Two-User MIMO Broadcast Channel with Partially Overlapping Correlation Eigenspaces

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Abstract— **The study of correlated MIMO broadcast channel is becoming an important subject due to the growing research interest in higher frequency and also massive MIMO. In this paper, we study a two-user MIMO broadcast channel where the two users have correlation matrices with eigenspaces partially overlapped with each other. It is neither identical nor fully overlapped. Thus no existing technique can be straight forward applied to this scenario. We show in such correlation structure, the overlapping between the two users can increase the degrees of freedom over TDMA. We leverage the overlapping eigenspaces in the system via pre-beamforming and combine the product superposition technique to obtain the new achievable degree of freedom region.**

I. Introduction

Spatial correlation of the channel effects the performance of MIMO broadcast channel. In most previous works, channels have been assumed to have identical correlation condition [1], [2]. Particularly, [1] showed that the transmit correlation is a detrimental impact on the sum capacity of multiuser MIMO system. [2] concluded that in the massive MIMO system, transmit correlation significantly decreases the system performance. But in practice, different users may have different scattering environments, such as in the street and on the top of the roof, thus the fading links may experience different spatial correlation. In such scenarios, the fundamental limits are unknown.

The scenario of different correlation matrices across links has raised an interesting question about the dependance of the broadcast performance on the relation between the correlation matrices. For the case of broadcast channel with correlation matrices having orthogonal (non-overlapping) eigenspaces, where Joint Spatial Division Multiplexing (JSDM) transmission scheme was proposed to provide gains by reducing the overhead needed for channel estimation [3]–[6]. For multiuser networks with orthogonal eigenspace correlation matrices, [7] showed transmit correlation helps in multicell network by partitioning the user space into clusters according to correlation. [8] also concluded transmit correlation benefits the sum rate in the downlink performance of a heterogeneous cellular network (HetNet) where both macro and small cells share the same spectrum. [9] presented how to exploit the transmit correlation in a two-tier system where a large number of small cells are deployed under a macro-cellular. For the case where users have non-orthogonal correlation eigenspaces, [10] proposed one achievable scheme via the method named product superposition for users having fully overlapped eigenspaces, which achieves gains over TDMA. These results show evidence that difference between transmit antenna correlation is a kind of potential gain resource in multiuser systems.

However, the performance of the broadcast channel with correlated channel in general is still unknown. For users with correlated channel, when the links between transmitter and users have different scatters, the null space of channel matrix for different users can be neither identical nor fully embedded. In such scenario, the eigenspace of these users will be partially overlapped. Due to the overlapping subspace, the pre-beamforming technique [3]–[6] for orthogonal eigenspaces cannot be directly applied and the product superposition scheme [10], which focused on the overlapping part, do not address the possible gains we can obtain from the non-overlapping subspace. Thus none of these known techniques fit to this scenario perfectly.

This paper studies this new scenario where the users have correlation matrices with partially overlapping eigenspaces. Our proposed schemes create multiuser gains from both the non-overlapping subspace and the overlapped part. We start with the two-user case where there is one dimension overlapped. Using the proposed transmit scheme, both users can decode their messages. After that, we consider the case of general correlation for both users with same number of antennas, where the degree of freedom gains are shown over the conventional transmission that employs TDMA. Then, we consider the case of arbitrary number of antennas, providing transmit schemes and calculate the degree of freedom gains.

Notations: For a matrix $\mathbf{U}$, $\mathbf{span}(\mathbf{U})$ represents the subspace including the linear sum of the columns of $\mathbf{U}$. For two subspaces $\mathcal{S}_1$ and $\mathcal{S}_2$, $\mathcal{S}_1 + \mathcal{S}_2$ represents the subspace of the linear sum of the vectors in $\mathcal{S}_1$ and $\mathcal{S}_2$. $\mathcal{S}_1 \backslash \mathcal{S}_2$ represents the subspace of the vectors in $\mathcal{S}_1$ which are orthogonal with the vectors in $\mathcal{S}_2$.

II. System Model

Consider a MIMO Broadcast channel with M transmit antennas serving K users, where user i is equipped with N_i antennas. The received signal at user i is

$$\mathbf{Y}_i = \mathbf{H}_i \mathbf{X} + \mathbf{W}_i, \qquad i = 1, \dots, K, \tag{1}$$

where $\mathbf{X} \in \mathbb{C}^{M \times T}$ is the transmitted signal and $\mathbf{W}_i \in \mathbb{C}^{N_i \times T}$ is the i.i.d. white Gaussian noise. The channel matrix $\mathbf{H}_i$ follows the block fading model. It remains constant during the coherence interval of T, which satisfies $T \geq 2\max(M, N_i)$, but changes independently across blocks [11]. Define $\mathbf{R}_i$ as the transmit correlation matrix of user i and $r_i = \text{rank}(\mathbf{R}_i)$. From the Kronecker model (a.k.a separable model), the channel $\mathbf{H}_i$ is given by $\mathbf{H}_i = \mathbf{G}_i\mathbf{R}_i^{\frac{1}{2}}$, where $\mathbf{G}_i \in \mathbb{C}^{N_i\times M}$ is a Gaussian random matrix with i.i.d. entries [12]. Let $\mathbf{R}_i^{\frac{1}{2}} = \mathbf{U}_i\mathbf{\Sigma}\mathbf{U}_i^H$ denote the eigen decomposition, where $\mathbf{\Sigma}_i \in \mathbb{C}^{r_i\times r_i}$ is a diagonal matrix that includes the non-zero eigenvalues of $\mathbf{R}_i^{\frac{1}{2}}$ and $\mathbf{U}_i \in \mathbb{C}^{M\times r_i}$ is the matrix whose columns are the eigenvectors of $\mathbf{R}_i^{\frac{1}{2}}$ corresponding to the non-zero eigenvalues. Therefore,

$$\mathbf{H}_i = \tilde{\mathbf{H}}_i\mathbf{U}_i^H, \tag{2}$$

where $\tilde{\mathbf{H}}_i = \mathbf{G}_i\mathbf{U}_i\mathbf{\Sigma}_i$, whose entries are independent but not identical Gaussian.

We assume there is no CSIR or CSIT, where $\mathbf{H}_i$ and $\tilde{\mathbf{H}}_i$ are not known at transmitter or receivers, while $\mathbf{\Sigma}_i$ and $\mathbf{U}_i$ are globally known. In this paper, correlation eigenspace refers to the span of eigenvectors of a correlation matrix.

We assume that there are K independent messages associated with rates $R_1(\rho), \dots, R_K(\rho)$ to be communicated from the transmitter to the K receivers at ρ signal-to-noise ratio. The degrees of freedom at receiver i achieving rate $R_i(\rho)$ is defined as

$$d_k = \lim_{\rho\to\infty} \frac{R_i(\rho)}{\log(\rho)}. \tag{3}$$

III. Two-User Broadcast Channel

In this section, we study the two-user broadcast channel with correlation. We start with a toy example and then extend it to general case. For each scenario, we propose a scheme achieving degree of freedom gains over TDMA.

A. Toy Example

For the broadcast channel defined as (1),(2) with two users, where $M = N_1 = N_2 = 2$, and

$$\mathbf{U}_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \tag{4}$$

$$\mathbf{U}_2 = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \tag{5}$$

the following degree of freedom region can be achieved

$$\begin{cases} d_1 \le \frac{2}{T}(T-2) \\ d_1 \le \frac{2}{T}(T-2) \\ d_1 + d_2 \le \frac{3}{T}(T-2) \end{cases} \tag{6}$$

Proof: The transmitted signal is

$$\mathbf{X} = \sqrt{\rho}\begin{bmatrix} 1 & 0 & \mathbf{x}_{\delta,1} \\ 0 & 1 & \mathbf{x}_{\delta,0} \\ 1 & 0 & \mathbf{x}_{\delta,2} \end{bmatrix}, \tag{7}$$

$\mathbf{x}_{\delta,1}$ and $\mathbf{x}_{\delta,2} \in \mathbb{C}^{1\times(T-2)}$ contain symbols intended for User 1 and User 2, $\mathbf{x}_{\delta,0} \in \mathbb{C}^{1\times(T-2)}$ contains symbols that both User 1 and User 2 can decode.

The received signal at User 1 is

$$\mathbf{Y}_1 = \sqrt{\rho}\tilde{\mathbf{H}}_1\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\mathbf{X} + \mathbf{W}_1 \tag{8}$$

$$= \sqrt{\rho}\tilde{\mathbf{H}}_1\begin{bmatrix} 1 & 0 & \mathbf{x}_{\delta,1} \\ 0 & 1 & \mathbf{x}_{\delta,0} \end{bmatrix} + \mathbf{W}_1. \tag{9}$$

User 1 estimates $\tilde{\mathbf{H}}_1$ during the first 2 time slots and can decode $x_{\delta,1}$ and $x_{\delta,0}$ during the remaining time slots achieving $2\times(T-2)$ degrees of freedom.

The received signal at User 2 is

$$\mathbf{Y}_2 = \sqrt{\rho}\tilde{\mathbf{H}}_2\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\mathbf{X} + \mathbf{W}_2 \tag{10}$$

$$= \sqrt{\rho}\tilde{\mathbf{H}}_2\begin{bmatrix} 1 & 0 & \mathbf{x}_{\delta,0} \\ 0 & 1 & \mathbf{x}_{\delta,2} \end{bmatrix} + \mathbf{W}_2. \tag{11}$$

User 2 estimates $\tilde{\mathbf{H}}_2$ during the first 2 time slots and can decode $\mathbf{x}_{\delta,0}$ and $\mathbf{x}_{\delta,2}$ during the remaining time slots achieving $2\times(T-2)$ degrees of freedom.

Because $\mathbf{x}_{\delta,0}$ is decoded by both User 1 and User 2, using time sharing, the degree of freedom pair $(\frac{2}{T}(T-2), \frac{1}{T}(T-2))$, $(\frac{1}{T}(T-2), \frac{2}{T}(T-2))$ can be achieved. Together with the single user degree of freedom bound [10], the degree of freedom region of (6) can be achieved, which is larger than the TDMA achievable region. ■

This toy example shows that when two users have partially overlapping correlation eigenspaces, we can achieve more degrees of freedom than TDMA.

B. Matched number of antennas: $M = N_1 = N_2$

1) Orthogonal eigenvectors:

Theorem 1: For the broadcast channel defined in (1) and (2) with two users, when the columns in $\mathbf{U}_1$ and $\mathbf{U}_2$ are either the same or orthogonal with each other and the number of duplicated columns is r_0, define $r_i^* = r_i - r_0,\ i = 1,2$ and without loss of generality, assume $r_1^* \ge r_2^*$. The following degree of freedom pair is achievable:

$$\begin{aligned} \mathcal{D}_1 = (&\frac{r_1^*+r_0}{T}(T-r_1^*-r_0) \\ &\frac{r_2^*}{T}(r_1^*-r_2^*) + \frac{r_2^*}{T}(T-r_1^*-r_0)), \end{aligned} \tag{12}$$

$$\begin{aligned} \mathcal{D}_2 = (&\frac{r_1}{T}(T-r_1^*-r_0), \\ &\frac{r_2^*}{T}(r_1^*-r_2^*) + \frac{r_2^*+r_0}{T}(T-r_1^*-r_0)), \end{aligned} \tag{13}$$

$$\mathcal{D}_3 = (\frac{(r_1^*+r_0)}{T}(T-r_1^*-r_0), \frac{r_0r_1^*}{T}), \tag{14}$$

$$\mathcal{D}_4 = (\frac{r_0r_2^*}{T}, \frac{(r_2^*+r_0)}{T}(T-r_2^*-r_0)). \tag{15}$$

Proof: Define $\mathbf{U}_0$ as the submatrix generated by the columns appearing in both $\mathbf{U}_1$ and $\mathbf{U}_2$. $\mathbf{U}_i^*$ represents the submatrix of $\mathbf{U}_i$ excluding the column vectors appearing in $\mathbf{U}_0$, i = 1,2. Let $\mathbf{U} = [\mathbf{U}_1^*\ \mathbf{U}_0\ \mathbf{U}_2^*]$, then $\mathbf{U}^H\mathbf{U} = \mathbf{I}$. We transmit the signal

$$\mathbf{X} = \sqrt{\rho}\mathbf{U}\bar{\mathbf{X}} = \sqrt{\rho}\mathbf{U}\begin{bmatrix} \mathbf{I}_{r_1^*} & 0 & \mathbf{X}_{\delta,1} \\ 0 & \mathbf{I}_{r_0} & \mathbf{X}_{\delta,0} \\ \mathbf{I}_{r_2^*}\ \mathbf{X}_2 & 0 & \mathbf{X}_{\delta,2} \end{bmatrix}, \tag{16}$$

where $\mathbf{X}_{\delta,1} \in \mathbb{C}^{r_1^* \times (T-r_1^*-r_0)}$ and $\mathbf{X}_{\delta,2} \in \mathbb{C}^{r_2^* \times (T-r_1^*-r_0)}$ contain symbols intended for User 1 and User 2, $\mathbf{X}_{\delta,0} \in \mathbb{C}^{r_0 \times (T-r_1^*-r_0)}$ contains symbols that both User 1 and User 2 can decode. $\mathbf{X}_2 \in \mathbb{C}^{r_2^* \times (r_1^*-r_2^*)}$ contains the symbols that intended for User 2 only.

The received signal at User 1 is

$$\mathbf{Y}_1 = \sqrt{\rho}\tilde{\mathbf{H}}_1[\mathbf{U}_1^* \ \mathbf{U}_0]^H \mathbf{X} + \mathbf{W}_1 \tag{17}$$

$$= \sqrt{\rho}\tilde{\mathbf{H}}_1 \begin{bmatrix} \mathbf{I}_{r_1^*} & 0 & 0 \\ 0 & \mathbf{I}_{r_0} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{I}_{r_1^*} & 0 & \mathbf{X}_{\delta,1} \\ 0 & \mathbf{I}_{r_0} & \mathbf{X}_{\delta,0} \\ \mathbf{I}_{r_2^*} \ \mathbf{X}_2 & 0 & \mathbf{X}_{\delta,2} \end{bmatrix} + \mathbf{W}_1 \tag{18}$$

$$= \sqrt{\rho}\tilde{\mathbf{H}}_1 \begin{bmatrix} \mathbf{I}_{r_1^*} & 0 & \mathbf{X}_{\delta,1} \\ 0 & \mathbf{I}_{r_0} & \mathbf{X}_{\delta,0} \end{bmatrix} + \mathbf{W}_1. \tag{19}$$

User 1 estimates $\tilde{\mathbf{H}}_1$ during the first $(r_1^* + r_0)$ time slots and can decode $\mathbf{X}_{\delta,1}$ and $\mathbf{X}_{\delta,0}$ during the remaining time slots achieving $(r_1^* + r_0)(T - r_1^* - r_0)$ degrees of freedom.

The received signal at User 2 is

$$\mathbf{Y}_2 = \sqrt{\rho}\tilde{\mathbf{H}}_2[\mathbf{U}_0 \ \mathbf{U}_2^*]^H \mathbf{X} + \mathbf{W}_2 \tag{20}$$

$$= \sqrt{\rho}\tilde{\mathbf{H}}_2 \begin{bmatrix} 0 & \mathbf{I}_{r_0} & 0 \\ 0 & 0 & \mathbf{I}_{r_2^*} \end{bmatrix} \begin{bmatrix} \mathbf{I}_{r_1^*} & 0 & \mathbf{X}_{\delta,1} \\ 0 & \mathbf{I}_{r_0} & \mathbf{X}_{\delta,0} \\ \mathbf{I}_{r_2^*} \ \mathbf{X}_2 & 0 & \mathbf{X}_{\delta,2} \end{bmatrix} + \mathbf{W}_1 \tag{21}$$

$$= \sqrt{\rho}\tilde{\mathbf{H}}_1 \begin{bmatrix} 0 & 0 & \mathbf{I}_{r_0} & \mathbf{X}_{\delta,0} \\ \mathbf{I}_{r_2^*} & \mathbf{X}_2 & 0 & \mathbf{X}_{\delta,2} \end{bmatrix} + \mathbf{W}_1. \tag{22}$$

User 2 estimates $\tilde{\mathbf{H}}_2$ during the first r_2^* and (r_1^*+1) to $(r_1^*+r_0)$ time slots and decode $\mathbf{X}_{\delta,0}$, $\mathbf{X}_{\delta,2}$ and $\mathbf{X}_2$ achieving $(r_2^*(r_1^* - r_2^*) + (r_0 + r_2^*)(T - r_1^* - r_0))$ degrees of freedom. Because $\mathbf{X}_{\delta,0}$ is decoded by both User 1 and User 2, the degree of freedom pair (12) and (13) are achievable via time sharing.

Next consider the product superposition transmit scheme proposed in [10]. Make User 1 achieving its single user bound. Transmit signals over the subspace of $\mathbf{span}([\mathbf{U}_1^* \ \mathbf{U}_0])$. The transmit signal is:

$$\mathbf{X} = \sqrt{\rho}[\mathbf{U}_1^* \ \mathbf{U}_0]\mathbf{X}_2\mathbf{X}_1, \tag{23}$$

where $\mathbf{X}_1$ and $\mathbf{X}_2$ has the following structure:

$$\mathbf{X}_1 = [\mathbf{I}_{r_1^*+r_0} \ \mathbf{X}_{\delta,1}], \tag{24}$$

$$\bar{\mathbf{X}} = \begin{bmatrix} \mathbf{I}_{r_0} & \mathbf{X}_{\delta,2} \\ 0 & \mathbf{I}_{r_1^*} \end{bmatrix}, \tag{25}$$

where $\mathbf{X}_{\delta,1} \in \mathbb{C}^{(r_1^*+r_0)\times(T-r_1^*-r_0)}$ and $\mathbf{X}_{\delta,2} \in \mathbb{C}^{r_0 \times r_1^*}$ contains symbol intended for User 1 and User 2. User 1 estimates its equivalent channel during the first $(r_1^* + r_0)$ time slots and decodes $\mathbf{X}_{\delta,1}$ during the remaining time slots achieving $(r_0+r_1^*)(T-r_1^*-r_0)$ degrees of freedom. User 2 estimates $\bar{\mathbf{H}}_2$ during the first r_0 time slots and decodes $\mathbf{X}_{\delta,2}$ achieving $r_0 r_1^*$ degrees of freedom. Thus this product superposition scheme achieves the degree of freedom pair (14). In the same way, if we make User 2 achieving its single user bound, we can achieve the degree of freedom pair (15). Thus completes the proof of Theorem 1. ■

2) Non-orthogonal Eigenvectors: In this section, we study the case where the columns in $\mathbf{U}_1$ and $\mathbf{U}_2$ do not satisfy the assumption of either the same or orthogonal but $\mathbf{span}(\mathbf{U}_1) \cap \mathbf{span}(\mathbf{U}_2) \neq \emptyset$.

Theorem 2: For the broadcast channel defined in (1) and (2) with two users. Define the subspace $\mathcal{S} = \mathbf{span}(\mathbf{U}_1) \cap \mathbf{span}(\mathbf{U}_2)$ and $\dim(\mathcal{S}) = r_0$ and $r_i^* = r_i - r_0, \ i = 1, 2$, the degree of freedom pair (12)(13)(14)(15) are achievable.

Proof: Define $\mathbf{V}_0 \in \mathbb{C}^{M \times r_0}$, whose columns are one set of basis of $\mathcal{S}$. Define the subspace $\mathcal{S}_i = \mathbf{span}(\mathbf{U}_i)\backslash\mathcal{S}$, $\dim(\mathcal{S}) = r_i^*$ and $\mathbf{V}_i \in \mathbb{C}^{M \times r_i^*}$, whose columns are one set of basis of $\mathcal{S}_i$, where $i = 1, 2$.

According to the definition, we have $\mathbf{span}(\mathbf{U}_1) = \mathcal{S} + \mathcal{S}_1$, thus there exists one non-singular matrix $\mathbf{T}_1$ such that $\mathbf{U}_1 = [\mathbf{V}_1 \ \mathbf{V}_0]\mathbf{T}_1$, thus the channel matrix $\mathbf{H}_1$ can be decomposed as follows:

$$\mathbf{H}_1 = \tilde{\mathbf{H}}_1\mathbf{U}_1^H = \tilde{\mathbf{H}}_1\mathbf{T}_1^H \begin{bmatrix} \mathbf{V}_1^H \\ \mathbf{V}_0^H \end{bmatrix} = \bar{\mathbf{H}}_1 \begin{bmatrix} \mathbf{V}_1^H \\ \mathbf{V}_0^H \end{bmatrix}, \tag{26}$$

where $\bar{\mathbf{H}}_1 = \tilde{\mathbf{H}}_1\mathbf{T}_1^H$. In the same way, there exists one non-singular matrix $\mathbf{T}_2$ such that:

$$\mathbf{H}_2 = \bar{\mathbf{H}}_2 \begin{bmatrix} \mathbf{V}_2^H \\ \mathbf{V}_0^H \end{bmatrix}, \tag{27}$$

where $\bar{\mathbf{H}}_2 = \tilde{\mathbf{H}}_2\mathbf{T}_2^H$. Define the matrix $\mathbf{V} = [\mathbf{V}_1 \ \mathbf{V}_0 \ \mathbf{V}_2]$. The transmitter sends the signal:

$$\mathbf{X} = \mathbf{V}(\mathbf{V}^H\mathbf{V})^{-1}\bar{\mathbf{X}}, \tag{28}$$

where $\bar{\mathbf{X}}$ has the same structure as (13). Because of the definition, $\mathbf{V}^H\mathbf{V}$ is invertible. The received signal at User 1 is:

$$\mathbf{Y}_1 = \sqrt{\rho}\bar{\mathbf{H}}_1[\mathbf{V}_1 \ \mathbf{V}_0]^H\mathbf{X} + \mathbf{W}_1 \tag{29}$$

$$= \sqrt{\rho}\bar{\mathbf{H}}_1[\mathbf{V}_1 \ \mathbf{V}_0]^H\mathbf{V}(\mathbf{V}^H\mathbf{V})^{-1}\bar{\mathbf{X}} + \mathbf{W}_1 \tag{30}$$

$$= \sqrt{\rho}\bar{\mathbf{H}}_1 \begin{bmatrix} \mathbf{I}_{r_1^*} & 0 & 0 \\ 0 & \mathbf{I}_{r_0} & 0 \end{bmatrix} \bar{\mathbf{X}} + \mathbf{W}_1 \tag{31}$$

$$= \sqrt{\rho}\bar{\mathbf{H}}_1 \begin{bmatrix} \mathbf{I}_{r_1^*} & 0 & \mathbf{X}_{\delta,1} \\ 0 & \mathbf{I}_{r_0} & \mathbf{X}_{\delta,0} \end{bmatrix} + \mathbf{W}_1. \tag{32}$$

The received signal at User 2 is

$$\mathbf{Y}_2 = \sqrt{\rho}\bar{\mathbf{H}}_2[\mathbf{V}_0 \ \mathbf{V}_2]^H\mathbf{X} + \mathbf{W}_2 \tag{33}$$

$$= \sqrt{\rho}\bar{\mathbf{H}}_2[\mathbf{V}_0 \ \mathbf{V}_2]^H\mathbf{V}(\mathbf{V}^H\mathbf{V})^{-1}\bar{\mathbf{X}} + \mathbf{W}_2 \tag{34}$$

$$= \sqrt{\rho}\bar{\mathbf{H}}_2 \begin{bmatrix} 0 & \mathbf{I}_{r_0} & 0 \\ 0 & 0 & \mathbf{I}_{r_2^*} \end{bmatrix} \bar{\mathbf{X}} + \mathbf{W}_2 \tag{35}$$

$$= \sqrt{\rho}\tilde{\mathbf{H}}_2 \begin{bmatrix} 0 & 0 & \mathbf{I}_{r_0} & \mathbf{X}_{\delta,0} \\ \mathbf{I}_{r_2^*} & \mathbf{X}_2 & 0 & \mathbf{X}_{\delta,2} \end{bmatrix} + \mathbf{W}_2. \tag{36}$$

We can achieve the same degree of freedom pair as (12) and (13). Then apply the product superposition transmit scheme. Make User 1 achieving its single user bound. Define the matrix $\mathbf{V} = [\mathbf{V}_1 \ \mathbf{V}_0]$ The transmitted signal is:

$$\mathbf{X} = \sqrt{\rho}\mathbf{V}(\mathbf{V}^H\mathbf{V})^{-1}\mathbf{X}_2\mathbf{X}_1, \tag{37}$$

where $\mathbf{X}_1$ and $\mathbf{X}_2$ has the following structure:

$$\mathbf{X}_1 = [\mathbf{I}_{r_1^*+r_0} \ \mathbf{X}_{\delta,1}], \tag{38}$$

$$\mathbf{X}_2 = \begin{bmatrix} \mathbf{I}_{r_0} & \mathbf{X}_{\delta,2} \\ 0 & \mathbf{I}_{r_1^*} \end{bmatrix}, \tag{39}$$

where $\mathbf{X}_{\delta,1} \in \mathbb{C}^{(r_1^*+r_0)\times(T-r_1^*-r_0)}$ and $\mathbf{X}_{\delta,2} \in \mathbb{C}^{r_0\times r_1^*}$ contains symbol intended for User 1 and User 2. This product superposition scheme achieves the same degree of freedom pair (14). In the same way, if we make User 2 achieving its single user bound, we can achieve the degree of freedom pair (15). Thus completes the proof of Theorem 2 ■

C. Unmatched number of antennas: $N_1, N_2 \leq M$

In this section, we study the case where the number of antennas do not match between transmitter and receivers. We will use the notations provided in the previous section and focus on the method of designing the pre-beamforming matrix $\mathbf{V}$.

1) $N_1 \leq r_1^*$ and $N_2 \leq r_2^*$: When the number of antennas of two users is small, we can transmit to two users at their single user degree of freedom bound. Generate submatrices $\mathbf{V}_1^*$ and $\mathbf{V}_2^*$ by selecting N_1 columns from $\mathbf{V}_1$ and N_2 columns from $\mathbf{V}_2$. Define the matrix $\mathbf{V} = [\mathbf{V}_1^* \ \mathbf{V}_2^*]$. The transmitted signal is:

$$\mathbf{X} = \sqrt{\rho}\mathbf{V}(\mathbf{V}^H\mathbf{V})^{-1}\bar{\mathbf{X}}, \tag{40}$$

where $\mathbf{X}$ has the following structure:

$$\bar{\mathbf{X}} = \begin{bmatrix} \mathbf{I}_{N_1\times N_1} & \mathbf{X}_{\delta,1} \\ \mathbf{I}_{N_2\times N_2} & \mathbf{X}_{\delta,2} \end{bmatrix}, \tag{41}$$

where $\mathbf{X}_{\delta,1} \in \mathbb{C}^{N_1\times(T-N_1)}$ and $\mathbf{X}_{\delta,2} \in \mathbb{C}^{N_2\times(T-N_2)}$ are the symbols intended for User 1 and User 2. The received signal at User 1 is:

$$\begin{aligned} \mathbf{Y}_1 &= \sqrt{\rho}\bar{\mathbf{H}}_1[\mathbf{V}_1 \ \mathbf{V}_0]^H\mathbf{X} + \mathbf{W}_1 && (42) \\ &= \sqrt{\rho}\bar{\mathbf{H}}_1[\mathbf{V}_1 \ \mathbf{V}_0]^H\mathbf{V}(\mathbf{V}^H\mathbf{V})^{-1}\bar{\mathbf{X}} + \mathbf{W}_1 && (43) \\ &= \sqrt{\rho}\bar{\mathbf{H}}_1\left[\mathbf{I}_{N_1} \quad \mathbf{X}_{\delta,1}\right] + \mathbf{W}_1. && (44) \end{aligned}$$

User 1 estimates $\bar{\mathbf{H}}_1$ during the first N_1 time slots and decodes $\mathbf{X}_{\delta,1}$ during the remaining time slots achieving $N_1(T-N_1)$ degrees of freedom, which meets the single user degree of freedom bound. Similarly, User 2 can achieve $N_2(T-N_2)$ degrees of freedom. Thus the degree of freedom pair $(\frac{N_1(T-N_1)}{T}, \frac{N_2(T-N_2)}{T})$ can be achieved.

2) $N_1 \geq r_1^*$ and $N_2 \leq r_2^*$: Generate submatrix $\mathbf{V}_2^*$ by selecting N_2 columns from $\mathbf{V}_2$. Define the matrix $\mathbf{V} = [\mathbf{V}_1 \ \mathbf{V}_2^*]$. The transmitted signal is:

$$\mathbf{X} = \sqrt{\rho}\mathbf{V}(\mathbf{V}^H\mathbf{V})^{-1}\bar{\mathbf{X}}, \tag{45}$$

where $\mathbf{X}$ has the following structure:

$$\bar{\mathbf{X}} = \begin{bmatrix} \mathbf{I}_{r_1^*} & \mathbf{X}_{\delta,1} \\ \mathbf{I}_{N_2} & \mathbf{X}_{\delta,2} \end{bmatrix}, \tag{46}$$

$\mathbf{X}_{\delta,1} \in \mathbb{C}^{r_1^*\times(T-r_1^*)}$ and $\mathbf{X}_{\delta,2} \in \mathbb{C}^{N_2\times(T-N_2)}$ contains symbol intended for User 1 and User 2. Thus the degree of freedom pair $(\frac{r_1^*}{T}(T-r_1^*), \frac{N_2}{T}(T-N_2))$ is achievable.

The other corner point can be achieved via the product superposition transmit scheme. Generate submatrix $\mathbf{V}_0^*$ by selecting $r_0^* = N_1 - r_0$ columns from $\mathbf{V}_0$. Define the matrix $\mathbf{V} = [\mathbf{V}_1 \ \mathbf{V}_0^*]$. The transmitted signal is:

$$\mathbf{X} = \sqrt{\rho}\mathbf{V}(\mathbf{V}^H\mathbf{V})^{-1}\mathbf{X}_2\mathbf{X}_1, \tag{47}$$

where $\mathbf{X}_1$ and $\mathbf{X}_2$ has the following structure:

$$\mathbf{X}_1 = [\mathbf{I}_{r_1^*+r_0^*} \ \mathbf{X}_{\delta,1}], \tag{48}$$

$$\bar{\mathbf{X}} = \begin{bmatrix} \mathbf{I}_{r_0^*} & \mathbf{X}_{\delta,2} \\ 0 & \mathbf{I}_{r_1^*} \end{bmatrix}, \tag{49}$$

where $\mathbf{X}_{\delta,1} \in \mathbb{C}^{(r_1^*+r_0^*)\times(T-r_1^*-r_0^*)}$ and $\mathbf{X}_{\delta,2} \in \mathbb{C}^{N_2\times r_1^*)}$ contains symbol intended for User 1 and User 2. User 1 can achieve $(r_1^*+r_0^*)(T-r_1^*-r_0^*)$ degrees of freedom. User 2 achieves $\min(N_2, r_0^*)r_1^*$ degrees of freedom. Thus the degree of freedom pair $(\frac{(r_1^*+r_0^*)}{T}(T-(r_1^*+r_0^*)), \frac{\min(N_2,r_0^*)r_1^*}{T}$ is achievable.

3) $N_1 \leq r_1^*$ and $N_2 \geq r_2^*$: Choose N_1 columns from $\mathbf{V}_1$, generating the submatrix $\mathbf{V}_1^*$. Define the matrix $\mathbf{V} = [\mathbf{V}_1^* \ \mathbf{V}_2]$.

The transmitted signal is:

$$\mathbf{X} = \sqrt{\rho}\mathbf{V}(\mathbf{V}^H\mathbf{V})^{-1}\bar{\mathbf{X}}, \tag{50}$$

where $\mathbf{X}$ has the following structure:

$$\bar{\mathbf{X}} = \begin{bmatrix} \mathbf{I}_{N_1} & \mathbf{X}_{\delta,1} \\ \mathbf{I}_{r_2^*} & \mathbf{X}_{\delta,2} \end{bmatrix}, \tag{51}$$

where $\mathbf{X}_{\delta,1} \in \mathbb{C}^{n_1\times(T-r_1^*)}$ and $\mathbf{X}_{\delta,2} \in \mathbb{C}^{N_2\times(T-N_2)}$ contains symbol intended for User 1 and User 2. Thus the degree of freedom pair $(\frac{N_1}{T}(T-N_1), \frac{r_2^*}{T}(T-r_2^*))$ is achievable.

4) $N_1 \geq r_1^*$ and $N_2 \geq r_2^*$: Define $r_0^* = \min(N_1 - r_1^*, N_2 - r_2^*)$. Choose r_0^* columns from $\mathbf{U}_0$, generating the submatrix $\mathbf{V}_0^*$. Define the matrix $\mathbf{V} = [\mathbf{V}_1 \ \mathbf{V}_0^* \ \mathbf{V}_2]$. The transmitter sends the signal:

$$\mathbf{X} = \mathbf{V}(\mathbf{V}^H\mathbf{V})^{-1}\bar{\mathbf{X}}, \tag{52}$$

where $\bar{\mathbf{X}}$ has the structure.

$$\bar{\mathbf{X}} = \begin{bmatrix} \mathbf{I}_{r_1^*} & 0 & \mathbf{X}_{\delta,1} \\ 0 & \mathbf{I}_{r_0} & \mathbf{X}_{\delta,0} \\ \mathbf{I}_{r_2^*}\ \mathbf{X}_2 & 0 & \mathbf{X}_{\delta,2}, \end{bmatrix} \tag{53}$$

where $\mathbf{X}_{\delta,1} \in \mathbb{C}^{r_1^*\times(T-r_1^*-r_0^*)}$ and $\mathbf{X}_{\delta,2} \in \mathbb{C}^{r_2^*\times(T-r_1^*-r_0^*)}$ contains symbols intended for User 1 and User 2, $\mathbf{X}_{\delta,0} \in \mathbb{C}^{{r_0}^{ast}\times(T-r_1^*-r_0^*)}$ contains symbol that both User 1 and User 2 can decode. $\mathbf{X}_2 \in \mathbb{C}^{r_2^*\times(r_1^*-r_2^*)}$ contains symbols intended for User 2 only. $(\frac{(r_1^*+r_0^*)}{T}(T-r_1^*-r_0^*), \frac{1}{T}(r_2^*(r_1^*-r_2^*)+r_2^*(T-r_1^*-r_0^*)))$ and $(\frac{r_1^*}{T}(T-r_1^*-r_0^*), \frac{1}{T}(r_2^*(r_1^*-r_2^*)+(r_2^*+r_0^*)(T-r_1^*-r_0^*)))$ are achievable via time sharing. Use the product superposition transmit scheme. Make User 1 achieving its single user bound. Define the matrix $\mathbf{V} = [\mathbf{V}_1 \ \mathbf{V}_0^*]$ The transmitted signal is:

$$\mathbf{X} = \sqrt{\rho}\mathbf{V}(\mathbf{V}^H\mathbf{V})^{-1}\mathbf{X}_2\mathbf{X}_1, \tag{54}$$

where $\mathbf{X}_1$ and $\mathbf{X}_2$ has the following structure:

$$\mathbf{X}_1 = [\mathbf{I}_{r_1^*+r_0^*} \ \mathbf{X}_{\delta,1}], \tag{55}$$

$$\bar{\mathbf{X}} = \begin{bmatrix} \mathbf{I}_{r_0^*} & \mathbf{X}_{\delta,2} \\ 0 & \mathbf{I}_{r_1^*} \end{bmatrix}, \tag{56}$$

where $\mathbf{X}_{\delta,1} \in \mathbb{C}^{(r_1^*+r_0^*)\times(T-r_1^*-r_0^*)}$ and $\mathbf{X}_{\delta,2} \in \mathbb{C}^{r_0^*\times r_1^*}$ contains symbol intended for User 1 and User 2. Thus this product superposition achieves the degree of freedom pair $(\frac{(r_1^*+r_0^*)}{T}(T-r_1^*-r_0^*), \frac{r_0^* r_1^*}{T})$. In the same way, if we make User 2 achieving its single user bound, we can achieve the degree of freedom pair $(\frac{r_0^* r_2^*}{T}, \frac{(r_2^*+r_0^*)}{T}(T-r_2^*-r_0^*))$.

IV. Numerical results

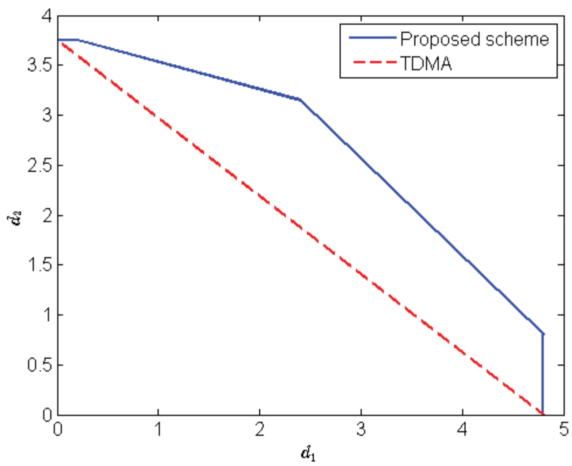

Fig. 1. Degrees of freedom region with $r_2^* = 1$

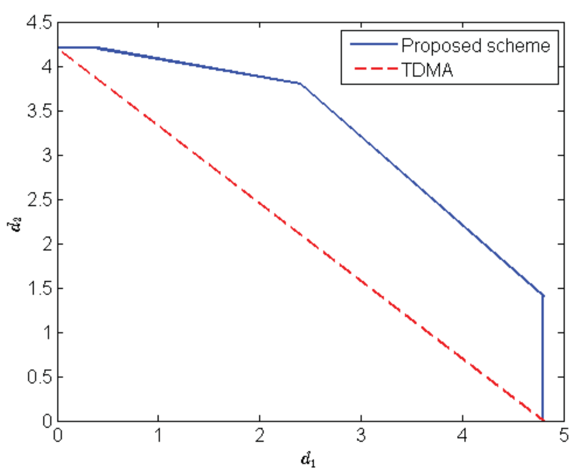

Fig. 2. Degrees of freedom region with $r_2^* = 2$

In this section, we compare the proposed scheme with TDMA transmission demonstrating the two-user achievable degrees of freedom region for different r_2^*, the dimension of the correlation matrix of User 2. We consider the case of $M = N_1 = N_2 = 10$ and $T = 20$. In Fig. 1, $r_1^* = 1$, the corner points (4.8,0.75) is achieved via pre-beamforming proposed in this paper and in Fig. 2, $r_2^* = 2$, the corner points (4.8,0.8) is achieved via product superposition. The corner points is achieved by pre-beamforming or product superposition depending on the coherence time and the dimension of the eigenspace. Comparing with the dimension of the eigenspace, when the coherence time is long, pre-beamforming has more gains, otherwise product superposition provides more achievable degree of freedom.

V. Conclusion

It has been shown that Correlation at the transmitter side can be beneficial in various scenarios in MIMO broadcast channel. In this paper, we study the case where the transmitter serves two users with partially overlapped eigenspaces of the channel correlation matrices. The proposed pilot-based signaling achieves degrees of freedom gains over TDMA transmission. This scheme transmits the pre-beamformed signal and allows them to decode their own symbol successfully. We also combined this pre-beamforming scheme with the product superposition technique, obtaining new achievable degree of freedom region. This achievable scheme is also promising to be extended to broadcast channel with multiple users.

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