Downlink Performance Analysis of Cell-Free Massive MIMO with Finite Fronthaul Capacity

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***Abstract*—In this work, we analyze the performance of the downlink of a cell-free massive multiple-input multiple-output (mMIMO) system considering finite capacity fronthaul links. We model the locations of the remote radio heads (RRHs) and the users as two independent binomial point processes (BPPs). Conditioned on the locations of the RRHs and users, and considering imperfect channel state information (CSI) and conjugate beamforming at the RRHs, we derive an achievable rate for a randomly selected user in the network. Further, based on the *dominant RRH* approach, we provide an approximate but accurate expression to analytically evaluate this rate averaged over the spatial realizations of RRH and user locations. From our analysis, we arrive at the following conclusions: (1) the achievable average system sum-rate is a strictly quasi-concave function of the number of users in the network, (2) for the same number of antennas in the system, the optimal number of antennas per RRH to maximize the average user rate as well as average system sum-rate depends on the quality of the CSI. While for a high-quality CSI a more collocated system is preferred, for low-quality CSI it is better to consider a more distributed RRH deployment.**

***Index Terms*—Cell-free massive MIMO, stochastic geometry, fronthaul capacity, binomial point process.**

I. Introduction

Massive multiple-input multiple-output (mMIMO) technology is poised to revolutionize the communication networks as it has been proven that under ideal conditions it eliminates the deleterious effect of channel fading and additive noise while negating the effect of network interference [1]. Further, having a large number of antennas at the base stations not only boosts spectral efficiency but also improves the energy efficiency of the overall network [2]. Traditionally, network densification has played a leading role in meeting the demand for higher network throughput. A similar trend is expected to continue for future wireless networks as well. This increasing density of nodes can be leveraged to implement mMIMO in a distributed manner as is envisioned in the form of cell-free mMIMO [3], [4]. The fundamental concept of cell-free mMIMO is similar to that of network MIMO, where a large number of geographically separated remote radio heads (RRHs) are centrally controlled by a baseband signal processing unit to serve users in its service region. Under the ideal assumptions such as unlimited fronthaul capacity, a fully distributed RRH setup is known to be better compared to semi-distributed or collocated setups in terms of user spectral efficiency. However, if one considers all the elements of a cell-free massive MIMO system such as the imperfect channel state information (CSI), finite capacity of fronthaul links, and beamforming based on local CSI at the RRHs, it is not clear whether it is better from the system performance perspective to have fewer RRHs with more antennas per RRH or vice versa. Motivated by this, in this work, our goal is to present a comprehensive analysis of cell-free mMIMO setup considering the finite capacity of fronthaul links and beamforming based on *local* imperfect CSI.

Related works: While the understanding of collocated mMIMO is quite mature, the research on cell-free mMIMO with realistic system assumptions is still evolving. The performance analysis of cell-free massive MIMO with imperfect CSI and power control is presented in [3], [4] for both downlink (DL) and uplink (UL). However, these works assume an unlimited capacity for the fronthaul links. In order to reduce the load on fronthaul links, a user-centric cell-free mMIMO approach and a compute-and-forward transmission approach are proposed in [5] and [6], respectively. However, these works do not characterize the impact of limited fronthaul capacity on system performance. The effect of limited fronthaul capacity for the UL of cell-free mMIMO is studied in [7], where authors characterize the effect of quantization error on user rate and propose a max-min power control algorithm so that each user gets a uniform rate. It is worth mentioning that performance analyses in these works are based on Monte Carlo simulations.

Another set of relevant prior works focus on devising compression algorithms while taking into account the limited fronthaul capacity in problems such as distributed antenna systems, coordinated multipoint, and cloud radio access networks. In [8], [9], authors provide information theoretic insights regarding the capacity of a backhaul-constrained distributed MIMO system. To make the analysis tractable, usually, a simplified system model is considered. For example, in [8] a linear modified Wyner model is considered where only two neighboring base stations cooperate to serve a user. In [9], authors have provided useful insights regarding backhaul-constrained capacity regions for a two transmitter and two receiver model. Extending the insights obtained from information theoretic analyses, in other notable works, authors use optimization framework to devise compression algorithms that efficiently utilize the fronthaul capacity constraints while maximizing a certain performance metric (e.g. sum-rate) (cf. [10], [11]). A comprehensive overview of such works can be found in [12]. While these works provide useful signal processing tools for efficient system design, it is necessary to have a mathematically tractable model of these systems to get a comprehensive understanding of the system performance. From this perspective, analytical evaluations of the downlink of distributed antenna systems are presented in [13]–[17] and the references therein. The performance evaluation is usually done through capacity bounds. Although these works provide useful insights without resorting to Monte Carlo simulations,

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they do not consider the limited fronthaul capacity or imperfect CSI in their analyses. Further, a few of these works present system analyses that require the global CSI to be present at the baseband unit (BBU). Since in cell-free mMIMO the channel information is likely to be available only at the RRHs, a distributed beam-forming approach such as sub-optimal conjugate beamforming (CB) is a more viable option. Hence, the inferences drawn in these works may not hold for cell-free mMIMO systems as envisioned.

Contributions of the work: We analyze the DL performance of a cell-free mMIMO system using tools from stochastic geometry. We consider a finite service region and model the locations of the RRHs and users as two independent binomial point processes (BPPs). We take into account the limited capacity of the fronthaul links between the BBU and the RRHs and imperfect CSI at the RRHs. Each RRH performs CB adhering to an average power constraint. For this system setup, we derive an achievable rate for a randomly selected user conditioned on the locations of the RRHs and users. Further, leveraging relevant distance distributions for a BPP, we also provide an approximate expression to analytically evaluate the user rate averaged over RRH and user locations. From our analyses, we infer that the average system sum-rate is a strictly quasi-concave function of the number of users and the optimal number of users to achieve the maximum system sum-rate increases with increasing fronthaul capacity. Further, in contrast to the established notion that fully distributed MIMO is superior to the collocated MIMO, our results suggest that in presence of high-quality CSI at the RRHs, a less distributed form of cell-free mMIMO is better, i.e. for an equal number of antennas in the system, it is better to deploy a fewer RRHs with more antennas per RRH.

II. System Model for Distributed Massive MIMO

We limit our attention to the DL of a cell-free mMIMO system. We assume that M RRHs equipped with N antennas each are uniformly distributed over a finite circular region of radius R_s centred at origin, i.e. $\mathcal{B}_{R_s}(\mathbf{o})$. Let $\Phi_r = \{\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_M\}$ be the set of the locations of these M RRHs. These RRHs collectively serve K single antenna users that are uniformly distributed over $\mathcal{B}_{R_s}(\mathbf{o})$. Let $\Phi_u = \{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_K\}$ be the set that contains these locations. Note that by construction, Φ_r and Φ_u form two independent BPPs. Further, the distance between a user at $\mathbf{u}_k$ and an RRH at $\mathbf{r}_m$ is denoted by d_{mk}. As assumed in the cell-free mMIMO literature, we consider that $M > K$. All the RRHs are connected to a BBU through a fronthaul network, where the capacity of each fronthaul link is C. Due to this limited capacity, the BBU employs a lossy compression scheme to forward user symbols to the RRHs.

A. Compression at the BBU

We first discuss the effect of compression on the user symbols. Let q_k be the symbol intended for the k-th user in the network, and $\mathbf{q} = [q_1, q_2, \ldots, q_K]^T$ be the signal vector consisting of all the symbols to be transmitted to the users. We consider that $\mathbf{q}$ is a circularly symmetric complex Gaussian random vector and $\mathbf{q} \sim \mathcal{CN}(\mathbf{0}_K, \rho_q \mathbf{I}_K)$, where $\rho_q = \mathbb{E}\left[|q_1|^2\right] = \mathbb{E}\left[|q_2|^2\right] = \ldots = \mathbb{E}\left[|q_K|^2\right]$. Using a lossy

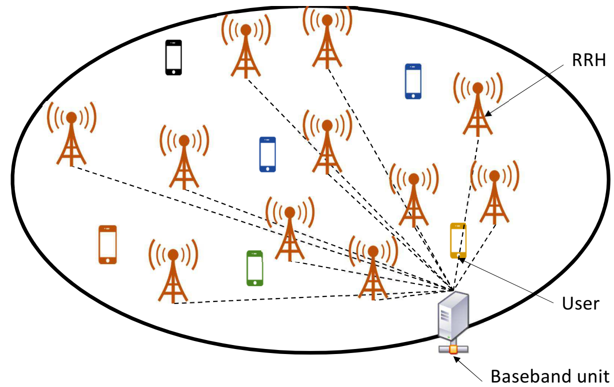

Fig. 1. A representative network diagram of the system, where RRHs with possibly multiple antennas are connected to a centralized BBU through limited capacity fronthaul links.

compression scheme, the BBU transmits $\hat{\mathbf{q}} = [\hat{q}_1, \hat{q}_2, \ldots, \hat{q}_K]^T$ over the fronthaul links to the RRHs. Similar to [8], we consider $\hat{\mathbf{q}} = \mathbf{q} + \tilde{\mathbf{q}}$, where $\tilde{\mathbf{q}} \sim \mathcal{CN}(\mathbf{0}_K, \rho_{\tilde{q}} \mathbf{I}_K)$ is the compression error vector and $\rho_{\tilde{q}} = \mathbb{E}\left[|\tilde{q}_1|^2\right] = \mathbb{E}\left[|\tilde{q}_2|^2\right] = \ldots = \mathbb{E}\left[|\tilde{q}_K|^2\right]$. Further, we assume that $\mathbf{q}$ and $\tilde{\mathbf{q}}$ are uncorrelated. Since both are Gaussian random vectors, they are independent as well. From the above exposition, it is clear that $\hat{\mathbf{q}} \sim \mathcal{CN}(\mathbf{0}_K, (\rho_{\tilde{q}} + \rho_q)\mathbf{I}_K)$. If $\mathbb{E}\left[|\hat{q}_k|^2\right]$ is same for all k and is fixed, then both $\rho_{\tilde{q}}, \rho_q$ can be argued to depend on the fronthaul capacity C, as discussed in the following lemma.

Lemma 1. *For a fronthaul capacity C and number of users K in the network, $\rho_q = \left(1 - 2^{-C/K}\right)\mathbb{E}\left[|\hat{q}_k|^2\right]$ and $\rho_{\tilde{q}} = 2^{-C/K}\mathbb{E}\left[|\hat{q}_k|^2\right]$.*

Proof: The amount of information that can be transmitted from the BBU to each RRH is upper limited by the fronthaul capacity C. Hence, we write $I(\hat{\mathbf{q}}; \mathbf{q}) \leq C \implies$

$$\mathrm{h}(\hat{\mathbf{q}}) - \mathrm{h}(\hat{\mathbf{q}}|\mathbf{q}) \leq C \implies \sum_{i=1}^{K} \mathrm{h}(\hat{q}_i) - \sum_{i=1}^{K} \mathrm{h}(\hat{q}_i|q_i) \leq C$$
$$\implies \log_2(\pi e(\rho_q + \rho_{\tilde{q}})) - \log_2(\pi e \rho_{\tilde{q}}) \leq C/K,$$

where $I(x; y)$ denotes the mutual information between two random variables x and y, $\mathrm{h}(x)$ denotes the differential entropy of a random variable x, and the last step follows from the fact that $\hat{q}_i$s and $\tilde{q}_i$s are complex Gaussian random variables. Ideally, the BBU would like to transmit the maximum information, which is ensured by introducing minimum error to each symbol. Hence, the minimum value of $\rho_{\tilde{q}}$ while satisfying the capacity constraint can be obtained by solving the following equation:

$$\log_2\left(1 + \rho_q/\rho_{\tilde{q}}\right) = C/K \implies \rho_q/\rho_{\tilde{q}} = 2^{C/K} - 1.$$

The expression in the lemma follows directly using the fact that $\rho_q + \rho_{\tilde{q}} = \mathbb{E}\left[|\hat{q}_k|^2\right]$. If we consider that $\mathbb{E}\left[|\hat{q}_k|^2\right] = 1$, then $\rho_q = \left(1 - 2^{-C/K}\right)$ and $\rho_{\tilde{q}} = 2^{-C/K}$. ■

Remark 1. *We consider that the BBU equally allocates the fraction of the fronthaul bandwidth to the symbols of each user in the network. Moreover, we consider the average effect of compression error on each symbol. More sophisticated* *information scheduling can be used to improve the overall system capacity, which is a promising direction for future work.*

B. Uplink channel estimation

Let $\mathbf{g}_{mk} = \sqrt{\beta_{mk}}\mathbf{h}_{mk}$ be the channel gain between the m-th RRH and the k-th user, where β_{mk} captures the large-scale channel gain and $\mathbf{h}_{mk} \sim \mathcal{CN}(\mathbf{0}_N, \mathbf{I}_N)$ captures the small-scale channel fluctuation. We consider that the large-scale channel gain β_{mk} is only due to the distance dependent pathloss, i.e. $\beta_{mk} = l(d_{mk})^{-1}$, where d_{mk} is the distance between the m-th RRH and the k-th user, and $l(\cdot)$ is a non-decreasing pathloss function presented in Section IV.

In order to obtain the channel estimates, we consider that each user uses a pilot from a set of K orthogonal pilot sequences of length τ_p symbol duration, which is assumed to be less than the coherence interval. Further, the transmit signal-to-noise ratio ($\mathtt{SNR}$) of each symbol in a pilot is ρ_p. Since we assume that these K sequences are orthogonal to each other, $\tau_p > K$ and $\boldsymbol{\psi}_i^H \boldsymbol{\psi}_j = \mathbf{1}(i = j)$, where $\mathbf{1}(\cdot)$ denotes the indicator function. The general case where number of users is more than the number of orthogonal pilots will be considered in the extended version of this work. Let the pilot used by the k-th user be $\boldsymbol{\psi}_k$. During the pilot transmission phase, the received signal matrix at the m-th RRH is

$$\mathbf{r}_m = \sqrt{\tau_p \rho_p} \sum_{k=1}^{K} \mathbf{g}_{mk} \boldsymbol{\psi}_k^T + \mathbf{W}_m \in \mathbb{C}^{N \times \tau_p},$$

where each element of $\mathbf{W}_m$ is $\mathcal{CN}(0,1)$. Let $\hat{\mathbf{g}}_m$ be the channel vector obtained after performing minimum-mean-squared-error (MMSE) channel estimation. In this case, the error $\tilde{\mathbf{g}}_{mk} = \mathbf{g}_{mk} - \hat{\mathbf{g}}_{mk}$ is uncorrelated to the estimated vector. Further, the estimate and the error vectors are [4]

$$\begin{aligned} \hat{\mathbf{g}}_{mk} &\sim \mathcal{CN}\left(\mathbf{0}_N, \gamma_{mk} \mathbf{I}_N\right), \\ \tilde{\mathbf{g}}_{mk} &\sim \mathcal{CN}\left(\mathbf{0}_N, \left(\beta_{mk} - \gamma_{mk}\right) \mathbf{I}_N\right), \end{aligned} \tag{1}$$

where $\gamma_{mk} = \frac{\tau_p \rho_p \beta_{mk}^2}{1 + \tau_p \rho_p \beta_{mk}}$.

C. Downlink data transmission

Since the BBU does not have the channel information, CB becomes the natural candidate for beamforming as it can be implemented in a distributed manner. Hence, the precoded symbol transmitted by the m-th RRH is given as

$$\mathbf{x}_m = \sum_{k=1}^{K} \sqrt{\rho_d \eta_{mk}} \hat{\mathbf{g}}_{mk}^* \hat{q}_k,$$

where ρ_d is the DL transmit $\mathtt{SNR}$, η_{mk} is normalization coefficient used by the m-th RRH for the k-th user to satisfy the average power constraint

$$\mathrm{Tr}(\mathbb{E}\left[\mathbf{x}_m \mathbf{x}_m^H\right]) \leq N \rho_d.$$

We observe that by setting $\eta_{mk} = 1/(\gamma_{mk} K)$ above constraint is satisfied with equality. The symbol received at a randomly selected user $o \in \{1, 2, \ldots, K\}$ is given as $r_o =$

$$\begin{aligned} &\sum_{m=1}^{M} \mathbf{g}_{mo}^T \mathbf{x}_m + w_o = \sum_{m=1}^{M} \mathbf{g}_{mo}^T \sum_{k=1}^{K} \sqrt{\rho_d \eta_{mk}} \hat{\mathbf{g}}_{mk}^* \hat{q}_k + w_o \\ &= \sum_{m=1}^{M} \mathbf{g}_{mo}^T \hat{\mathbf{g}}_{mo}^* \hat{q}_o \sqrt{\rho_d \eta_{mo}} + \sum_{\substack{k=1, k\neq o \\ m=1}}^{K,M} \sqrt{\rho_d \eta_{mk}} \mathbf{g}_{mo}^T \hat{\mathbf{g}}_{mk}^* \hat{q}_k + w_o \\ &= \sum_{m=1}^{M} \|\hat{\mathbf{g}}_{mo}\|^2 \sqrt{\rho_d \eta_{mo}} q_o + \sum_{m=1}^{M} \|\hat{\mathbf{g}}_{mo}\|^2 \sqrt{\rho_d \eta_{mo}} \tilde{q}_o \\ &\quad + \sum_{m=1}^{M} \sqrt{\rho_d \eta_{mo}} \tilde{\mathbf{g}}_{mo}^T \hat{\mathbf{g}}_{mo}^* \hat{q}_0 \\ &\quad + \sum_{\substack{k=1 \\ k\neq o}}^{K} \sqrt{\rho_d} \left(\sum_{m=1}^{M} \sqrt{\eta_{mk}} \mathbf{g}_{mo}^T \hat{\mathbf{g}}_{mk}^* \right) \hat{q}_k + w_o, \end{aligned} \tag{2}$$

where the last step follows from replacing $\mathbf{g}_{mk} = \hat{\mathbf{g}}_{mk} + \tilde{\mathbf{g}}_{mk}$ and $q_k = \hat{q}_k + \tilde{q}_k$. In the following lemma, we provide an expression for an achievable rate (a lower bound on capacity). Note that in favor of simpler exposition, we ignore the constant pre-log factors such as bandwidth, and fraction of DL transmission duration in a time division duplex setup.

Lemma 2. *An average achievable rate of a randomly selected user is given by*

$$R_o = \mathbb{E}_{\Phi_r, \Phi_u}\left[\log_2\left(1 + \mathtt{SINR}_o\right)\right], \tag{3}$$

where $\mathtt{SINR}_o =$

$$\frac{\rho_d \frac{N^2}{K} \left(\sum\limits_{m=1}^{M} \sqrt{\gamma_{mo}} \right)^2 \mathbb{E}\left[|q_0|^2\right]}{\rho_d \frac{N^2}{K} \big(\sum\limits_{m=1}^{M} \sqrt{\gamma_{mo}} \big)^2 \mathbb{E}\left[|\tilde{q}_0|^2\right] + \rho_d N \sum\limits_{m=1}^{M} \beta_{mo} + 1}. \tag{4}$$

The corresponding system sum-rate is KR_o.

Proof: Please refer to Appendix A. ■

In the cell-free mMIMO literature, the usual approach to evaluate the above expression is through numerical simulations. However, in this work, using the properties of the BPP, we present an analytical approach to evaluating this expression.

III. Average Rate Evaluation

The *exact* analytical evaluation of (3) is challenging as it requires an $(M+1)$-fold integration to average it over the locations of all RRHs and the o-th user. Notice that each term in (4) have either of the following terms:

$$I_1 = \sum_{m=1}^{M} \sqrt{\gamma_{mo}}, \quad I_2 = \sum_{m=1}^{M} \beta_{mo}. \tag{5}$$

Further, note that γ_{mk} is an increasing function of β_{mk}, which is a decreasing function of d_{mk}. Hence, γ_{mk} is also a decreasing function of d_{mk} and can be expressed as $\gamma_{mk}(d_{mk}) = \frac{\tau_p \rho_p l(d_{mk})^{-2}}{1 + \tau_p \rho_p l(d_{mk})^{-1}}$. Due to pathloss either of the terms is likely to be dominated by contributions from a few nearest RRHs. Hence, we approximate I_1 and I_2 as the sum of exact contribution from the nearest RRH and the mean contribution from the rest of the RRHs conditioned on the distance between the o-th user and its nearest RRH, i.e. for I_1 we write

$$I_1 = \sum_{m=1}^{M} \sqrt{\gamma_{mo}} = \sqrt{\gamma_{oo}} + \mathbb{E}\Bigg[\sum_{m=1, m\neq o}^{M} \sqrt{\gamma_{mo}} \Bigg| d_{oo}, r_o \Bigg], \tag{6}$$

where d_{oo} is the distance between o-th user and its nearest RRH, and r_o are the distance between the o-th user and the center of the service region $\mathbf{o}$. Similarly, I_2 can also be expressed as the sum of the dominant term and conditional expectation of rest of the terms. As we will see in the sequel, due to this approximation, we are able to evaluate (3) with maximum four integrals as opposed to the $(M+1)$-fold integration for the exact expression. It is worth mentioning that this approach has been used for DL coverage probability analysis in cellular systems (cf. [18]). To derive the final result, we need a few important distance distributions in a BPP, which are presented next.

A. Relevant distance distributions in a BPP

Let R_o be the distance of the o-th user from the center of the circle $\mathcal{B}_{R_s}(\mathbf{o})$. The cumulative distribution function (CDF) and probability density function (PDF) of the distance of o-th user, which is a randomly and uniformly distributed point in $\mathcal{B}_{R_s}(\mathbf{o})$, from the center $\mathbf{o}$ is given as

$$F_{R_o}(r) = \frac{r^2}{R_s^2}, \qquad f_{R_o}(r) = \frac{2r}{R_s^2} \quad 0 \le r \le R_s. \quad (7)$$

Now, we present the distance distribution between the o-th user to a randomly distributed RRH in $\mathcal{B}_{R_s}(\mathbf{o})$.

Lemma 3. *Conditioned on the distance R_o, the* CDF *of the distance between the o-th user and the m-th RRH is given as* $F_{D_{mo}}(d|r_o) =$

$$\frac{d^2}{R_s^2}\,\mathbf{1}(0 \le d < R_s - r_o) + \mathbf{1}(R_s - r_o \le d \le R_s + r_o) \left(\frac{d^2}{\pi R_s^2}\left(\theta^* - \frac{\sin(2\theta^*)}{2}\right) + \frac{1}{\pi}\left(\phi^* - \frac{\sin(2\phi^*)}{2}\right)\right),$$

and corresponding PDF *is given as*

$$f_{D_{mo}}(d|r_o) = \frac{2d}{R_s^2}\,\mathbf{1}(0 \le d < R_s - r_o) + \mathbf{1}(R_s - r_o \le d \le R_s + r_o)\frac{2d}{\pi R_s^2}\theta^*$$

where $\theta^* = \arccos\left(\frac{d^2+r_o^2-R_s^2}{2r_o d}\right), \phi^* = \arccos\left(\frac{R_s^2+r_o^2-d^2}{2r_o R_s}\right)$.

Proof: We provide the sketch of the proof of this lemma. Please refer to [19, Lemma 1] for the detail proof. Without loss of generality, consider that o-th user is located at $\mathbf{u}_o = (r_o, 0)$. Then, condition on $\mathbf{u}_o$ (equivalently r_o), a uniformly distributed point in $\mathcal{B}_{R_s}(\mathbf{o})$ can lie either in the circle $\mathcal{B}_{R_s - r_o}(\mathbf{u}_o)$ or in the region $\mathcal{B}_{R_s}(\mathbf{o}) \setminus \mathcal{B}_{R_s - r_o}(\mathbf{u}_o)$. In the CDF expression of the lemma both this conditions are captured by the indicator function and corresponding conditional CDFs are presented. The expression for the PDF is obtained by taking the derivative of the CDF with respect to d along with some algebraic manipulation. ■

Now, using the results from order statistics, we present the conditional distance distribution between the o-th user and its nearest RRH.

Lemma 4. *Conditioned on the distance R_o, the* CDF *of the distance D_{oo} between the o-th user and its nearest RRH is given as* $F_{D_{oo}}(d_{oo}|r_o) =$

$$\mathbb{P}\left[D_{oo} \le d_{oo}|r_o\right] = 1 - (1 - F_{D_{mo}}(d_{oo}|r_o))^M,$$

and the corresponding PDF *is given as*

$$f_{D_{oo}}(d_{oo}|r_o) = M f_{D_{mo}}(d_{oo}|r_o)(1 - F_{D_{mo}}(d_{oo}|r_o))^{M-1},$$

where $f_{D_{mo}}, F_{D_{mo}}$ are presented in Lemma 3.

Note that conditioned on the distance D_{oo} , rest of the RRHs in $\mathcal{B}_{R_s}(\mathbf{o})$ are uniformly and randomly located in $\mathcal{B}_{R_s}(\mathbf{o}) \setminus \mathcal{B}_{d_{oo}}(\mathbf{u}_o)$, where d_{oo} is a realization of D_{oo}. In the following lemma, we present the distribution of the distance between a randomly located RRH in the above region and the o-th user.

Lemma 5. *Conditioned D_{oo} and R_o, the* PDF *of the distance $\hat{D}_{mo}$ between a randomly located RRH in $\mathcal{B}_{R_s}(\mathbf{o}) \setminus \mathcal{B}_{d_{oo}}(\mathbf{u}_o)$ and the o-th user is given as*

$$f_{\hat{D}_{mo}}(d|d_{oo}, r_o) = \frac{f_{D_{mo}}(d|r_o)}{1 - F_{D_{mo}}(d_{oo}|r_o)}, d_{oo} \le d \le r_o + R_s.$$

Proof: We provide the sketch of the proof for this lemma. For the detailed proof, please refer to [19, Lemma 3]. Conditioned on D_{oo}, rest of the RRHs are uniformly distributed in $\mathcal{B}_{R_s}(\mathbf{o}) \setminus \mathcal{B}_{d_{oo}}(\mathbf{u}_o)$. Hence, the distribution of the distance $\hat{D}_{mo}$ follows the lower truncated distribution of D_{mo}, which is captured in the above expression. ■

Next, using the above distance distribution, we present the approximate expression to evaluate (3).

B. Approximate evaluation of average achievable user rate

In the following lemma, using the fact that conditioned on D_{oo}, distances between o-th user and rest of the RRHs in the network are independent and identically distributed, we present an expression to evaluate the expectation term in (6).

Lemma 6. *Conditioned on R_o and the distance D_{oo} to the nearest RRH, the expectation term in* (6) *is given as*

$$\mathbb{E}\left[\sum_{\substack{m=1\\m\neq o}}^{M}\sqrt{\gamma_{mo}}\middle|d_{oo}, r_o\right] = \sum_{\substack{m=1\\m\neq o}}^{M}\mathbb{E}\left[\sqrt{\gamma_{mo}}\middle|d_{oo}, r_o\right]$$

$$=(M-1)\int_{r=d_{oo}}^{r_o+R_s}\frac{\sqrt{\tau_p\rho_p}l(r)^{-1}}{\sqrt{1+\tau_p\rho_p l(r)^{-1}}}f_{\hat{D}_{mo}}(r|d_{oo}, r_o)\mathrm{d}r, \quad (8)$$

where $f_{\hat{D}_{mo}}(r|d_{oo}, r_o)$ is presented in Lemma 5.

Hence, $I_1 \approx \hat{I}_1(d_{oo}, r_o) = \sqrt{\gamma_{mo}} + (M-1)$

$$\times\int_{r=d_{oo}}^{r_o+R_s}\frac{\sqrt{\tau_p\rho_p}l(r)^{-1}}{\sqrt{1+\tau_p\rho_p l(r)^{-1}}}f_{\hat{D}_{mo}}(r|d_{oo}, r_o)\mathrm{d}r. \quad (9)$$

Similarly, for a given realization of R_o and D_{oo}, $I_2 \approx \hat{I}_2(d_{oo}, r_o) =$

$$\beta_{oo} + (M-1)\int_{r=d_{oo}}^{r_o+R_s} l(r)^{-1} f_{\hat{D}_{mo}}(r|d_{oo}, r_o)\mathrm{d}r. \quad (10)$$

Using the above result, next, we present an approximate expression to evaluate the achievable average user rate.

Proposition 1. *The average achievable rate of a randomly selected user can be approximately evaluated as*

$$R_o = \mathbb{E}_{\Phi_r,\Phi_u}\left[\log_2\left(1 + \mathtt{SINR}_o\right)\right]$$

$$\approx \mathbb{E}_{D_{oo},r_o}\left[\log_2\left(1+\mathtt{SINR}_o^{\mathrm{Apx}}(d_{oo},r_o)\right)\right],$$

where $\mathtt{SINR}_o^{\mathrm{Apx}}(d_{oo}, r_o)$ *is presented in* (11) *at the top of the next page, the* PDF*s of* D_{oo} *and* R_o *are presented in Lemma 4 and* (7)*, respectively. The corresponding system sum-rate is* KR_o*.*

This completes the technical part of this paper. Next, using the above analyses, we provide a few system design insights.

IV. Results and Discussion

In this section, we study the effect of different system parameters on the average user rate and the average system sum-rate. We verify the accuracy of the approximate theoretical expression for the lower bound on average rate through Monte Carlo simulations. We have considered $R_s = 1000$ m. The pathloss function between any two nodes at a distance r is

$$l(r) = r^{3.7}\mathbf{1}(r>1)+\mathbf{1}(r\le 1).$$

We consider the DL SNR $\rho_d = 100$ dB. The reason behind this high SNR is to ensure that the system is limited by interference due to inter-user interference, channel estimation and compression error. Further, we take $\tau_p = 168$, which corresponds to the number of resource elements in a resource block in LTE. The choice of other system parameters are indicated at necessary places.

1) The effect of fronthaul capacity: In Fig. 2, the average system sum-rate is presented as a function of the number of users K for different fronthaul capacities. We have kept a high pilot transmission SNR $\rho_p = 100$ dB corresponding to an almost perfect CSI scenario to highlight the effect of fronthaul capacity on the system performance. As evident from the figure, the average system sum-rate is strictly quasi-concave function of the number of users. Further, for a given number of RRHs, the optimum number of users that should be multiplexed to maximize the average rate increases with the increasing fronthaul capacity. When C is unlimited, the maximum average rate is obtained by serving all the users simultaneously. Hence, it is intuitive that with increasing C, the optimum number of users that should be served increases.

2) Distributed vs. collocated: In Fig. 3, we present the average user rate for different number of antennas at each RRH while keeping the total number of antennas in the service region fixed, i.e. $MN = 128$. We consider an ideal fronthaul of unlimited capacity to study the effect of CSI error on system performance. We observe that for high ρ_p (i.e. high-quality CSI) as we move towards a more collocated setup, average user rate increases. On the other hand, with low ρ_p (i.e. low-quality CSI), the average user SE is a quasi-concave function of the number of antennas per RRH. We observe that in case of high-quality CSI, both the mean desired power and mean interference power increase monotonically with increasing number of antennas per RRH. However, the rate of growth of the desired power is higher than that of interference power. Therefore, the average user SE increases monotonically. On the other hand, in case of low-quality CSI, although the received interference power increases monotonically with increasing number of antennas, the average desired power shows a concave behaviour. This also gets reflected in the average user SE curves in case of low-quality CSI.

V. Conclusion

In this work, we have analyzed the DL performance of a cell-free mMIMO system under the assumptions of a finite fronthaul capacity, distributed CB-based precoding using local imperfect CSI at each RRH. Using relevant distance distributions for a BPP, we present an approximate analytical expression for an achievable user rate averaged over the realizations of RRHs and user locations. Our results suggest that under the assumption of finite fronthaul capacity, there exists an optimal number of users that maximize the average system sum-rate. Further, in the presence of high-quality CSI, for the same number of antennas in the system, it is preferable to have fewer RRHs with more antennas per RRH. A possible future extension of this work involves the consideration of more sophisticated compression schemes and extension to an infinite network where instead of all the RRHs, a few RRHs are grouped together to serve a user.

Appendix

A. Proof of Lemma 2

From (2), we write r_o as

$$\begin{aligned} r_o = &\underbrace{\sqrt{\rho_d}\sum_{m=1}^{M}\sqrt{\eta_{mo}}\mathbb{E}\left[\|\hat{\mathbf{g}}_{mo}\|^2\right]q_o}_{T_1:\text{ Desired signal}} \\ &+\underbrace{\sqrt{\rho_d}\sum_{m=1}^{M}\sqrt{\eta_{mo}}\left(\|\hat{\mathbf{g}}_{mo}\|^2-\mathbb{E}\left[\|\hat{\mathbf{g}}_{mo}\|^2\right]\right)q_o}_{T_2:\text{ Beamforming uncertainity}} \\ &+\underbrace{\sum_{m=1}^{M}\|\hat{\mathbf{g}}_{mo}\|^2\sqrt{\rho_d\eta_{mo}}\tilde{q}_o}_{T_3:\text{ quantization error}}+\underbrace{\sum_{m=1}^{M}\sqrt{\rho_d\eta_{mo}}\tilde{\mathbf{g}}_{mo}^T\hat{\mathbf{g}}_{mo}^*\hat{q}_o}_{T_4:\text{ estimation error}} \\ &+\underbrace{\sum_{\substack{k=1\\k\neq o}}^{K}\sqrt{\rho_d}\Big(\sum_{m=1}^{M}\sqrt{\eta_{mk}}\mathbf{g}_{mo}^T\hat{\mathbf{g}}_{mk}^*\Big)\hat{q}_k}_{T_5:\text{ inter user interference}}+w_o, \end{aligned} \tag{12}$$

where we consider that the o-th user has the average channel statistics with respect to each RRH in the network. From (12), it can be shown that the desired signal term is uncorrelated to the rest of the terms. An achievable rate (lower bound on the capacity) is obtained by using the fact that mutual information is minimized when the uncorrelated signals to the desired signal is replaced by independent Gaussian noise [20] with variance equal to the sum of variances of undesired signals, i.e. T_2, T_3, T_4, T_5, and w_o. Hence, the SINR corresponding to this lower bound on capacity is given as

$$\mathtt{SINR}_o = \frac{\mathbb{E}\left[|T_1|^2\right]}{\sum_{i=2}^{5}\mathbb{E}\left[|T_i|^2\right]+1}.$$

In this case, note that $\mathbb{E}\left[T_i\right]=0$ for all i. Further,

$$\mathbb{E}\left[|T_1|^2\right] = \rho_d\frac{N^2}{K}\Big(\sum_{m=1}^{M}\sqrt{\gamma_{mo}}\Big)^2\mathbb{E}\left[|q_o|^2\right],$$

$$\mathbb{E}\left[|T_2|^2\right] = \rho_d\frac{N}{K}\sum_{m=1}^{M}\gamma_{mo}\mathbb{E}\left[|q_o|^2\right],$$

$$\texttt{SINR}_o^{\text{Apx}}(d_{oo}, r_o) = \frac{\rho_d \frac{N^2}{K}(\hat{I}_1(d_{oo}, r_o))^2 \mathbb{E}\left[|q_o|^2\right]}{\rho_d \frac{N^2}{K}(\hat{I}_1(d_{oo}, r_o))^2 \mathbb{E}\left[|\tilde{q}_o|^2\right] + \rho_d N \hat{I}_2(d_{oo}, r_o) + 1}, \tag{11}$$

where $\hat{I}_1(d_{oo}, r_o)$ and $\hat{I}_2(d_{oo}, r_o)$ are given in (9) and (10), respectively.

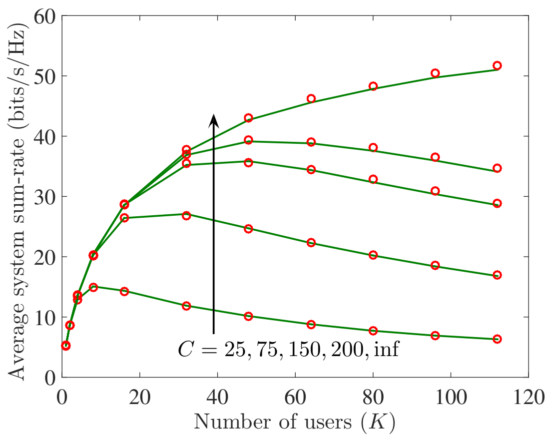

Fig. 2. The effect of fronthaul capacity on system sum-rate. The solid lines are obtained using the analytical expression in Proposition 1, markers are Monte Carlo simulation results obtained using Lemma 2. The fronthaul capacity is in bits/s/Hz. $M = 16, N = 8, \tau_p = 168, \rho_p = 100$ dB.

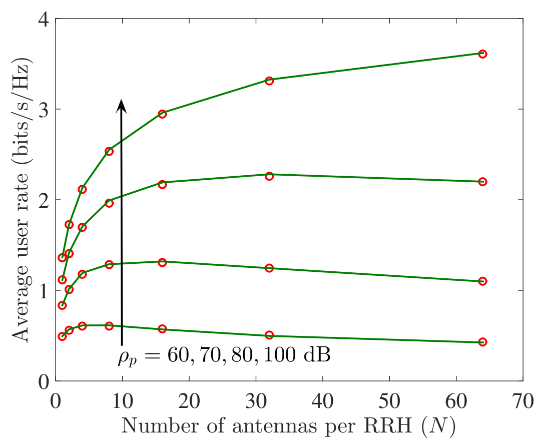

Fig. 3. The effect of number of antennas per RRH on average user rate. Solid lines and markers represent analytical and simulation results, respectively. We have considered $MN = 128, \tau_p = 168, K = 8$.

$$\mathbb{E}\left[|T_3|^2\right] = \left(\rho_d \frac{N}{K}\sum_{m=1}^{M}\gamma_{mo} + \rho_d \frac{N^2}{K}\Big(\sum_{m=1}^{M}\sqrt{\gamma_{mo}}\Big)^2\right)\mathbb{E}\left[|\tilde{q}_o|^2\right],$$

$$\mathbb{E}\left[|T_4|^2\right] = \rho_d \frac{N}{K}\sum_{m=1}^{M}(\beta_{mo} - \gamma_{mo})\mathbb{E}\left[|\hat{q}_o|^2\right],$$

$$\mathbb{E}\left[|T_5|^2\right] = \rho_d \frac{N(K-1)}{K}\sum_{m=1}^{M}\beta_{mo}.$$

Substituting these values, we obtain the expression presented in the lemma. ■

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