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Interactions Among Control Channels in Dynamical Networks

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Abstract—**Control-channel interactions in a linear diffusive network model are studied, with the aim of highlighting the role of the network's topology in such interactions. Specifically, the influence of a built controller on the infinite and finite zero structure of a second control channel is characterized. The analysis shows how the network's graph topology, the relative positions of the control channels, and the specifics of the built controller influence the zero structure of the second control channel. In particular, it is shown that the some control architectures can introduce undesirable non-minimum-phase dynamics at remote locations, while others are guaranteed to maintain or promote minimum-phase dynamics.**

I. Introduction

Many modern engineered networks, ranging from terrestrial-scale infrastructures to the Internet-of-things, use multiple feedback control systems in tandem [1]. These control systems may act on different parts of the network, operate at various temporal and spatial scales, and/or regulate different aspects of the networks' dynamics. Some of these control systems may be coordinated, while others are decentralized or even antagonistic. In such pervasively-controlled networks, interactions among the control systems can modulate wide-area dynamics, and thus have profound consequences on the network's global function. For example, in the bulk power transmission network, the tuning of some wide-area controls may dictate whether fault resolution by other control systems is successful or causes poorly-damped oscillations [2]–[4].

As control systems in engineered networks become more sophisticated, interactions among them are becoming increasingly impactful and also difficult to predict from experience [5]. At their essence, however, one would expect the interactions to depend on the topology of the network being controlled, and the positions of the control channels relative to this topology. Here, we undertake a study of the interactions among control channels in a representative dynamical-network model, with the aim of gaining topological insights into the problem. Specifically, for a linear diffusive network model, we study how the deployment and tuning of one control system influences the transfer function seen across a second control channel, with the main aim of developing topological insights into the influence.

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The research described here contributes to an extensive research effort on the dynamics and control of complex networks, which spans across the physics, natural sciences, and controls/circuits literature. This research direction was initially focused on the emergence of global behaviors from local interactions in networks (e.g., synchronization of coupled oscillators) [6]. Subsequently, the controls community has extensively studied control design for autonomous but communicating agents, to achieve global coordination. A common theme in both directions has been to tie global stability and performance to the graph topology of the network. Relevant to the the study here, recently this research has been extended toward understanding sparse control of already-built dynamical networks, including observability and controllability [7]–[9], input-output behaviors (finite- and infinite- zeros) [10]–[13], predatory control [14], and resource optimization subject to a budget constraint [15]. These efforts on sparse control give insight into the ability of control channels to modulate the network's intrinsic dynamics. However, the studies have not addressed how one control system in a network affects the performance of other control channels, and hence their design. The results presented here begin to address this question. *Ab initio* design of decentralized controllers for large-scale system has also been extensively studied [16]. Relative to this literature, the work presented here instead considers the implications of one control on other channels, reflecting that diverse control capabilities are often built piecemeal as needs arise in network operations rather than as a global solution.

Here, control-channel interactions are characterized for a standard linear diffusive model defined on a digraph [10], [11], [17]. To do this, the model is enhanced to explicitly represent a linear feedback control system that is built into the network. The dependence of another SISO channel's transfer function on this control system is examined. Specifically, we focus on characterizing the infinite and finite zero structure of the second channel, which dictate its input-output behavior, limit control performance, and guide controller design [18]. The analysis shows that the positions of the two control channels relative to the network graph determine the zero structure. Specifically, it is shown that particular controller positions and architectures maintain or promote minimum-phase dynamics, but other control schemes may inadvertently cause nonminimum phase dynamics at remote locations; whether or not nonmimum-phase dynamics may result has much to do with the lengths and strengths of paths between the input and output, and the location of the built controller relative to these paths.

The remainder of the article is organized as follows. The model and problem formulation are given in Section II. The main results on control-channel interactions are presented in Section III. Finally, some examples are presented in Section IV. *Due to space constraints, proofs and some details are excluded, see [19].*

II. Modeling and Problem Formulation

A linear diffusive network model defined on a digraph is considered. Since our interest is in analysis of input-output properties, the model is augmented to represent a single-input single-output (SISO) channel of interest (see also [10], [11], [17]). Here, the model is further enhanced to explicitly represent an in-built linear feedback controller, which may use measurement data from parts of the network to set one or more inputs. This enhanced model is used to study the effect of an existing or in-built control on another SISO channel's transfer function and, specifically, finite-zero structure.

A network with n components or nodes, labeled as $1, 2, ..., n$, is considered. Each node j is associated with a scalar state x_j. The nodes' states are nominally governed by a linear dynamical model with diffusive state matrix A, and are further modulated by an in-built feedback control. The in-built feedback control is assumed to be a linear state-space dynamical system which processes a scalar combination of state-variable measurements (e.g., a single state variable or a difference of them) to set an additive input, typically at a single node. Formally, the feedback is modeled as an additive vector input signal to the state dynamics, which we denote as $\mathbf{P}$. For most of the analyses pursued here, $\mathbf{P}$ is assumed to have only one nonzero entry. Specifically, the control vector $\mathbf{P}$ is assumed to be determined from the state vector $\mathbf{x}$ according to a Laplace-domain relationship of the following form: $\mathbf{P} = \mathbf{e}_q H_c(s)\mathbf{z}^T\mathbf{x}$, where the vector $\mathbf{z}$ indicates what combination of state variables are being used in feedback, $\mathbf{e}_q$ is 0–1 indicator vector which shows the single node q that is being actuated by the in-built controller, and $H_c(s)$ is the transfer function of the in-built controller. The nominal model for the dynamical network as a whole, including the in-built controller, is thus:

$$\begin{aligned} \dot{\mathbf{x}} &= A\mathbf{x} + \mathbf{P} \\ \mathbf{P} &= \mathbf{e}_q H_c(s)\mathbf{z}^T\mathbf{x} \end{aligned} \tag{1}$$

where $\mathbf{x} = \begin{bmatrix} x_1 \dots x_n \end{bmatrix}^T$ is the **full state** of the network. The state matrix A is assumed to be a M-matrix: its off-diagonal entries are assumed to be nonnegative, while the diagonal entries are negative and satisfy $A_{l,l} \leq -\sum_{j=1, j\neq l}^{n} A_{l,j}$. This model encompasses many of the canonical models for synchronization/consensus, diffusion, and spread in dynamical networks (e.g., [20], [21]). It is worth stressing that the matrix A need not be symmetric. Since the state matrix encodes the topology of the network, it is referred to as the *graph matrix*. While the primary focus is on built controllers that actuate a single node, we alternatively also consider the common circumstance that the built controller incorporates local proportional controllers at multiple nodes. In this case, the control vector takes the form $\mathbf{P} = -\sum_{j \in V_b} \mathbf{e}_j k_j x_j$, where V_b is a subset of the nodes in the network.

The goal of this paper is to study how the deployment and tuning of the in-built controller affects the characteristics of another channel of interest. As in [11], a single-input single-output channel is considered, which is defined by an additive input at a single network node i, and an output which measures the state at a single node (labeled n without loss of generality). The full model of the system, with the SISO channel of interest included, is thus given by:

$$\begin{aligned} \dot{\mathbf{x}} &= A\mathbf{x} + \mathbf{P} + \mathbf{e}_i u \\ \mathbf{P} &= \mathbf{e}_q H_c(s)\mathbf{z}^T\mathbf{x} \\ y &= \mathbf{e}_n^T\mathbf{x} \end{aligned} \tag{2}$$

where u is the scalar input signal acting on the node i, and the output y is equal to the state of node n. The focus of our study is to characterize the transfer function of the channel of interest (i.e., the transfer function from u to y). The channel of interest may be another control channel in the network, or may capture other input-output behaviors of interest (e.g., a disturbance response that is a concern to network operators).

Since topological results are sought, it is convenient to associate a graph with the network dynamics. Specifically, a weighted digraph G with n vertices is defined, where each vertex $l = 1, 2, \dots, n$ in the graph corresponds to the network node l. Formally, an arc (directed edge) is drawn from vertex l to vertex j in the graph (l,j distinct) if and only if $A_{j,l} \neq 0$, and is assigned a weight of $A_{j,l}$. The vertices corresponding to the input and output network nodes are referred to as the input and output vertices. The state matrix A can be viewed as (the transpose of) a grounded Laplacian matrix associated with the directed graph. In this paper, the notation d_{ab} is used the directed distance from vertex a to vertex b in the digraph G.

We aim to characterize the influence of local network controls on the input-output properties of a channel of interest. Specifically, the transfer function from u to y is characterized in terms of: 1) the network graph G, 2) the structure of the in-built controller P, and 3) the position of the channel of interest. The poles of this transfer function depend solely on the native network dynamics and the in-built control (not the particular channel considered), and their analysis and design are traditional controls concepts. Here, the finite zeros of the channel, including particularly the presence or absence of nonminimum-phase zeros, are analyzed. It is well known that the zeros are invariants which place essential limits on feedback control and specify channel response characteristics [18]. Particularly, the presence of nonminimum-phase zeros place essential limits on control performance (e.g., reference tracking or disturbance rejection error). Thus, the analyses pursued here explore how one controller in a network limits and modulates other control channels.

III. Results

First, results connecting the network's native structure (i.e., the uncontrolled network) with its zeros are reviewed and extended (III.A). Then, the main results on control channel interactions are presented, focusing first on controls that can make the channel of interest minimum phase (III.B), and then on controls that either make the channel nonminimum phase (III.C) or do not influence the phase properties (III.D). The results developed in this section assume that the network graph is strongly connected; we focus on this case to avoid trivial cases where the built controller does not have any influence on the channel of interest.

A. Native Graph Structure and Zeros: Review and New Result

Two relationships between a network's native graph topology and the zeros of an input-output channel are briefly reviewed [10]–[13], because they are a basis for the analysis of controlled networks. A further result, concerned with networks that comprise multiple sparsely-interfaced subnetworks, is also developed. The results in this section assume that no control has been implemented, i.e. $\mathbf{P} = 0$.

A main outcome of previous research is that the presence/absence of nonminimum-phase zeros is closely dependent on the lengths and strengths of paths between the input and the output [10]–[13], [17]. In particular, if there is a single path between the input and output, or the shortest path is dominant, the network is necessarily minimum phase. In contrast, if an alternate path between the input and output is sufficiently long and strong, the network is guaranteed to be nonminimum phase. These results are an important starting point because they: 1) motivate studying whether implementation/tuning of controls can alter the zero structure arising from the network topology, and 2) give insight into what control schemes may remove or cause nonminimum-phase behaviors. Two main results of this sort are the following:

1) Consider the input-output system (2) without controller, i.e. for $\mathbf{P} = 0$. The system is minimum phase if there is a single directed path from the input vertex to the output vertex. Also, the system is minimum phase if there are multiple input-output paths provided that at least one path of minimum length is made sufficiently strong, in these sense the edge weights on this path are sufficiently scaled up.

2) Consider the input-output system (2) without controller, i.e. for $\mathbf{P} = 0$. The system has closed right half plane (CRHP) zeros if short paths between the input and output are sufficiently weak. Precisely, consider the case where the distance between the input and output on the network graph is d_{in}. If there is at least one input-output path of length $\bar{d}_{in} \geq d_{in} + 3$, and all paths of length less than $d_{in} + 3$ are made sufficiently weak (i.e., at least one edge on each of these paths is scaled down), the system is nonminimum phase.

Formal statements and proofs of the above results, and bounds on edge weights that guarantee minimum-phase or nonminimum-phase dynamics, can be found in [11], [17].

Many large-scale dynamical networks are interconnections of multiple subnetworks, which may have distinct operational paradigms or control authorities. For these networks, there is interest in characterizing zeros of network channels in terms of properties of the assimilated subnetworks. Such analyses are also a starting point toward understanding control-channel interactions among multiple network authorities. With this motivation in mind, a result on interconnections of subnetworks is given in the following theorem. In particular, we show that subnetworks that are interconnected by a single link preserve minimum-phase behaviors in a certain sense.

The theorem requires some notation for singly-interconnected subnetworks. Formally, the network input-output model (2) without controller ($\mathbf{P} = 0$) is considered. Without loss of generality, two subnetworks in the network model, comprising nodes $1, \ldots, n_1$ and $n_1 + 1, \ldots, n$, are considered. The state matrix A is partitioned in a commensurate way, as $A = \begin{bmatrix} A_{aa} A_{ab} \\ A_{ba} A_{bb} \end{bmatrix}$ where A_{aa} is an $n_1 \times n_1$ matrix. Similarly, the state vector is partitioned as $\mathbf{x} = \begin{bmatrix} \mathbf{x_a} \\ \mathbf{x_b} \end{bmatrix}$, where $\mathbf{x_a}$ has n_1 entries. The network is said to be singly interconnected, if the network graph has a single (possibly bi-directional) edge between the vertices $1, \ldots, n_1$ and the vertices $n_1 + 1, \ldots, n$. For a singly-interconnected network, there is only one pair (j, k), where $j = 1, \ldots, n_1$ and $k = n_1 + 1, \ldots, n$ such that $A_{j,k}$ and $A_{k,j}$ are nonzero. For a singly-interconnected network, the network input-output model (2) can be expressed in terms of the following two interconnected subsystem models:

Subsystem S_1, which has two inputs (i.e. u and z_2) and one output (i.e. z_1) as $S_1 : \dot{\mathbf{x}}_\mathbf{a} = A_{aa}\mathbf{x_a} + \mathbf{e}_i u + \mathbf{e}_j A_{j,k} z_2$ and $z_1 = \mathbf{e}_j^T \mathbf{x_a}$.

Subsystem S_2, which has one input (i.e. z_1) and two outputs (i.e. z_2 and y) as $S_2 : \dot{\mathbf{x}}_\mathbf{b} = A_{bb}\mathbf{x_b} + \mathbf{e}_{k-n_1} A_{k,j} z_1$; $z_2 = \mathbf{e}_{k-n_1}^T \mathbf{x_b}$ and $y = \mathbf{e}_{n-n_1}^T \mathbf{x_b}$

We find it convenient define the transfer function in subsystem S_1 from the input u to the output z_1 as $H_1(s) = \frac{Z_1(s)}{U(s)}$. Similarly, we define the transfer function in subsystem S_2 from input z_1 to the output i.e. y as $H_2(s) = \frac{Y(s)}{Z_1(s)}$. We also find it convenient to define the induced subgraphs of G on vertices $1, \ldots, n_1$ and $n_1 + 1, \ldots, n$ as G_1 and G_2, respectively. For a singly interconnected network, the graph G contains a single edge between subgraphs G_1 and G_2, see Fig. 1.

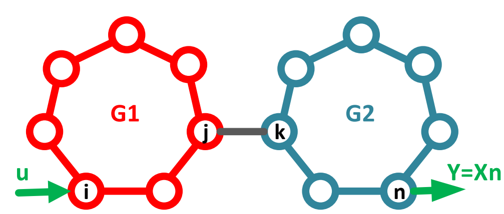

Fig. 1: A network comprising two subnetworks that are connected by a single line.

Theorem 1: Consider a network input-output model which is made up of singly-interconnected subnetworks. The zeros of the full input-output dynamic model (2) are contained within: 1) the zeros of the transfer function $H_1(s)$, 2) the zeros of the transfer function $H_2(s)$, and 3) the (stable) poles of subsystems S_1 and S_2. Further, all right half plane zeros of the transfer functions H_1 and H_2 are necessarily zeros of the full input-output dynamic model (2).

Theorem 1 shows that connecting two minimum phase dynamic networks by an bi-directional edge results in minimum phase behavior in the interconnected network. In other words, a network with two connected areas will maintain minimum-phase characteristics of the individual ones, provided that only a single edge connects the two areas. On the other hand, if two areas are connected by more than one edge, non-minimum phase behaviors may result even if the transfer functions for each area are minimum phase.

B. Control Schemes that Promote Minimum-Phase Dynamics

Control strategies are developed that yield minimum-phase dynamics on channels of interest. First, the following two simple theorems demonstrate that a high gain controller acting between the input and output nodes of a channel can be used to make the channel minimum phase. The first theorem considers a simple proportional controller, and the second addresses proportional-derivative control.

Theorem 2: Consider the network input-output model (2). Assume that a proportional controller is applied across the input-output channel of interest, i.e. $\mathbf{P} = \mathbf{e}_n k(x_i - x_n)$. Then, 1) the relative degree of the system (2) is two, and 2) the model is asymptotically stable and minimum phase for all sufficiently large feedback gains k (i.e., for all $k \geq \widehat{k}$, for some $\widehat{k}$).

Theorem 3: Consider the network input-output model (2). Assume that a proportional-derivative controller is applied across the input-output channel of interest, i.e. $\mathbf{P} = \mathbf{e}_n \left[k(x_i - x_n) + q\frac{d}{dt}(x_i - x_n)\right]$. Then, 1) the relative degree of the system with model (2) is equal to two, and 2) the model is asymptotically stable and minimum phase for all sufficiently large feedback gains k (i.e., for all $k \geq \widehat{k}$ for some $\widehat{k}$) and any $q < 1$.

The above theorems show that, by applying a strong feedback of the state difference across a channel of interest, that channel can be made minimum phase.

The following results show that local controllers with sufficiently large gain applied remotely to an input-output channel of interest can also be used to achieve minimum-phase dynamics on that channel. These results require some further terminology regarding the network input-output model. The term *special input-output path* is used to refer to a path of minimum length (least number of edges) between the input and output in graph G. As defined before, the notation d_{in} is used for the length of the special input-output path, i.e. for the distance between the input vertex i and output vertex n. Additionally, we define a modified system based on a subgraph of G. Specifically, we consider the uncontrolled input-output model, with a subset of vertices deleted. Formally, let us consider a subset of vertices $V_b \subset \{1, \ldots, n\}$, which does not include the input and output vertices (i and n). Let us also define the vectors $\mathbf{e}_r^{(V_b)}$ as a modified version of the vector $\mathbf{e}_r$, where the entries $i \in V_b$ are omitted. Similarly, $A^{(V_b)}$ is defined as a submatrix of A obtained by deleting the rows and columns specified in V_b. Then, the **deletion subsystem** is defined as:

$$\begin{aligned} \dot{\mathbf{x}}^{(V_b)} &= A^{(V_b)}\mathbf{x}^{(V_b)} + \mathbf{e}_i^{(V_b)}\widehat{u} \\ \widehat{y} &= {\mathbf{e}^{(V_b)}}_n^T \mathbf{x}^{(V_b)}, \end{aligned} \tag{3}$$

where $\mathbf{x}^{(V_b)}$, $\widehat{u}$, and $\widehat{y}$ are the state, input, and output, respectively. The deletion system (3) is associated with a weighted directed **deletion graph** $G^{(V_b)} = G - V_b$. Also, we define $d_{in}^{(V_b)}$ as the distance between the input and output vertices (i.e. from vertex i to n) in graph $G^{(V_b)}$.

First, a key theorem is presented that characterizes the finite zeros of the network input-output model, when local proportional controllers are applied at a subset of network nodes.

Theorem 4: Consider the network input-output model (2). Assume that local proportional controllers are applied at network nodes in the set V_b, i.e. $\mathbf{P} = -\sum_{j \in V_b} \mathbf{e}_j k_j(x_j)$. Also, suppose that the input-output channel is remote from the local controllers ($i, n \notin V_b$), and that $d_{in}^{(V_b)} = d_{in}$. When the gains k_j ($j \in V_b$) are scaled up, a subset of the zeros of (2) approach the zeros of the deletion system (3), while all other zeros are in open left half plane (OLHP).

The theorem immediately permits us to define local proportional control schemes that make the input-output model minimum phase, as formalized in the following corollaries.

Corollary 1: Consider the network input-output model (2). Assume that local proportional controllers are applied at network nodes in the set V_b, i.e. $\mathbf{P} = -\sum_{j \in V_b} \mathbf{e}_j k_j(x_j)$, and consider an input-output channel that is remote from the local controllers ($i, n \notin V_b$). When the gains k_j ($j \in V_b$) are sufficiently scaled up, the system (2) is minimum phase if $d_{in}^{(V_b)} = d_{in}$ and the deletion system (3) is minimum phase.

Corollary 2: Consider the network input-output model (2). Say that local proportional controllers are applied at the network nodes specified in the set V_b, i.e. $\mathbf{P} = -\sum_{j \in V_b} \mathbf{e}_j k_j(x_j)$. Consider an input-output channel that is remote from the controllers ($i, n \notin V_b$). When the gains k_j ($j \in V_b$) are sufficiently scaled up, the system (2) is minimum phase if $d_{in}^{(V_b)} = d_{in}$ and the network graph after removing all vertices specified in V_b (i.e. $G^{(V_b)}$) has a single path between the input and output vertices.

Remark: Per Corollary 2, one way to make a system minimum phase is to put high gain local controllers at all vertices adjacent to special input-output path.

C. Controls that Cause Nonminimum-Phase Dynamics

In many large-scale networks, there is a significant concern that the actions of a control authority may make other regulation and control tasks difficult, or alter properties of remote input-output channels in undesirable ways. For instance, operators of the electric power grid have recognized that newly-integrated fast controls may alter performance of other controls, or cause unexpected disturbance responses [13]. These concerns suggest that, while controllers are typically designed to achieve desirable internal properties, they may incidentally alter the input-output characteristics of remote channels in undesirable ways (e.g., cause the channel to become nonminimum phase, or increase susceptibility to disturbances). Here, we identify conditions under which the built control causes the network's input-output channel to become non-minimum-phase.

First, a simple result is given which shows that a high-gain nonminimum-phase controller or a low-gain unstable controller necessarily makes all remote channels nonminimum phase. While it is not typical to use nonminimum-phase or unstable controllers, there is some motivation for studying this case. First, the presence of delays and other unmodeled dynamics in the controller implementation may introduce non-minimum-phase characteristics in the feedback block, whose impacts across the network are of interest. Also, in some special cases, unstable or nonminimum-phase controllers are indeed needed (see e.g. [22]).

Lemma 5: Consider the input-output model (2). Assume that the controller $\mathbf{P} = \mathbf{e}_q H_c(s) x_j$ is applied. Consider any input-output channel that is either remote from the built controller (i.e. $j \neq i, n$ and $q \neq i, n$), or alternately the channel across the built controller (i.e. $j = i$ and $q = n$). Also assume that $d_{ij} + d_{qn} \neq d_{in} + d_{qj}$. Then:

- If the controller transfer function $H_c(s) = kH_u(s)$ is non-minimum phase, then the input-output model (2) is non-minimum phase for all sufficiently large feedback gains k (i.e., for all $k \geq \widehat{k}$ for some $\widehat{k} > 0$).
- If the controller transfer function $H_c(s) = kH_u(s)$ is unstable, then the input-output model (2) is non-minimum phase for all sufficiently small feedback gains k (i.e., for all $0 < k \leq \widehat{k}$ for some $\widehat{k} > 0$).

The main concept underlying the result is that the zeros of remote channels approach the controller's zeros when a high-gain feedback is used. Also, the zeros of remote channels approach the controller's poles when a low-gain feedback is used. Note that in cases discussed in previous lemma, the system becomes unstable, but we have seen cases that by increasing or decreasing the gain, first nonminimum-phase behavior appears and then instability occurs.

D. Control Schemes that Do Not Alter Channel Phase Characteristics

Often, it is important to ascertain whether a control scheme can alter a channel's phase characteristics, and specifically whether the control will preserve minimum phase dynamics on the channel. The following theorems identify control schemes that are guaranteed to maintain minimum-phase dynamics, i.e. not to change a minimum-phase channel to a nonminimum-phase channel. For this development, we say that the *phase property of the network input-output model is maintained*, to indicate that the model remains strictly minimum phase (respectively strictly nonminimum phase) upon inclusion of the controller P when the uncontrolled model is strictly minimum phase (respectively strictly nonminimum phase).

First, it is shown that low-gain proportional control schemes maintain the phase property of channels, provided that they do not alter the network structure in a certain sense.

Theorem 6: Consider the network input-output model (2), and assume that a controller $\mathbf{P} = k\mathbf{e}_q x_j$ or $\mathbf{P} = k\mathbf{e}_q\,(x_j - x_q)$ is applied. Also, consider the network graph G as well as a modified graph $\tilde{G}$ where the directed edge $j \to q$ is added to G. Consider any input-output channel. If the distance between the input and output vertices in the original and modified graphs is identical, then the phase property of the network input-output model is maintained for any sufficiently small gain k (i.e. for all $k < f$ for some threshold $f < \infty$).

Remark: The conditions for maintaining the phase property in Theorem 6 is necessarily satisfied, if the proportional controller is local ($q = j$), or acts across a link that is already present in the network. Likewise, it is necessarily satisfied if the channel of interest is local ($i = n$), or is placed across a network link.

Remark: Low-gain controllers necessarily only move system eigenvalues, and hence poles of any defined channel, by a small amount. In contrast, they can introduce/remove zeros or cause zeros to jump in general; the graph-theoretic condition in Theorem 6 is sufficient to guarantee that this does not happen, and hence that the phase property is maintained.

The following lemma shows that, if there is a directed edge from input vertex to output vertex, any proportional controller acting between the input vertex and another vertex maintains minimum-phase characteristic of the system.

Lemma 7: Consider the network input-output model (2), and assume that a proportional controller $\mathbf{P} = \mathbf{e}_q k(x_j - x_q)$ is applied. Consider any input-output channel across a network link (i.e. there is a directed edge from the input to the output vertex in the network graph G), which is adjacent to the built controller (i.e. $q = i$). Then the phase property of the network input-output model is maintained.

The following theorem identifies further conditions under which a proportional controller cannot influence the phase property of the network input-output model, no matter what the gain. In particular, in the case where there is only a single path between the input and output, we show in the following theorem that a proportional controller does not influence the phase property of the network provided that it does not introduce any further input-output paths: indeed, both the uncontrolled and controlled models are both minimum phase. We notice that this result builds on Lemma 3 in [17], which addressed the single-path case in the uncontrolled model.

Theorem 8: Consider the input-output model (2). Assume that either a local proportional controller $P = -k\mathbf{e}_q x_q$, or a controller of the form $\mathbf{P} = k\mathbf{e}_q(x_j - x_q)$, is applied, where k is positive. Consider any input-output channel such that there is only a single path between the input and output in the network graph. Then the input-output model (2) is minimum phase if adding the directed edge $j \to q$ in G does not create another directed path from the input to the output.

IV. Example

The graph-theoretic results on control channel interactions are illustrated in an example. The example is concerned specifically with using remote controllers to improve zero locations, in addition to improving system stability. We consider an input-output model (2) with 8 nodes, with input and output at nodes 1 and 3, respectively.

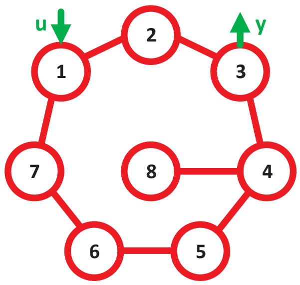

Fig. 2: Graph G associated with a network with 8 nodes.

The following state matrix A is considered (see Fig. 2 for an illustration of the network graph):

$$A = \begin{bmatrix} -1.15 & 0.05 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0.05 & -0.2 & 0.05 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.05 & -1.15 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -3.1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -2.1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2.1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & -2.1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1.1 \end{bmatrix}$$

The network input-output model has two zeros in the ORHP, so it is non-minimum phase. Based on Corollary 2, consider a local proportional controller at node 7 ($\mathbf{P} = -\mathbf{e}_7 k_7(x_7)$). We note that the corresponding vertex 7 is not on the special input-output path. From Corollary 2, we would expect a high-gain controller to promote minimum-phase dynamics. Indeed, by increasing the gain of the controller, we see in Fig. 3 that the real part of the dominant zero (i.e. the zero with maximum real part) changes from positive to negative, and so the input-output model becomes minimum phase with a high gain.

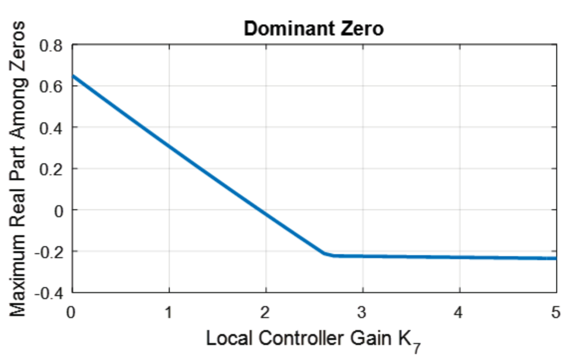

Fig. 3: The dependences of the dominant zero location (the largest real part among the zeros) on local controller gain.

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