Joint Constellation and Code Design for the Gaussian Multiple Access Channel

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***Abstract*—The joint design of both transmit constellation and low-density parity-check codes (LDPC) for the two-user, symbol-synchronous, binary-input Gaussian multiple access channel is considered. A transmission scheme is proposed to approach the symmetric capacity without the use of time-sharing or rate-splitting by joint decoding of the noisy sum of two LDPC codewords. This scheme relies on an extension of the classic belief propagation (BP) algorithm which allows for the simultaneous decoding of two LDPC codewords. We use a Gaussian approximation (GA) of the message distribution to investigate the convergence of the decoding process and derive a linear programming technique for joint code design. We also implement a superposition modulation scheme to achieve higher rate. This code design is applied to different input constellation choices which attain the symmetric capacity in different SNR regimes. It is shown that, quite surprisingly, in the moderate SNR regime the best performance is obtained by an asymmetric constellation.**

I. Introduction

Joint decoding is a fundamental ingredient of multi-terminal transmission schemes and generally provides substantial improvements over time-sharing and rate-splitting. Despite its importance from both a practical and theoretical perspective, the study of optimal codes for joint decoding has not been represented well in the literature as only a few good code designs are currently known.

The study of codes for the MAC has focused primarily on two models: the real adder channel and the GMAC. The real adder channel is a noiseless channel with binary inputs in which the output is the real sum of the inputs. For this model, correct decoding is possible only when any two input codewords are always distinguishable from their sum. The construction of block codes and decoding for the two-user real adder channel is studied in [1] and is extended in [2] to the real adder channel with any number of transmitters. Convolutional codes for GMACs are investigated in [3] which shows that non-uniquely decodable binary convolutional code pairs exist with a sum rate larger than the time-sharing rate. The design of LDPC codes for the GMAC is first considered in [4], although only one construction is mentioned. The authors of [5] introduce the concept of a "MAC node" for the Tanner graph when describing the decoding of LDPC codes for the GMAC. This node is a third type of node, together with variable and check nodes, which receives the channel output and the bit-reliability for a symbol of one transmitter and produces the bit-reliability of a symbol of the other transmitter. In [6], the authors propose a soft demapping method for multilevel modulation on the GMAC based on LDPC codes and investigate the role of symbol mapping in this setting. Spatially coupled codes for the binary adder channel with erasures are studied in [7] where it is shown that threshold saturation as in the point-to-point erasure channel also occurs in this model. In [8] spatially-coupled codes for the GMAC are studied, and is shown that threshold saturation occurs for the joint decoding of two codewords: this result naturally leads to the design of codes which are universal with respect to the channel parameters. Although very powerful, the approach of [8] has not so far been complemented by numerical evaluations.

In this paper, we consider the joint optimization of both the transmitter constellation and LDPC codes for a GMAC: this is, to our knowledge, the first time that these two design problems have been jointly considered. We consider an extension of the BP algorithm for the decoding of the sum of two LDPC codewords corrupted by additive white Gaussian noise at the receiver. We derive convergence conditions for this decoding algorithm and propose a numerical optimization tool for code design based on linear programming. This code construction is applied to three input constellations which maximize the symmetric rate in three different signal-to-noise ratio (SNR) regimes: (i) antipodal input–optimal at low SNR, (ii) inputs that maximize the minimum distance in the received constellation–optimal at high SNR, and (iii) an asymmetric constellation–optimal at moderate SNR. We show that, quite surprisingly, in the moderate SNR regime, an asymmetric constellation outperforms symmetric input constellations. Simulations results are presented to show the effectiveness of the proposed construction.

II. Channel Model and Code Description

We study the two-user, symbol-synchronous GMAC in which the channel output is obtained as

$$Y^N = X_1^N + X_2^N + Z^N, \quad (1)$$

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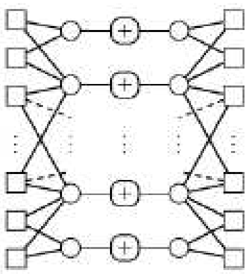


Fig. 1. Tanner graph of MAC considered in this work: check nodes are indicated as squares, variable nodes as circles, and the MAC nodes are indicated with boxes containing the symbol $+$.

where Z^N is an i.i.d. sequence drawn from $\mathcal{N}(0, \sigma_z^2)$ and all additions are over the reals. The channel input X_k, $k \in \{1, 2\}$ is binary and uniformly distributed, i.e.

$$\Pr(X_k) = \begin{cases} 1/2 & \text{for } X_k = x_k(0), \\ 1/2 & \text{for } X_k = x_k(1), \end{cases} \tag{2}$$

where the support $\{x_k(0), x_k(1)\}$ is subject to the power constraint $x_k^2(0) + x_k^2(1) \leq 2$.

The user $k \in \{1, 2\}$ wishes to communicate the messages $W_k \in \{1, \ldots, 2^{NR_k}\}$ to the receiver. The receiver produces message estimates such that the probability of error vanishes as the blocklength N goes to infinity.

In the following, we assume that X_1^N and X_2^N are two binary LDPC codewords in which the symbol $0/1$ is mapped to $x_k(0)/x_k(1)$. Under this assumption, we jointly optimize the codes for two users and their respective input constellation so as to approach the symmetric capacity. For the GMAC, the symmetric capacity is obtained as an upper bound on the symmetric rate R_{sym} as

$$R_{\text{sym}} \leq \max_{x_k(0), x_k(1)} \min\left\{ I(Y; X_1|X_2), I(Y; X_2|X_1), \frac{1}{2} I(Y; X_1, X_2) \right\} \tag{3}$$

where the maximization is over the power constraint.

The joint decoding of LDPC codes for the GMAC can be represented using a modified Tanner graph as shown in Fig. 1: unlike a classical Tanner graph, the Tanner graph in Fig. 1 also contains a *MAC node*. This node takes as inputs the channel output and the bit-reliability of one user and produces the bit-reliability of the other user. The degree of the variable nodes from each user connected through the MAC node is described by the joint variable node distribution. This degree distribution, together with the check node degree distribution, describes a 2-user code ensemble for the GMAC. More specifically, we define $\rho_r^{(k)}$ and $\Gamma_r^{(k)}$ as the check node degree distribution of type r edges for user k from the edge and the node perspective, respectively, as

$$\Gamma^{(k)}(\chi_k) = \sum_r \Gamma_r^{(k)} \chi_k^r, \quad \rho^{(k)}(\chi_k) = \sum_r \rho_r^{(k)} \chi_k^{r-1} \tag{4}$$

for $k \in \{1, 2\}$. Similarly, we define $\lambda_{l_1, l_2}^{(1)}$ as the fraction of user 1 edges both connected to variable nodes with l_1 outgoing user 1 edges and connected through the MAC node to a variable node from user 2 with l_2 outgoing edges. The polynomial $\lambda_{l_1, l_2}^{(2)}$ is defined in a symmetric manner for user 2. Finally, Λ_{l_1, l_2} is defined as the fraction of variable nodes with l_1 outgoing edges of user 1 and l_2 outgoing edges of user 2 as

$$\Lambda(\chi_1, \chi_2) = \sum_{l_1, l_2} \Lambda_{l_1, l_2} \chi_1^{l_1} \chi_2^{l_2}, \tag{5a}$$

$$\lambda^{(1)}(\chi_1, \chi_2) = \sum_{l_1, l_2} \lambda_{l_1, l_2}^{(1)} \chi_1^{l_1 - 1} \chi_2^{l_2}, \tag{5b}$$

$$\lambda^{(2)}(\chi_1, \chi_2) = \sum_{l_1, l_2} \lambda_{l_1, l_2}^{(2)} \chi_1^{l_1} \chi_2^{l_2 - 1}. \tag{5c}$$

Further, the design rate for user 1 is obtained as

$$r^{(1)} = 1 - \frac{\sum_i \rho_i^{(k)}/i}{\sum_{i,j} \lambda_{i,j}^{(1)}/i}, \tag{6}$$

similarly for design rate of user 2, $r^{(2)}$.

Note that the code specified in (4)–(6) generalizes the constructions in [4], [5], [9] which only consider the case where Λ is a diagonal matrix, i.e., where nodes of degree l in the first code collide with nodes of degree l it the second code over the MAC node. As in these works, we assume in the following that the codes have the same check node distribution $\rho^{(1)}(\chi) = \rho^{(2)}(\chi)$, however, no further assumptions are made on the codes.

III. Constellation Design

A closed-form expression of the symmetric capacity of the binary input GMAC is currently not available but can be obtained numerically, as for Bernoulli(0.5)-distributed channel inputs the channel output distribution is given as

$$P_Y(y) = \sum_{(i,j) \in \{0,1\}^2} \frac{1}{4\sqrt{2\pi\sigma_Z^2}} e^{-\frac{1}{2\sigma_Z^2}(y - x_1(i) - x_2(j))^2},$$

and, thus, the mutual information terms in (3) can be precisely evaluated using numerical integration.

By performing the numerical optimization on the r.h.s. of (3) with binary uniform inputs, we obtain that three different constellations, up to reasonable numerical precision, maximize the symmetric capacity for different SNR regimes, as indicated in Fig. 2. The optimal choice for user 1 is $[x_1(0)\ x_1(1)] = [-1\ \ +1]$ for all SNRs while the optimal $[x_2(0)\ x_2(1)]$ is obtained as

- **Antipodal constellation**–low SNR:

$$[x_2(0)\ x_2(1)] = [-1\ \ +1]. \tag{7}$$

- **Maximum minimum distance constellation**–high SNR:

$$[x_2(0)\ x_2(1)] = [-1/2\ \ +1/2], \tag{8}$$

- **Asymmetric constellation**–moderate SNR:

$$[x_2(0)\ x_2(1)] = [0.1571\ \ +1.4055]. \tag{9}$$

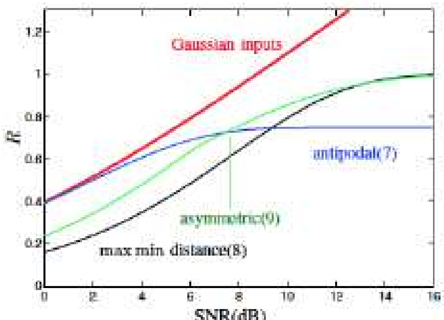


Fig. 2. The attainable symmetric rate for the input constellations (7), (8), and (9) and for Gaussian inputs.

We call (8) the maximum minimum distance constellation is because the minimum distance between the constellation points is maximized. Fig. 2 also plots the capacity under a power constraint but not a constellation constraint, in which case Gaussian inputs are optimal. Note that the constellation in (8) does not meet the power constraint with equality. The fact that the asymmetric constellation in (9) attains the capacity in the moderate SNR regime shows there exists a choice of input constellations which aids joint decoding. Indeed any constellation of the form $[x_1(0)\ x_1(1)] = [-1\ +1]$ and $[x_2(0)\ x_2(1)] = [\Delta\ \Delta + 1.2484]$ for Δ such that the power constraint is satisfied attains the same rate performance as (9).

IV. Gaussian Approximation of Density Evolution

In this section, we study the convergence of the BP algorithm through density evolution (DE): we assume that the message at the output of a variable node has a Gaussian distribution while the message at the output of a MAC node has a distribution given by the mixture of two Gaussian densities. Under these assumptions, we determine the convergence of a for the BP algorithm as a function of the code distribution in (5). In the BP algorithm, there are eight types of messages: we will denote them as

$$m_{\rm ij}^{(k)}, \quad i,j \in \{{\rm v},{\rm c},{\rm m}\},\ i \neq j, \quad k \in \{1,2\}, \tag{10}$$

where the subscript indicates that the message is from the type of node i to type of node j and the superscript indicates the user k. All possible messages $m_{\rm ij}^{(k)}$ are shown in Fig. 3 where $\mathfrak{L}$ indicates the log-likelihood ratio of the channel output.

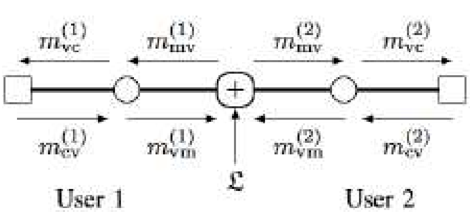


Fig. 3. Messages employed in the MAC BP algorithm.

To simplify the analysis of the BP algorithm, we study the evolution of mutual information during BP decoding by means of the mutual information transfer function. Here we assume that the message is a Gaussian random variable with mean μ. Thus, the mutual information between the message $m_{ij}^{(k)}$ and the input constellation point $[x_k(0), x_k(1)]$ for user k is defined as $I(x_k; m_{ij}^{(k)})$ in (13) such that $0 \le I(x_k; m_{ij}^{(k)}) \le 1$. Herein, we have

$$p_Y(\tau|X=x) = \frac{1}{\sqrt{2\pi}\sigma_Y} \exp\left(-\frac{\tau - \mu x - c}{2\sigma_Y^2}\right), \tag{11}$$

for

$$c = \frac{1-(x_k(0))^2}{x_k(0)-x_k(1)}\mu, \quad \sigma_Y^2 = \frac{2-2x_k(0)x_k(1)}{x_k(0)-x_k(1)}\mu.$$

Note that the messages $m_{\rm vc}^{(k)}$, $m_{\rm cv}^{(k)}$, $m_{\rm vm}^{(k)}$ follow from the standard approximation of the BP evolution while the MAC node update rule must be analyzed separately. The relationship between the messages in Fig. 3 is given as

$$m_{\rm mv}^{(1)} = \tag{12}$$
$$\log\left(\frac{e^{-\frac{1}{2\sigma^2}(Y-x_1(0)-x_2(0))^2} e^{m_{\rm vm}^{(2)}} + e^{-\frac{1}{2\sigma^2}(Y-x_1(0)-x_2(1))^2}}{e^{-\frac{1}{2\sigma^2}(Y-x_1(1)-x_2(0))^2} e^{m_{\rm vm}^{(2)}} + e^{-\frac{1}{2\sigma^2}(Y-x_1(1)-x_2(1))^2}}\right),$$

where $m_{\rm mv}^{(2)}$ is defined in a symmetric manner.

Since we wish to track the mean of the messages, let $F_{ij}^{(k)}(\sigma^2, \mu_{\rm vm})$ for $i,j \in \{0,1\}$, $k \in \{1,2\}$, be defined as the mean of the messages from MAC to variable node. Here, σ^2 denotes the noise variance of the channel, and $\mu_{\rm vm}$ is the mean of the messages from variable to MAC node for the other user. The quantity $F_{00}^{(1)}(\sigma^2, \mu_{\rm vm})$ is defined in (14) at the top of this page. The other terms are defined in an analogous manner. Using these definitions, the mutual information $I_{\rm mv}^{(k)}$ can be evaluated as

$$I_{\rm mv}^{(k)} = \sum_{[i,j]\in[0,1]^2} \frac{1}{4} J\left(F_{ij}^{(k)}(\sigma^2, J^{-1}(I_{\rm vm}^{(3-k)}))\right), \tag{15}$$

where the function $J(\cdot,\cdot)$ is defined in (13).

Let now $I_{\rm vc}^{(k,l)}$ be the mutual information between x_k and the variable-to-check message and $I_{\rm cv}^{(k,l)}$ be the mutual information between x_k and the check-to-variable message in the l-th iteration for user k, respectively. Similarly, let $I_{\rm vm}^{(k,l)}$ be the mutual information between x_k and the messages from variable to MAC nodes, and $I_{\rm mv}^{(k,l)}$ be the mutual information between x_k and the messages from MAC to variable nodes, respectively. Then the mutual information between x_1 and the messages from variable to check nodes for user 1 is given as

$$I_{\rm vc}^{(1,l)} = \sum_i \sum_j \lambda_{i,j} J\left(J^{-1}(I_{\rm mv}^{(1,j,l-1)}) + (i-1)J^{-1}(I_{\rm cv}^{(1,l-1)})\right), \tag{16}$$

where

$$I_{\rm mv}^{(1,j,l-1)} = \sum_{[a,b]\in[0,1]^2} \frac{1}{4} J\left(F_{ab}^{(1)}\left(\sigma^2, jJ^{-1}(I_{\rm cv}^{(2,l-1)})\right)\right), \tag{17a}$$

$$I_{\rm cv}^{(k,l)} = 1 - \sum_m \rho_m J\left((m-1)J^{-1}(1 - I_{\rm vc}^{(k,l)})\right). \tag{17b}$$

The term $I_{vc}^{(2,l)}$ can be derived similarly as (17b). The conver-

$$I(x_k; m_{ij}^{(k)}) \triangleq J(x_k, \mu) = \frac{1}{2} \sum_{x \in x_k(0), x_k(1)} \int_{-\infty}^{\infty} p_Y(\tau | X = x) \log_2 \left(\frac{2 \cdot p_Y(\tau|X = x)}{p_Y(\tau|X = x_k(0)) + p_Y(\tau|X = x_k(1))} \right) \mathrm{d}\tau, \quad k \in \{1, 2\} \tag{13}$$

$$F_{00}^{(1)}(\sigma^2, \mu_{\mathrm{vm}}) \triangleq \mathbb{E}\left[m_{\mathrm{mv}}^{(1)} | X_1 = x_1(0), X_2 = x_2(0)\right] = -\mu_{\mathrm{vm}} + \frac{x_2(0)x_2(1) - x_1(0)x_1(1)}{\sigma^2} \tag{14}$$

$$+ \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-z^2} \log \left[\frac{1 + e^{\sqrt{4\mu_{\mathrm{vm}} + \frac{2(x_2(0) - x_2(1))^2}{\sigma^2}} z + \mu_{\mathrm{vm}} + \frac{1 - x_2(0)x_2(1)}{\sigma^2}}}{1 + e^{-\sqrt{4\mu_{\mathrm{vm}} + \frac{2(x_2(0)-x_2(1))^2}{\sigma^2}} z - \mu_{\mathrm{vm}} - \frac{1 - x_1(0)x_2(1) - x_2(0)x_2(1) + x_1(1)x_2(1) + x_1(0)x_2(0) - x_2(0)x_1(1)}{\sigma^2}}} \right] \mathrm{d}z$$

gence conditions for the l-th iteration then become

$$I_{\mathrm{vc}}^{(1,l)} > I_{\mathrm{vc}}^{(1,l-1)}, \quad I_{\mathrm{vc}}^{(2,l)} > I_{\mathrm{vc}}^{(2,l-1)}, \quad \forall I_{\mathrm{vc}} \in (0, 1). \tag{18}$$

From these conditions on the two-dimensional recursion in (16) and (17), we obtain approximate conditions for the convergence of the BP algorithm, which will be exploited for code design in Sec. VI.

Using the GA of the DE, we can formulate the numerical optimization of the symmetric rate as in (6):

$$\max_{\Lambda_{l_1,l_2}} R_{\mathrm{sym}} = 1 - \frac{\sum_{l_1,l_2} l_1 \Lambda_{l_1,l_2}}{\sum_{l_1} l_1 \Gamma_{l_1}^{(1)}}, \tag{19}$$

subject to convergence conditions

$$\lambda_2 < \exp\left(\frac{1}{2\sigma}\right) \left(\sum_{m=2}^{d_c} (m-1)\rho_m \right)^{-1}, \tag{20a}$$

$$F^{(k)}(\lambda_{i,j}, I_{\mathrm{vc}}) > I_{\mathrm{vc}}, \quad \forall I_{\mathrm{vc}} \in [0, 1], \tag{20b}$$

$$x_k^2(0) + x_k^2(1) \leq 2, \tag{20c}$$

for $k \in \{1, 2\}$, $\lambda_2 = \sum_i \lambda_{i,2} = \sum_j \lambda_{2,j}$ and with

$$F^{(k)}(\lambda_{i,j}, I_{\mathrm{vc}}) = \sum_{i=j} \lambda_{i,j} J(J^{-1}(I_{\mathrm{mv}}^{(k)}) + (i-1)J^{-1}(I_{\mathrm{cv}}^{(k)})).$$

Note that (20a) is the stability constraint, (20b) is the DE for GA in (16) while (20c) enforces the power constraint for the binary input constellation.

V. Higher Order Constellations

In this section, we construct a superposition coded modulation based on the binary constellation we introduced in Sec II to increase the input cardinality and symmetric rate.

For instance a 4-PAM input can be obtained as the superposition of two binary constellations, that is

$$X_1^N = \sqrt{P_{11}} X_{11}^N + \sqrt{P_{12}} X_{12}^N, \tag{21}$$

$$X_2^N = \sqrt{P_{21}} X_{21}^N + \sqrt{P_{22}} X_{22}^N, \tag{22}$$

where X_{ij} is a binary LDPC codewords mapped to the binary constellation and under the power constraint

$$P_{11} + P_{12} \leq 1, P_{21} + P_{22} \leq 1. \tag{23}$$

BP and capacity region can be derived in a similar way, but to simplify the optimization procedure, here we only consider the antipodal constellation, with

$$P_{11} = P_{12}, P_{21} = P_{22}, R_{11} = R_{12} = R_{L1}, R_{21} = R_{22} = R_{L2}, \tag{24}$$

where R_{ij} is the rate of codeword X_{ij}, and R_{Li} is the different power level. Since we mainly focus on antipodal constellations, we also define

$$R_{sym1} = R_{11} + R_{21}, R_{sym2} = R_{12} + R_{22}. \tag{25}$$

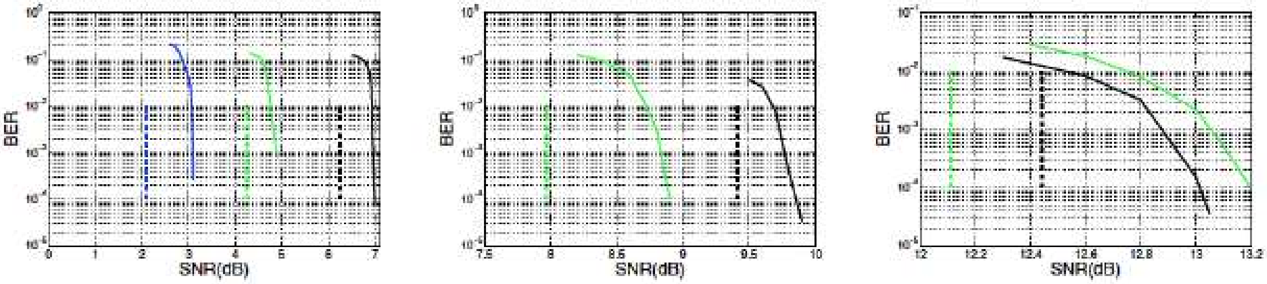
VI. Simulation Results

For simulations, the transmission blocklength is $N = 10^4$, and the code optimization in (19), (20) is performed by using CVX and linear programming. The check node degree distribution is fixed to a single degree while the variable node degree distribution is maximized for a maximum degree distribution equal to 200. The parity check matrix is constructed by using the PEG algorithm in [10]. At decoding, the messages are exchanged through the MAC node at each iteration; the maximum number of iterations is set to 300. For message passing schedules, different update rate of two users are employed to verify that scheduling has no influence on the overall performance.

Fig. 4 presents the error probabilities of the proposed code design versus the SNR for the input constellations in Sec. III. The simulations verify that the antipodal constellation in (7) has the best performance at low rate, i.e., $R \leq 0.5$, while no code can be successfully decoded for rates larger than 0.7 with this input constellation. Instead, for higher rates with $R > 0.8$, the maximum minimum distance constellation in (8) attains the best performance, and the gap to capacity decreases as the SNR increases. For the range of rates in $0.7 \leq R \leq 0.8$, the asymmetric constellation in (9) attains a better performance than both the constellations in (7) and (8). This is rather surprising as it shows that there exists a specific constellation choice which aids joint decoding, at least for the proposed code design. Note that the constellation in (9) arises from the maximization of an information theoretic quantity under a large blocklength assumption. However, the performance gain of this constellation can also be verified in the finite blocklength regime based on numerical simulations as shown in Fig. 4. The code polynomials for the relevant codes in Fig. 4 are shown in Tab. I. Although the optimization algorithm allows for any joint variable node distribution, the best performance is obtained when the matrix Λ in (5a) has a diagonal structure.

In Fig. 5 the distance between the performance of our code designs and the theoretical performance in Fig. 2 is displayed. It can be seen that we are able to attain the theoretical performance to within 1dB for most SNRs.

Fig. 6 shows the BER performance for a superposition modulation antipodal constellation with same total rate $R_{total} = 2$,



(a) $R = 0.5$ (b) $R = 0.75$ (c) $R = 0.93$

Fig. 4. Error probability for (i) blue – antipodal constellation, (ii) green – asymmetric constellation, (iii) black – max min distance constellation. Dashed lines indicate the theoretical performance in Fig. 2 for the selected rates and solid lines indicate the results from numerical simulations, respectively.

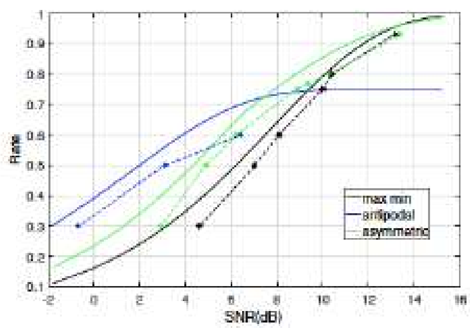


Fig. 5. Gap to the capacity for error probability $P_e = 10^{-4}$. The solid lines indicate the maximal achievable symmetric rates from Fig. 2, and the stars indicate the attainable rates for our code designs.

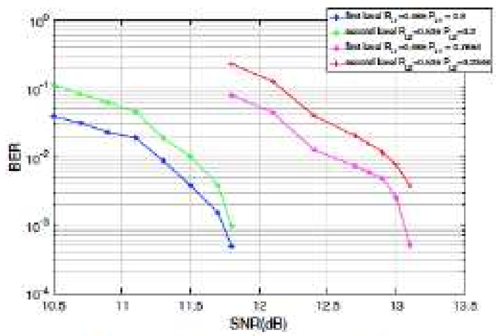


Fig. 6. BER performance for $n = 10^4$, $R_{sym1} + R_{sym2} = 2$

but different power allocations. The first level with larger power $P_{L1} = 0.8$ is about 1.5 dB better than the other one with $P_{L1} = 0.7654$. In this case, we cannot guarantee the power allocation with $P_{L1} = 0.8$ can give us the best BER, and the optimization of the power allocation would be an interesting issue.

TABLE I
LDPC DEGREE DISTRIBUTION FOR THE TWO-USER GMAC AND THE RELEVANT SCENARIOS IN FIG. 6

R	0.5	0.75	0.93
	antipodal	asymmetric	max. min. dist.
$\Gamma(x)$	x^{10}	x^{20}	x^{60}
$\Lambda(x)$	$0.6322x_1^2x_2^2$ $0.3236x_1^3x_2^3$ $0.0220_1^{18}x_2^{18}$ $0.0105x_1^{19}x_2^{19}$ $0.0117x_1^{100}x_2^{100}$	$0.3639x_1^2x_2^2$ $0.5213x_1^3x_2^3$ $0.0496x_1^{12}x_2^{12}$ $0.0325x_1^{13}x_2^{13}$ $0.0176x_1^{33}x_2^{33}$ $0.0151x_1^{74}x_2^{74}$	$0.1688x_1^2x_2^2$ $0.6899x_1^3x_2^3$ $0.0280x_1^{12}x_2^{12}$ $0.1133x_1^{13}x_2^{13}$

VII. Conclusion

This paper proposes an implementation of joint decoding for LDPC codes for the two-user, symbol-synchronous, binary-input GMAC that maximizes the symmetric rate. We focus, in particular, on a joint code and constellation design and show that it is possible to construct good irregular LDPC codes which attain the theoretical performance to within 1dB for most SNRs. We show that at low SNR the best performance is attained by using antipodal inputs at both encoders. For high SNR, the best performance is attained by a constellation choice which maximizes the minimum distance among the received constellation points. For the moderate SNR regime, we show the interesting result that an asymmetric constellation is able to outperform symmetric constellations. We also extend the results to larger constellation alphabets, but only for the antipodal constellation case.

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