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A Bayesian kernel approach to modeling resilience-based network component importance

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ABSTRACT

The resilience of infrastructure networks is an increasingly important consideration in infrastructure planning and risk management. One aspect of resilience-based planning is determining which components in the network are most important to the resilience of the network. This work makes use of a resilience-based component importance measure, the resilience worth, and proposes to model this measure under uncertainty using a Bayesian kernel technique. Such a technique can be useful in modeling component importance as it enables the probability distribution for the importance measure to be updated using data and prior information with a Bayesian kernel model. The proposed approach is applied to study the importance of locks and dams along the Mississippi River Navigation System. The highest predictive overall accuracy is achieved with a uniform prior distribution, and using the posterior distribution and a multicriteria decision analysis technique, we identify the five locks and dams with the largest impact on the system's resilience.

1. Introduction and motivation

Critical infrastructure networks, including roadway, railway, inland waterway, and electric power networks, are vital to the regional and national economy as well as the community that relies on them. These networks are also prone to disruptive events that can disable their operation. Such disruptions can be the result of natural disasters (e.g., in 2012, Hurricane Sandy disabled the physical infrastructure networks in the heavily populated NY/NJ area for several days [51]), human-made attacks (e.g., in 2013, shooters armed with assault rifles did extensive damage to 17 transformers in southern California [24]), and failures (e.g., the American Society of Civil Engineers has assigned US public infrastructure a grade of D + [2]). To reduce the effects of these events, most research efforts have been devoted to developing traditional measures of protection [10,31] that can be expensive and degrade typical performance. However, given the inevitability of these events, recent attention has been placed on the ability "to withstand and rapidly recover from all hazards [52], where the combination of "withstanding" and "recovering" from disruptions constitutes *resilience*. The Department of Homeland Security [17] announced a set of grant programs targeting different areas prone to willful attacks or natural disasters, aiming to provide resources helpful in supporting the National Preparedness Goal (NPG) in ensuring "a secure and *resilient* Nation with the capabilities required across the whole community to prevent, protect against, mitigate, respond to, and recover from the threats and hazards that pose the greatest risk" [16].

Resilience is broadly defined as the ability of a system to absorb the shock of a disruptive event and bounce back from adverse effects. Models and measures of resilience have increasingly been seen in the literature [29]. Historical references to resilience appeared in the ecology literature [27], with other fields more recently adopting the terms, including psychology [9,47], business [26], economic impacts [42,43], and engineering [21,28,38]. A paradigm in the civil infrastructure field is the "resilience triangle," which integrates robustness (initial impact) and rapidity (speed of recovery) for a disruptive event [12,15,48,54]. In the network domain, Najjar and Gaudiot [34] and Rosenkrantz et al. [44] have proposed topological measures for resilience in networks.

This work adopts the paradigm for resilience based on system performance $\varphi(t)$, as shown in Fig. 1 [25]. Three dimensions of resilience are depicted in Fig. 1:(i) reliability, or the ability of the network to meet performance expectations prior to a disruption, (ii) vulnerability, or the ability of disruptive event e^j to impact the system performance in an adverse manner, and (iii) recoverability, or the ability and speed of a network to recover after e^j. A time-dependent measure of resilience, or the ratio of recovery over loss, accompanies this paradigm, provided in Eq. (1) [4,8,35].

$$Я_{\varphi}\left(t_r|e^j\right) = \frac{\varphi\left(t_r|e^j\right) - \varphi\left(t_d|e^j\right)}{\varphi(t_0) - \varphi\left(t_d|e^j\right)} \quad (1)$$

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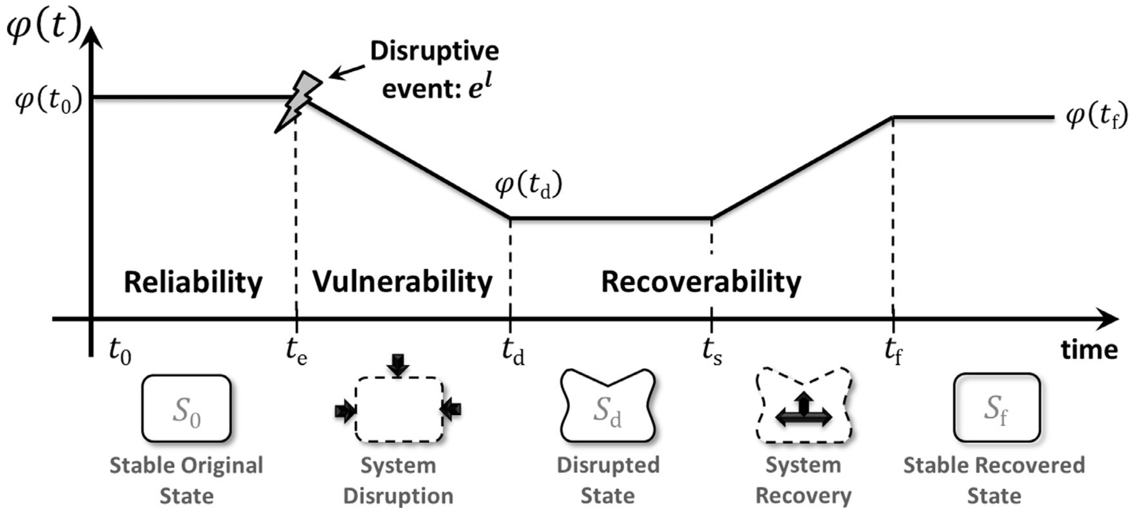

Fig. 1. Graphical depiction of network performance, $\varphi(t)$, across several state transitions before and after disruptive event e^j.

A means to identify areas for investment in infrastructure networks is to focus on those components found to be most critical to the operation of the network. The field of reliability engineering identifies these most critical components through component importance measures, widely used to find the weakest components in a system and candidates for investment [30]. Starting with the Birnbaum [11] importance measure, which measures how the change in reliability of component i influence a change in the reliability of the system as $I_i^B = \partial R_s / \partial R_i$, many reliability-based component importance measures have been proposed. These include the risk reduction worth (RRW), an index that quantifies the potential damage to a system caused by a particular component, and the reliability achievement worth (RAW) of a component, the maximum proportion increase in system reliability generated by that component [37].

Barker et al. [4] proposed two stochastic importance measures for identifying network components that contribute to the resilience of the network. As these measures are stochastic, a distribution for the importance measure is produced for each network component. Naturally, this distribution could be improved over time as more disruption data about a network are collected, including how the disruption impacts the performance of the network (vulnerability, in reference to Fig. 1) and how much time is required to restore the network to a desired performance level (recoverability).

Baroud et al. [7] proposed a simulation technique that deploys stochastic importance measures to identify the impact of individual components on the recovery of a disrupted infrastructure network. In particular, this previous work was concerned with identifying the best strategy to recover disrupted links of the river of an inland waterway based on different resilience-based importance measures of these links as well as other factors such as cost of implementation of the strategy. In this paper we focus on modeling and predicting resilience-based importance measures by exploring the possibility of updating the distribution for the importance measure using data and prior information with a Bayesian kernel method. Locks and dams along an inland waterway can differ in many aspects, such as their physical characteristics (e.g., geographic location, age, capacity), their operations (e.g., number of lockages, vessels, tons of commodity flowing), and their past performances (e.g., number of lock outages, average delay, percentage of vessels delayed). As such, identifying the most important lock/dam based on these multiple criteria by simply examining data or relying solely on operators' expertise is challenging. Further, the ranking of these components can be based on different attributes such as probability of failure or impact on system recovery time and effectiveness. The objective of this work is to rank the importance of locks/dams according to their impact on the overall resilience of the waterway system. The proposed approach combines decision maker expertise with historical data to predict resilience-based importance metrics. Ultimately, when this approach is implemented in practice, decision makers would inform the prior distribution based on their knowledge and preferences.

In contrast to the preliminary results of this work found in Baroud and Barker [6], this paper focuses on the prediction accuracy and the interpretability of Bayesian methods to model resilience-based importance measures. Section 2 provides some background to the stochastic resilience-based importance measures and Bayesian kernel methods. Section 3 develops the Bayesian kernel approach to updating network resilience, and Section 4 illustrates the use of the method in an inland waterway network application whose locks and dams are disrupted. Finally, concluding remarks are found in Section 5.

2. Methodological background

This section offers a review of importance measures and of Bayesian kernel methods, both of which are integrated in this work.

2.1. Resilience-based importance measures

Component importance measures have been commonly used in the reliability literature, examples of such measures include [50]: (i) Birnbaum importance, or $\partial R_S / \partial R_i$ where R_S and R_i are system and component i reliability, respectively, which describes the probability that component i is critical to the functioning of the system, (ii) reliability achievement worth (RAW), or the maximum proportion increase in system reliability generated by a given component, (iii) risk reduction worth (RRW), an index that quantifies the potential damage to a system caused by a particular component, and (iv) Fussell-Vesely, an index quantifying the maximum decrement in system reliability caused by a particular component. Several other discussions of importance measures include those by Ramirez-Marquez and Coit [36], Zio et al. [53], and Rocco and Ramirez-Marquez [39], among others, and they generally calculate these measures as a ratio of the measure of component contribution to system reliability and a measure of system reliability itself.

Resilience-based importance measures calculate the contribution of a component to the resilience of the network as a function of its vulnerability (i.e., initial degradation in network performance) and recoverability (i.e., time required for the recovery of network performance). A number of such measures have been developed by analogy to the reliability-based importance measures [4,18].

In this section, we review one particular measure known as the "resilience worth", $\mathrm{W}Я_{\varphi,i}(t|e^j)$ in Eq. (2), which serves as an index quantifying how the time to full network service restoration is improved when a component is assumed to be invulnerable. The resilience worth measure provides an analogous perspective to the reliability achievement worth (RAW) from the reliability engineering field.

$$W\text{Я}_{\varphi,i}\left(t|e^j\right) = \frac{T_{\varphi\left(\mathbf{x}(t_0)|V_i^j\right)} - T_{\varphi\left(\mathbf{x}(t_0)|V_i^j=0\right)}}{T_{\varphi\left(\mathbf{x}(t_0)|V_i^j\right)}} \tag{2}$$

Eq. (2) defines the resilience worth of component i as a function of the magnitude of and time required to recover from disruptive event, e^j. In particular, the resilience worth is computed during the recovery process at time $t\in(t_s, t_f)$ and the ratio is a function of the time to recovery, $T_{\varphi(\mathbf{x}(t_0)|V_i^j)}$, where $\varphi(\mathbf{x}(t_0)|V_i^j)$ describes the network performance conditioned on the vulnerability of the ith component as a function of the disruptive event or its ability to maintain service after the disruption, $V_i^j(e^j) = V_i^j$ where $V_i^j \in [0,1]$. For example, $\varphi(t)$ would represent commodity flows through an inland waterway network at time t and $x(t_0)$ represents the as-planned performance state prior to the onset of disruptive event e^j. As such, $T_{\varphi(\mathbf{x}(t_0)|V_i^j)}$ measures the total time spent from time t_s when recovery activities are started to time t_f when system service is completely restored, $\text{Я}_{\varphi}(t|e^j) = 1$, and $T_{\varphi(\mathbf{x}(t_0)|V_i^j=0)}$ assumes the specific case where the ith component is invulnerable. V_i^j represents the percentage reduction in the component performance state at the onset of the event. Decreasing performance $\varphi(t)$ occurs until t_d when the new disrupted state is reached, Eq. (3). A complete reduction in the functionality of the link occurs when $V_i^j = 1$, and $V_i^j = 0$ when the event does not impact the functionality of link i.

$$x_i(t_d) = \left(1 - V_i^j\right)x_i(t_0) \tag{3}$$

Parameter V_i^j is a stochastic term, with Eq. (4) describing the likelihood of V_i^j lying in [a,b] $\in$ [0,1].

$$P\left(a < V_i^j \le b\right) = \int_a^b f\left(v_i^j\right)dv_i^j \tag{4}$$

Given the information describing the disruptive event and its impact on the individual components as well as the overall system, recoverability is the time required to recover the functionality of a link. As the initial effect of e^j would impact recovery time, recovery time for the ith component is described as a function of V_i^j, or $U_i^j(V_i^j(e^j)) = U_i^j(V_i^j)$. Similar to the initial impact, recovery can also be stochastic. Eq. (5) provides the probability that component i recovers prior to time $t\in(t_s, t_f)$. It is assumed that $x_i(t) = x_i(t_d)$ until the recovery time is met, suggesting a step function to recovery.

$$P\left(t_s < U_i^j\left(V_i^j\right) \le t\right) = \int_{t_s}^{t_r} f\left(u_i^j\left(V_i^j\right)\right)dv_i^j \tag{5}$$

As shown in Fig. 1, vulnerability, V_i^j, and recoverability, $U_i^j(V_i^j)$, combine to describe network performance $\varphi(t)$ for $t\in(t_s, t_f)$ as a function of component state variables $\varphi(t) = \varphi(\mathbf{x}(t))$. If V_i^j and $U_i^j(V_i^j)$ are both stochastic terms, as presented above, a probability distribution for the time to full network service resilience can be constructed. And since the time to full network service resilience is stochastic, then $W\text{Я}_{\varphi,i}(t|e^j)$ can be modeled using a probability distribution. Prior studies have considered simulation methods that rely on assumptions of the severity of the event, the time to full network service resilience, and the component vulnerability to model the resilience worth. In this paper, we propose to combine probabilistic assumptions with data-driven methods to improve the predictive accuracy and interpretability of modeling the resilience worth importance measure. We propose to use the beta Bayesian kernel method, which is reviewed in the following section.

2.2. Bayesian kernel methods

Bayesian kernel methods integrate (i) the Bayesian property of improving predictive accuracy as data are dynamically obtained, with (ii) the kernel function which adds specificity to the model and can make nonlinear data more manageable. Kernel-based approaches to data classification have revolutionized data mining [46]. Kernel functions map input data that are potentially not easily classified with a linear classifier to a higher dimensional space, where algorithms (e.g., least squares regression, support vector machines) enable classification or regression [14,45]. More recently, kernel functions have been integrated with Bayesian methods [3].

Given that Bayesian methods make use of previous data to estimate posterior probability distributions of the parameter of interest that follows a specific prior distribution, the integration of Bayesian and kernel methods allows for a classification algorithm that provides probabilistic outcomes (i.e., probability of a data point belonging to a particular class) as opposed to deterministic outcomes (i.e., purely the classification of a data point to a particular class). Several extensions of Bayesian kernel models have assumed both Gaussian and non-Gaussian distributions for this classification probability to be estimated. In particular, for the non-Gaussian case, models were developed with a beta conjugate prior to model binary classification by estimating the probability of a data point belonging to one classification [32], while another used a Poisson Bayesian kernel model to estimate the frequency of disruptive events [5,19]. In the beta Bayesian kernel model, the prior and posterior distribution of the parameter of interest, θ_i, is a beta distribution with parameters (α, β) and (α^*, β^*), respectively. The relationship between prior and posterior parameters in Eq. (6) is used to classify the observations of an unknown data point i represented by the vector $\mathbf{x}_i$. The probability that data point i is positively labeled follows the beta distribution where y_i represents the unknown classification of data point i and $\mathbf{y}$ is a vector of m known classifications (the training set).

$$\begin{aligned}\alpha^* &= \alpha + \frac{m_-}{m}\sum_{\{j|y_j=1\}} k\left(\mathbf{x}_i, \mathbf{x}_j\right)\\ \beta^* &= \beta + \frac{m_+}{m}\sum_{\{j|y_j=-1\}} k\left(\mathbf{x}_i, \mathbf{x}_j\right)\end{aligned} \tag{6}$$

The kernel function is $k(\mathbf{x}_i, \mathbf{x}_j)$, m_+ is the number of positive labels, and m_- is the number of negative labels in the training set of size m. The ratios representing the proportions of each class ensure an unbiased estimation of the posterior parameters in the presence of imbalanced data sets [32]. The model in Eq. (6) is considered to be a weighted Bayesian kernel model where $\frac{m_-}{m}$ and $\frac{m_+}{m}$ are weighting parameters. The kernel function $k(\mathbf{x}_i, \mathbf{x}_j)$ is determined by the model user. In this paper, a radial basis kernel function is used, Eq. (7).

$$k\left(\mathbf{x}_i, \mathbf{x}_j\right) = \exp\left(-\frac{\mathbf{x}_i - \mathbf{x}_j{}^2}{2\sigma^2}\right) \tag{7}$$

With such a model, the probability distribution of the parameter of interest could be derived. And a point estimate of that parameter could be the expected value of the posterior probability distribution or any other conditional expected value representing a more extreme case.

3. Bayesian kernel approach to modeling resilience importance

Previous work computes the resilience worth by assuming that the time to full network resilience is stochastic and follows a particular probability distribution. $W\text{Я}_{\varphi,i}(t|e^j)$ is then computed by means of simulation. This paper incorporates data-driven tools to the modeling of resilience worth, providing a similar approach to the non-Gaussian Bayesian kernel models discussed above and applies it to model the resilience worth of the components of a network. The outcome of the model, $W\text{Я}_{\varphi,i}(\mathrm{t}|e^j)$, is a value between 0 and 1, where 0 represents a non-impactful component and 1 represents a highly impactful component. Therefore, a suitable conjugate prior in this case is the beta distribution for which the range of the random variable is [0, 1]. Eq. (8) is a representation of the beta prior probability distribution with parameters $\alpha > 0$ and $\beta > 0$, where $W\text{Я}$ is the resilience worth described in Eq. (2) and

B(α, β) is the beta function. Bayesian kernel methods provide a more accurate estimate of the resilience worth as the posterior probability distribution relies on prior information pertaining to the component's characteristics and historical data of disruptions.

$$P(\mathrm{W}Я) = \frac{\mathrm{W}Я^{\alpha-1}(1-\mathrm{W}Я)^{\beta-1}}{\mathrm{B}(\alpha,\beta)} \tag{8}$$

Prior parameters are often assumed to be given or otherwise chosen to be noninformative. In this case, such parameters can be determined from expert elicitation to incorporate an opinion and prior knowledge on the importance of network components. Data describing past disruptive events and component characteristics are then embedded into the kernel matrix and integrated in the computation of the posterior parameters.

There are two ways to analyze the outcome of the resilience worth from the beta Bayesian kernel model. One possibility is to analyze components using a point estimate (e.g., the expected value of the posterior distribution) and examine the resilience worth of a component based on this estimate. The point estimate for the resilience worth importance measure is found in Eq. (9) where the larger the estimate, the more impactful the component.

$$\overline{\mathrm{W}Я}^{*} = \frac{\alpha + \frac{m_-}{m}\sum_{\{j|y_j=1\}} k\left(\mathbf{x}_i,\mathbf{x}_j\right)}{\alpha + \frac{m_-}{m}\sum_{\{j|y_j=1\}} k\left(\mathbf{x}_i,\mathbf{x}_j\right) + \beta + \frac{m_+}{m}\sum_{\{j|y_j=-1\}} k\left(\mathbf{x}_i,\mathbf{x}_j\right)} \tag{9}$$

Another possibility is to analyze components using the entire probability distribution in Eq. (8) instead of only the point estimate. Doing so takes advantage of the entire distribution of resilience worth (e.g., not only central tendency but also the tails of the distribution). We later discuss an approach for comparing the distributions of different locks and dams with a decision analysis technique applied to stochastic ranking.

As the objective of this paper is to predict importance measures and assess the model interpretability, the main outcome used in this analysis is the importance of each component. If the decision maker is interested in identifying the most impactful attributes contributing to these measures, further analysis within the kernel matrix would be required. Each entry in the kernel represents a similarity measure between the test data point and each data point in the training set. The similarity measure corresponds to the summation of kernel function values across all the attributes. To learn more about the role of the attributes in the calculation of importance measures, the decision maker could look into the individual kernel function value for each attribute instead of the overall summation across attributes to determine the covariate (or set of covariates) that most significantly impacts the classification of the component.

4. Illustrative example: inland waterway network resilience

The framework discussed above is applied to analyze the resilience worth of locks on the Mississippi River Navigation System, which is modeled as a network of nodes representing ports and locks/dams, and links representing sections of the river. The system performance upon which the importance measures are calculated is assumed to be the total amount of commodity being shipped through the entire navigation system in a given year. If a disruption results in a river section or lock/dam closure, its impact would be measured in terms of a decrease in total commodity flow regardless of the directionality. The importance of a waterway component is then calculated based on the time required for the system to recover from such a disruption as a function of the vulnerability of that component. Prior studies have considered resilience-based importance measures to analyze the impact of links (i.e., sections of the river) on the recoverability of the waterway after a disruption [6]. The risk and impact of lock outages play a different role than closures of river sections. As a result, the two analyses can result in different investment strategies and corresponding disruption losses. This article, however, is not concerned with identifying risk management and recovery strategies, but rather the focus is given to effectiveness of using data-driven Bayesian techniques to identify important locks based on (i) the predictive accuracy of the Bayesian approach, and (ii) the interpretability of resilience-based importance measures. As such, we assume that decision makers have some prior knowledge on the importance measures of each lock/dam instead of calculating or simulating their values based on system performance, and we update this knowledge with data using the Bayesian kernel model.

The Mississippi River Navigation System has 29 locks acting as key connectors between different ports nationwide. The data, retrieved from the database collected by the US Army Corps of Engineers [49], contain detailed information on each lock's characteristics including the river mile, the total number of vessels passing by the lock, the total tonnage, the frequency and average delay for the vessels and tows experiencing delay time due to the lock's closure, and the yearly frequency of closure for each lock. A sample of the data is presented in Table 1. No prior data are available for the resilience worth, but we assume that such data can be elicited from risk managers or government officials. Given the characteristics of each lock and dam, an individual can be asked to classify each lock and dam as either impactful or non-impactful.

We focus our analysis of the waterway system on the two main components of the model: (i) the prior distribution and its impact on model results, and (ii) the interpretation of the posterior distribution of the resilience worth to infer ranking of the locks and dams.

4.1. Prior distribution impact analysis

In this section, we explore the predictive ability, the sensitivity, and the interpretability of the prior distribution.

4.1.1. Predictive accuracy

The ultimate goal of quantifying and analyzing the resilience of infrastructure systems is to develop risk and recovery management strategies, and an ordered ranking of important system components can assist these strategies. However, some critical characteristics of these statistical methods for quantifying and predicting resilience metrics is the accuracy of such models, their interpretability, and their flexibility. In the following analysis, we address the ability of Bayesian kernel methods to address these characteristics. This class of models offers a great deal of benefits by integrating the Bayesian property with kernel functions. The Bayesian property is used to account for uncertainty, to improve predictive accuracy as new information becomes available, and to incorporate decision maker expertise and knowledge. The kernel function takes into account the influence of multiple factors on the resilience of an infrastructure system through the integration of covariates into the Bayesian model.

We first consider the predictive accuracy of the beta Bayesian kernel model by assessing the ability of the model to classify correctly each lock as either impactful or not impactful. Table 2 provides a summary of predictive accuracy metrics which are (i) the ability to correctly classify a lock as impactful (true positive rate, *TP*), (ii) the ability to correctly classify a lock as not impactful (true negative rate, *TN*), and (iii) the accuracy score, $ACC = \sqrt{TP \times TN}$. These accuracy metrics were computed for the Bayesian kernel model under multiple assumptions for the prior distribution.

The predictive accuracy metrics are used to assess the ability of the model to correctly classify new data points given their attribute information. Since the outcome of the model is a probability distribution of the parameter of interest, we will use a point estimate that is the average of the posterior distribution, shown in Eq. (10). If the estimate of resilience worth is greater than or equal to a threshold (e.g., 0.5), the lock is classified as impactful, indicating a resilience worth of 1. If the estimate is below 0.5, the lock would be classified as non-impactful with $\mathrm{W}Я = 0$.

$$\bar{\theta}_i = \frac{\alpha^*}{\alpha^* + \beta^*} \tag{10}$$

Table 1
Example data describing lock and dam characteristics.

Lock & Dam	Closure frequency	River Mile	Vessels	Tonnage	Lockages	...
L&D 3	0	797	9397	6747	4406	...
L&D 13	6	523	2810	14,545	3155	...
L&D 2	0	815	4478	6735	2893	...
L&D 20	23	343	2508	20,828	3582	...
L&D 22	40	301	2280	22,476	3486	...
L&D 8	6	679	4333	10,277	2620	...
...	...	...	...	...	...	...

Table 2
Predictive accuracy of the beta Bayesian kernel model (BK).

	BK			GLM			Bayesian GLM		
	TP	TN	ACC	TP	TN	ACC	TP	TN	ACC
Uniform	0.80	**0.96**	**0.87**	0.76	0.74	0.72	**0.88**	0.88	**0.87**
Jeffrey's Prior	0.80	**0.96**	**0.87**						
Empirical ($\alpha = 1$)	0.85	0.75	0.76						
Empirical ($\beta = 1$)	0.81	0.75	0.72						

The different priors considered for the Bayesian kernel model are the following, (i) a uniform distribution where both prior parameters are equal to 1, (ii) Jeffrey's prior [13] where both prior parameters are equal to 0.5, and (iii) two variations of an empirical prior. Empirical ($\alpha = 1$) assumes that $\alpha = 1$ and calculates β using the method of moments by assuming that the mean of the prior distribution is equal to the proportion of positively classified data points in the training set, and Empirical ($\beta = 1$) assumes that $\beta = 1$ and calculates α using the same method of moments. The objective of considering these priors is to assess the impact of eliciting a prior distribution on the predictive accuracy of the model. This is often a crucial and impactful factor in Bayesian modeling techniques, however, it is also challenging and expensive to perform these elicitations, and as such, it would be helpful to form an idea on the impact of prior information on future prediction accuracy.

A cross-validation technique is used to assess the predictive accuracy of the models: 50% of the data is used to train the model, an additional 20% is used to tune the parameter in the kernel function, and the model is tested on the remaining 30% of the data. Table 2 provides a summary of the TP, TN, and ACC metrics for the beta Bayesian kernel (BK) technique, as well as two other classical statistical techniques, the Generalized Linear Models (GLM) [33] and the Bayesian GLM [22]. More specifically, for GLM, we fit the data with a logistic regression model which assumes that the outcome variable follows a binomial distribution and computes the logit of the probability of success which is our parameter of interest, θ_i, as a linear function of the attributes, shown in Eq. (11).

$$\operatorname{logit}(\theta_i) = \beta_0 + \beta_1 x_1 + \dots \beta_d x_d \tag{11}$$

For Bayesian GLM, we consider a Bayesian version of logistic regression that models the coefficients as random variable and updates their prior distribution accordingly, the default case for which has the prior distribution for all the covariates following a Cauchy distribution with center 0 and scale 2.5 [23]. As a result, the model prediction is in the form of the probability of a lock being impactful as opposed to a deterministic outcome such as the case in the logistic regression.

The Uniform and Jeffrey's prior provide the same results for the BK as the form of these distributions is quite similar and in this case the slight difference in the prior parameters did not impact the outcome of the model. The best overall accuracy of the model is provided by the BK method with either a Uniform or a Jeffrey's prior at an accuracy rate of 87%, the same as the Bayesian GLM. The Bayesian GLM also resulted in the best overall true positive rate at 88%, though the model did not effectively capture the points in the negative class. The BK resulted in higher rates of true negatives, with the best rate being 96% for the BK under both the Uniform and Jeffrey's priors. The BK models with the empirical priors performed similarly to the logistic regression, with the exception of a better true positive rate for the BK models. We analyze the prior distributions further in the next section.

The selection of a model to predict future resilience worth for the locks and dams of the Mississippi River Navigation System rely heavily on the risk attitude of the decision maker. For example, if the decision maker is risk averse, they may not able to tolerate a low rate of true positives, as an underestimation of the importance of the lock and dam might lead to an underinvestment in preparedness or recovery strategies that might result in potentially large losses that a risk averse decision maker would not tolerate. In such a case, a Bayesian logistic regression is the model selected. If the decision maker is risk neutral, they would be interested in a predictive model with a good overall accuracy without any particular preference toward the true positive or true negative rate, as such, they would be indifferent between the BK and the Bayesian logistic regression. Finally, if the decision maker is risk seeking with a strong preference to minimize the budget allocated for rehabilitation or recovery, making them willing to tolerate an underestimation of the resilience worth, then they would want to use a model that accurately predicts the locks and dams that are not so impactful in order to avoid unnecessary costs. In that case, the decision maker would select a BK model that predicts low resilience worth components with an accuracy of 96%.

4.1.2. Sensitivity analysis on the prior distributions

The predictive accuracy analysis has so far considered point estimates. In this section, we look at the impact of priors on the shape of the posterior distribution. Using the beta Bayesian kernel model and a uniform beta distribution for the prior, we compute the posterior distribution parameters α^* and β^* across all the locks in the data for 200 iterations of randomly selected training and testing sets. At each iteration, we obtain the posterior probability distribution of the resilience worth for each lock and dam and use its expected value as a point estimate. As a result, we have 200 realizations of the resilience worth estimate for each lock and dam, and we plot the distribution of these estimates in a histogram in Fig. 2. Note that the distribution is dispersed around a range of values going from approximately 0.25 to 0.4. Variability is mainly due to the data set being small. Also, the median of the distribution reflects the actual number of positive classification originally in the data. With such information, risk managers can identify the degree to which the lock and dam is impactful with the probabilistic outcome rather than a simple classification of 0 or 1. This helps in a more accurate allocation of recovery resources.

We can examine the impact of different priors on the posterior probability distribution of the resilience worth to identify whether or not

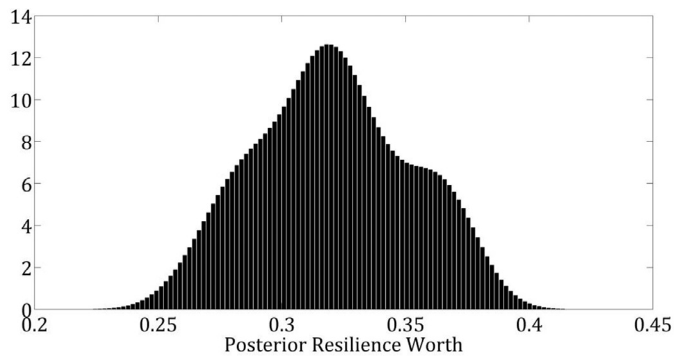

Fig. 2. Probability distribution of the posterior expected value of the resilience worth.

the model is robust to potential noise that can result from the elicitation process. Mentioned previously, it would be ideal to integrate the opinion and knowledge of experts in the field with historical data to obtain the best predictive results. However, eliciting such qualitative information in the form of probability distributions is often challenging. To account for uncertainty, we look at the impact of the structure of the prior on the posterior distribution. Fig. 3 depicts the histogram of the posterior probability distribution for the resilience for a number of priors including (a) uniform, (b) Jeffrey's prior, (c) and (d) the empirical priors discussed previously, and (e) and (f) a couple of skewed priors. The uniform and Jeffery's prior result in less variability in the posterior distribution, whereas the empirical prior results in more breadth and thicker tails which means there is more variability in the prediction when we use empirical priors relative to uniformly structured priors.

The skewed priors are considered to be extreme cases where the elicitation results in misleading information that contradicts the historical

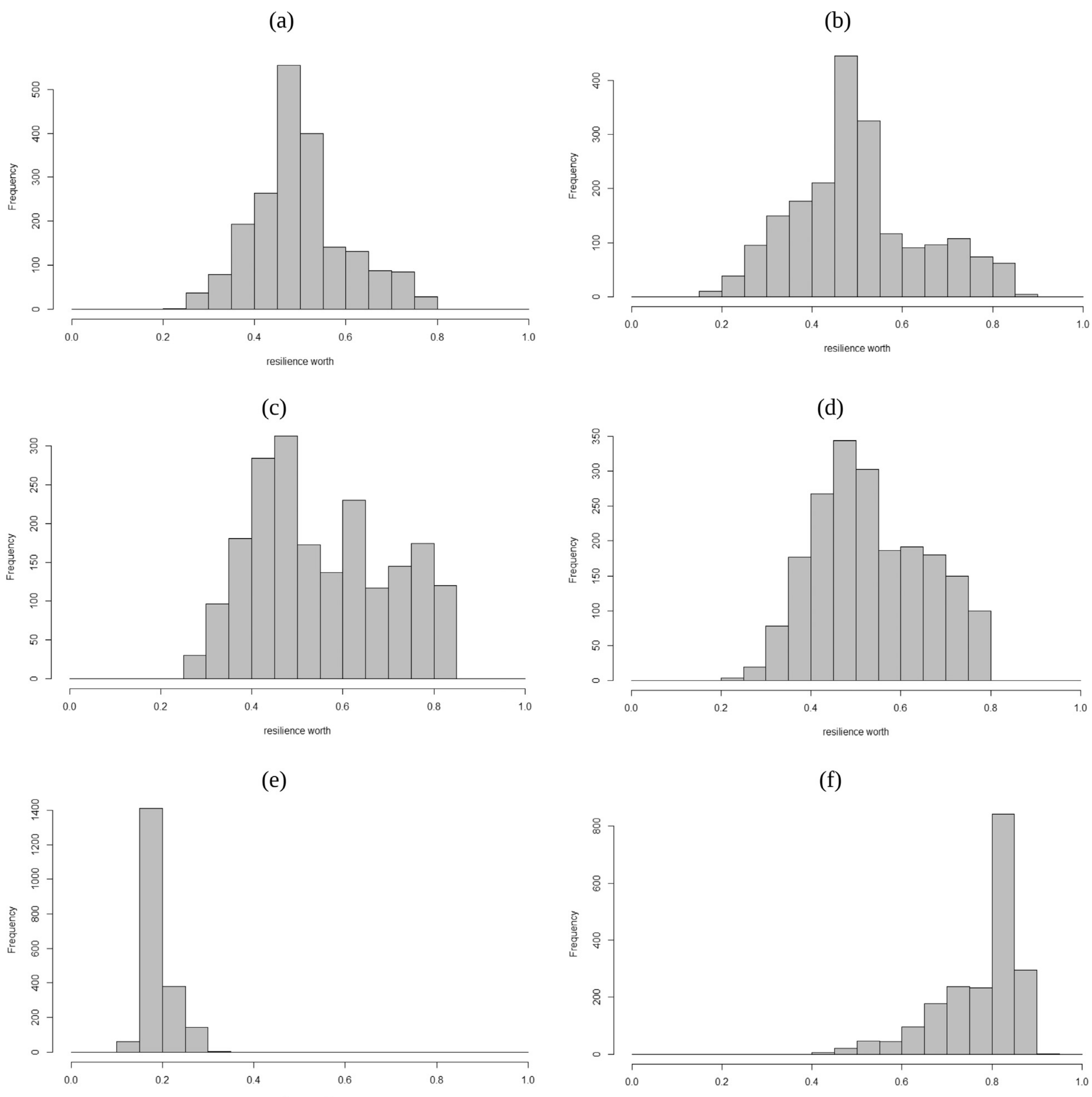

Fig. 3. Resilience worth posterior frequency distribution using the BK model with (a) a uniform prior, (b) Jeffrey's prior, (c) an empirical prior where $\alpha = 1$, (d) an empirical prior where $\beta = 1$, (e) a prior skewed to the right, and (f) a prior skewed to the left.

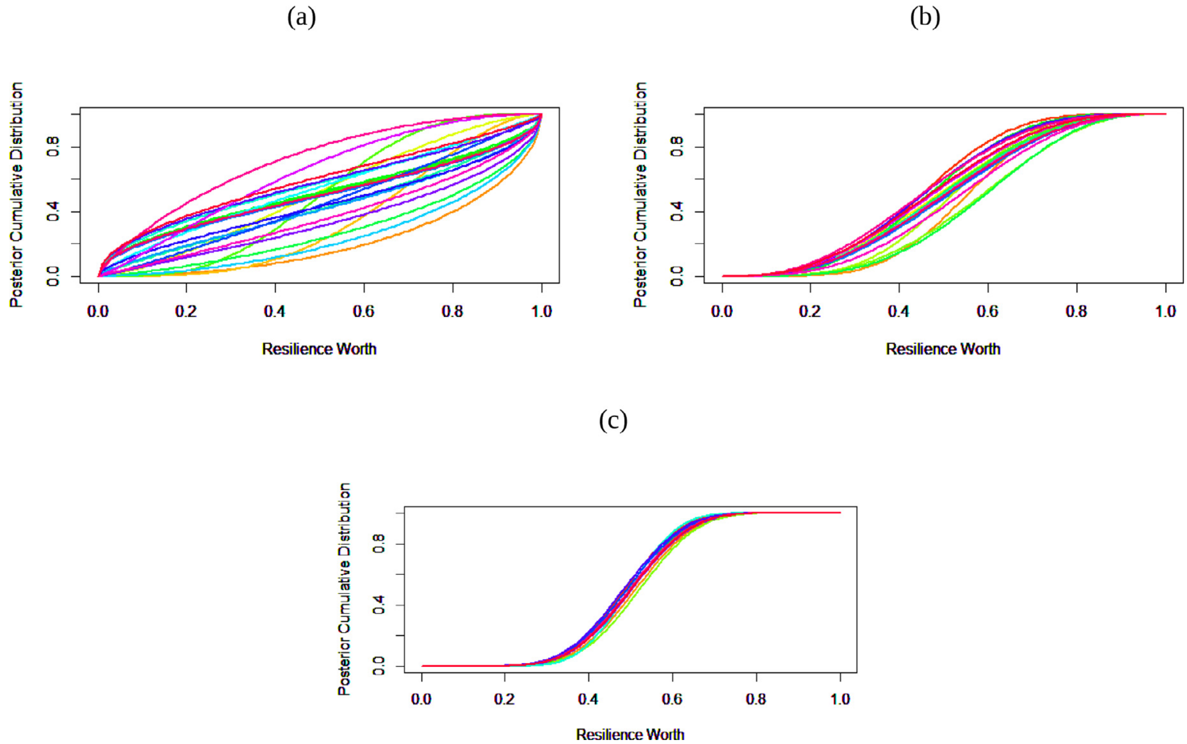

Fig. 4. Resilience worth posterior cumulative distribution with varying predictive accuracies, (a) ACC = 0.65, (b) ACC = 0.68, and (c) ACC = 0.80.

data. The data are skewed to the right with 30% of the locks and dams considered to be impactful on the resilience of the inland waterway, which is why because of the weights in the computation of the posterior parameters, the BK model is able to account for this imbalance. However, a prior that is strongly defined to be extremely skewed to either the left or the right will result in a similar shape for the posterior regardless of the shape of the historical data, as shown in plots (e) and (f) in Fig. 3.

4.1.3. Accuracy and interpretability using different priors

Before implementing the model into a decision making framework, we must also check the trade-off between its accuracy and its interpretability. In this section, we examine the accuracy of the model using the entire posterior distribution of WЯ.

We plot the cumulative posterior distribution of all 29 locks and dams considered in this study with three different prior distributions that consider different levels of variability in the prior and, as such, result in different predictive accuracy levels. The three prior distribution considered to produce the posteriors in Fig. 4 are (a) $\alpha = \beta = 0.5$, (b) $\alpha = \beta = 3$, and (c) $\alpha = \beta = 10$.

Examining the plots in Fig. 4, we notice that it is easier to distinguish the different resilience worth probability distributions for the different locks and dams in (a) than it is in (b) and (c), where in (c), all distributions seem to overlap. The reason behind this effect is the amount of variance assumed in the prior distribution that gets transferred to the estimation of the posterior distribution. The less variability there is in the posterior, the higher the accuracy is: plot (a) has an accuracy of 65%, while accuracy improves to 80% in plot (c). However, the interpretability diminishes considerably, making it more difficult for decision makers to visualize the importance of different locks and dams on the inland waterway system. If a high predictive accuracy is desired, a stochastic ranking approach would enable the ranking of probability distributions that may not otherwise be distinguishable. We present a multicriteria decision analysis technique in the following section to address this challenge.

4.2. Analysis of the Bayesian kernel posteriors using stochastic ranking techniques

The average resilience worth for every lock is listed in Table 3 and ranked by order of importance from the largest to the smallest. A number of the locks and dams have multiple chambers (mostly a main and an auxiliary lock chamber). As such, we calculate the resilience worth for each chamber in any lock. More than half of the locks and dams are considered to be critical components that will contribute to the recovery of the waterway navigation system in the event of a large scale disruption.

Included in this table are a subset of the characteristics of each lock and dam, noting that the strength of the BK model is its ability to account for the non-linear relationship between the attributes and the response variables in a semi-parametric way without establishing a direct link between the variables. Results in Table 3 suggest that the five locks and dams with the highest resilience worth are 18, 24, 17, 25, and 12. We then look at these five most impactful locks/dams judging by the expected value, and we notice from their cumulative probability distributions in Fig. 6 that it is difficult to distinguish their actual ranking of importance, which can be the case for locks and dams with similar characteristics or geographical locations. Such cases arise when disruptions occur in a particular region and result in the closure of a number of similar locks and dams. Since we are able to construct the posterior probability distribution, we propose to use a multicriteria decision analysis technique to perform stochastic ranking of the posterior distribution of the resilience worth of these locks and dams. Consider the comparison of alternatives a and b, which are compared with measure X, a random variable. Fig. 5 illustrates how we can compare the percentiles of the cumulative distribution function (CDF) of X_a and X_b using a multicriteria decision analysis technique to perform stochastic ranking of these two alternatives [40]. In this application, we consider the Copeland score (CS) method which is, in general, a multicriteria ranking technique [1], and we use it here as a stochastic ranking tool.

Table 3
Ranking of all the locks based on the resilience worth point estimate.

Rank	Lock & Dam	*W*Я	Closures	River Mile	Average delayed tows
1	L&D 18	0.658	35	273	1.84
2	L&D 24	0.571	4	738	1.85
3	L&D 17	0.555	105	186	1.95
4	L&D 25	0.537	23	343	2.59
5	L&D 12	0.535	40	301	1.23
6	L&D 15-1	0.528	4	201	1.9
7	L&D 14-1	0.513	6	679	1.74
8	L&D 13	0.513	20	186	1.15
9	L&D 21	0.511	16	325	1.95
10	L&D 22	0.507	15	583	2.43
11	L&D 16	0.507	0	615	1.33
12	L&D 10	0.502	2	714	0.96
13	L&D 20	0.5	47	411	2.27
14	L&D 3	0.5	0	797	0.71
15	L&D 27-1	0.5	6	523	2.06
16	L&D 27-4	0.5	0	815	1.71
17	L&D 19	0.5	5	753	3.42
18	L&D 5A	0.5	84	241	0.66
19	L&D 15-4	0.5	21	437	1.46
20	L&D 9	0.499	36	364	0.99
21	L&D 7	0.498	1	702	1.01
22	L&D 11	0.495	1	493	0.91
23	L&D 6	0.479	29	493	0.99
24	L&D 14-4	0.47	0	854	0
25	L&D 4	0.467	33	457	0.73
26	L&D 5	0.454	1	729	0.77
27	St Anthony Falls—Upper	0.438	4	483	0.02
28	L&D 1	0.433	3	647	0.25
29	Mel Price L&D 4	0.431	107	556	0.13
30	L&D 8	0.402	0	848	1.18
31	L&D 2	0.379	93	483	0.86
32	St Anthony Falls—Lower	0.312	0	853	0.2

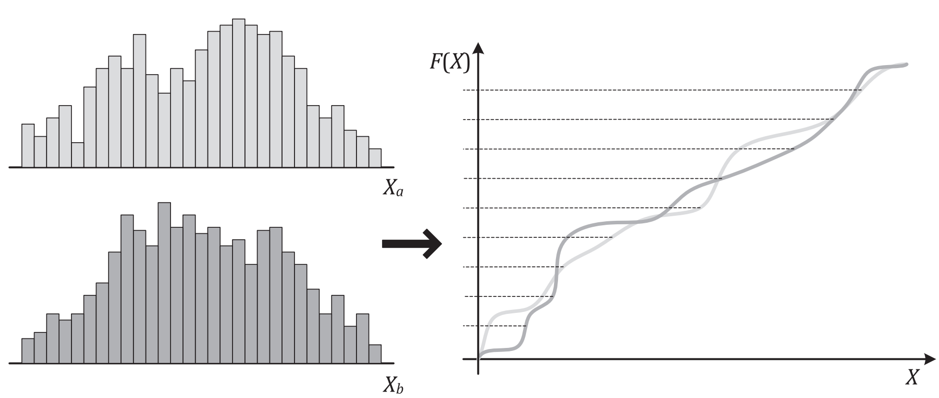

Fig. 5. Comparison of distributions of performance for alternatives a and b as a multicriteria decision problem [40].

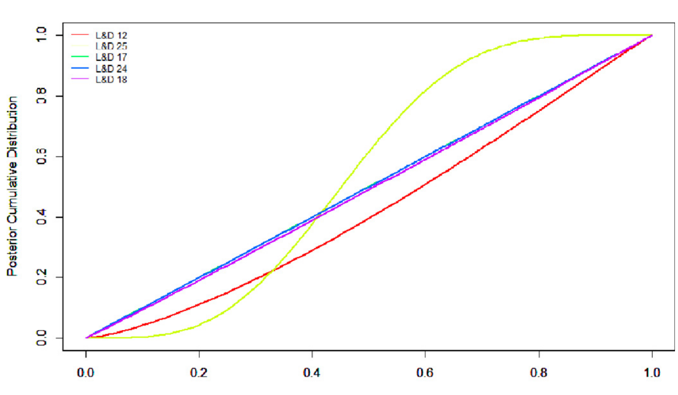

Fig. 6. Cumulative posterior probability distributions of the five most impactful locks and dams of the navigation system.

The Copeland score method is considered to be a nonparametric technique in that each criterion is given an equal weight. Other multicriteria decision analysis techniques that can accommodate importance weights applied to different criteria include the Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE), Ordered Weighted Averaging, and the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), among others. Rocco et al. [41,41] and Floyd et al. [20] discuss PROMETHEE, Ordered Weighted Averaging, and TOPSIS, respectively, for stochastic ranking where (i) the alternatives to be ranked exhibit uncertainty, (ii) this uncertainty is manifested in probability distributions (histograms), and (iii) rather than comparing the central tendency of these distributions, the entirety of the distribution is considered across multiple percentiles that are treated as criteria in the multicriteria technique.

The CS is computed based on pairwise comparisons between alternatives in a set and is defined as the difference between the number of times an alternative a is better (with respect to criterion q_k) than the

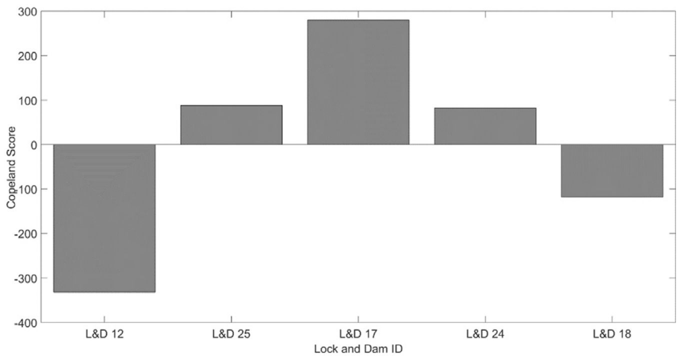

Fig. 7. Copeland score of the five most impactful locks and dams of the navigation system.

Table 4
Lock and dam repair order.

$W\text{Я}$ ranking	Posterior expected value	Posterior Copeland score
1	L&D 18	L&D 17
2	L&D 24	L&D 25
3	L&D 17	L&D 24
4	L&D 25	L&D 18
5	L&D 12	L&D 12

other alternatives and the number of times that alternative a is worse (with respect to the same criterion q_k) to the other alternatives. $C_k(a,b)$ provides a value based on a comparison between alternative a and alternative b for attribute q_k, $k = 1, \ldots, \Omega$, performed according to the rule in Eq. (12). As applied here, the CS method will be used to compare different components according to the CDF of their WЯ importance measure, where the criteria represent different percentiles of the distribution. Since we would like to identify the most critical components (largest WЯ), a maximum $C_k(a,b)$ is desired. $C_0(a, b)$ is initialized at zero, and Eq. (13) iterates through all Ω criteria (percentiles).

$$C_k(a,b) = \begin{cases} C_{k-1}(a,b) + 1 & q_k(a) > q_k(b) \\ C_{k-1}(a,b) - 1 & q_k(a) < q_k(b) \\ C_{k-1}(a,b) & q_k(a) = q_k(b) \end{cases} \tag{12}$$

Eq. (13) shows that the CS of alternative a is obtained by adding $C_i(a, b)$ over all b, each representing the other alternatives [1]. The component with the largest CS value is assumed to stochastically dominate all other components with respect to the set of criteria [4].

$$\mathrm{CS}(a) = \sum_{b \neq a} C_{\Omega}(a,b) \tag{13}$$

Using this stochastic ranking technique, the locks and dams can be ranked according to their Copeland score with approximated percentiles of the CDF for resilience worth as criteria (the top five of which appear in Fig. 7). Table 4 shows the ranking of the locks and dams based on (i) the posterior expected value and (ii) the posterior Copeland score. Note that each method results in a different ranking, the reason for which is that the Copeland score represents the entire distribution (lower and upper tails) while the expected value is only a point estimate of the average resilience worth.

In case of a disruptive event impacting several components in the system, determining the component resilience worth helps decision makers in identifying the best strategy to recover the disrupted critical infrastructure by ordering the component repairs according to their resilience worth.

Note that the Copeland score method is a nonparametric decision analysis technique that weights all percentiles equally. A different decision analysis technique (e.g., PROMETHEE, TOPSIS) could allow for different weights on the percentiles (e.g., upper 10% to more effectively account for risk).

5. Concluding remarks

This paper applies a beta Bayesian kernel model to analyze the resilience of critical infrastructure networks by estimating the resilience worth of each component in the network using prior information as well as historical data on the component's characteristics. The methodology is applied to an inland waterway transportation network, the Mississippi River Navigation System, and the resilience worth of locks and dams is estimated to rank components depending on how impactful they are to the rest of the network. Resilience worth is a resilience-based component importance measure derived from the concept of reliability achievement worth in the reliability engineering field.

The performance of the model is first analyzed, whereby the predictive accuracy of the Bayesian kernel model was compared to traditional statistical methods (GLM and Bayesian GLM) under different assumptions of the prior distribution. The metrics considered to evaluate the predictive accuracy were the rate of true positives, the rate of true negatives, and the overall accuracy. The best overall models were the Bayesian kernel model with either a Uniform or a Jeffrey's prior and the Bayesian logistic regression. Depending on the decision maker's risk preference, a model can be selected based on either the highest rate of true positives (for a risk averse decision maker who cannot tolerate an underestimation of the resilience worth) or the highest rate of true negatives (for a risk taking decision maker whose main objective is to minimize the amount of resources spent and wants to accurately identify the locks and dams that are not impactful to the resilience of the system to avoid unnecessary investment costs). The posterior distribution is sensitive to extremely skewed prior distributions that dictate the structure and form of the posterior regardless of the historical data, though the model is robust when the parameters have different values under the same distribution form, such as the case with the Uniform, Jeffrey's priors, and empirical priors.

Also, the accuracy of the model might compensate for its interpretability particularly when considering the posterior distribution of the resilience worth for all the components of the inland waterway system. Visualizing the different ranking of importance of the locks and dams becomes more challenging as we aim toward a higher level of accuracy. However, this should not pose a problem, as stochastic ranking techniques, such as the Copeland score method, can be used to distinguish and rank overlapping probability distributions. Results show that while the expected value can be used as an estimator, a more comprehensive metric is the Copeland score which considers the entire posterior distribution and accounts for more uncertainty and all possible disruption scenarios.

Such an analysis can assist risk managers and decision makers in allocating resources and determining the ranking order of the repair activities in case of an event resulting in multiple disrupted components. A main assumption in this study is that the directionality of commodity flow does not impact these importance measures. The direction in which commodity is flowing can result in a wide range of impacts on the disrupted component as well as on the overall system recovery. For instance, if a lock was closed for hours or days, the impact on the rest of the network as well as its recovery will depend on the traffic level in each direction, potentially resulting in a different importance measure for the component.

While the resilience worth is a key factor in determining the recovery strategy, it is equally important to account for the overall cost and time of recovery of the strategy. Future research is involved in determining the optimal recovery strategy by taking into account the tradeoff of the Bayesian kernel estimates of the component importance, the time, and the cost of recovery.

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