic. For tasks that are simple and not demanding of working-memory, paying too much attention may interfere with automated performance (Beilock et al., 2002). Alternatively, it is also possible that lower performing individuals' performance is less automated than higher preforming individuals and thus they recruit task-related network more. Future studies may include measures of automaticity (e.g. Logan and Klapp, 1991) and/or a secondary task where attentional demands can be allocated to (e.g. Beilock et al., 2002) to address these possibilities.

Second latent variable. The PLS results from the second LV provided evidence for differences between high and low math anxious groups in the relation between brain activity and performance, accounting for 23% of the covariance (P = 0.02) among addition and 22% covariance (P = 0.008) among subtraction problems (Figure 3). Across addition and subtraction, 13 low,

13 In addition to regions that overlapped in the negative association between brain activity and performance across addition and subtraction in low, compared to high, math anxious individuals, there are distinct regions specific to the association between brain activity and performance for addition and subtraction separately (Supplementary Results; Supplementary Tables S5 and S6). For addition problems, there is a negative relation between brain activity and performance for low, compared to high, math anxious individuals in right insula and a positive relation between brain activity and performance for low compared to high math anxious individuals in left anterior cingulate. For subtraction problems, there is a negative relation between brain activity and performance for low, compared to high, math anxious group in right inferior frontal gyrus, right superior/medial frontal gyrus/paracentral lobule, left middle frontal gyrus, and right cingulate gyrus/anterior cingulate cortex, and a positive relation between brain activity and performance for low, compared to high, math anxious

¹² Task PLS analysis (analysis not including behavioral performance) results are included in Supplementary Results.





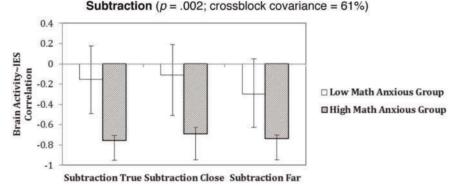


Fig. 2. Results from first latent variable. The first LV revealed similar activation patterns between high and low math anxious individuals, including increased activity in the default mode network associated with better performance across addition and subtraction. The DMN included the medial prefrontal cortex, superior/middle/ inferior temporal gyrus, parahippocampal gyrus, posterior cingulate cortex, and cuneus (Supplementary Table S3). Activation clusters from positive relation between performance and brain activity are depicted in red/yellow color; activation clusters from negative relation between performance and brain activity are depicted in blue/light blue color. The graphs depict the directionality of correlation between brain and behavior scores for each condition and group. The y-axis represents the correlation of brain activity and inverse efficiency score (IES; higher IES indicates worse performance). Negative bars indicate positive correlation between performance and brain activities. Error bars are 95% confidence intervals around the mean correlation between brain and behavior scores.

compared to high, math anxious individuals' activity decreased as performance increased in the left inferior frontal gyrus (left

group in right fusiform gyrus/inferior temporal gyrus, left precentral gyrus/inferior frontal gyrus, right supramarginal gyrus and left supramarginal gyrus/inferior parietal lobule. These findings suggest that lower math anxious individuals perform better on addition problems when they activate negative emotional processing brain region such as right insula (Simmons et al., 2004) less and recruit attentional network that modulate cognitive or emotional processing, such as anterior cingulate (Casey et al., 1997; Bush et al., 2000). For subtraction problems, lower math anxious group performs better when they activate inhibitory control network such as right inferior frontal gyrus (Aron et al., 2004) less and rely on regions that are known to relate to digit identification, numerical processing, or arithmetic fact retrieval, such as inferior temporal gyrus, inferior parietal lobule, and supramarginal gyrus (Dehaene et al., 1996; Menon et al., 2000; Rickard et al., 2000).

IFG) and right superior parietal lobule (right SPL; Supplementary Table S6). These patterns of PLS results remained similar, accounting for 22% of the covariance (P = 0.003) among addition and 20% of covariance (P = 0.006) among subtraction problems, when basic arithmetic ability (WJ-III fluency) and scanner variability were controlled for by regressing them from brain and behavioral scores.

To confirm whether activity in each of these regions was significantly related to performance for each of high and low math anxious groups, we ran post-hoc ROI analyses for these specific regions (Supplementary Results; Supplementary Figure S1). From the ROI analysis, we found that for low math anxious group, reduced activity in the left IFG was associated with better performance for addition and subtraction (addition: r = -0.37; P = 0.036; subtraction: r = -0.51; P = 0.003) and reduced activity

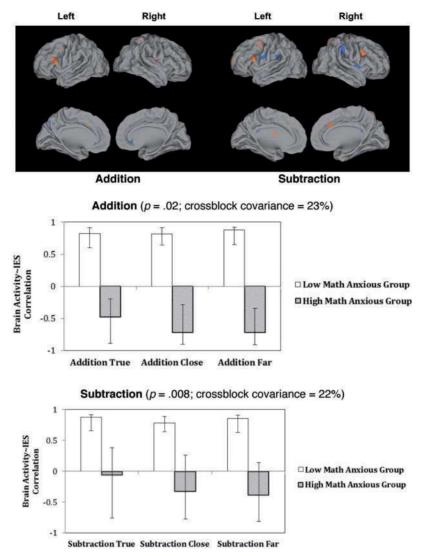


Fig. 3. Results from second latent variable. The second LV revealed different activation patterns between high and low math anxious individuals. Low math anxious individuals showed decreased fronto-parietal network activity associated with better performance across addition and subtraction. This network included the left inferior frontal gyrus and right superior parietal lobule (Supplementary Table S6). Activation clusters from negative relation between performance and brain activity for low—compared to high—math anxious individuals are depicted in red/vellow color; activation clusters from positive relation between performance and brain activity for low-compared to high-math anxious individuals are depicted in blue/light blue color. The graphs depict the directionality of correlation between brain and behavior scores for each condition and group. The y-axis represents the correlation of brain activity and inverse efficiency score (IES; higher IES indicates worse performance). Positive bars indicate negative correlation between performance and brain activities. Error bars are 95% confidence intervals around the mean correlation between brain and behavior scores.

in the right SPL14 was associated with better performance for subtraction (r = -0.42; P = 0.016); for high math anxious group, the brain activity ~ behavioral performance relation was not significant in these regions (left IFG for addition: r = 0.20; P = 0.45; left IFG for subtraction: r = 0.20; P = 0.47; right SPL for subtraction: r = 0.32; P = 0.22). Fisher's Z transformation was used to examine the differences in correlation coefficients between groups. The relation between brain activity and performance was negative for low-compared to high-math anxious group in these regions (left IFG for addition: z = -1.77, P = 0.04; left IFG for subtraction: z = -2.29, P = 0.01; right SPL for subtraction: z = -2.33, P = 0.01).

14 For addition problems, only seven voxels of the right superior parietal lobule were significant and did not meet the cluster threshold of 10 voxels in the current analysis.

Discussion

The current study examined the neural representations underlying simple arithmetic performance across high and low math anxious individuals. We found that high and low math anxious individuals performed the same behaviorally, similar to previous behavioral studies that show that math anxiety is minimally related to performance of overlearned, simple arithmetic in young adults (Ashcraft et al., 1998; Faust et al., 1996). Utilizing a multivariate approach, PLS, we identified the neural representations associated with performance for simple arithmetic. While high and low math anxious individuals performed similarly on simple arithmetic problems, they exhibit differences in attention-related, fronto-parietal network activation (inferior frontal gyrus and superior parietal lobule) associated with performance. For low math anxious individuals, performance on simple arithmetic improves when they recruit the

fronto-parietal network less. Low math anxious individuals' simple arithmetic performance may be more efficient as they perform more or less automatically. In contrast, high math anxious individuals' performance was less dependent on the reduction in recruitment of the fronto-parietal network.

Why is the math performance of high math anxious individuals less related to a reduction in fronto-parietal network recruitment than that of low math anxious individuals? One possibility is that some high math anxious individuals recruit cognitive control resources to regulate negative emotional responses associated with math or math-related situations, even when the math task at hand aren't necessarily difficult or working memory demanding. In other words, these individuals may attempt to reduce their worries and negative thoughts by recruiting fronto-parietal network when processing or manipulating numerical information. Similarly, using complex arithmetic problems that high math anxious individuals typically underperform, Lyons and Beilock (2012a) demonstrated that high math anxious individuals' increased fronto-parietal brain activity during anticipation mitigates upcoming hard math task performance deficits. Future studies that separate out the anticipatory phase from task phase, including math problems with different levels of difficulty, which may help elucidate the functional role of fronto-parietal network during anticipation (and/or task) phase(s) in high math anxious individuals' performance on various math problems. Further, it would be useful to implement measures of state anxiety or negative affect (e.g. Denny et al., 2014) to identify patterns of brain activity associated with regulation of negative emotional responses in high math anxious individuals.

Another possibility why high math anxious individuals exhibit less consistent pattern of relationship between frontoparietal network activity and performance could be due to their use of a mix of strategies to solve simple arithmetic problems. If math anxiety is associated with increased avoidance of math problems, it may be that across the life span, more mathavoidant high math anxious individuals may transition to automatic strategies (e.g. fact retrieval) less frequently¹⁵ (Tenison et al., 2016) and their performance may be dependent on working memory and attentional resources, similar to the patterns of neural activity observed during mental arithmetic in younger children (Rivera et al., 2005) or during working memory tasks in

15 One may wonder whether high math anxious individuals were slower or less accurate to complete simple arithmetic problems if they were using nonretrieval strategies more often than low math anxious individuals. On average, high math anxious individuals were \sim 13 ms slower and 1.14% less accurate on addition and subtraction problems than low math anxious individuals, and these differences were not statistically significant (Ps > 0.53). It is possible that high math anxious individuals use retrieval-based strategies less frequently or less efficiently, but given that these individuals were asked to complete overlearned simple addition and subtraction problems, they may still rely on retrieval-based strategies to a greater extent than they would for complex arithmetic problems, at a rate that is comparable to that of low math anxious individuals at a behavioral level. It is also possible that they were able to complement their reduced frequency or efficiency of retrieval-based strategies by relying on brain networks differently from lower math anxious individuals, as a result of using a mix of strategies (retrieval-based and procedural-based) and/or reduced neural efficiency in executing retrieval-based strategies. To better distinguish between these interpretations, future research may utilize trial-by-trial strategy report in order to identify brain activities associated with performance on problems that are solved by retrieval- or procedural-based strategies in high and low math anxious groups.

older adults (Reuter-Lorenz et al., 2000). Thus, the weaker relationship between brain activity and behavioral performance in high math anxious individuals might reflect some compensatory effort to achieve performance equivalent to low math anxious counterparts, in line with the prediction of attentional control theory (Eysenck et al., 2007). Future studies may benefit from an application of trial-by-trial strategy reports (asking participants to report how they solved each problem, for example, by remembering or calculating; e.g. Grabner et al., 2009; Ramirez et al., 2016) to assess patterns of brain activity associated with differential strategy use between high and low math anxious

Third, it is possible that some high math anxious individuals' increased reliance on frontoparietal network when solving simple arithmetic might also reflect their increased effort and/ or decreased efficiency when processing numerical information. For example, when asked to determine the larger of two numbers (e.g. 7 and 3), high math anxious individuals exhibit an increased numerical distance effect, which is thought to be indicative of a less precise representation of numerical magnitudes (Maloney et al., 2011; Núñez-Peña and Suárez-Pellicioni, 2014). If high math anxious individuals, compared to low math anxious individuals, exhibit reduced fluency in processing and manipulating numbers, such differences in proficiency in numerical processing might alter the patterns of brain activity during simple arithmetic as well. Future studies that include more basic, lower-level numerical tasks (e.g. numerical comparison) may help identify whether the differences in the brain-behavior relationship observed between high and low math anxious individuals may arise from variability in one's efficiency in using arithmetic strategies and/or fluency in numerical processing.

Finally, it is also possible that high math anxious individuals are more variable as a group, in the profile of brain-behavior relation, compared to low math anxious individuals, similar to the observation that there is a wide range of variability in cognitive functioning among young adults with anxiety or depressive disorder (Castaneda et al., 2008). Further elucidating the neurobiological mechanisms underlying math anxiety may provide a better understanding of how high and low math anxious individuals may approach the math problems differently. Understanding whether high math anxious individuals' altered fronto-parietal network activity during simple arithmetic reflects either emotional regulation efforts and/or less automated or variable strategy use may help develop targeted/different interventions for boosting math performance among these individuals. For example, students experiencing high levels of math anxiety may benefit from classroom practices that help students regulate their negative emotions, develop their fluency in retrieval of simple arithmetic facts, and/or a combination of both.

Our results contribute to the existing knowledge of neural correlates of math anxiety. Multiple brain regions have been implicated in math-anxiety-related responses in adults and young children. Lyons and Beilock (2012b) demonstrated that activity in dorso-posterior insula during math task anticipation is associated with self-reported math anxiety among highly math anxious adults. Pletzer et al. (2015) showed that math anxiety is associated with reduced deactivation in default mode network during number comparison and number bisection tasks. Sarkar et al. (2014) showed that transcranial direct current stimulation applied to dorsolateral prefrontal cortex improved simple arithmetic performance in high math anxious individuals, compared to sham stimulation. These studies with adult population, including the current study, suggest that math anxiety may relate to aberrant activity in cortical structures engaged in top-down emotional processing (Ochsner et al., 2009).

Among young children, math anxiety appears to relate to responses in the amygdala, a subcortical brain region known to be generally associated with bottom-up fear and anxiety in humans and non-human animals (LeDoux, 2007). Young et al. (2012) showed that in 7-9-year-old children, math anxiety is associated with increased activity in right amygdala (and its enhanced connectivity to ventromedial prefrontal cortex and reduced connectivity to posterior parietal regions) and reduced activity in posterior parietal and dorsolateral prefrontal cortex regions during complex addition and subtraction. Supekar et al. (2015) showed that among third graders, 8 weeks of one-to-one math tutoring on arithmetic facts reduce self-reported levels of math anxiety and amygdala responses during simple addition. Given these different neural circuitries associated with math anxiety in adults and young children, it is possible that the experience of math anxiety changes across development, as individuals gain more experience with numerical information. Young children's anxiety about math may be more stimulusdriven than adults who, after years of experience with number knowledge, may experience dysfunctional top-down perception of numerical information if they are anxious about math. A development of longitudinal studies that examine the neural representations of math anxiety across development may be crucial for examining this possibility and designing targeted interventions for different populations.

One of the limitations of the current work is that our sample size was smaller for high math anxious group than low math anxious group. Future studies with a larger sample of high math anxious individuals may be able to increase the power for detecting the relationship between brain activity and behavioral performance. Another limitation is that the current findings may not generalize to population with all ranges of math anxiety, since we have examined a sample with extreme scores of math anxiety. Future studies may reveal whether there exists a different pattern of relationship between brain activity and performance for individuals who are moderately anxious about math. Finally, given the correlational nature of the current evidence, it is possible that other sociocontextual factors may differentially influence the relation between brain activity and performance for high and low math anxious individuals.

In conclusion, we provide evidence that high and low math anxious individuals show differential patterns of neural activity related to behavioral performance, even for simple arithmetic problems typically mastered in early elementary school. These findings point to the possibility that performance differences on cognitively demanding math problems between high and low math anxious individuals may arise from the way that these individuals approach the most fundamental math problems. Low math anxious individuals' abilities to develop automaticity in simple arithmetic may contribute to boosting their ability to perform well on complex math problems.

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Supplementary data

Supplementary data are available at SCAN online.

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