

OCCUPANT-LOCATION-CATERED CONTROL OF IOT-ENABLED BUILDING HVAC SYSTEMS

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ABSTRACT

Catered control strategies for building HVAC systems, which react to the changing location of an occupant of the building, are studied. A structured Markov Jump-Linear System (MJLS) model is developed which captures the diffusive heat-exchange process in the building, stochastic occupant movement, and feedback control enacted by the HVAC system. A statistical analysis of the closed-loop dynamics is undertaken, and used to evaluate stability and control performance. The tuning of simple proportional-integral-derivative control schemes to optimize comfort is also discussed, and demonstrated in an example.

1. INTRODUCTION AND MOTIVATION

The pervasive deployment of cyber and sensing technologies, and the networking of these devices to form an “Internet-of-Things, are enabling new paradigms for thermal regulation of buildings [1–3]. In particular, these changes enable customized controls that improve Heating-Ventilation-Air-Conditioning (HVAC) system performance while also reducing energy use and cost. One direction of particular interest is to design occupant-location-catered control schemes for temperature regulation. The main idea is to design HVAC controls to react to the occupants’ current locations, so that desired comfort levels are maintained while excess energy costs are avoided. Such controls can readily be deployed in new cyber-enabled buildings, using a combination of fixed sensor networks, handheld mobile devices, and software-defined networking/radio technologies [4–7]. However, to be effective, models and control designs are required that account for the building’s thermal dynamics, and weigh the benefits and drawbacks of changing HVAC outputs in reaction to the occupant’s location profile.

In this study, the analysis and design of occupant-location-catered building temperature controllers is pursued, focusing on the base case that one occupant’s location is considered

and a centralized HVAC system is controlled (i.e., there is one control zone). The problem is approached as a feedback-control task for a Markov jump-linear system (MJLS), which captures heat flow within the building along with the occupant’s stochastic movement among the buildings rooms. A simple proportional-integral-derivative (PID) controller is considered, which uses temperature measurements at the occupant’s location to set the HVAC control input. Statistics of the closed-loop dynamics, and hence stochastic stability, are evaluated via a two-moment analysis of the MJLS. Also, tuning of the control gains is pursued, to optimize a performance metric which reflects occupant comfort and energy consumption.

The research described here connects to a wide literature on modeling, monitoring, and control of building thermal processes, (e.g. [8–13]). Our work also relates to a research thrust on statistical analysis of occupant locations in buildings, to support efficient operation [14–16]. Relative to these studies, the main contribution of this work is to use real-time sensing of occupant locations to enable refined building thermal control. Methodologically, the research described here also draws on and contributes to the literature on Markov jump linear systems (MJLS) [18–22], which have been a focus of controls and signal-processing communities. Specifically, the closed-loop stability analysis and control optimization pursued here directly uses the two-moment analysis of MJLS. Beyond this standard analysis, the special diffusion-network structure of the heat-flow dynamics and the specialized form of the Markov variations are exploited to obtain refined characterizations of the moments and their asymptotes [21, 22].

2. MODELING AND PROBLEM FORMULATION

A standard linear diffusive model for heat transfer in a building, which is often referred to as an resistive-capacitive (RC) network model in the literature, is considered [8, 9, 17]. The model tracks the the temperatures in a network of rooms, which evolve due to heat transfer processes. Formally, a network with $n - 1$ rooms, labeled $1, 2, \dots, n - 1$, is considered. Each room i has associated with it a temperature $x_i(t)$, which evolves with the continuous time variable t . Each temperature

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$x_i(t)$ is governed by the following differential equation:

$$\dot{x}_i(t) = \frac{1}{m_i} \left(\sum_{j=1}^{n-1} w_{ij}(x_j - x_i) + w_{in}(x_n - x_i) + \gamma_i u(t) \right), \quad (1)$$

where m_i is the thermal capacitance of room i , w_{ij} is the thermal conductance between room i and j (where $w_{ij} = w_{ji}$), x_n is the fixed outside temperature, w_{in} is the thermal conductance between room i and the outside, $u(t)$ is the scalar heat output of the HVAC system, and γ_i is the fraction of that heat output that enters room i (where $\sum_i \gamma_i = 1$). It is assumed in this work that $u(t)$ may be either positive or negative, reflecting that heating or air conditioning may be used. The thermal model is a significant abstraction from reality, but sufficient for a basic evaluation of an occupant-location-based control scheme. The dynamics can be expressed in matrix form as $\dot{\mathbf{x}} = A\mathbf{x}(t) + Bu(t)$, where $\mathbf{x} = [x_1 \ \cdots \ x_{n-1} \ x_n]^T$. The matrix A is an $n \times n$ matrix where $A_{ij} = \frac{w_{ij}}{m_i}$ for $i = 1, \dots, n-1, j = 1, \dots, n, j \neq i$, the diagonal elements $A_{ii} = 1, \dots, n-1$ are chosen such that the row-sums of A are zero, and $A_{ij} = 0$ for $i = n, j = 1, \dots, n$. B is an $n \times 1$ vector with i th entry equal to γ_i for $i = 1, \dots, n-1$, and final entry equal to zero.

The movement of a single occupant among the buildings rooms is considered. The occupant's room location is modeled as a Markov process. The HVAC controller is assumed to be alerted to the occupant's location (room) at clocked intervals, specifically at the times $t = kT$ for $k = 0, 1, 2, \dots$. Further, temperature measurements at the occupant's current location are assumed to be available to the HVAC controller at these clocked intervals (with minimal delay), for the purpose of feedback control. Since a Markov model is assumed for the occupant's room location, a finite-state Markov chain can be used to model the room location $s[k]$ at the data-transmission times $t = kT$. We notice that $s[k]$ may take on values among $1, \dots, n-1$ at each time step, corresponding to the $n-1$ room locations. The $(n-1) \times (n-1)$ transition matrix for the room-location Markov chain is denoted by $P = [p_{ij}]$ and is assumed throughout this work to be ergodic. Also, we define the temperature observation $y[k]$ as the temperature of the occupied room sampled at time $t = kT$, i.e. $y[k] = x_{s[k]}(kT)$. For the statistical analysis developed in this article, it is more convenient to express the observed temperature as a time-varying projection of the state $\mathbf{x}(t)$. Specifically, we have that $y[k] = \vec{v}^T[k]\mathbf{x}(kT)$, where $\vec{V}^T[k] = [\vec{e}^T 0]$, and $\mathbf{e}[k]$ 0-1 indicator vector for the occupant's room location $s[k]$.

The HVAC controller aims to regulate the temperature of the room occupied by the occupant at a desired reference temperature y_{ref} , by adjusting the thermal input $u(t)$. Here, a simple PID control scheme is considered. A simple scheme of this sort is appealing, given the need for portable, robust, and easy-to-implement solutions. Data transmission and computation for networked control schemes are typically clocked,

and data rates for building controls are sufficiently fast compared to the thermal dynamics of the building. With this in mind, it is natural to apply a zero-order-hold control, for which the control input is updated after each data transmission and held constant in between. The following PID control scheme of this form is proposed:

$u(t) = u[k]$ for $kT \leq t < (k+1)T$, where:

$$u[k] = K_p(y_{ref} - y[k]) + K_d(y[k] - y[k-1]) + K_i \sum_0^k (y_{ref} - y[k]) \times T$$

and where K_p , K_d and K_i are proportional, derivative, and integral gains.

The main focus of this work is to develop a statistical analysis of the closed-loop performance of the HVAC system, and to develop techniques for tuning the control gains to optimize a combined comfort- and cost- based performance metric.

3. CONTROLLER ANALYSIS AND DESIGN

The closed-loop dynamics are first reformulated as an autonomous state equation whose state matrix switches according to the occupant-location Markov chain, i.e. as an autonomous MJLS. This reformulation requires defining an

extended state vector $\vec{\xi}$, as: $\vec{\xi}[k] = \begin{bmatrix} \vec{\theta}[k] \\ \vec{\theta}[k-1] \\ Acc[k] \end{bmatrix}$ where

$\vec{\theta}[k] = \vec{x}[k] - y_{ref} \vec{1}$ is a shift of the temperature vector relative to the reference (goal) temperature, $\vec{1}$ is a vector with all unity entries, $Acc[k] = \sum_{m=1}^k z[m]$ is an accumulator for the integral controller, and $z[k] = \vec{v}^T \vec{\theta}$ indicates the temperature of the occupied room in the shifted coordinates.

The extended state vector $\vec{\xi}[k]$ is governed by a discrete-time Markovian jump linear process, which can be obtained by solving the continuous-time dynamics over intervals of duration T for each possible underlying occupant-location state:

$$\vec{\xi}[k+1] = G(i) \vec{\xi}[k] \quad (2)$$

In this equation, i is the state of the occupant-location Markov chain, and the $(2n+1)(2n+1)$ matrix $G(i)$ is given by $G(i) = G_A(i) + G_p(i) + G_d(i) + G_i(i)$, where $G_A(i) =$

$$\begin{bmatrix} \bar{A} & 0_{n \times n} & 0_{n \times 1} \\ I_{n \times n} & 0_{n \times n} & 0_{n \times 1} \\ \vec{v}^T(i) & 0_{1 \times n} & 1 \end{bmatrix}, G_p(i) = -K_p \Phi_{\bar{\beta}} \begin{bmatrix} S_2(i) & 0_{n \times n} & 0_{n \times 1} \\ 0_{n \times n} & 0_{n \times n} & 0_{n \times 1} \\ 0_{1 \times n} & 0_{1 \times n} & 0 \end{bmatrix},$$

$$G_d(i) = \frac{-K_d}{T} \Phi_{\bar{\beta}} \begin{bmatrix} S_2(i) & -S_2(i) & 0_{n \times 1} \\ 0_{n \times n} & 0_{n \times n} & 0_{n \times 1} \\ 0_{1 \times n} & 0_{1 \times n} & 0 \end{bmatrix}, \text{ and } G_i(i) = -K_i T$$

$$\Phi_{\bar{\beta}} \begin{bmatrix} S_2(i) & 0_{n \times n} & 1_{n \times 1} \\ 0_{n \times n} & 0_{n \times n} & 0_{n \times 1} \\ 0_{1 \times n} & 0_{1 \times n} & 0 \end{bmatrix}. \text{ In these expressions, we have}$$

that $\bar{A} = e^{AT}$, and $\Phi_{\bar{\beta}}$ is a $(2n+1) \times (2n+1)$ diagonal matrix whose j th diagonal element is equal to j th entry of the vector $\Phi_{\beta} = \Phi B$ for $j = 1, \dots, n-1$ and zero otherwise. Here, Φ is given by $\Phi = \int_0^T e^{A(T-\tau)} d\tau$. Also, S_2 is an

$n \times n$ matrix whose i th column is a unity vector while all other entries are 0. The matrix $G_A(i)$ in the above expression describes the internal heat-exchange dynamics and the evolution of the accumulator state $Acc[k]$, while $G_p(i)$, $G_d(i)$ and $G_i(i)$ capture the effects of the proportional, derivative, and integral controls, respectively. If the sampling time T is short enough, a simpler approximation which encodes the graph structure of the room network can also be developed, see [23] for details.

A two-moment analysis of the closed-loop model can be developed by considering the Kronecker product vectors $\psi_1[k] = \vec{v}^T[k] \otimes \xi[k]$ and $\psi_2[k] = \vec{v}^T[k] \otimes \xi[k]^{\otimes 2}$, which contain products of the extended state vector entries with an indicator of the underlying Markov chain's status. Per the standard analysis of MJLS, the *first-moment vector* $E(\psi_1[k])$ and the *second-moment vector* $E(\psi_2[k])$ are governed by the following linear dynamics or recursions: $E(\psi_1[k+1]) =$

$$H_1 E(\psi_1[k]), \text{ where } H_1 = \begin{bmatrix} p_{11}G(1) & \cdots & p_{1b}G(b) \\ \vdots & \ddots & \vdots \\ p_{b1}G(1) & \cdots & p_{bb}G(b) \end{bmatrix}$$

and $b = n - 1$ is the number of states in the underlying Markov chain (the number of indoor rooms). Likewise, we have that $E(\psi_2[k+1]) = H_2 E(\psi_2[k])$, where

$$H_2 = \begin{bmatrix} p_{11}G(1)^{\otimes 2} & \cdots & p_{1b}G(b)^{\otimes 2} \\ \vdots & \ddots & \vdots \\ p_{b1}G(1)^{\otimes 2} & \cdots & p_{bb}G(b)^{\otimes 2} \end{bmatrix}, \text{ and the notation}$$

$(Q)^{\otimes 2}$ refers to the self-Kronecker product of the matrix Q . The entries in the first and second moment vectors identify conditional moments of the extended state vector. For example, the first moment vector can be written

$$\text{as } E(\psi_1[k]) = \begin{bmatrix} E(\xi[k]|s[k]=1)P(s[k]=1) \\ \vdots \\ E(\xi[k]|s[k]=b)P(s[k]=b) \end{bmatrix}, \text{ where}$$

$E(\xi[k]|s[k]=j)$ is the conditional expectation of the extended state given the observer's location, and $P(s[k]=j)$ is the observer's location probability. From these conditional moment vectors, the first two moments of the temperature in the occupant's current room as well as of the input can be directly computed. Thus, the occupant's comfort and the energy use in the closed loop can be characterized.

Toward characterizing the closed-loop performance, first conditions are found such that the expected squared deviation of the temperature at the occupant's current location from the goal temperature $E((y(t) - y_{ref})^2)$ remains bounded over time. Persistent boundedness of the deviation corresponds exactly with the stability of the first- and second- moment recursions (dynamics). Conditions for stability can be developed via an eigenanalysis of the recursion matrices H_1 and H_2 , which draws on the diffusive structure of the heat-flow dynamics. The analysis centrally depends on a lemma which indicates that H_1 and H_2 necessarily have unity eigenvalues:

Lemma 1:

The first moment recursion matrix H_1 and second moment recursion matrix H_2 each have at least one eigenvalue equal to 1.

Lemma 1 provides a starting point toward a moment-stability analysis. Qualitatively, two outcomes can result: either the eigenvalues of H_1 and H_2 are strictly inside the unit circle except for a single eigenvalue at 1, thus guaranteeing boundedness of the squared deviation, or the moment-recursions are unstable. The main outcome of our analysis is that, for any building heat-flow model where all rooms are thermally connected, boundedness of the expected squared deviation is guaranteed when sufficiently small negative proportional-derivative feedback is used. This result is formalized in the following theorem.

Theorem 1:

Consider the closed-loop heat-flow dynamics in the case that the rooms are thermally connected, and a proportional-derivative control is used (i.e. $K_i = 0$). For all sufficiently small negative feedback ($0 \leq K_p \leq \bar{K}_p$ and $0 \leq K_d \leq \bar{K}_d$ for some $\bar{K}_p > 0$ and $\bar{K}_d > 0$), the first and second moment dynamics are stable. Further, the expected squared deviation $E((y(t) - y_{ref})^2)$ is bounded for all $t \geq 0$.

The proofs of Lemma 1 and Theorem 1 can be found in the extended document [23].

Beyond assuring moment boundedness, it is natural to select the gains to optimize the a performance metric which captures: 1) the effectiveness of the control in meeting the reference temperature at the occupant's current location and 2) the energy (effort) expended for control. The following quadratic cost metric captures these factors: $J = \frac{1}{T_2 - T_1} \sum_{k=T_1}^{T_2-1} \alpha_1 z[k]^2 + \alpha_2 u[k]^2$ Where $z[k] = y_{ref} - y[k]$ is the temperature deviation of the occupied room from the desired, $[T_1, T_2]$ is the time interval of interest, and α_1 and α_2 are weighting factors. Our primary interest is to characterize time-average cost metric J over a long horizon $[T_1, T_2]$. In the case that the closed-loop is two-moment stable and the underlying Markov chain is ergodic, it is easy to argue that the time average cost metric J approaches the steady-state ensemble average of the squared deviation cost, in a probability-1 sense. Thus, the performance of the closed-loop control system can equivalently be evaluated as $\tilde{J} = \lim_{k \rightarrow \infty} \alpha_1 E[z[k]^2] + \alpha_2 E[u[k]^2]$, which can be expressed as: $\tilde{J} = \lim_{k \rightarrow \infty} E[\alpha_1 z[k]^2 + \alpha_2 u[k]^2 | s[k] = i] Pr(s[k] = i)$. The summands in the expression for \tilde{J} can be computed from entries in the first and second moment vectors, evaluated in steady-state. These steady-state entry values are contained in the right eigenvectors of H_1 and H_2 associated with their unity eigenvalues (appropriately normalized), which we label \vec{V}_1 and \vec{V}_2 . The eigenvectors are normalized so that the entries corresponding to the outside environment

are scaled to equal the outside temperature. Precisely, the entries $(2n+1)(i-1)+n$, $i=1, \dots, b$ of \bar{V}_1 are identical; the vector \bar{V}_1 is scaled so that those entries equal the outside temperature in shifted coordinates (i.e., the temperature deviation relative to the desired). Similarly, \bar{V}_2 is scaled so that the entries corresponding to the outside environment are equal to the square of the outside temperature in the shifted coordinates. In terms of the normalized eigenvectors, the cost metric can be computed for the two-moment-stable ergodic case as

$$\tilde{J} = \sum_{i=1}^b \alpha_1 F_1 + \alpha_2 F_2, \text{ Where } F_1 = \bar{V}_{2\bar{i}+(i-1)\bar{n}+i} \text{ and } F_2 = (K_p)^2 + \left(\frac{K_d}{T}\right)^2 + 2\frac{K_p K_d}{T} \bar{V}_{2\bar{i}+(i-1)\bar{n}+i} + \left(\frac{K_d}{T}\right)^2 \bar{V}_{2\bar{i}+(n+i-1)\bar{n}+n+i} + (K_i T)^2 \bar{V}_{2\bar{i}-1} + \left(-2\frac{K_d}{T}\right) \left(K_p + \frac{K_d}{T}\right) \bar{V}_{2\bar{i}+(i-1)\bar{n}+n+i} + (-2K_d K_i) \bar{V}_{2\bar{i}+(n+i)\bar{n}} + \left(2\left(K_p + \frac{K_d}{T}\right) T K_i\right) \bar{V}_{2\bar{i}+(i)\bar{n}}, \text{ and where } \bar{n} = (2n+1) \text{ and } \bar{i} = (i-1)\bar{n}^2.$$

3.1. Example

A building with 6 rooms, which are connected as shown in Figure 1, is considered. The following parameters are assumed: $m_i = 1$ for $i = 1, \dots, 6$, $w_{ij} = 1$ for each connected pair of rooms, $w_{in} = 0.1$ for all i , $P_{ii} = 0.998$ for $i = 1, \dots, 6$, $P_{ij} = 0.0004$ for $i \neq j$, $x_o = 50, y_{ref} = 70$, $x_i[0] = 50$ for $i = 1, \dots, 6$, $\gamma_3 = 1$, and $\gamma_i = 0$ otherwise. The occupant begins the simulation in room 2.

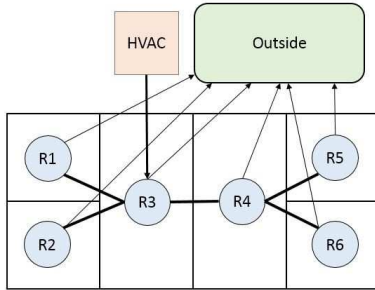


Fig. 1. Building diagram for the heat-flow model.

Tuning of the control parameters is considered. Specifically, the control parameters K_p and K_i are varied within the intervals $K_p \in [-0.1, 2]$ and $K_i \in [-0.05, 0.2]$, for $k_d = 0.02$. The boundedness of the mean-square deviation of the closed-loop dynamics (equivalently, stability of the first two moments) is determined, and the steady-state performance metric $E[\tilde{J}]$ is also evaluated. For the performance analysis, the cost function constants are chosen as $\alpha_1 = 1$ and $\alpha_2 = 1$. Figure 2 shows the performance metric and bounded deviation region as a function of K_p and K_i for $K_d = 0.02$. We emphasize that the performance metric is meaningful only when the first two moments are stable (the expected squared deviation is bounded), therefore unstable control gains are indicated with a dark color. For the stabilizing designs, the cost

is shown on a logarithmic scale, as indicated by the color bar.

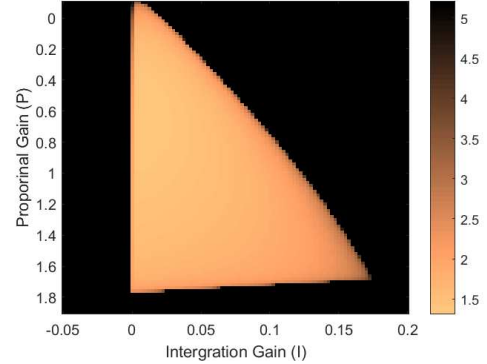


Fig. 2. Performance metric and expected square deviation boundedness vs. K_p and K_i for $K_d = 0.02$. The dark shaded areas correspond to unstable moment recursions (unbounded expected squared deviation from the reference)

We note that the optimal performance metric value is typically achieved close to but not exactly at the boundary of the moment-recursion-stability region. Also, using a larger derivative term widens the range of K_p and K_i parameters for which the closed-loop system is stable, and placing the heater in a central room (3 or 4) permits a larger stability region as compared to a heater placed in a side room (details not shown – see extended document [23]).

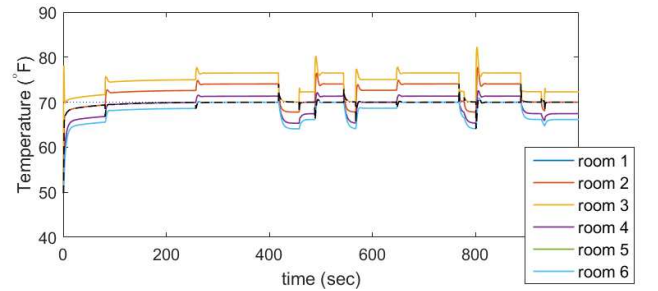


Fig. 3. Sample temperature dynamics for the optimal design ($K_p = 0.62$, $K_i = 0.005$ and $K_d = 0.02$). Occupied room is determined by black dashed line

Figure 3 show simulations of the optimal occupant-location-based control. The controller is able to regulate the temperature of occupied room near the desired reference, even though the temperatures in the other rooms deviate from the reference. This suggests that an occupant-location-catered control is effective in maintaining comfort. Simulations also suggest that the control does not significantly increase energy cost compared to a fixed scheme, provided that the occupant changes locations relatively slowly.

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