Sensitivity Analysis of Convergence Bids in Nodal Electricity Markets

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***Abstract*—Recent reports from Independent System Operators (ISOs) have raised some concerns about the impact of convergence bids (CBs) on nodal electricity markets. In particular, in some cases, there are concerns about cases where CBs are profitable for some market participants without increasing market efficiency significantly or even decreasing market efficiency. The latter occurs when CBs create price divergence instead of price convergence across the day-ahead and real-time markets. Accordingly, in this paper, we investigate the sensitivity of nodal electricity market price to CBs and seek to build an analytical foundation to explain under what conditions placing a CB at a bus in a nodal electricity market can create price divergence at that bus. Illustrative test cases are discussed to provide intuitions and engineering implications of the results on sensitivity analysis.**

***Keywords*: Convergence bidding, virtual bidding, nodal electricity markets, transmission line congestion, price sensitivity.**

NOMENCLATURE

DAM parameters and variables:

$\mathbf{x}$, $\mathbf{y}$	Vectors of physical supply and demand bids
$\mathbf{v}$, $\mathbf{w}$	Vectors of supply and demand CBs
$\mathbf{p}$	Vector of all bids of all types
$\mathbf{\Phi}$, $\mathbf{\Psi}$	Incidence matrices for $\mathbf{p}$ and $\mathbf{x}$ to system buses
$\mathbf{K}$	Incidence matrix for $\mathbf{p}$ to $\mathbf{x}$
$\boldsymbol{\pi}$, $\boldsymbol{\mu}$, λ	Locational, shadow, and reference prices
$\bar{\mathbf{D}}$	Index matrix for congested transmission lines

RTM parameters and variables:

$\mathbf{z}$	Vector of physical supply bids
$\boldsymbol{l}$	Vector of actual demands at time of operation
$\mathbf{\Theta}$, $\mathbf{\Omega}$	Incidence matrices for $\mathbf{z}$ and $\boldsymbol{l}$ to system buses
$\boldsymbol{\sigma}$, $\boldsymbol{\eta}$, δ	Locational, shadow, and reference prices
$\bar{\mathbf{R}}$	Index matrix for congested transmission lines

System parameters:

$\mathbf{S}$	Shift factor matrix of the power grid
$\mathbf{c}$	Vector of transmission line capacities
$\mathbf{\Delta}$	Vector of price differences: $\boldsymbol{\pi} - \boldsymbol{\sigma}$
α, β	Coefficients of the cost or utility functions

I. INTRODUCTION

Wholesale electricity markets in North America and elsewhere are often set up as two-settlement markets, with a day-ahead market (DAM) and a real-time market (RTM), e.g., see [1]–[4]. Ideally, and to assure market efficiency, there must be no difference between the prices in the DAM and the RTM. Otherwise, some generation resources may practice market power and withhold a portion of their capacities to increase the DAM or RTM prices to gain more profit [3], [5]–[7].

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Nevertheless, in practice, there is always a *gap* between the two sets of prices. For example, Fig. 1(b) shows the distribution of the price difference in trading hub SP15 in Southern California across 24 hours and 30 days in March 2016 [8]. Here, the price difference is calculated as the DAM price minus the RTM price. There are several days and hours (such as 2 PM on March 14) where the DAM price is much higher than the RTM price and there are also several days and hours (such as 9 AM on March 14) where the RTM price is much higher than the DAM price. Fig. 1(c) shows similar data at two nodes within SP15 on March 8 and 14. We can see that price gap can be less or more severe at different nodes due to locational issues such as transmission line congestion.

A. Convergence Bidding

To eliminate the above price gap, Convergence bids (CBs), a.k.a., Virtual bids (VBs), have been introduced to electricity markets [5]–[7]. Note that, CB is the term that is used by the California ISO and VB is the term that is used by the Pennsylvania-Jersey-Maryland (PJM) Interconnection and some other ISOs. CBs allow market participants to arbitrage between the DAM and RTM, exempting them from physically consuming or producing energy [6]. CBs are similar to what is known as future trading in traditional commodity and financial markets [5]. Similar to physical bids, CBs have two types: supply CBs and demand CBs. Supply (demand) CB is a bid to sell (buy) energy in DAM without any obligation to produce (consume) energy. If the CB is cleared in the DAM, then the bidder is credited (charged) at the DAM price and charged (credited) at the RTM price. Therefore, the difference between the earning in the DAM (RTM) and the cost in the RTM (DAM) will be the payment to the CB bidder.

From an ISOs perspective, if participants make profit through CBs, it should automatically help closing the price gap [9]. For example, when DAM price is greater (less) than the RTM price, the participants can make profit by submitting supply (demand) CB into DAM. Increasing supply (demand) CBs results in decreasing (increasing) the DAM price due to the virtual surplus of supply (demand) in the DAM. As a result, more (less) demand needs to be cleared in the RTM leading to increase (decrease) in the RTM price [5], [9], [10]. Therefore, while market participants make profit out of their CBs, they also help in reducing the price difference between the DAM and RTM; thus, solving the aforementioned price gap problem.

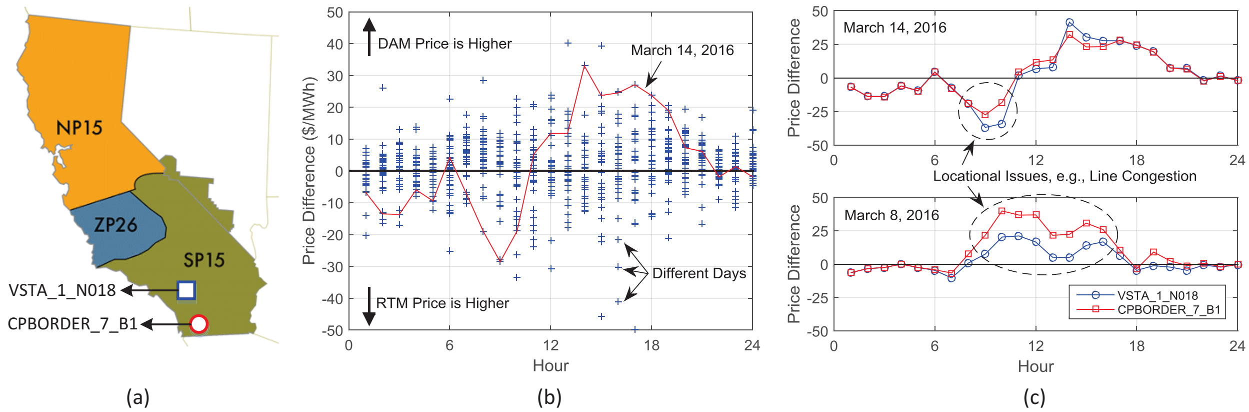


Fig. 1. Examples of the price gap, i.e., the DAM price minus the RTM price, in the California ISO market during March 2016: (b) the full month for trading hub SP15 in Southern California; (c) two sample days at two nodes within SP15.

B. Motivation

The concept of CBs is relatively new in the ISO markets. For example, California ISO put CBs into effect in 2011 [11]. So far, most ISOs have adopted this concept with almost no rudimentary changes from traditional commodity and financial markets [10]. However, there are recent ISO reports raising some concerns about CBs, arguing that CBs may not have performed well in ISO markets and it is generally difficult for ISOs to even analyze how CBs may have actually affected price convergence and market efficiency in ISO markets. For example, here is a related quote from the California ISO 2015 Annual Report on Market Issues and Performance [12]:

> "However, the degree to which convergence bidding has actually increased market efficiency has not been assessed. In some cases, virtual bidding may be profitable for some market participants without increasing market efficiency significantly or even decreasing market efficiency."

Here is another quote from the PJM Interconnection 2015 Report on Virtual Transactions in the Energy Markets [10]:

> "In considering when and to what degree virtual trading offers benefits to PJM markets, it is important to account for these distinctions before definitively concluding that the generally accepted principles of market efficiency as demonstrated by trading in other financial and commodity marketplaces hold equally well to PJMs energy markets."

The above citations and quotes exemplify the current state of uncertainty and debate about the advantages or disadvantages of CBs in nodal electricity markets. With this in mind, our study seeks to address the above open problem by analyzing how a CB may affect the price gap between DAM and RTM.

C. Literature Review

The literature on CBs in *non-electricity* markets is rich, c.f. [13]–[15]. However, the literature on CBs in *electricity* markets has emerged only recently, and there is limited studies on addressing the issues related to CBs in this market. The common approach so far has been to use *historical market data* from different ISOs to conduct *statistical analysis* on market prices to show that, on average, CBs do help with price convergence over months/years long-term [2], [16]–[18]; however, as we discussed in Section I.B, the California ISO and PJM are currently skeptical about the benefits of CBs, e.g., see Section I.B. Also, it is yet to be investigated how CBs may affect the price gap at each market operation time.

As for the few studies that take a rather analytical approach to CBs, so far, most of them have focused on cases where the CBs are somewhat *abused*, either by a market player, e.g., when submitted strategically in conjunction with Financial Transmission Rights (FTR)s [9], [19], or by an adversary, e.g., in a cyber-physical attack [20]. As another example a data-driven approach combined with a game-theoretic analysis was done in [21], In contrast, in this paper, the focus is on investigating CBs when they are used as intended, yet they may demonstrate counter-intuitive results.

There are a few recent studies that have pointed out the complexities around CBs in electricity markets and the fact that CBs in electricity markets cannot be evaluated in the same way that they are often assessed in other markets [10]–[12]. However, so far, no prior study has provided any analytical method to explain such complexities and their root causes.

D. Summary of Contributions

In this paper, we focus on one of the primary factors that influences the performance of CBs in electricity markets, i.e., transmission line congestion. Our goal is to provide in-depth sensitivity analysis to understand how the price gap between DAM and RTM is affected by the CBs under different grid operational conditions in congested nodal electricity markets. Our analysis is *not* statistical; thus, it is inherently different from the existing literature on CBs in electricity markets, e.g., in [2], [16]–[18]. Instead, we look at the basic formulation of CBs in nodal electricity markets and obtain *closed-form* sensitivity models to explain how a CB may influence the DAM and RTM prices at a bus where it is cleared. Finally, built upon the fundamental sensitivity analysis, several case studies are presented to show that the impact of CBs in the nodal electricity market may cause price convergence (intuitive result) or price divergence (counter-intuitive result).

II. Sensitivity Analysis of Two-Settlement Market Prices to Convergence Bids

In this section, we investigate the sensitivity of the DAM and RTM prices to CBs in order to understand how CBs may influence the price difference in a two-settlement nodal electricity market.

A. Electricity Market Model:

Consider the following DAM market clearing optimization problem in presence of convergence bids [2], [6]:

$$\begin{aligned} \textbf{min} \quad & 0.5\,\mathbf{p}^T\mathbf{A}\mathbf{p} + \mathbf{b}^T\mathbf{p} && \text{(1a)}\\ \textbf{s.t.} \quad & \mathbf{1}^\mathrm{T}\,\mathbf{p} = 0 \qquad\qquad : \lambda && \text{(1b)}\\ & -\mathbf{c} \le \mathbf{S}\,\boldsymbol{\Phi}\,\mathbf{p} \le \mathbf{c} \quad : \boldsymbol{\mu}^-, \boldsymbol{\mu}^+ && \text{(1c)}\\ & \mathbf{p}^{\min} \le \mathbf{p} \le \mathbf{p}^{\max} && \text{(1d)} \end{aligned}$$

where the optimization variables are

$$\mathbf{p} \triangleq \begin{bmatrix}\mathbf{x} & \mathbf{y} & \mathbf{v} & \mathbf{w}\end{bmatrix}^\mathrm{T}. \tag{2}$$

In (1), $\mathbf{A}$ is a positive diagonal matrix comprising α and $-\alpha$ components of all supply and demand bids in the DAM, respectively. Both physical and convergence bids are taken into consideration. Moreover, $\mathbf{b}$ is the vector comprising of β and $-\beta$ components of all supply and demand bids, respectively. Equality (1b) represents the system balance constraint ensuring total generation matches total load. Also, the Lagrange multiplier associated with (1b) provides the reference price. The transmission line flow limit constraints in two directions are expressed in (1c). Also, the Lagrange multipliers associated with (1c) indicate the shadow prices. The last inequality in (1d) expresses each participant's upper and lower operating capacity limits. Finally, by using the reference and shadow prices, the LMPs can be obtained as

$$\boldsymbol{\pi} = \lambda\mathbf{1} - \mathbf{S}^T\boldsymbol{\mu}, \qquad \text{where } \boldsymbol{\mu} = \boldsymbol{\mu}^+ - \boldsymbol{\mu}^-. \tag{3}$$

The RTM market clearing optimization problem can also be formulated as [2], [6]:

$$\begin{aligned} \textbf{min} \quad & 0.5\,\mathbf{z}^T\mathbf{C}\,\mathbf{z} + \mathbf{d}^T\mathbf{z} && \text{(4a)}\\ \textbf{s.t.} \quad & \mathbf{1}^T\mathbf{z} + \mathbf{1}^T\mathbf{x} = \mathbf{1}^T\boldsymbol{l} \qquad : \delta && \text{(4b)}\\ & -\mathbf{c} \le \mathbf{S}(\boldsymbol{\Psi}\,\mathbf{x} + \boldsymbol{\Theta}\,\mathbf{z} - \boldsymbol{\Omega}\,\boldsymbol{l}) \le \mathbf{c} \quad : \boldsymbol{\eta}^-, \boldsymbol{\eta}^+ && \text{(4c)}\\ & \mathbf{z}^{\min} \le \mathbf{z} \le \mathbf{z}^{\max} && \text{(4d)} \end{aligned}$$

where the optimization variables are the elements of vector $\mathbf{z}$. Similar to the DAM LMPs, the RTM LMPs are obtained by using the Lagrange multipliers in (4b) and (4c) as

$$\boldsymbol{\sigma} = \delta\mathbf{1} - \mathbf{S}^T\boldsymbol{\eta}, \qquad \text{where } \boldsymbol{\mu} = \boldsymbol{\eta}^+ - \boldsymbol{\eta}^-. \tag{5}$$

We must note two key differences between (1) and (4). First, demand bids are not allowed at the RTM, instead, ISOs use the forecasted load as constant at problem (4), c.f. [22]. Second, as in practice, the RTM clearing process is based on only physical bids but *not* CBs [2], [6]. Note that, even though CBs do not appear in the RTM optimization in (4), because they *do* affect the cleared physical supply bids in the DAM i.e. $\mathbf{x}$, they indirectly have impact on the LMPs of the RTM.

B. Closed-Form Sensitivity Analysis

We are now ready to present a formal theorem to explain how the cleared energy of a CB can affect price difference between the DAM and RTM at the bus where the CB is placed.

Theorem 1. *Consider a CB at bus i. Without loss of generality, suppose it is a supply CB, whose cleared energy bid is denoted by $\boldsymbol{v}_i$. (a) The price gap $\boldsymbol{\Delta}_i = \boldsymbol{\pi}_i - \boldsymbol{\sigma}_i$ at bus i is a piecewise linear function of the cleared CB ($\boldsymbol{v}_i$). (b) The slope of such function, i.e., the right-sided partial derivative, is obtained as*

$$\begin{aligned} \frac{\partial \boldsymbol{\Delta}_i}{\partial \boldsymbol{v}_i} &= \frac{\partial \boldsymbol{\pi}_i}{\partial \boldsymbol{v}_i} - \frac{\partial \boldsymbol{\sigma}_i}{\partial \boldsymbol{v}_i}\\ &= \frac{-1}{\boldsymbol{1}^T\boldsymbol{h}} - \frac{1}{\boldsymbol{1}^T\boldsymbol{e}}\frac{1}{\boldsymbol{1}^T\boldsymbol{h}}(\boldsymbol{1}^T\hat{\boldsymbol{K}}\boldsymbol{h} - \boldsymbol{r}\hat{\boldsymbol{K}}\boldsymbol{h}) \qquad (6)\\ &= -\frac{1}{\boldsymbol{1}^T\boldsymbol{h}}\frac{1}{\boldsymbol{1}^T\boldsymbol{e}}(\boldsymbol{1}^T\boldsymbol{e} + \boldsymbol{1}^T\hat{\boldsymbol{K}}\boldsymbol{h} - \boldsymbol{r}\hat{\boldsymbol{K}}\boldsymbol{h}), \end{aligned}$$

where

$$\boldsymbol{h} \triangleq \boldsymbol{\Lambda 1} - \boldsymbol{\Lambda X}^T(\boldsymbol{X\Lambda X}^T)^{-1}\boldsymbol{X\Lambda 1}, \tag{7}$$

$$\boldsymbol{e} \triangleq \boldsymbol{\Gamma 1} - \boldsymbol{\Gamma Y}^T(\boldsymbol{Y\Gamma Y}^T)^{-1}\boldsymbol{Y\Gamma 1}, \tag{8}$$

$$\boldsymbol{r} \triangleq \boldsymbol{1}^T\boldsymbol{\Gamma Y}(\boldsymbol{Y\Gamma Y}^T)^{-1}\bar{\boldsymbol{R}}\boldsymbol{S}\hat{\boldsymbol{\Psi}}, \tag{9}$$

and $\boldsymbol{\Lambda}$, $\boldsymbol{\Gamma}$, $\hat{\boldsymbol{K}}$, $\boldsymbol{X}$, $\boldsymbol{Y}$, $\bar{\boldsymbol{R}}$, and $\hat{\boldsymbol{\Psi}}$ are constant matrices that depend on cleared bids and admittance and congestion status of lines.

Note that, if the CB is a demand bid, then we can replace $\mathbf{v}_i$ with $-\mathbf{w}_i$ in (6). The proof of Theorem 1 is as follows.

Proof. Suppose bus i is taken as the reference bus, the price gap at bus i is obtained as $\boldsymbol{\Delta}_i = \lambda - \delta$. Let $\mathbf{v}_{-i}$ denote the set of all supply CBs other than $\mathbf{v}_i$. We can now define:

$$\mathbf{p}_{-i} \triangleq \begin{bmatrix}\mathbf{x} & \mathbf{y} & \mathbf{v}_{-i} & \mathbf{w}\end{bmatrix}^\mathrm{T} \tag{10}$$

as the optimal solution of all variables in (1) other than $\mathbf{v}_i$. We also define $\mathbf{A}_{-i}$, $\mathbf{b}_{-i}$, $\mathbf{p}_{-i}^{\min}$, $\mathbf{p}_{-i}^{\max}$, and $\boldsymbol{\Phi}_{-i}$ by removing row i and/or column i from $\mathbf{A}$, $\mathbf{b}$, $\mathbf{p}^{\min}$, $\mathbf{p}^{\max}$, and $\boldsymbol{\Phi}$.

Let us now decompose vector $\mathbf{p}_{-i}$ into vector $\bar{\mathbf{p}}_{-i}$ for entries that are *binding* by any of the two inequality constraints in (1d) and vector $\hat{\mathbf{p}}_{-i}$ for entries that are *not* binding by either of these two constraints. Similarly, we define $\bar{\mathbf{A}}_{-i}$, $\hat{\mathbf{A}}_{-i}$, $\bar{\mathbf{b}}_{-i}$, $\hat{\mathbf{b}}_{-i}$, $\bar{\mathbf{p}}_{-i}^{\min}$, $\hat{\mathbf{p}}_{-i}^{\min}$, $\bar{\mathbf{p}}_{-i}^{\max}$, $\hat{\mathbf{p}}_{-i}^{\max}$, $\bar{\boldsymbol{\Phi}}_{-i}$, and $\hat{\boldsymbol{\Phi}}_{-i}$. We also decompose vector $\boldsymbol{\mu}$ into vector $\bar{\boldsymbol{\mu}}$ for the Lagrange multipliers corresponding to the *binding* constraints in (1c). Let $\bar{\mathbf{D}}$ denote a row-reduced identity matrix, i.e., an identity matrix with the same size of matrix $\mathbf{S}$ whose rows that correspond to the non-binding transmission line capacity constraints are eliminated. Finally, we define $\hat{\boldsymbol{\mu}}$ as the Lagrange multipliers which are *not* binding by any of the transmission line capacity constraints. Note that, due to complimentary slackness, we have $\hat{\boldsymbol{\mu}} = \mathbf{0}$. Using convex optimization theory [23, Chapter 4], we can show that problem (1) is *equivalent* to the following problem:

$$\begin{aligned} \min_{\hat{\mathbf{p}}_{-i}} \quad & 0.5\,\hat{\mathbf{p}}_{-i}^T\,\hat{\mathbf{A}}_{-i}\,\hat{\mathbf{p}}_{-i} + \hat{\mathbf{b}}_{-i}^T\,\hat{\mathbf{p}}_{-i} && \text{(11a)}\\ \textbf{s.t.} \quad & \mathbf{1}^T\hat{\mathbf{p}}_{-i} + \mathbf{1}^T\bar{\mathbf{p}}_{-i} + \mathbf{v}_i = 0 \qquad : \lambda && \text{(11b)}\\ & \bar{\mathbf{D}}\mathbf{S}\,(\hat{\boldsymbol{\Phi}}_{-i}\,\hat{\mathbf{p}}_{-i} + \bar{\boldsymbol{\Phi}}_{-i}\,\bar{\mathbf{p}}_{-i}) = \bar{\mathbf{D}}\mathbf{c} \quad : \bar{\boldsymbol{\mu}}. && \text{(11c)} \end{aligned}$$

Here, $\mathbf{v}_i$ and $\bar{\mathbf{p}}_{-i}$ are fixed at their optimal values but $\hat{\mathbf{p}}_{-i}$ is variable. The objective function includes only those terms that depend on $\hat{\mathbf{p}}_{-i}$. Since bus i is the reference bus, $\mathbf{S\Phi p} = \mathbf{S\Phi}_{-i}\mathbf{p}_{-i}$. Also, we kept only those line capacity constraints that are binding at the optimal solution of problem (1).

Since (11) is a convex quadratic program, it can be solved by equivalently solving the following system of linear equations, namely the KKT conditions [23], over $\hat{\mathbf{p}}_{-i}$, λ and $\bar{\boldsymbol{\mu}}$, as follow:

$$\boldsymbol{\Lambda}^{-1}\hat{\mathbf{p}}_{-i} + \hat{\mathbf{b}}_{-i} = \begin{bmatrix}\mathbf{1}^T\\ -\mathbf{X}\end{bmatrix}^T \begin{bmatrix}\lambda\\ \bar{\boldsymbol{\mu}}\end{bmatrix} \tag{12a}$$

$$\begin{bmatrix}\mathbf{1}^T\\ \mathbf{X}\end{bmatrix}\hat{\mathbf{p}}_{-i} = \mathbf{n} - \begin{bmatrix}1\\ \mathbf{0}\end{bmatrix}\mathbf{v}_i. \tag{12b}$$

where

$$\mathbf{X} \triangleq \bar{\mathbf{D}}\mathbf{S}\hat{\boldsymbol{\Phi}}_{-i}, \quad \boldsymbol{\Lambda} \triangleq \hat{\mathbf{A}}_{-i}^{-1}, \quad \mathbf{n} \triangleq \begin{bmatrix}-\mathbf{1}^T\bar{\mathbf{p}}_{-i}\\ \bar{\mathbf{D}}\mathbf{c} - \bar{\mathbf{D}}\mathbf{S}\,\bar{\boldsymbol{\Phi}}_{-i}\,\bar{\mathbf{p}}_{-i}\end{bmatrix}. \tag{13}$$

The coefficients in (12) hold as long as the set of binding constraints do not change at the solution of problem (1). If a binding constraint becomes unbinding or an unbinding constraint becomes binding, then some or all matrices $\boldsymbol{\Lambda}$, $\hat{\mathbf{b}}_{-i}$, $\mathbf{X}$, and $\mathbf{n}$ may change, but keeping the relationship between variables, i.e., λ and $\mathbf{v}_i$, linear. Thus, the overall relationship is piecewise linear. From (12) and (7), we have:

$$\partial\lambda/\partial\mathbf{v}_i = -1/\mathbf{1}^T\mathbf{h}. \tag{14}$$

The analysis of the RTM prices is similar. We can first show that problem (4) is *equivalent* to the following problem:

$$\min_{\hat{\mathbf{z}}} \quad 0.5\,\hat{\mathbf{z}}^T\hat{\mathbf{C}}\,\hat{\mathbf{z}} + \hat{\mathbf{d}}^T\hat{\mathbf{z}} \tag{15a}$$

$$\text{s.t.} \quad \mathbf{1}^T\hat{\mathbf{z}} + \mathbf{1}^T\bar{\mathbf{z}} + \mathbf{1}^T\hat{\mathbf{x}} + \mathbf{1}^T\bar{\mathbf{x}} = \mathbf{1}^T l \qquad :\delta \tag{15b}$$

$$\bar{\mathbf{R}}\mathbf{S}(\bar{\boldsymbol{\Psi}}\,\bar{\mathbf{x}} + \hat{\boldsymbol{\Psi}}\,\hat{\mathbf{x}} + \bar{\boldsymbol{\Theta}}\,\bar{\mathbf{z}} + \hat{\boldsymbol{\Theta}}\,\hat{\mathbf{z}} - \boldsymbol{\Omega}\,l) = \bar{\mathbf{R}}\mathbf{c} \quad :\bar{\boldsymbol{\eta}}, \tag{15c}$$

where $\hat{\mathbf{x}} = \hat{\mathbf{K}}\hat{\mathbf{p}}_{-i}$ and $\bar{\mathbf{x}} = \bar{\mathbf{K}}\bar{\mathbf{p}}_{-i}$. Again, since (15) is a convex quadratic program, we can solve it by equivalently solving its corresponding KKT conditions [23], which in this case are a system of linear equations over $\hat{\mathbf{z}}$, δ and $\bar{\boldsymbol{\eta}}$:

$$\boldsymbol{\Gamma}^{-1}\hat{\mathbf{z}} + \hat{\mathbf{d}} = \begin{bmatrix}\mathbf{1}^T\\ -\mathbf{Y}\end{bmatrix}^T \begin{bmatrix}\delta\\ \bar{\boldsymbol{\eta}}\end{bmatrix} \tag{16a}$$

$$\begin{bmatrix}\mathbf{1}^T\\ \mathbf{Y}\end{bmatrix}\hat{\mathbf{z}} = \mathbf{m} - \begin{bmatrix}\mathbf{1}^T\\ \bar{\mathbf{R}}\mathbf{S}\hat{\boldsymbol{\Psi}}\end{bmatrix}\hat{\mathbf{K}}\hat{\mathbf{p}}_{-i} \tag{16b}$$

where $\mathbf{Y} \triangleq \bar{\mathbf{R}}\mathbf{S}\hat{\boldsymbol{\Theta}}$, $\boldsymbol{\Gamma} = \hat{\mathbf{C}}^{-1}$, and $\mathbf{m}$ is defined as

$$\mathbf{m} \triangleq \begin{bmatrix}\mathbf{1}^T l - \mathbf{1}^T\bar{\mathbf{z}} - \mathbf{1}^T\bar{\mathbf{K}}\bar{\mathbf{p}}_{-i}\\ \bar{\mathbf{R}}\mathbf{S}(\Omega l - \bar{\boldsymbol{\Psi}}\bar{\mathbf{K}}\bar{\mathbf{p}}_{-i} - \bar{\boldsymbol{\Theta}}\,\bar{\mathbf{z}})\end{bmatrix}. \tag{17}$$

Finally, by obtaining $\hat{\mathbf{p}}_{-i}$ as a function of $\mathbf{v}_i$ from (12), and using the KKT conditions of RTM in (16), the sensitivity of δ with respect to $\mathbf{v}_i$ can be obtained:

$$\frac{\partial\delta}{\partial\mathbf{v}_i} = \frac{\partial\delta}{\partial\hat{\mathbf{p}}_{-i}}\cdot\frac{\partial\hat{\mathbf{p}}_{-i}}{\partial\mathbf{v}_i} = \frac{1}{\mathbf{1}^T\mathbf{e}}\frac{1}{\mathbf{1}^T\mathbf{h}}(\mathbf{1}^T\hat{\mathbf{K}}\mathbf{h} - \mathbf{r}\hat{\mathbf{K}}\mathbf{h}) \tag{18}$$

where $\mathbf{e}$ and $\mathbf{r}$ are defined in (8) and (9). Note that, the coefficient in (18) depends on the set of binding constraints in not only the RTM optimization problem in (4) but also the DAM optimization problem in (1). If a binding constraint becomes unbinding or an unbinding constraint becomes binding, then some or all vectors $\mathbf{e}$, $\mathbf{h}$, and $\mathbf{r}$ may change, but keeping the relationship between δ and $\mathbf{v}_i$ linear.

TABLE I
GENERATORS BIDS PARAMETERS

			Bids					
			α_1	β_1	α_2	β_2	α_3	β_3
Scenario	1	DAM	0.1	8	-	-	0.3	10
		RTM	0.7	2	1.7	3	1.9	4
	2	DAM	0.1	8	-	-	0.3	10
		RTM	0.7	2	1.7	3	1.9	4
	3	DAM	0.1	8	-	-	0.3	10
		RTM	0.7	2	1.7	3	**0.1**	**9**

Since both λ and δ are piecewise linear function of $\mathbf{v}_i$, their difference, i.e., $\boldsymbol{\Delta}_i$ is also a piecewise linear function of $\mathbf{v}_i$. The slope of such function is derived as in (6) by subtracting (18) from (14). This concludes the proof. □

The above theorem explains how a CB may change the price difference between the DAM and RTM of the bus where it is placed. Given the sensitivity model for price gap in (6), can ISOs guarantee that a profitable CB helps the system efficiency by closing the price gap under different grid operational conditions? First, what ISOs expect from the sensitivity of the price gap needs to be understood. Recall from Section I that ISOs assume that increasing a supply (demand) CB at a bus decreases (increases) the DAM price and increases (decreases) the RTM price at that bus. In fact, ISOs believe that

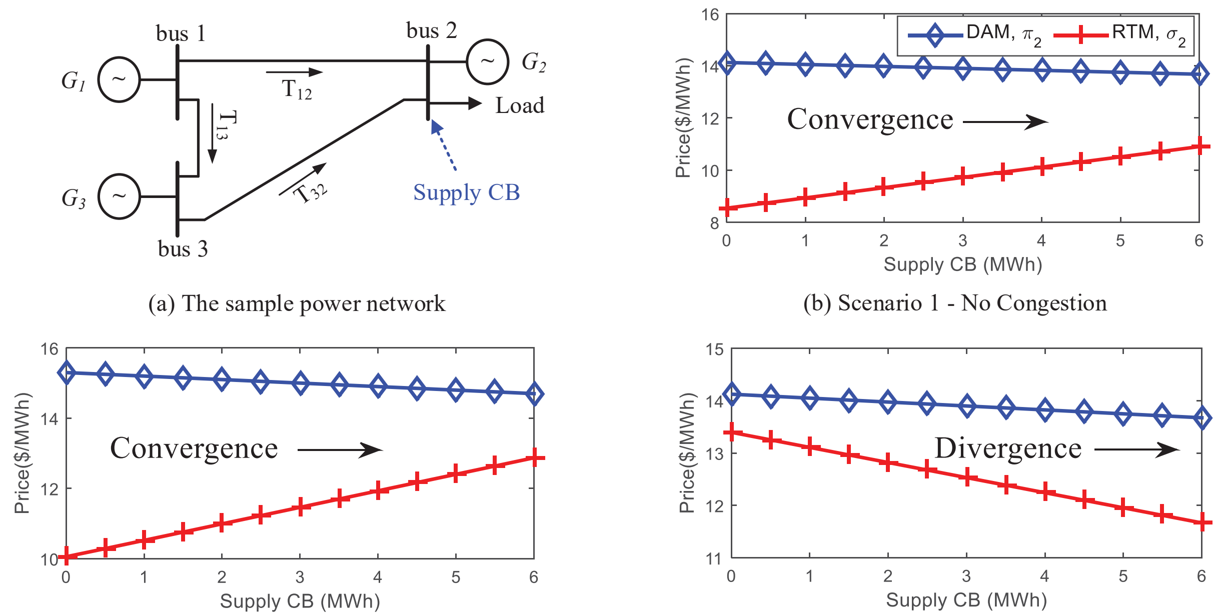
$$\frac{\partial\boldsymbol{\Delta}_i}{\partial\mathbf{v}_i} = \frac{\partial\boldsymbol{\pi}_i}{\partial\mathbf{v}_i} - \frac{\partial\boldsymbol{\sigma}_i}{\partial\mathbf{v}_i} < 0 \tag{19}$$

Therefore, if $\boldsymbol{\Delta}_i > 0$, the market participants can earn profit by submitting supply CBs; on the other hand, from (19), the supply CBs close the price gap among DAM and RTM. The same argument can be done when $\boldsymbol{\Delta}_i < 0$ and the demand CBs are submitted. However, as we show in Section III, this argument does not always hold in nodal electricity markets. In fact, the impact of a CB on the price gap of the bus where it is placed depends on the coefficients of the piece-wise linear functions in Theorem 1. Indeed, under each network operating condition; depending on the coefficients in (6); placing a CB may enforce convergence (desirable) or divergence (undesirable) of the DAM and RTM prices. Accordingly, compared to the impact of CBs in financial markets, the impact of CBs in nodal electricity markets is much more complicated. Unfortunately, it appears that the CB-related studies were not aware of such complex issues, and they could not address the concerns raised by ISOs on CBs performance, as we pointed out in Section I.B.

III. CASE STUDIES

In this section, we discuss a few illustrative examples to demonstrate the fundamental concepts that our proposed analysis can help explain. Consider the three-bus power network in Fig. 2(a). Generators G_1 and G_3 participate in both the DAM and RTM, while generator G_2 participates only in the RTM. All generators have quadratic cost functions in form of $0.5\alpha_i\mathbf{x}_i^2 + \beta_i\mathbf{x}_i$, and their values are shown in Table I. The reactance for all transmission lines is 0.1 Ohm. The resistance is negligible. The load at bus 2 procures 75 MWh from the DAM. Its actual load is realized as 90 MWh at the RTM.

Three scenarios are studied under different grid conditions and transmission line capacities. The scenarios are as follow:



(a) The sample power network

(b) Scenario 1 - No Congestion

(c) Scenario 2 – Congestion at both DAM and RTM

(d) Scenario 3 – Congestion only at RTM

Fig. 2. An example in a three-bus network to illustrate the intuitive (convergence) and counter-intuitive (divergence) results of convergence bidding.

TABLE II
LINE FLOWS WITHOUT CBS

			Line flow (MW)		
			T_{12}	T_{13}	T_{32}
Scenario	1	DAM	45.4	15.8	29.6
		RTM	52.4	18.2	34.3
	2	DAM	41.5	**8**	33.5
		RTM	46.9	**8**	38.9
	3	DAM	45.4	15.8	29.6
		RTM	**50.0**	16.1	33.9

Scenario 1: The transmission lines have sufficiently large capacity, such that no transmission line can be congested. If no CB is placed to the market, then the cleared market prices are $\boldsymbol{\pi}_1 = \boldsymbol{\pi}_2 = \boldsymbol{\pi}_3 = \$14.12$ and $\boldsymbol{\sigma}_1 = \boldsymbol{\sigma}_2 = \boldsymbol{\sigma}_3 = \$8.54$.

Scenario 2: The capacity of the transmission line between buses 1 and 3 (T_{13}) is 8 MW. All other parameters are the same as in Scenario 1. In this scenario, and in the absence of the CB, transmission line T_{13} is congested at both DAM and RTM, as shown in Table II. Accordingly, LMPs are different across different buses in both markets: $\boldsymbol{\pi}_1 = \$12.95$, $\boldsymbol{\pi}_2 = \$15.30$, $\boldsymbol{\pi}_3 = \$17.65$; and $\boldsymbol{\sigma}_1 = \$5.8$, $\boldsymbol{\sigma}_2 = \$10.05$, $\boldsymbol{\sigma}_3 = \$14.31$.

Scenario 3: The capacity of the transmission line between buses 1 and 2 (T_{12}) is 50 MW. The bid components of G_3 submitted at the RTM are also changed as shown in Table I. All other parameters are the same as in Scenario 1. In the absence of the CB, *no* transmission line is congested in the DAM, and we have: $\boldsymbol{\pi}_1 = \boldsymbol{\pi}_2 = \boldsymbol{\pi}_3 = \$14.12$; however, the transmission line between buses 1 and 2 *is* congested at the RTM, as shown in Table II, note the bold underlined numbers. Therefore, we have: $\boldsymbol{\sigma}_1 = \$5.4$, $\boldsymbol{\sigma}_2 = \$13.4$, and $\boldsymbol{\sigma}_3 = \$9.4$.

A. Numerical Results

In all scenarios, and in the absence of any CB, we have $\boldsymbol{\pi}_2 > \boldsymbol{\sigma}_2$, i.e., the DAM price is higher than the RTM price at bus 2. Therefore, placing a *supply* CB at bus 2 is *profitable* for the market participant. Of concern is whether or not such profitable supply CB can also help reducing the gap between the DAM and the RTM prices at bus 2, i.e., $\boldsymbol{\pi}_2 - \boldsymbol{\sigma}_2$.

The outcome of placing a supply CB at bus 2 and increasing its amount is shown in Fig. 2(b), (c), and (d) for Scenarios 1, 2, and 3, respectively. In Scenarios 1 and 2, placing a profitable supply CB at bus 2 results in price *convergence* at bus 2. However, under Scenario 3, placing a profitable supply CB at bus 2 results in price *divergence* at bus 2. This is *counter-intuitive* and against what ISOs expect from a CB [10].

B. Analytical Explanations Using Theorem 1

In this section, we use the analytical foundation that we developed in Theorem 1 to explain the numerical results that we observed earlier in the three scenarios.

Scenario 1: Since in this scenario, neither DAM nor RTM experience congestion, we have $\bar{\mathbf{D}} = \bar{\mathbf{R}} = \mathbf{0}$. From this, together with definition of $\mathbf{X}$ and $\mathbf{Y}$, we have $\mathbf{X} = \mathbf{Y} = \mathbf{0}$. By substituting these terms in (7), (8), and (9), we have:

$$\frac{\partial \boldsymbol{\Delta}_i}{\partial \mathbf{v}_i} = -\frac{1}{\mathbf{1}^T \boldsymbol{\Lambda} \mathbf{1}} \frac{1}{\mathbf{1}^T \boldsymbol{\Gamma} \mathbf{1}} (\mathbf{1}^T \boldsymbol{\Gamma} \mathbf{1} + \mathbf{1}^T \hat{\mathbf{K}} \boldsymbol{\Lambda} \mathbf{1}) < 0 \qquad (20)$$

where the inequality is due to $\boldsymbol{\Lambda}$ and $\boldsymbol{\Gamma}$ being diagonal positive semi-definite matrices and $\hat{\mathbf{K}}$ comprising basis vectors. In fact, if the grid is not congested, then the electricity market reduces to a typical two-settlement financial market, in which CBs always improve market efficiency by reducing the price gap. In other words, what the ISOs often assume when they work with CBs is true for a nodal electricity market without transmission line congestion. For instance, in Scenario 1, we have $\boldsymbol{\Delta}_2 = 5.59 > 0$, and $\partial \boldsymbol{\Delta}_2 / \partial \mathbf{v}_2 = -0.47 < 0$, which results in price convergence between DAM and RTM, as shown in Fig. 2(b).

Scenario 2: In this scenario, the congested transmission line at both DAM and RTM is T_{13}; therefore, $\bar{\mathbf{R}} = \bar{\mathbf{D}}$. Also, all marginal, i.e., price-maker, bids in the DAM are of type physical supply; i.e. $\hat{\mathbf{K}} = \mathbf{I}$ and $\hat{\mathbf{\Psi}} = \hat{\mathbf{\Phi}}_{-i}$. Thus, we have

$$\mathbf{r}\hat{\mathbf{K}}\mathbf{h} = \mathbf{r}\mathbf{h} = \mathbf{1}^T\mathbf{\Gamma}\mathbf{Y}(\mathbf{Y}\mathbf{\Gamma}\mathbf{Y}^T)^{-1} \times (\mathbf{X}\mathbf{\Lambda}\mathbf{1} - \mathbf{X}\mathbf{\Lambda}\mathbf{X}^T(\mathbf{X}\mathbf{\Lambda}\mathbf{X}^T)^{-1}\mathbf{X}\mathbf{\Lambda}\mathbf{1}) = 0. \quad (21)$$

Also, we can prove that $\mathbf{1}^T\mathbf{h}$ is always greater than zero:

$$\mathbf{1}^T\mathbf{h} = \|\mathbf{\Lambda}^{0.5}\mathbf{1} - \mathbf{\Lambda}^{0.5}\mathbf{X}^T(\mathbf{X}\mathbf{\Lambda}\mathbf{X}^T)^{-1}\mathbf{X}\mathbf{\Lambda}\mathbf{1}\|_2^2 > 0 \quad (22)$$

And similarly, $\mathbf{1}^T\mathbf{e} > 0$. Therefore, the sensitivity of the price gap to the supply CB is less than zero as expressed in (23):

$$\frac{\partial \mathbf{\Delta}_i}{\partial \mathbf{v}_i} = -\frac{\mathbf{1}^T\mathbf{h} + \mathbf{1}^T\mathbf{e}}{(\mathbf{1}^T\mathbf{h})(\mathbf{1}^T\mathbf{e})} \leq 0 \quad (23)$$

The above inequality explains the desirable results in Scenario 2. In fact, the conditions of this scenario guarantees that the supply CB results in price convergence. In particular, in this scenario, we have $\mathbf{\Delta}_2 = 5.25 > 0$, and $\partial\mathbf{\Delta}_2/\partial\mathbf{v}_2 = -0.57 < 0$, which supports the outcome of the supply CB on the price gap as shown in Fig. 2(c).

Scenario 3: In this scenario, we have $\mathbf{\Delta}_2 = 0.73 > 0$. Also, from (6) in Theorem 1, $\partial\mathbf{\Delta}_2/\partial\mathbf{v}_2 = 0.21 > 0$. This is in contrast to what ISOs expect from CBs as expressed in (19). In other words, despite the fact that submitting a supply CB at bus 2 is reasonable for an independent CB market participant, the outcome to the market is in form of price *divergence* and against what is considered desirable by an ISO.

In summary, from Scenario 1, 2 and 3, it can be concluded that whether or not a CB causes price convergence between DAM and RTM in a congested nodal electricity market, depends on the sensitivity of the price gap to the CB, which relies on the grid conditions and transmission line congestion configuration. Therefore, while CBs always act as intended and results in price convergence in other financial market or nodal electricity market without congestion, but they may not act as expected in a nodal electricity market with congestion.

IV. Conclusions and Future Work

This paper was motivated by the current state of uncertainty and debate about the impact of CBs in nodal electricity markets, which have been recently reported by multiple ISOs. To address this open problem, in this paper, a fundamental sensitivity analysis has been introduced to understand how a CB may affect the DAM and RTM prices in a transmission-constrained nodal electricity market. Based upon the proposed sensitivity model and intuitive case studies, it is shown that the transmission line congestion can influence the impact of convergence bidding in nodal electricity markets in a way that is possible to degrade market efficiency. Specifically, under certain conditions, placing a CB at a bus can result in divergence (not convergence) between the DAM and RTM prices in that bus, which is counter-intuitive and undesirable.

The results in this paper can be extended in several directions. For example, while we studied the impact of CBs on price convergence (divergence) on the same bus where the CB was placed, one can similarly study the impact also on price convergence (divergence) at buses other than where CBs are placed. One may also investigate insightful sufficient grid operational conditions to guarantee price convergence (divergence) when a CB is placed at a bus. Also, the analysis could be extended to explain the collective impact of a group of several CBs that are placed at different locations on the price gap of all system buses. Such extended analysis would be beneficial to ISOs to understand how it is possible to shape the price difference caused by CBs across the power system.

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