Full-Duplex Relays under Multilevel Coding: Correlation Design via Modulation Labeling

Ahmed Attia Abotabl and Aria Nosratinia
Department of Electrical Engineering, University of Texas at Dallas, Richardson, TX 75083-0688 USA,
E-mail: ahmed.abotabl@utdallas.edu;aria@utdallas.edu.

***Abstract*—Unlike the half-duplex relay, the performance of the full-duplex relay is highly sensitive to the correlation between the source and relay codebooks. Linear coding complicates the design of correlated codebooks, for example in multilevel coding (MLC) linear codes at each layer can only have correlation zero or one, leading to a performance penalty that has been characterized in earlier work. In this paper, we propose a new design technique that significantly reduces the correlation penalty of linear codes via intelligent labeling for the modulation. The basic idea is as follows: the chain rule for mutual information, which is the backbone of MLC, is not-unique in two ways: the labeling of modulation constellation as well as the ordering of the chain rule. Our optimization at each level pushes the mutual information terms involving new information (for the relay) or beamforming information to zero or one. In effect this finds a suitable decomposition of overall correlation to a set of binary correlations at individual levels of MLC. Simulations show that point-to-point LDPC codes in combination with the proposed correlation design lead to excellent performance.**

I. Introduction

Full-duplex transmissions have recently seen a lot of hardware and signal processing advances that put the full-duplex transmission back on the map [1], [2]. Full-duplex relay channel is one important example where the full-duplex transmission have higher rate than the half-duplex. Earlier results in coding for the relay channel focused on the half-duplex relay channel or binary signaling for the full-duplex relay [3], [4]. The main focus of this work is coding for the bandwidth limited decode-and-forward full-duplex relay channel.

Several contributions for the bandwidth limited relay channel focused on the two way relay channel. Ravindran et. al [5] studied LDPC codes with higher order modulations for the two way relay channel. Chen and Liu [6] analyzed different coded modulation transmissions for the two way relay channel. Chen et. al [7] studied multilevel coding in the two-way relay channel. Multilevel coding was also studied in the context of compute-and-forward [8]. Superposition multilevel coding which is a key technique in this paper was studied in the context of the broadcast channel in [9], [10]. However, unlike the two-way relay channel, the conventional relay channel considers a direct link. Recent contributions in the bandwidth limited relay channel used lattice codes [11] and multilevel coding [12], [13] which is the main technique of this paper.

This work was supported in part by the NSF grants 1546969 and 1711689.

978-1-5386-3531-5/17/$31.00

The key advantage of multilevel coding [14], [15] is that it uses binary codes whose design is by now very well understood. Moreover, the multiple binary encoders that feed the bit-levels of the modulation can operate independently, but for optimal performance some coupling between the bit-level decoders is necessary, e.g., successive decoding.

In decode-and-forward full-duplex relay channel, the source has two tasks at each transmission block, assist the relay in transmitting its message to the destination sending new information to the relay to be sent to the following block. This simultaneous transmission is realized via superposition coding. Superposition coding of coded modulation was proposed by the authors of the present paper in [12]. The authors used multilevel coding at the source and the relay to breakdown the superposition of coded modulation into a multiple binary superpositions. Each binary input to the mapper has a superposition coding that contributes to the source assistance to the relay and to the new information to be sent to the relay. It was shown that using linear codes at the source and the relay is equivalent to restricting each binary level such that it either provides assistance or send new information to the relay. This restriction greatly simplifies the encoding and decoding but it severely deteriorates the performance specially for small constellations.

In this paper, we first analyze the effect of this restriction on the overall performance and explain it in terms of the tradeoff between the beamforming gain (or the correlation between the source and the relay) and the new information to be sent to the relay. Second, we show that by an intelligent design of the labeling, this loss in performance can be avoided. Third, we show via simulations that the residual rate penalty after implementing our method is small, and also highlight the error-rate performance of our system using practical point-to-point LDPC codes.

II. Preliminaries

In the point-to-point channel, binary component multilevel coding is implemented by splitting the data stream and independently encode each sub-stream (See Fig. 1). The mutual information between the channel input and output is

$$I(X;Y) = I(B_1, B_2, \ldots, B_m; Y) = \sum_{i=1}^{m} I(B_i; Y|B^{i-1})$$

with the definition $B^{i-1} \triangleq [B_1, B_2, \ldots, B_{i-1}]$, and using the chain rule for mutual information and the one-to-one relationship between X and $[B_1, B_2, \ldots, B_m]$. Fig. 1 shows a diagram of multilevel coding with multistage decoding in the point-to-point channel.

We consider the three nodes relay channel and denote the signal transmitted from the source and the relay in block t by $X_1^{(t)}$ and $X_2^{(t)}$ respectively. The received signals at the relay and destination are $Y_2^{(t)}$ and $Y_3^{(t)}$ respectively and h_{12}, h_{13} and h_{23} are the channel coefficients from the source to the relay, the source to destination and relay to destination respectively.

III. Multilevel Decode and Forward

The decode-and-forward multilevel implementation proposed in [12] with binary additive superposition coding is illustrated in Fig 2. The relay transmission resembles simple point-to-point multilevel transmission to the destination. The relay-destination codewords are linear codes which have a uniform distribution. Superposition coding is involved in the source transmission. The source transmission has two components, the beamforming component C_i and the new information to be sent to the relay R_i. The two components are superimposed using an XOR operation to produce B_i which is the input the to the mapper at level i. The destination decodes the transmitted message using multistage decoding where in each level, the destination operates in a usual manner while taking into account the output of the decoders in the preceding levels.

Since the distribution of C_i is uniform, the distribution of R_i determines how much of level i is assigned to beamforming gain and how much of the level is to send new information to the relay. For example, when $P(r_i = 0) = 1$ this means that $B_i = C_i$, and hence, this level provides maximum beamforming gain to the relay transmission. As $P_{R_i}(r_i = 0)$ increases, B_i becomes

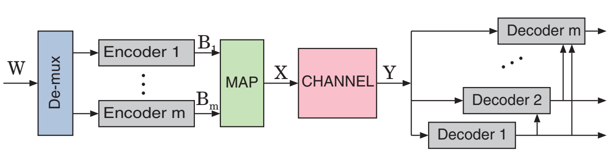

Fig. 1. MLC and MSD in point to point channel.

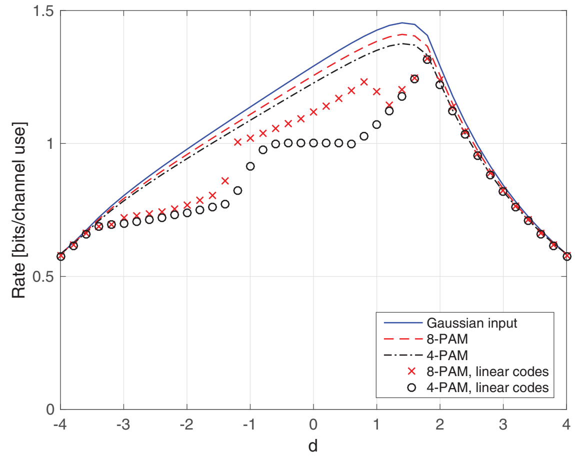

Fig. 3. General versus linear codes for $P_1 = P_2 = 13$dB

more independent from C_i and has more meaningful information about r_i. Eventually, when $P_{R_i}(r_i = 0) = 0.5$, B_i is independent from C_i, and hence, zero beamforming gain is provided through level i. Moreover, since the relay knows C_i, it can decode R_i so that it can be re-encoded and transmitted in the following block.

The transmission rate of this technique is

$$R \leq \max_{P_{B_i|C_i} P_{C_i}} \min\{I(B^m; Y_2|C^m), I(B^m, C^m; Y_3)\}$$

A sufficient condition for this to be capacity optimal is:

$$P^*_{B_i|C^m}(b_i|c^m) = P^*_{B_i|C_i}(b_i|c_i) \qquad \forall i$$

where P^* denotes the optimal distribution.

When R_i is also a linear code this means that $P_{R_i}(r_i = 0)$ is either 0 (for a zero rate code) or 0.5 (for a code rate greater than 0). This is the same as allowing B_i to be one of two things, either $B_i = C_i$ or $B_i = R_i$. The transmission rate of multilevel decode-and-forward is shown in Fig. 3 where we assume that the source, relay and destination are all on one line. The distance between the source and the destination is fixed to 4 while the distance between the source and the relay d is changing. The power of the source and the relay are denoted by P_1 and P_2 respectively. For comparison, we use the achievable rates for the Gaussian input relay channel which is obtained by optimizing over the correlation ρ between X_1 and X_2.

As shown in Fig. 3, restricting R_i to be uniformly distributed or in other words restricting B_i to be either equal to C_i or R_i severely deteriorates the performance. However, this restriction allows a very simple transmission at the source node and simple decoding at both the relay and destination nodes. Therefore, solving the rate-loss problem that result from this restriction is of great importance as it will result in a system with a comparable complexity of point-to-point transmission.

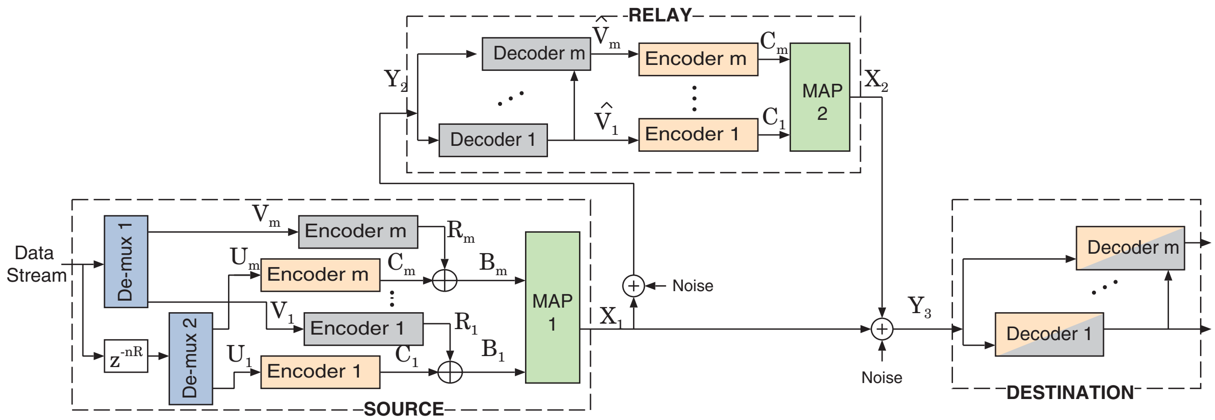

Fig. 2. MLC and MSD in the Relay channel with level by level decoding

In the following section, we present our solution to this problem that shows effectiveness in obtaining much better performance and yet the same simple encoding.

IV. Labeling Design

In the following we illustrate why the labeling design can affect the total transmission rate in contrast with the point-to-point transmission where the labeling design does not affect the total transmission rate.

In order to explain our proposed design, we define the vector $[\rho_1,\ldots,\rho_m]$ where ρ_i is the bit-wise correlation between C_i and R_i. Note that since the levels do not have the same power, the contribution of the bit-wise correlation ρ_i in the total source-relay correlation ρ depends on the level index i. Linear codes constrain the feasible set of the vector $[\rho_1,\ldots,\rho_m]$ to a binary vector. It is tempting to think that the design variable is the bit-wise correlation vector however, the real design variable is $p_{B_i|C_i}(b_i|c_i)p_{C_i}(c_i)$ with the restriction of using linear codes. The bit-wise correlation vector is a function of $p_{B_i|C_i}(b_i|c_i)$ and $p_{C_i}(c_i)$ therefore, it is not optimal to optimize over the bit-wise correlation because it is a function of the design variable. The following example illustrates the sub-optimality of the optimization over the bit-wise correlation.

Example 1: Assume a relay channel with $d_{13}=4$ and $d_{12}=1.77$ with 4-PAM constellation at the source and the relay. For $P_1=P_2=10$, the optimal correlation is $\rho^*=0.2$. For a 4-PAM constellation with natural labeling, this can be obtained if we sit $\rho_1=0$ and $\rho_2=1$. On the other hand, sitting $\rho_1=0$ and $\rho_2=0$ will result in higher transmission rate even though the total correlation becomes zero. This is because when $\rho_2=1$, the least significant bit does not send any new information to the relay, and hence, activating the first term in (**??**). On the other hand, sitting $\rho_2=0$ results in higher correlation mismatch but will still achieve higher rates than the former case.

Now, we formalize this idea. First, linear codes make

$$I(B_i;Y_2|B^{i-1},C^m)\neq 0 \quad \text{when } \rho_i=0$$
$$I(B_i;Y_2|B^{i-1},C^m)=0 \quad \text{when } \rho_i=1$$

and the transmission rate becomes

$$R\le \max_{\prod_{i=1}^m P_{B_i|C_i}P_{C_i}} \min\{\sum_{i=1}^m I(\rho_i)I(B_i;Y_2|C^m,B^{i-1}),$$
$$\sum_{i=1}^m I(C_i;Y_3|C^{i-1})+I(B_i;Y_3|B^{i-1},C^m)\} \qquad (1)$$

where $I(\rho_i)=1-\rho_i$, and since ρ_i takes only binary values, $I(\rho_i)$ also takes binary values. Clearly, canceling out some terms in the first summation in (1) by sitting $\rho_i=1$ for some levels, will increase the second term in (1) since the source assistance to the relay increases for larger correlation.

The problem described above is the trade-off between the source-relay rate and dedicating some levels for full correlation. A better trade-off between the source-relay rate and the correlation can be realized by re-assigning the point-to-point capacity of each level so that the levels to be assigned for correlation already have the smallest possible capacity, and hence, assigning this level for correlation does not highly affect the source-relay transmission rate while attaining highest correlation possible. Changing the levels point-to-point capacity can be realized by changing the labeling. Fig. 4 shows the capacity of each level of 4-PAM constellation for two different labelings. This observation shows that dedicating level i for correlation will have a different impact on the source-relay rate depending on the mapping rule. However, ρ_i which is another important metric as will be illustrated is not always inversely proportional with the point-to-point capacity of level i.

The following example shows how the mapping rule can change the transmission rate.

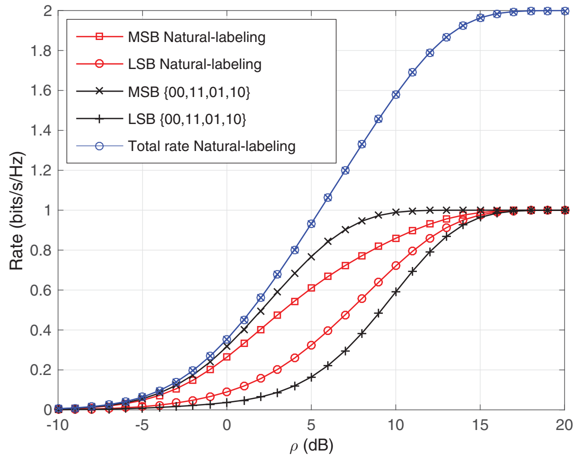

Fig. 4. Maximum achievable rate for 4-PAM under different mappings

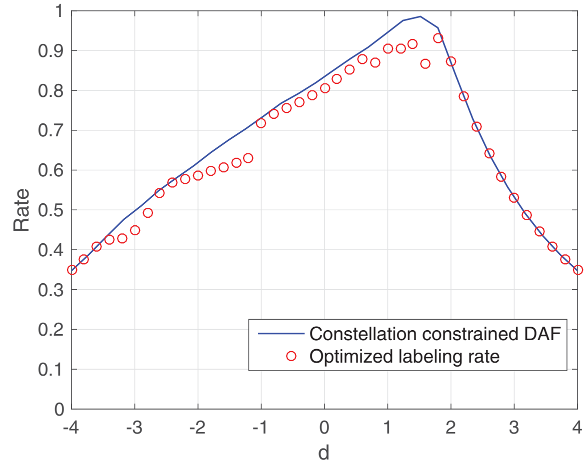

Fig. 5. MLC rate with optimized labeling, $P_1 = P_2 = 10$dB

$[\rho_1, \rho_2]$	[0,0]	[0,1]	[1,0]	[1,1]
Natural mapping {00,01,10,11}	0	0.2	0.8	1
Gray mapping {00,01,11,10}	0	0.19	0.79	1
Custom mapping {00,11,01,10}	0	0.41	0.51	1

TABLE I
TOTAL CORRELATION ACHIEVED BY LINEAR CODES

Example 2: For a 4-PAM constellation, the signals X_1 and X_2, for any mapping can be expressed as

$$X_1 = \alpha_1 B_1 + \beta_1 B_2 - \gamma_1 + \delta_1 B_1 B_2 \tag{2}$$
$$X_2 = \alpha_2 C_1 + \beta_2 C_2 - \gamma_2 + \delta_2 C_1 C_2 \tag{3}$$

for some constants $\alpha_i, \beta_i, \gamma_i$ and δ_i. After some mathematical manipulations, the total correlation for linear codes is

$$\rho = \frac{1}{20}(\alpha_1\alpha_2 + \frac{1}{2}\alpha_1\delta_2 + \frac{1}{2}\alpha_2\delta_1)\rho_1 + (\beta_1\beta_2 + \frac{1}{2}\beta_1\delta_2 + \frac{1}{2}\beta_2\delta_1)\rho_2 + \frac{\delta_1\delta_2}{16}(\rho_1\rho_2 + \rho_1 + \rho_2)$$

Table. I gives the corresponding values of ρ as a function of ρ_1 and ρ_2 for the three different labelings. The Table shows that different mappings have different possible sets of correlation and according to the channel conditions, each mapping can achieve higher transmission rates.

Choosing the best labeling at each location of the channel will result in the rate in Fig. 5, for 4-PAM constellation. The figure shows that the optimizing over the labeling have an excellent performance. Clearly, there is no optimal mapping for all values of d. Each mapping generates possible values of ρ and the set of levels will have different point-to-point transmission rate from the source to the relay. The optimization of the total transmission rate from the source to the destination is a complicated optimization problem, however, for a given channel parameters exhaustive search for the optimal value mapping is very easy specially for small constellation sizes.

Remark 1: When $d = -1$ and $d = 1$, the broadcast phase of the channel is equivalent. However, the multiple-access phase when $d = -1$ is worse than the multiple-access phase when $d = 1$. Therefore, when $d = -1$, the relay requires more beamforming gain provided from the source which means more correlation. In this case, the mapping is chosen such that the the error in correlation is minimized. On the other hand, when $d = 1$, the relay needs less beamforming and higher source-relay transmission rate might be more important. Therefore, the mapping is designed such that the levels that will be dedicated to correlation will have a small effect on the source-relay transmission rate.

Remark 2: The design in this paper is general and include the fading channels. This is because the expressions of the mutual informations in case of fading are averaged over the channel gains, $I(X;Y) = E_h[I(X;Y|h)]$ where $E[X]$ is the expectation of X. The only difference in the design will be the set of curves in Fig. 4 which should be generated according to an averaging over the fading coefficients. Once the curves are obtained the same design and analysis follow.

V. SIMULATIONS

We assume that $P_1 = P_2 = P$. We consider the same setting of the example in [16]. The source, relay and the destination nodes are aligned where the distance between the source and the destination is d_{13}, the distance between the source and the relay is variable and is given by d, and hence, the distance between the relay and destination is $d_{23} = d_{13} - d$. $d_{13} = 4$ in the simulations. Therefore, $h_{ij} = (1/d_{ij})^{\alpha/2}$ where we assume that $\alpha = 4$. The noise power spectral density at the source and the relay is assumed to be 1.

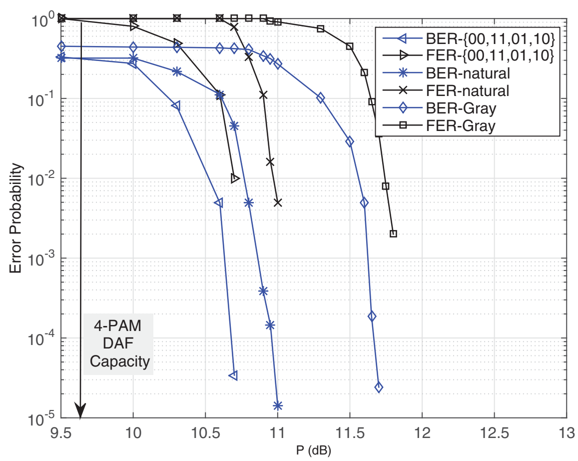

Fig. 6. Performance of Multilevel coding for 4-PAM with $d = 1$

The DVB-S2 LDPC codes are used as component codes for each of the levels at the source and the relay to examine the performance of the proposed solution for linear multilevel transmission. The blocklength of the component codes is $n = 64$k. Both the relay and destination used belief propagation decoding at each level where the maximum number of iterations is set to 20. The LLR calculations at the relay node and the destination node are shown in the Appendix. Fig. 6 shows the system performance under three different labelings where the gap to capacity is in the order of 1.5dB.

Appendix A
LLR calculations

The decoding process in the relay node during the transmission of block t takes into account the knowledge of the vector c^m at block t which is an estimate of the vector b^m at block $t-1$. During decoding level, the receiver also knows the transmitted signal at all levels preceding i. The following is the LLR calculations of level i at the relay at block t.

$$LLR_r = \log \frac{P(y_2|c^m, b^{i-1}, b_i = 0)}{P(y_2|c^m, b^{i-1}, b_i = 1)} \tag{4}$$

where

$$P(y_2|c^m, b^{i-1}, b_i) = \frac{1}{P(c^m, b^{i-1}, b_i)} \sum_{b^m_{i+1}} P(y_2|c^m, b^m)$$

Whereas in the destination, assume that the destination will first decode the signal from the relay and then decode the signal from the source.

The LLR of level i of the relay at the destination is

$$LLR_{RD} = \log \frac{P(y_3|c^{i-1}, c_i = 0)}{P(y_3|c^{i-1}, c_i = 1)} \tag{5}$$

where

$$P(y_3|c^{i-1}, c_i) = \frac{1}{P(c^{i-1}, c_i)} \sum_{b^m, c^m_{i+1}} P(y_3|b^m, c^m)$$

while the next step is to decode the signal of the source given the transmitted signal from the relay according to

$$LLR_{SD} = \log \frac{P(y_3|c^m, b^{i-1}, b_i = 0)}{P(y_3|c^m, b^{i-1}, b_i = 1)} \tag{6}$$

where

$$P(y_3|c^m, b^{i-1}, b_i) = \frac{1}{P(c^m, b^{i-1}, b_i)} \sum_{b^m_{i+1}} P(y_3|b^m, c^m)$$

References

[1] A. Sabharwal, P. Schniter, D. Guo, D. Bliss, S. Rangarajan, and R. Wichman, "In-band full-duplex wireless: Challenges and opportunities," *IEEE J. Select. Areas Commun.*, vol. 32, no. 9, Sept. 2014.

[2] E. Ahmed, A. Eltawil, and A. Sabharwal, "Rate gain region and design tradeoffs for full-duplex wireless communications," *IEEE Trans. Wireless Commun.*, vol. 12, no. 7, pp. 3556–3565, July 2013.

[3] A. Chakrabarti, A. de Baynast, A. Sabharwal, and B. Aazhang, "Low density parity check codes for the relay channel," *IEEE J. Select. Areas Commun.*, vol. 25, no. 2, pp. 280–291, Feb. 2007.

[4] P. Razaghi and W. Yu, "Bilayer low-density parity-check codes for decode-and-forward in relay channels," *IEEE Trans. Inform. Theory*, vol. 53, no. 10, pp. 3723–3739, Oct. 2007.

[5] K. Ravindran, A. Thangaraj, and S. Bhashyam, "LDPC codes for network-coded bidirectional relaying with higher order modulation," *IEEE Trans. Commun.*, vol. 63, no. 6, pp. 1975–1987, June 2015.

[6] Z. Chen and H. Liu, "Spectrum-efficient coded modulation design for two-way relay channels," *IEEE J. Select. Areas Commun.*, vol. 32, no. 2, pp. 251–263, Feb. 2014.

[7] Z. Chen, B. Xia, Z. Hu, and H. Liu, "Design and analysis of multi-level physical-layer network coding for gaussian two-way relay channels," *IEEE Trans. Commun.*, vol. 62, no. 6, pp. 1803–1817, June 2014.

[8] B. Hern and K. Narayanan, "Multilevel coding schemes for compute-and-forward with flexible decoding," *IEEE Trans. Inform. Theory*, vol. 59, no. 11, pp. 7613–7631, Nov. 2013.

[9] A. A. Abotabl and A. Nosratinia, "Broadcast coded modulation: Multilevel and bit-interleaved construction," *IEEE Trans. Commun.*, vol. 65, no. 3, pp. 969–980, Mar. 2017.

[10] ——, "Bit-additive superposition coding for the bandwidth limited broadcast channel," in *IEEE International Symposium on Information Theory (ISIT)*, July 2016, pp. 165–169.

[11] N. Ferdinand, M. Nokleby, and B. Aazhang, "Low-density lattice codes for full-duplex relay channels," *IEEE Trans. Wireless Commun.*, vol. 14, no. 4, pp. 2309–2321, Apr. 2015.

[12] A. A. Abotabl and A. Nosratinia, "Multilevel coding for the full-duplex relay channel," in *IEEE Global Communications Conference (GLOBECOM)*, Dec. 2015, pp. 1–6.

[13] A. Abotabl and A. Nosratinia, "Multi-level coding and multi-stage decoding in MAC, broadcast, and relay channel," in *IEEE International Symposium on Information Theory*, June 2014, pp. 96–100.

[14] H. Imai and S. Hirakawa, "A new multilevel coding method using error-correcting codes," *IEEE Trans. Inform. Theory*, vol. 23, no. 3, pp. 371–377, 1977.

[15] U. Wachsmann, R. F. H. Fischer, and J. Huber, "Multilevel codes: theoretical concepts and practical design rules," *IEEE Trans. Inform. Theory*, vol. 45, no. 5, pp. 1361–1391, 1999.

[16] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. Inform. Theory*, vol. 51, no. 9, pp. 3037–3063, Sept. 2005.