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Sensing-Motion Co-Planning For Reconstructing a Spatially Distributed Field Using a Mobile Sensor Network

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Abstract— **We investigate the problem of simultaneous parameter identification and mapping of a spatially distributed field using a mobile sensor network. We first develop a parametrized model that represents the spatially distributed field. Based on the model, a recursive least squares algorithm is developed to achieve online parameter identification. Next, we design a global state observer, which uses the estimated parameters, together with data collected by the mobile sensor network, to real-timely reconstruct the whole spatial-temporal varying field. Since the performance of the parameter identification and map reconstruction algorithms depends on the trajectories of the mobile sensors, we further develop a Lyapunov redesign based online trajectory planning algorithm for the mobile sensor network so that the mobile sensors can use local real-time information to guide them to move along information-rich paths that can improve the performance of the parameter identification and map construction. Lastly, a cooperative filtering scheme is developed to provide the state estimates of the spatially distributed field, which enables the recursive least squares method. To test the proposed algorithms in realistic scenarios, we first build a CO_2 diffusion field in a lab and construct a sensor network to measure the field concentration over time. We then validate the algorithms in the reconstructed CO_2 field in simulation. Simulation results demonstrate the efficiency of the proposed method.**

I. INTRODUCTION

The state estimation and prediction of spatially distributed fields described by partial differential equations (PDEs) plays key roles in services such as chemical containment detection, pollution control, and search and rescue missions [1]–[3]. A typical spatially distributed field is the dispersion of gases from a gas source into an ambient environment, which results in a plume. Mapping or estimation of the resulting plume constitutes the first step in performing monitoring tasks and controlling the plume. Most of related earlier works regarding state estimation of spatially distributed fields are model-based [3]–[5]. By incorporating the dynamics of the process modeled by a PDE, the field concentration can be estimated using a large number of static sensors spreading in the whole domain. It seems clear that endowing nodes in a sensor network with mobility drastically expands the spectrum of the network's capabilities [6]. This leads to recent flourishing progress in the use of mobile sensor networks (MSNs) to improve parameter identification and state estimation of spatially distributed systems [5]–[8].

Various observer designs have been proposed in the literature to map the fields using MSNs [6], [9]–[11]. As a natural enhancement of the state observers, a common and powerful tool is the Kalman-Bucy filter [10]. The series of publications [9], [12], [13] establish a general theoretical framework for distributed filtering and state estimation. Since the sensors are not assigned to fixed spatial positions, the measurements along certain trajectories yield more information about the field than those at other trajectories, which makes the mobile sensor trajectory design important. The optimal trajectory designs based on different criteria can be seen in [9], [12], [13]. However, most of existing studies assume the parameters of PDE models are known or can be estimated offline [2], [12], [14], [15], with few exceptions that investigate online parameter identification [7], [14], [16]. In many realistic scenarios, it is common that some parameters in the PDEs such as the diffusion coefficient and decay rate may be unknown or inaccurate. Hence, simultaneous parameter identification while a MSN is exploring a spatially distributed field becomes necessary [7], [16]. In this setting, our most recent efforts [7], [16] proposed a cooperative filtering scheme for performing online parameter estimation for advection-diffusion processes. Under this cooperative filtering scheme, the diffusion coefficient can be estimated recursively without intensive computational loads to solve the PDEs in the entire spatial domain [16], [17].

In this paper, we develop a sensing-motion co-planning scheme for a MSN tasked with reconstructing a spatially distributed process. We first propose a parametrized model that represents the spatially distributed field. Base on this model, the proposed scheme consists of three parts: first, a recursive least squares (RLS) algorithm is presented for the parameter identification of the parametrized model. Next, using the estimated parameters together with data collected by the MSN, we design a global state estimator to real-timely reconstruct the whole spatially distributed field. Additionally, a Lyapunov-based trajectory design is provided for the motion control of the MSN. To enable the RLS, a cooperative Kalman filtering is further developed to provide the necessary state estimates of the spatially distributed field along the trajectory of the MSN. The convergence analysis shows that the proposed scheme can achieve the boundedness of parameter and state estimation errors. Simulation results based on a real CO_2 diffusion field show satisfactory performance.

The problem is formulated in Section II. Section III presents the parameterization of the PDE model. Section IV shows the sensing-motion co-planning scheme. Section V illustrates the convergence analysis. Section VI discusses the cooperative Kalman filter. Simulation is presented in Section VII. Conclusions and future work follow in Section VIII.

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II. PROBLEM FORMULATION

In this section, we formulate the problem of sensing-motion co-planning using MSNs.

A. The model

We assume that the dynamics of a spatially distributed system is described by the following two-dimensional (2D) partial differential equation defined on a domain $\Omega = [0, L_x] \times [0, L_y] \in \mathbb{R}^2$:

$$\frac{\partial z(r,t)}{\partial t} = f(z(r,t), \nabla z(r,t), \nabla^2 z(r,t)), \ r \in \Omega, \tag{1}$$

where $z(r,t)$ is the concentration function, ∇ represents the gradient operator, and ∇^2 represents the Laplacian operator. $f(\cdot)$ is an unknown nonlinear function. The meaning of Equation (1) is that there is a net flow of substance from the regions with higher concentration of the substance to the ones with lower concentration. This type of nonlinear PDEs in Equation (1) is widely used to described physical and engineering phenomena such as heat process, population dynamics, chemical reactors, fluid dynamics, etc., [15].

In practical applications such as environmental monitoring, the domain Ω is much larger than sensor dimensions so that the boundary can be modeled as a flat surface [2], [7]. Hence, the initial and Dirichlet boundary conditions for Equation (1) are assumed as [2], [7] $z(r,0) = z_0(r), z(r,t) = z_b(r,t),\ r \in \partial\Omega$, where $z_0(r)$ and $z_b(r,t)$ are the arbitrary initial condition and Dirichlet boundary condition, respectively.

B. Sensor dynamics

Consider a formation of N coordinated sensing agents moving in the field, each of which carries a sensor that takes point measurements of the field $z(r,t)$. We consider the sensing agents with single-integrator dynamics given by $\dot{r}_i(t) = u_i(t), i = 1,2,...,N$, where $r_i(t)$ and $u_i(t) \subseteq \mathbb{R}^2$ are the position and the velocity of the ith agent, respectively. In most applications, the sensor measurements are taken discretely over time. Let the moment when new measurements are available be t_k, where k is an integer index. Denote the position of the ith agent at the moment t_k be r_i^k and the field value at r_i^k be $z(r_i^k, k)$. The measurement of the ith agent can be modeled as $p(r_i^k, k) = z(r_i^k, k) + n_i$, where n_i is assumed to be i.i.d. Gaussian noise. We have the following assumption for the sensing agents.

Assumption II.1 *Each agent can measure its position r_i^k and concentration value $p(r_i^k, k)$, and share these information with other agents.*

Under Assumption II.1, the object is to construct a map of the spatially distributed field in a real-time fashion. Since the original PDE model (1) is difficult to identify due to the complex nonlinear structures, we will first parametrize the model in Equation (1). Based on this parametrized model, we construct a co-planning scheme for parameter identification and mapping of the spatially distributed field and design the error-minimum trajectory for the MSN.

III. THE PARAMETERIZATION OF THE PDE MODEL

This section introduces the parametrized model that we adopt in this research. Under Assumption II.1, the object is to construct a map of the process (1) using the discrete measurements taken by mobile agents over time as input. Hence, we need to discretize the PDE (1) using some numerical methods. Suppose the current time step is t_k. The temporal variations of the concentration can be approximated with finite difference as,

$$\frac{\partial z(r,t)}{\partial t}|_{t=t_k} \approx \frac{z(r,k+1) - z(r,k)}{t_s}, \tag{2}$$

where t_s is the sampling interval.

Applying the above finite difference to Equation (1) gives,

$$z(r,k+1) \approx z(r,k) + t_s f(z(r,k), \nabla z(r,k), \nabla^2 z(r,k)). \tag{3}$$

Exact solutions for the nonlinear PDE (3) are difficult to obtain due to diverse nonlinearity, different structures, and complex boundary conditions [15]. Therefore, we parameterize the nonlinear function $f(\cdot)$ in Equation (3) by assuming that the unknown nonlinear function $f(\cdot)$ takes a form of polynomial. Severals works have illustrated that the polynomial expression of $f(\cdot)$ can be a good approximation of the original model in Equation (1) [15]. The polynomial form of Equation (3) is given by the model,

$$\begin{aligned} z(r,k+1) = z(r,k) + \sum_{i=1}^{M} \theta_i \left(t_s \phi_i(z(r,k), \nabla z(r,k), \nabla^2 z(r,k)) \right) \\ + e(r,k), \end{aligned} \tag{4}$$

where M denotes the order of the polynomial, θ_i is the coefficient of the ith polynomial term, and $\phi_i(z(r,k), \nabla z(r,k), \nabla^2 z(r,k))$ is the corresponding monomial, which is the product of different spatial derivatives $z(r,k)$, $\nabla z(r,k)$, and $\nabla^2 z(r,k)$. $e(r,k)$ is the approximation modeling error. $e(r,k)$ is a higher order term of the space sampling interval, which allows us to assume it as an independent noise sequence with zero mean and finite variance [16], [17].

We observe that Equation (4) is just a semi-discrete representation of the original continuous PDE (1). That is because a direct differentiation process of higher-order spatially derivative terms such as $\nabla z(r,k)$ and $\nabla^2 z(r,k)$ tend to amplify the effects of the noise [15]. Therefore, different from existing lumped models that discretize each time and spatial derivative term [2], [15], we only consider the time derivative discretization in our work. We will employ a cooperative Kalman filter to directly estimate the spatial derivative terms $\nabla z(r,k)$ and $\nabla^2 z(r,k)$ along the trajectory of the MSN. This part of work will be introduced in Section VI. Here, we denote $\hat{z}(r,k+1)$, $\hat{z}(r,k)$, $\nabla \hat{z}(r,k)$, and $\nabla^2 \hat{z}(r,k)$ as the estimated states from the cooperative Kalman filter, which can be specified as follows, $\hat{z}(r,k+1) = z(r,k+1) + \xi_1, \hat{z}(r,k) = z(r,k) + \xi_2, \nabla \hat{z}(r,k) = \nabla z(r,k) + \xi_3, \nabla^2 \hat{z}(r,k) = \nabla^2 z(r,k) + \xi_4$. Since the cooperative Kalman filter converges as proved in [7], [16], all the error terms ξ_1, ξ_2, ξ_3, and ξ_4 can be assumed as Gaussian noises with zero mean and bounded covariances.

Substituting the states $\hat{z}(r,k+1)$, $\hat{z}(r,k)$, $\nabla\hat{z}(r,k)$, and $\nabla^2\hat{z}(r,k)$ into (4), we have the following equation, $\hat{z}(r,k+1) = \hat{z}(r,k) + \sum_{i=1}^{M}\theta_i\left(t_s\phi_i(\hat{z}(r,k),\nabla\hat{z}(r,k),\nabla^2\hat{z}(r,k))\right) + h(r,k)$, where $h(r,k) = \xi(r,k) + e(r,k)$. For notation simplification, we denote $\xi(r,k)$ as the combination of the Gaussian noises ξ_1, ξ_2, ξ_3, and ξ_4, i.e., $\xi(r,k) = \sum_{i=1}^{4}\xi_i$. $h(r,k)$ combines two Gaussian noises $\xi(r,k)$ and $e(r,k)$, which can be assumed as a Gaussian noise with zero mean and finite variance. Then, we can rewrite the dynamic of $\hat{z}(r,k+1)$ in a vector form,

$$\hat{z}(r,k+1) = \hat{\Phi}(r,k)\Theta + h(r,k), \tag{5}$$

where $\Theta^T = [1,\theta_1,\cdots,\theta_M]$, which is the parameter vector that we will identify, and $\hat{\Phi}(r,k) = [\hat{z}(r,k), t_s\phi_1(\hat{z}(r,k),\nabla\hat{z}(r,k),\nabla^2\hat{z}(r,k)),\cdots,t_s\phi_M(\hat{z}(r,k),\nabla\hat{z}(r,k),\nabla^2\hat{z}(r,k))]$. In Equation (5), $\hat{z}(r,k+1)$ and $\hat{\Phi}(r,k)$ will be determined by the cooperative Kalman filtering in Section VI along the formation center of the MSN, Θ is the parameter that needs to be identified. In the following section, a co-planning scheme for parameter identification and mapping of the spatially distributed field will be designed based on the model (5).

IV. Simultaneous Parameter Identification and Mapping plus Trajectory Design for MSNs

A. RLS parameter identification

Based on the parametrized spatially distributed system (5), the proposed parameter identification algorithm uses the discrete measurements taken by mobile agents over time as input. Within a mobile sensor network, consider the N agents as a group. Let $r_c^k = [r_{c,x}^k, r_{c,y}^k]^T$ be the center of the formation at t_k, i.e., $r_c^k = \frac{1}{N}\sum_{i=1}^{N} r_i^k$. The illustration of r_c^k for four agents is shown in Fig. 1. By running the cooperative Kalman filter in real-time, only the states $\hat{z}(r_c^k,k)$, $\nabla\hat{z}(r_c^k,k)$, and $\nabla^2\hat{z}(r_c^k,k)$ at the formation center can be provided by combing measurements from the sensing agents in the group. Thus, we need to analyze the dynamics of the field value along the formation center r_c^k. Therefore, we replace r in Equation (5) with r_c^k, which results in,

$$\hat{z}(r_c^k,k+1) = \hat{\Phi}(r_c^k,k)\Theta + h(r_c^k,k), \tag{6}$$

where $\hat{\Phi}(r_c^k,k) = [\hat{z}(r_c^k,k), t_s\phi_1(\hat{z}(r_c^k,k),\nabla\hat{z}(r_c^k,k),\nabla^2\hat{z}(r_c^k,k)),\cdots,t_s\phi_M(\hat{z}(r_c^k,k),\nabla\hat{z}(r_c^k,k),\nabla^2\hat{z}(r_c^k,k))]$.

Similarly, the system dynamics at the formation center r_c^k at time step t_{k-1} can be written as,

$$\hat{z}(r_c^k,k) = \hat{\Phi}(r_c^k,k-1)\Theta + h(r_c^k,k-1). \tag{7}$$

Let $\hat{\Theta}(k)$ be the estimate of Θ at time instant k. Given an initial estimate of $\hat{\Theta}(0)$, a direct application of the RLS method can iteratively update the estimate of Θ. The proposed RLS algorithm can be stated as follows:

$$\begin{aligned}\hat{\Theta}(k) &= \hat{\Theta}(k-1) + R(k)\hat{\Phi}^T(r_c^k,k)\cdot\\ &\quad (\hat{z}(r_c^k,k) - \hat{\Phi}(r_c^k,k-1)\hat{\Theta}(k-1)),\\ R^{-1}(k) &= R^{-1}(k-1) + \hat{\Phi}^T(r_c^k,k)\hat{\Phi}(r_c^k,k),\end{aligned} \tag{8}$$

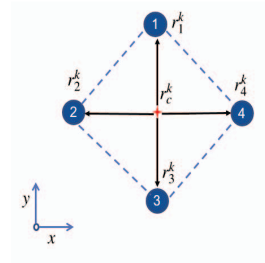


Fig. 1. The illustration of the formation center for a four-agent group.

where $R(k)$ is the error covariance matrix and $\hat{\Phi}(r_c^k,k-1) = [\hat{z}(r_c^k,k-1), t_s\phi_1(\hat{z}(r_c^k,k-1),\nabla\hat{z}(r_c^k,k-1),\nabla^2\hat{z}(r_c^k,k-1)),\cdots,t_s\phi_M(\hat{z}(r_c^k,k-1),\nabla\hat{z}(r_c^k,k-1),\nabla^2\hat{z}(r_c^k,k-1))]$.

We should point out that to run the recursive update law in Equation (8), we require $\hat{\Phi}(r_c^k,k-1)$ and $\hat{\Phi}(r_c^k,k)$, which can be obtained from the cooperative Kalman filter introduced in Section VI.

B. Simultaneous identification and mapping

In Section IV.A, we have shown how to identify Θ based on Equation (5). With the identified parameter $\hat{\Theta}(k)$ using Equation (8), we develop a state estimator for the field values over the whole area so that a map of the field can be generated simultaneously with the parameter identification. In the following, we show how to obtain this state estimator.

Let us first define $\bar{z}(r_c^k,k+1) = \hat{\Phi}(r_c^k,k)\hat{\Theta}(k)$, where $\hat{\Theta}(k)$ is the estimate of Θ using Equation (8), and the bar notation $\bar{z}(r_c^k,k+1)$ represents the output from a state estimator at the formation center r_c^k. By modeling $\hat{z}(r_c^k,k)$ as a Dirac measure concentrated at the formation center, we have $\hat{z}(r_c^k,k) = \bar{z}(r_c^k,k) + \sigma(r-r_c^k)(\hat{z}(r_c^k,k) - \bar{z}(r_c^k,k))$, where $\sigma(.)$ is an impulse function. Similarly, we further model the other states $\nabla\hat{z}(r_c^k,k)$ and $\nabla^2\hat{z}(r_c^k,k)$ as Dirac measurements,

$$\begin{aligned}\nabla\hat{z}(r_c^k,k) &= \nabla\bar{z}(r_c^k,k) + \sigma(r-r_c^k)(\nabla\hat{z}(r_c^k,k) - \nabla\bar{z}(r_c^k,k)),\\ \nabla^2\hat{z}(r_c^k,k) &= \nabla^2\bar{z}(r_c^k,k) + \sigma(r-r_c^k)(\nabla^2\hat{z}(r_c^k,k) - \nabla^2\bar{z}(r_c^k,k)).\end{aligned}$$

By combining the above Dirac measurements, we can construct the Luenberger observer as follows,

$$\begin{aligned}\bar{z}(r_c^k,k+1) &= \bar{z}(r_c^k,k) + \sigma(r-r_c^k)(\hat{z}(r_c^k,k) - \bar{z}(r_c^k,k))\\ &+ \sum_{i=1}^{M}\hat{\theta}_i\left(t_s\phi_i(\bar{z}(r_c^k,k),\nabla\bar{z}(r_c^k,k),\nabla^2\bar{z}(r_c^k,k))\right)\\ &+ \sigma(r-r_c^k)\sum_{i=1}^{M}\hat{\theta}_i(t_s\phi_i(\hat{z}(r_c^k,k),\nabla\hat{z}(r_c^k,k),\nabla^2\hat{z}(r_c^k,k))\\ &- t_s\phi_i(\bar{z}(r_c^k,k),\nabla\bar{z}(r_c^k,k),\nabla^2\bar{z}(r_c^k,k))\\ &= \bar{\Phi}(r_c^k,k)\hat{\Theta}(k) + \sigma(r-r_c^k)(\hat{z}(r_c^k,k) - \bar{z}(r_c^k,k))\\ &+ \sigma(r-r_c^k)\sum_{i=1}^{M}\hat{\theta}_i(t_s\phi_i(\hat{z}(r_c^k,k),\nabla\hat{z}(r_c^k,k),\nabla^2\hat{z}(r_c^k,k))\\ &- t_s\phi_i(\bar{z}(r_c^k,k),\nabla\bar{z}(r_c^k,k),\nabla^2\bar{z}(r_c^k,k)),\end{aligned} \tag{9}$$

where $\bar{\Phi}(r_c^k,k) = [\bar{z}(r_c^k,k), t_s\phi_1(\bar{z}(r_c^k,k),\nabla\bar{z}(r_c^k,k),\nabla^2\bar{z}(r_c^k,k)),\cdots,t_s\phi_M(\bar{z}(r_c^k,k),\nabla\bar{z}(r_c^k,k),\nabla^2\bar{z}(r_c^k,k))]$. Since the cooperative Kalman filter is able to provide the states $\hat{z}(r_c^k,k)$, $\nabla\hat{z}(r_c^k,k)$, and $\nabla^2\hat{z}(r_c^k,k)$ along the moving trajectory, these states can be treated as the "measurements" for the state estimator. By replacing r_c^k in terms $\bar{z}(r_c^k,k+1)$ and $\bar{\Phi}(r_c^k,k)$ with r in (9) , the global state estimator can be readily obtained to estimate the field value over the entire spatial domain [2]. The proposed global state estimator takes the form,

$$\begin{aligned}\bar{z}(r,k+1) &= \bar{\Phi}(r,k)\hat{\Theta}(k) + \sigma(r-r_c^k)(\hat{z}(r_c^k,k)-\bar{z}(r_c^k,k)) \\ &\quad + \sigma(r-r_c^k)\sum_{i=1}^{M}\hat{\theta}_i t_s(\phi_i(\hat{z}(r_c^k,k),\nabla\hat{z}(r_c^k,k),\nabla^2\hat{z}(r_c^k,k)) \\ &\quad - \phi_i(\bar{z}(r_c^k,k),\nabla\bar{z}(r_c^k,k),\nabla^2\bar{z}(r_c^k,k)),r\subseteq\Omega, \quad (10)\end{aligned}$$

where $\bar{\Phi}(r,k) = [\bar{z}(r,k),t_s\phi_1(\bar{z}(r,k),\nabla\bar{z}(r,k),\nabla^2\bar{z}(r,k)),\cdots,$ $t_s\phi_M(\bar{z}(r,k),\nabla\bar{z}(r,k),\nabla^2\bar{z}(r,k))]$. We observe that Equation (9) constitutes a finite-dimensional measurement representation of (10). The state estimator (10) is in the form of a Luenberger observer [2] with some output injection terms.

C. The trajectory design for the MSN

Many results have shown that the performance of state estimation and parameter identification depends on the trajectories of sensing agents [2], [9]. In this paper, the trajectory design of the MSN is based on the state estimation error at the formation center, that is, $e(r_c^k,k)=\hat{z}(r_c^k,k)-\bar{z}(r_c^k,k)$. The goal of the design scheme is to provide the sensing agents with control signals that move towards the direction associated with larger state estimation error. We apply the following stable motion control laws for the mobile agents:

$$\begin{aligned}r_{c,x}^{k+1} &= r_{c,x}^k - k_x e(r_c^k,k)e_x(r_c^k,k),\\ r_{c,y}^{k+1} &= r_{c,y}^k - k_y e(r_c^k,k)e_y(r_c^k,k), \quad (11)\end{aligned}$$

where $r_c^k=[r_{c,x}^k,r_{c,y}^k]$, k_x and k_y are user-defined positive gains, $e_x(r_c^k,k)$ and $e_y(r_c^k,k)$ are the error gradients at the formation center r_c^k specified as follows,

$$\begin{aligned}e_x(r_c^k,k) &\triangleq \frac{\partial\hat{z}(r_c^k,k)}{\partial r_{c,x}^k} - \frac{\partial\bar{z}(r_c^k,k)}{\partial r_{c,x}^k},\\ e_y(r_c^k,k) &\triangleq \frac{\partial\hat{z}(r_c^k,k)}{\partial r_{c,y}^k} - \frac{\partial\bar{z}(r_c^k,k)}{\partial r_{c,y}^k}. \quad (12)\end{aligned}$$

V. The convergence analysis

In this section, we provide the convergence proof of the co-planning scheme. We have the following proposition.

Proposition V.1 Consider the discretized model in Equation (5) with the parameter Θ unknown. Apply the proposed co-planning scheme, which consists of the RLS algorithm (8), the state estimator (10), and the motion control (11). The error signal $e(r_c^k,k)=\hat{z}(r_c^k,k)-\bar{z}(r_c^k,k)$ and the parameter estimation errors $\delta\Theta=\Theta-\hat{\Theta}$ are bounded for all $k\geq 0$.

Proof: Let us denote the error signal at the formation center r_c^k and at time step $k+1$ as $e(r_c^k,k+1)=\hat{z}(r_c^k,k+1)-\bar{z}(r_c^k,k+1)$. By combing Equations (5) and Equation (9), the time variance of the error $e(r_c^k,k)$ can be obtained, which satisfies the following equation:

$$\begin{aligned}e(r_c^k,k+1)-e(r_c^k,k) &= \sum_{i=1}^{M}(\theta_i-\hat{\theta}_i)t_s\cdot \quad (13)\\ &\Big(\phi_i(\hat{z}(r_c^k,k),\nabla\hat{z}(r_c^k,k),\nabla^2\hat{z}(r_c^k,k))\Big)-e(r_c^k,k).\end{aligned}$$

Define a Lyapunov function as $V(r_c^k,k)=\frac{1}{2}(e^2(r_c^k,k)+\delta\Theta R^{-1}(k)\delta\Theta)$, where $\delta\Theta=\Theta-\hat{\Theta}(k)$. Then the difference of the discrete Lyapunov function can be written as

$$\begin{aligned}&V(r_c^{k+1},k+1)-V(r_c^k,k)=e(r_c^k,k)e_r(r_c^k,k)\cdot\\ &\left(r_c^{k+1}-r_c^k\right)+e(r_c^k,k)\left(e(r_c^k,k+1)-e(r_c^k,k)\right)\\ &-\delta\Theta R^{-1}(t)\big(\hat{\Theta}(k)-\hat{\Theta}(k-1)\big), \quad (14)\end{aligned}$$

where $e_r(r_c^k,k)=[e_x(r_c^k,k),e_y(r_c^k,k)]$ represents the error gradients in a vector form defined in Equation (12).

It is obvious that the first term in Equation (14) is an indefinite term which must somehow be made negative. The desired control law for mobile agents in Equation (11) can guarantee this indefinite term in the Lyapunov function be negative semidefinite. Substituting the control laws in Equation (11) into Equation (14), we can get that

$$\begin{aligned}&V(r_c^{k+1},k+1)=V(r_c^k,k)-k_x e^2(r_c,t)e_x^2(r_c,t)\\ &-k_y e^2(r_c,t)e_y^2(r_c,t)+e(r_c^k,k)\left(e(r_c^k,k+1)-e(r_c^k,k)\right)\\ &-\delta\Theta R^{-1}(t)\big(\hat{\Theta}(k)-\hat{\Theta}(k-1)\big). \quad (15)\end{aligned}$$

Then, substituting the update law in Equations (8) and the dynamics of the error (13) into (15) yields

$$\begin{aligned}&V(r_c^{k+1},k+1)=V(r_c^k,k)-k_x e^2(r_c^k,k)e_x^2(r_c^k,k)\\ &-k_y e^2(r_c^k,k)e_y^2(r_c^k,k)+e(r_c^k,k)\left(e(r_c^k,k+1)-e(r_c^k,k)\right)\\ &-\delta\Theta\hat{\Phi}^T(r_c^k,k)e(r_c^k,k)\\ &=V(r_c^k,k)-k_x e^2(r_c^k,k)e_x^2(r_c^k,k)-k_y e^2(r_c^k,k)e_y^2(r_c^k,k) \quad (16)\\ &-e^2(r_c^k,k)-\delta\Theta\hat{\Phi}^T(r_c^k,k)e(r_c^k,k)\\ &\sum_{i=1}^{M}(\theta_i-\hat{\theta}_i)t_s\left(\phi_i(\hat{z}(r_c^k,k),+\nabla\hat{z}(r_c^k,k),\nabla^2\hat{z}(r_c^k,k))\right)e(r_c^k,k).\end{aligned}$$

By definition, we know that

$$\delta\Theta\hat{\Phi}^T(r_c^k,k)=\sum_{i=1}^{M}(\theta_i-\hat{\theta}_i)t_s\left(\phi_i(\hat{z}(r_c^k,k),\nabla\hat{z}(r_c^k,k),\nabla^2\hat{z}(r_c^k,k))\right).$$

Using the update law in Equation (8), we can get that, $V(r_c^{k+1},k+1)=V(r_c^k,k)-k_x e^2(r_c^k,k)e_x^2(r_c^k,k)-k_y e^2(r_c^k,k)e_y^2(r_c^k,k)-e^2(r_c^k,k)$, which implies $V(r_c^{k+1},k+1)\leq V(r_c^k,k)$. Then we conclude that the parameter estimation errors $\delta\Theta$ and the prediction errors $e(r_c^k,k)$ are bounded. ■

VI. Cooperative Kalman filter

In this section, we develop a cooperative Kalman filter that provides necessary information needed to enable the RLS algorithm. From Equation (8), we know that we require $\hat{\Phi}(r_c^k,k)$ and $\hat{\Phi}(r_c^k,k-1)$, which consist of the states $z(r_c^k,k)$, $\nabla z(r_c^k,k)$, $\nabla^2 z(r_c^k,k)$, $z(r_c^k,k-1)$, $\nabla z(r_c^k,k-1)$, and $\nabla^2 z(r_c^k,k-1)$. While the MSN is moving in the field, the field value along the trajectory of the formation center r_c evolves according to

$$\dot{z}(r_c,t) = \nabla z(r_c,t)\cdot \dot{r}_c + \frac{\partial z(r_c,t)}{\partial t}, \tag{17}$$

Substituting the PDE (1) into (17), we obtain

$$\dot{z}(r_c,t) = \nabla z(r_c,t)\cdot \dot{r}_c + f(z(r,t),\nabla z(r,t),\nabla^2 z(r,t)). \tag{18}$$

We also derive the total time derivative of $\nabla z(r_c,t)$ as

$$\dot{\nabla} z(r_c,t) = H(r_c,t)\cdot \dot{r}_c + \frac{\partial \nabla z(r_c,t)}{\partial t}, \tag{19}$$

where $H(r_{ci},t)$ is the Hessian matrix.

To construct the cooperative Kalman filter, define the information state as $X(k) = [z(r_c^k,k),\nabla z(r_c^k,k),z(r_c^k,k-1),\nabla z(r_c^k,k-1)]^T$. In practice, sensors take measurements discretely with sampling interval t_s. By discretizing Equations (18) and (19), we obtain the state equation as the information state evolves according to the following equation:

$$X(k+1) = A(k)X(k) + U(k) + e(k), \tag{20}$$

where $e(k) = [e(r_c^k,k),0,e(r_c^k,k-1),0]^T$ represents the model error terms in Equation (3). We denote the covariance matrix of $e(k)$ as $E[e(k)e(k)^T] = W$. The matrices $A(k)$ and $U(k)$ are defined by

$$A(k) = \begin{bmatrix} 1 & (r_c^{k+1}-r_c^k)^T & 0 & 0 \\ 0 & I_{2\times 2} & 0 & 0 \\ 0 & 0 & 1 & (r_c^{k+1}-r_c^k)^T \\ 0 & 0 & 0 & I_{2\times 2} \end{bmatrix},$$

$$U(k) = \begin{bmatrix} \sum_{i=1}^{M} \hat{\theta}_i \left(t_s \phi_i(z(r_c^k,k),\nabla z(r_c^k,k),\nabla^2 z(r_c^k,k))\right) \\ H(r_c^k,k)(r_c^{k+1}-r_c^k) \\ \sum_{i=1}^{M} \hat{\theta}_i \left(t_s \phi_i(z(r_c^k,k-1),\nabla z(r_c^k,k-1),\nabla^2 z(r_c^k,k-1))\right) \\ H(r_c^k,k-1)(r_c^{k+1}-r_c^k) \end{bmatrix},$$

where $H(r_c^{k-1},k)$ is the Hessian matrix and $\hat{\theta}_i$ can be obtained by the RLS algorithm in Equation (8).

A measurement equation is also required for the cooperative Kalman filter. By applying the formation control, r_i^k and r_i^{k-1} can be controlled to be close to r_c^k. Therefore, the concentration can be locally approximated by a Taylor series up to second order as $z(r_i^k,k) \approx z(r_c^k,k) + (r_i^k - r_c^k)^T \nabla z(r_c^k,k) + \frac{1}{2}(r_i^k - r_c^k)^T H(r_c^k,k)(r_i^k - r_c^k)$, $z(r_i^{k-1},k-1) \approx z(r_c^k,k-1) + (r_i^{k-1} - r_c^k)^T \nabla z(r_c^k,k-1) + \frac{1}{2}(r_i^{k-1} - r_c^k)^T H(r_c^k,k-1)(r_i^{k-1} - r_c^k)$. Let $P(k)$ be the vector that contains all measurements from all the agents at time k and $k-1$. Then, the measurement equation can be modeled as,

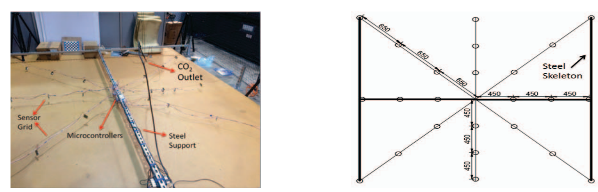
$$P(k) = C(k)\cdot X(k) + D(k)\hat{H}(k) + D(k)\varepsilon(k) + n(k), \tag{21}$$

where $\hat{H}(k)$ represents the estimate of the Hessian $H(k) = [H(r_c^k,k),H(r_c^k,k-1)]^T$ at the center r_c^k in a vector form and $\varepsilon(k)$ represents the error in the estimation of the Hessian matrices. Denote $E[n(k)n(k)^T] = R$ and $E[\varepsilon(k)\varepsilon(k)^T] = Q$. $D(k)$ is a matrix with its first N rows defined by $[\frac{1}{2}((r_i^k - r_c^k)\otimes(r_i^k - r_c^k))^T \; 0]$ and last N rows defined by $[0 \; \frac{1}{2}((r_i^k - r_c^{k-1})\otimes(r_i^k - r_c^{k-1}))^T]$, where $i = 1,2,\cdots,N$ and $\otimes$ is the Kronecker product. $C(k)$ is a matrix with its first N rows defined by $[1 \; (r_i^k - r_c^k)^T \; 0 \; 0]$ and last N rows defined by $[0 \; 0 \; 1 \; (r_i^{k-1} - r_c^k)^T]$ for $i = 1,2,\cdots,N$. Given (20) and (21), the equations for the cooperative Kalman filter can be readily constructed [11]. For more details, please refer to our previous works [7], [16], [17].

VII. Simulation

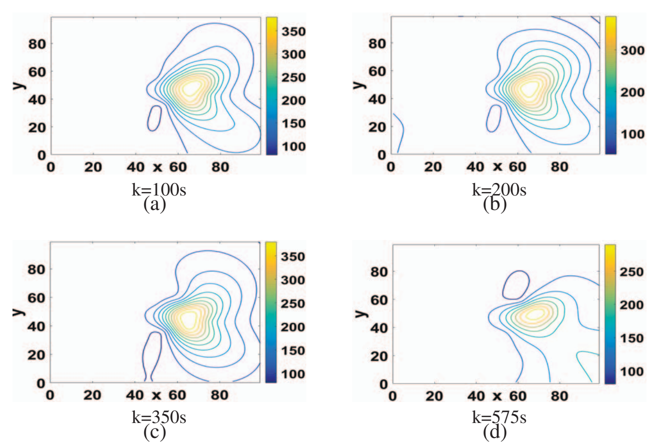
A. Generating and visualizing a real diffusion field

In this section, we validate the proposed algorithm in simulation based on a reconstructed CO_2 diffusion field, which is a typical advection-diffusion process. Note that even though the co-planning scheme is based on the nonlinear PDE (1), in this paper, we only consider the validation in a diffusion process, which is a linear PDE, because it is really hard to construct a controllable nonlinear spatially distributed field in a lab environment. More simulation and experiment validations will be included in our further work. The area of the test-bed is $3.5 \times 3.5m^2$. To measure the concentration of the CO_2 gas over the area, a sensor grid is assembled. As shown in Fig. 2, the sensor grid consists of 24 CO_2 sensors, which is attached to 8 arms. We then use MATLAB to reconstruct the diffusion process by interpolating the field values collected by the sensor grid at every discrete time instant. The diffusion process obtained from the real field is shown in Fig. 3. The contours in Fig. 3 represent the level curves of the diffusion field. The diffusion procedure of CO_2 begins at step $k = 0s$ and ends at $k = 575s$. For more details about the experiment, please refer to our previous paper [16].



(a) (b)

Fig. 2. The illustration of the sensor grid.



(a) (b) (c) (d)

Fig. 3. The diffusion field collected by the sensor grid.

B. Simulation results

In the following, we validate the co-planning scheme with four sensing agents deployed in the reconstructed CO_2

field in simulation. The agents are controlled to move along the direction associated with higher state estimation error while keeping a constant formation as shown in Fig. 4. In Fig. 4, the colored stars represent the four sensing agents, the dotted colored line represents the designed trajectory for the center of the MSN. While the MSN is exploring the field, it also achieves online parameter identification by implementing the cooperative Kalman filter and update law in (8). The estimation results of the diffusion coefficient are shown in Fig. 5. As we can observe from Fig. 5 that, the estimate of the parameter can converge to a stabilized value. Based on the state estimator in (10), we show the results of mapping the diffusion field using the estimated $\hat{\Theta}$ in Fig. 4. The mapping result of the state estimator is shown to be a little different from the true values, which is illustrated in Fig. 3. This is expected, because this difference is due to the unknown initial conditions of the field and limited measurements along the trajectory of the MSN. To illustrate the efficacy of the proposed approach, we further calculate the root mean squared error (RMSE) of the state estimation error, which is shown in Fig. 6. As expected, RMSE is gradually reduced as the estimator learns about the process.

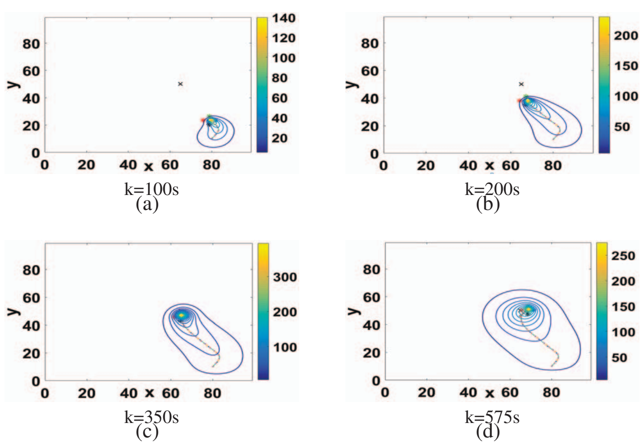


Fig. 4. Evolution of the mapping field using the co-planning scheme.

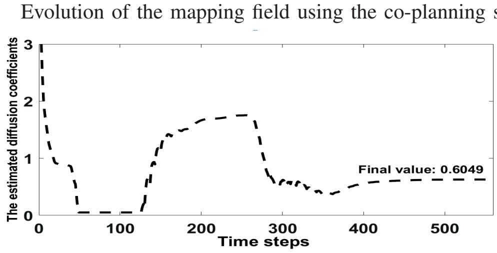


Fig. 5. The estimated diffusion coefficients.

VIII. Conclusions and Future Work

We propose a novel co-planning scheme for parameter identification and state estimation of spatially distributed processes. By designing a RLS algorithm and a global state estimator, the proposed scheme can deal with the use of MSNs for simultaneous parameter and state estimation of spatially distributed fields. The trajectory design based on Lyapunov redesign method is also proposed. Theoretical justifications are provided for the convergence. Simulation results based on

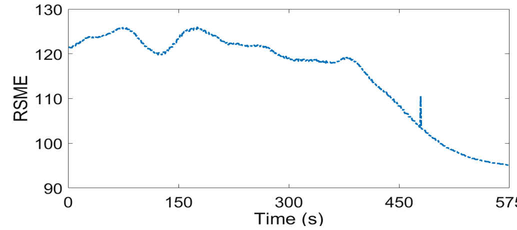


Fig. 6. The RMSE of the state estimation error.

a real CO_2 field show satisfactory performance. Future work includes extending the proposed algorithm to PDEs with spatially varying parameters and experimental validation.

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