PHYSICS OF PLASMAS 25, 053107 (2018)

Interaction of an electron with coherent dipole radiation: Role of convergence and anti-dephasing

A. P. L. Robinson[1] and A. V. Arefiev[2]
[1]Central Laser Facility, STFC Rutherford-Appleton Laboratory, Didcot OX11 0QX, United Kingdom
[2]Department of Mechanical and Aerospace Engineering, University of California at San Diego, La Jolla, California 92093, USA

(Received 30 January 2018; accepted 30 April 2018; published online 21 May 2018)

The impact of longitudinal electric fields that are present in intense focusing and defocusing electromagnetic pulses on electron acceleration is investigated. These fields are typically much weaker than the transverse fields, but it is shown that they can have a profound effect on electron energy gain. It is shown that the longitudinal electric field of a defocusing pulse is directed backward along the trajectory of an accelerated electron, which leads to a continuous net energy gain. At the same time, the effect of the transverse oscillating electric field in a defocusing pulse is to reduce the electron energy over multiple oscillations. In contrast to a well-known interaction with a plane wave, the electron is able to retain a substantial amount of energy following its interaction with a defocusing pulse. The roles of the transverse and longitudinal electric fields are reversed in a focusing pulse, which leads to a reduction in the energy retention. The present analysis underscores the importance of relatively weak oscillating electric fields in focusing and defocusing pulses. *Published by AIP Publishing.* https://doi.org/10.1063/1.5024049

I. INTRODUCTION

The foundational status that fast electrons have in ultra-intense laser-plasma interactions has made the subject of fast electron generation an on-going relevance. One mechanism which continues to be discussed is "Direct Laser Acceleration" (DLA), which is generally understood to mean the acceleration of electrons in underdense plasma when the electron experience both the laser field and some self-consistently generated "plasma" fields.[1–4] The specific case of acceleration of electrons in a ponderomotively evacuated ion channel has been a major topic of study.[5–11]

Plasma fields are often held to be necessary in order to ensure net energy gain from the interaction, and the Lawson-Woodward Theorem is usually cited to support this argument. The Lawson-Woodward (L-W) Theorem is usually stated in different ways by different authors. Esarey *et al.*[12] state that the L-W theorem ensures that a charged particle cannot gain energy from an electromagnetic field in vacuum, provided that a number of conditions apply. Others[13] state the L-W theorem as follows: a charged particle cannot be accelerated by a plane electromagnetic wave alone. The latter statement of the L-W theorem is probably the better way to state the theorem. However, this statement of the L-W theorem also changes its relevance to laser-plasma interactions, as plane waves will not always be good approximations to strongly focussed laser pulses, even though plane waves are often used in analyses of DLA.

The question of net energy gain from strongly diffracting electromagnetic pulses was addressed by Troha *et al.*[13] who used an exact solution to Maxwell's equations corresponding to a coherent pulse emitted from an electric dipole. Troha *et al.* employed numerical integration of the equations of motion to show that an electron gained net energy from this pulse when the radius of curvature of the wavefronts was small, with the energy gain vanishing as the radius of curvature became increasingly large (i.e., in the limit that the L-W theorem applies). Clearly, the results of Troha *et al.* show that "plasma fields" are not absolutely essential for net energy gain, provided that one does not employ plane waves.

This raises a number of questions, including (i) Does it matter whether a coherent pulse of radiation is focussing or defocussing? and (ii) what role "plasma fields" play in interactions with strongly diffracting pulses? The role of "plasma fields" in interactions with plane waves has been the subject of a number of investigations. One important aspect of electric fields in plane wave interactions is that electric fields can alter the dephasing rate of the electron, which can lead to electrons achieving "superponderomotive" energies when the dephasing rate is strongly reduced. In the case of a strongly diffracting pulse, does this mean that superponderomotive net energy gain is possible?

In this paper, we show that (i) the net energy gain is highly dependent on whether the pulse is focussing or de-focussing, with de-focussing required for high net energy gain, and (ii) superponderomotive energy gain is indeed possible when a diffracting radiation pulse interacts with a pre-accelerated electron. We also employ a coherent dipole radiation pulse for the electromagnetic pulse. We examine the interaction in greater detail than Troha *et al.*, and we show that the net energy gain is due to the radial component of the electric field.

In terms of the organization of this paper, we have adopted the following structure: In Sec. II, we present arguments for why a defocussing pulse is necessary for high net energy gain. In Sec. III, we first provide a very brief recap of the key results of electron motion in a plane EM wave and how anti-dephasing can produce trajectories which achieve superponderomotive energies. In Sec. IV A, we provide the details of the electromagnetic field that we use in our numerical calculations. In Sec. IV B, we describe the numerical method employed. In Sec. V, we present the results of our calculations and proceed to analyze and discuss these results. The conclusions of this paper are then summarized in Sec. VI.

II. FOCUSSING VERSUS DE-FOCUSSING OF THE DIPOLAR PULSE

As already mentioned, the analysis of DLA is often simplified by treating the laser pulse as a plane electromagnetic wave. This simplification is frequently used in semi-analytical models to make the problem tractable. The rationale for neglecting focusing or defocusing of the laser pulse is the well-known stabilizing effect of the plasma on laser beam propagation where the plasma effectively functions as an optical waveguide. However, full elimination of focusing and defocusing requires a precise match of laser and plasma parameters. It is therefore worth examining the role that these effects can play in electron dynamics.

The goal of this section is to examine the role of focusing and defocusing by examining dynamics of a single electron irradiated in a vacuum by an electromagnetic wave. In order to make the analysis easier, we assume that both focusing and defocusing are relatively weak.

Our basic setup is shown in Fig. 1. We are considering a linearly polarized laser pulse propagating along the x-axis in the positive direction. We assume that the laser pulse has a limited width R along the z-axis, while the laser magnetic field is polarized along the y-axis. This implies that the laser electric field has a longitudinal component E_x in addition to the transverse component E_z.

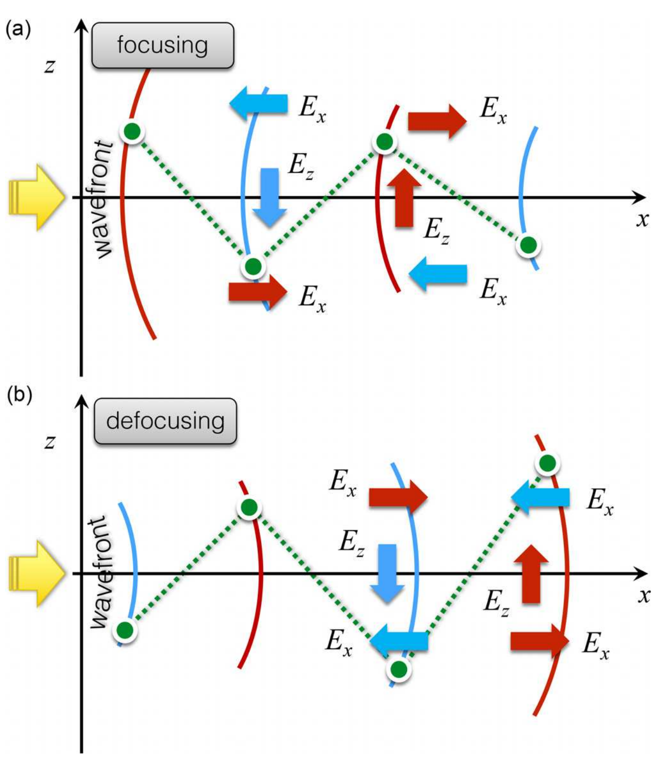

FIG. 1. Electric field structure in focusing and defocusing laser pulses. Both pulses are propagating to the right. Circles indicate the positions of the turning points along the electron trajectory that is schematically indicated by dotted lines.

Here, we are only interested in the local field structure. Neglecting corrections introduced by the finite width of the pulse, we have the following structure for the transverse electric and magnetic fields

$$E_z = E_0 \sin(2\pi x/\lambda - \omega t + \psi), \tag{1}$$

$$B_y = -E_z, \tag{2}$$

where E_0 is the amplitude, ψ is a phase, λ is the wavelength, and $\omega \equiv 2\pi c/\lambda$ is the frequency of the laser pulse.

A longitudinal electric field must also be present. Its structure is set by the condition $\nabla \cdot \mathbf{E} = 0$, which yields

$$\frac{\partial E_x}{\partial x} = -\frac{\partial E_z}{\partial z}. \tag{3}$$

Since the transverse spatial scale is R, the amplitude of the longitudinal electric field is relatively small

$$|E_x| \sim E_0 \lambda / R \ll E_0. \tag{4}$$

We assume that the (x, y)-plane is the plane of symmetry for the transverse fields, which implies that the amplitude of the transverse fields peaks at $z = 0$. It is then evident from Eq. (3) that E_x is asymmetric, with $E_x = 0$ at $z = 0$.

The sign of E_x is determined by the Poynting vector, $\mathbf{S} = (c/4\pi)\mathbf{E} \times \mathbf{B}$. If the laser pulse is focusing, then the transverse component of the Poynting vector, $S_z = (c/4\pi)E_x B_y$, should be directed towards the (x, y)-plane. This means that

$$E_x E_z > 0 \text{ for } z > 0, \tag{5}$$

$$E_x E_z < 0 \text{ for } z < 0, \tag{6}$$

where we took into account that $B_y = -E_z$. On the other hand, the Poynting vector should be directed away from the (x, y)-plane if the laser pulse is defocusing. This then leads to the following conditions:

$$E_x E_z < 0 \text{ for } z > 0, \tag{7}$$

$$E_x E_z > 0 \text{ for } z < 0. \tag{8}$$

Figure 1 schematically illustrates both cases.

In order to determine how the longitudinal field of the pulse impacts electron energetics, we examine the shape of the electron trajectory. We do this by neglecting corrections associated with focusing and defocusing. This means that we are considering an electron trajectory in a plane wave of relativistic amplitude. Such a wave pushes the electron forward, while inducing transverse oscillations. Transverse stopping points that we denote for convenience as $z = z_*$ correspond to the peak amplitude of the transverse electric field (see Sec. 3 of Ref. 6 for illustrations). Since the electron is negatively charged, its upward motion along the z axis is stopped by a positive transverse electric field, whereas its downward motion is stopped by a negative transverse electric field (see Fig. 1). The sign of the transverse electric field at each stopping point can then be determined by the following simple condition:

$$z_* E_z > 0. \tag{9}$$

As evident from Fig. 1, the sign of the longitudinal electric field at each turning point is uniquely defined by the sign of the transverse electric field. It follows directly from Eqs. (5), (6), and (9) that the longitudinal electric field of a focusing laser pulse is always positive at a turning point. In contrast to that, the longitudinal electric field of a defocusing laser pulse is always negative at a turning point. Since the electron is moving forward along the electron trajectory, a negative longitudinal field would increase the energy of the electron, whereas a positive longitudinal field would decrease the energy. We therefore arrive to an important conclusion: longitudinal electric fields of a focusing laser pulse decrease the total electron energy, while longitudinal electric fields of a defocusing laser pulse increase the total electron energy.

This observation allows us to readily estimate how much energy the electron would lose or gain from the longitudinal electric field after each laser period depending on the focusing or defocusing pulse configuration. The electron is moving in the forward direction with a velocity close to the speed of light c if the normalized laser amplitude $a_0 \equiv |e|E_0/m_e \omega c$ is highly relativistic, i.e., $a_0 \gg 1$. We can then estimate the change in energy induced by a longitudinal electric field as

$$\Delta\epsilon \approx -|e| E_x c \Delta t, \tag{10}$$

where Δt is the time it takes to perform one transverse oscillation. In a co-moving frame of reference, the period of one oscillation is $2\pi/\omega$. Taking into account that the electron is moving forward with a relativistic factor γ, we find that in the laboratory frame of reference, the period of oscillations is longer by a factor of γ, with $\Delta t \sim \gamma c/\omega$. We use the estimate for E_x given by Eq. (4) to obtain a relative change in the electron energy because of the longitudinal electric field

$$\frac{|\Delta\epsilon|}{\gamma m_e c^2} \approx a_0 \lambda / R. \tag{11}$$

As already stated, this change is negative if the pulse is focusing and positive if it is defocusing.

It is worth pointing out that R is the characteristic scale over which the fields are changing in the transverse direction. There are two characteristic scales: the width of the beam and the radius of curvature of its wavefronts. The parameter R represents the smallest of the two. In the case of a Gaussian beam, R represents the width of the beam over longitudinal distances less than the Rayleigh length from the focal spot.

Finally, we can employ the qualitative picture that has been developed in this section to also gain insight into the role of focusing and defocusing in the work performed by transverse electric fields. Let us consider a part of the trajectory where the electron begins its upward motion at $z=0$ and then returns to $z=0$ after being turned around at a stopping point. In a plane wave, this motion is symmetric, with the electron gaining the same amount of energy from E_z as it lost traveling to the stopping point. This energy exchange takes place in a positive field. We now recall that the electron is moving forward while performing this transverse oscillation. In a focusing laser pulse, the transverse electric field increases as the electron returns to the $z=0$ location, whereas the field decreases in a defocusing pulse. We therefore conjecture that the work done by the transverse electric field of a defocusing laser pulse gradually reduces electron energy. In contrast to that, a focusing laser pulse should gradually increase the electron energy.

We now summarize our qualitative findings in Table I that shows how the electron energy changes as a result of the interaction with different components of the electric field in focusing and defocusing laser pulses.

It must be emphasized that these are relatively small energy changes that occur with each oscillation as compared to the primary energy oscillations of the order of $\Delta\epsilon \approx a_0^2 m_e c^2$ that occur due to the oscillations of the transverse electric field at the electron location.

III. ANTI-DEPHASING IN THE PLANE WAVE CASE

In order to aid the discussion of the main points in this paper, it is useful to review the main results from the plane wave case. Suppose that an electron, which initially has momentum along the x-axis ($\mathbf{p}_0 = p_0\hat{\mathbf{x}}$, where $p_0 > 0$), undergoes an interaction with a plane wave: $\mathbf{A} = [0, A_0(\tau)\cos(\omega_L \tau), 0]$, where $\tau = t - x/c$ and A_0 is a slowly varying envelope function. This problem has been dealt with in detail in various publications. Let tilde denote a momentum that is normalized by m_{ec} and lowercase a denote a normalized vector potential. The results can then be summarized as follows:

$$\tilde{p}_y = a_y, \tag{12}$$

$$\tilde{p}_x = \frac{a_y^2}{2\Xi} + \tilde{p}_0, \tag{13}$$

$$\gamma = \sqrt{1 + a_y^2 + \left(\frac{a_y^2}{2\Xi} + \tilde{p}_0\right)^2}. \tag{14}$$

In the above equations, the term Ξ is a constant of motion, namely,

$$\Xi = \gamma - \tilde{p}_x. \tag{15}$$

In the case where the electron is initially at rest, $\Xi = 1$, and we obtain what we might term the "vacuum" or "ponderomotive" case: $\gamma = 1 + \frac{a_y^2}{2}$. When $p_0 > 1$, it can be seen that $\Xi < 1$ (in the extreme limit $\Xi \to \frac{1}{2\tilde{p}_0}$). Therefore, when $p_0 > 0$, $p_x > \frac{a_y^2}{2}$, which is what we term "superponderomotive." The quantity, Ξ, is essentially a dephasing rate. Pre-acceleration of the electron or interaction

TABLE I. Summary of electron energy changes in focusing and defocusing cases.

	Transverse electric field E_z	Longitudinal electric field E_x
Focusing pulse	$\Delta\epsilon > 0$	$\Delta\epsilon < 0$
Defocusing pulse	$\Delta\epsilon < 0$	$\Delta\epsilon > 0$

with electric fields during interaction with the laser pulse can reduce the dephasing rate and produce a superponderomotive trajectory. Note that since $a_y \to 0$ at some point (assuming a finite laser pulse), there will be no net energy gain in the case considered above. This is why the dipole radiation case is interesting, as it is known that this does lead to net energy gain. The question is to what extent the role of anti-dephasing applies to the dipole radiation case.

IV. NUMERICAL METHOD

A. Coherent dipole radiation

Here, we provide a brief description of the dipole radiation field that we use in our calculations and was previously used by Troha *et al*. If we adopt the Lorenz gauge condition, then the vector potential in vacuum is governed by

$$\nabla^2 \mathbf{A} - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} = 0. \tag{16}$$

Any *cartesian* component of the vector potential is therefore governed by a scalar wave equation. We can therefore immediately state that a valid solution is

$$A_z = \Gamma f(\eta)/r, \tag{17}$$

where $\eta = r - ct$ and $f(\eta)$ is an arbitrary pulse profile. Note that r is the radius in spherical polar coordinates. As the Lorenz gauge applies, the corresponding scalar potential is

$$\phi = \frac{c^2\Gamma z}{r^2}\left[\frac{h}{r} + \frac{f}{c}\right], \tag{18}$$

where $\partial h/\partial t = f$. Having obtained a solution (for any f) in terms of the potentials, we can calculate the cartesian components of the $\mathbf{E}$ and $\mathbf{B}$ fields via $\mathbf{B} = \nabla \times \mathbf{A}$ and $\mathbf{E} = -\nabla\phi - \partial\mathbf{A}/\partial t$. These are

$$E_z = \frac{c^2\Gamma}{r}\left[\frac{z^2}{r^2}\left(\frac{3}{r}\left(\frac{h}{r}+\frac{f}{c}\right) - \frac{1}{c}\frac{\partial f}{\partial \eta}\right) - \left(\frac{1}{r}\left(\frac{h}{r}+\frac{f}{c}\right) - \frac{1}{c}\frac{\partial f}{\partial \eta}\right)\right], \tag{19}$$

$$E_y = \frac{c^2\Gamma yz}{r^3}\left(\frac{3}{r}\left(\frac{h}{r}+\frac{f}{c}\right) - \frac{1}{c}\frac{\partial f}{\partial \eta}\right), \tag{20}$$

$$E_x = \frac{c^2\Gamma xz}{r^3}\left(\frac{3}{r}\left(\frac{h}{r}+\frac{f}{c}\right) - \frac{1}{c}\frac{\partial f}{\partial \eta}\right), \tag{21}$$

$$B_z = 0, \tag{22}$$

$$B_x = -\frac{y\Gamma}{r^2}\left[\frac{f}{r} - \frac{\partial f}{\partial \eta}\right], \tag{23}$$

$$B_y = \frac{x\Gamma}{r^2}\left[\frac{f}{r} - \frac{\partial f}{\partial \eta}\right]. \tag{24}$$

B. Particle Pusher

The relativistic equations of motion for a single electron were numerically integrated via the Boris method.[14] This was done using a time step of 0.1 fs for 1.6×10^6 time steps (it was determined that the solution has converged for this choice). The fields derived in the preceding section treat the divergent/defocussing case, but the fields can be derived for the convergent case as well. For the pulse profile, f, we used

$$f(\eta) = \exp\left[-\frac{(\eta - r_0)^2}{2\sigma^2}\right]\cos(\omega\eta). \tag{25}$$

The frequency, ω, was set equal to $2\pi/\lambda$, where $\lambda = 1\mu$m. The pulse envelope width parameter, σ, was set to $\sigma = 5\mu$m throughout. The amplitude fixing constant, Γ, was set to $\Gamma = r_0 m_e c a_0/e$, where the normalized vector potential was set to $a_0 = 10$. The electron was initially placed at $\mathbf{x}(t=0) = [x_0, 0, 0]$, with initial momentum given by $\mathbf{p} = [p_0, 0, 0]$. We denote the initial Lorentz factor of the electron by γ_0. The value of r_0 was set to $r_0 = x_0 - 20\mu$m. In this set-up, the equivalent plane-wave calculation would result in a maximum γ of 51, which we denote as γ_{pl}.

V. RESULTS AND DISCUSSION

A. Overview of results

The first set (set A) of calculations was done for the case where the electron was initially at rest. The initial position along the x-axis was varied. The Lorentz factor of the electron at the end of each calculation for this set is plotted in Fig. 2.

In the second set (set B) of calculations, the electron initially had a Lorentz factor of 2.23 with its momentum directed along the +x-axis. The initial position along the x-axis was varied. The Lorentz factor of the electron at the end of each calculation for this set is plotted in Fig. 3. Note that sets A and B only consider a divergent/defocussing pulse.

In the third and fourth sets of calculations, we examined the effect of a focussing or convergent pulse (in which the pulse is travelling inwards towards the origin). This involved amending the field equations appropriately and setting the pulse to be at a larger radius than the electron (via

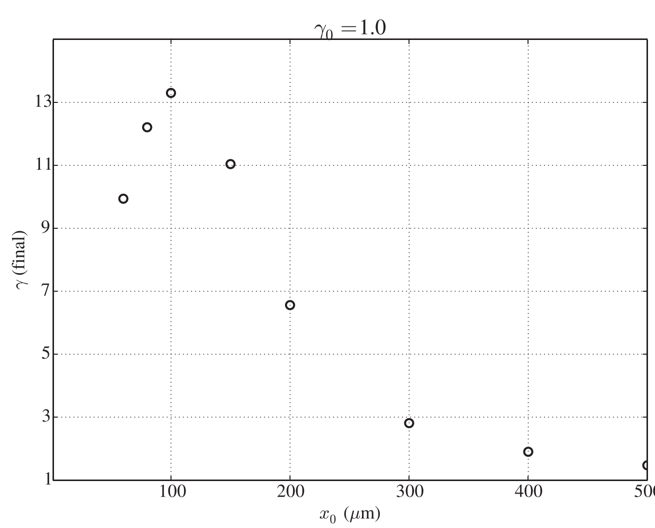

FIG. 2. The final value of the Lorentz factor of the electron at the end of the calculation for a set of calculations with γ_0=1 (Set A).

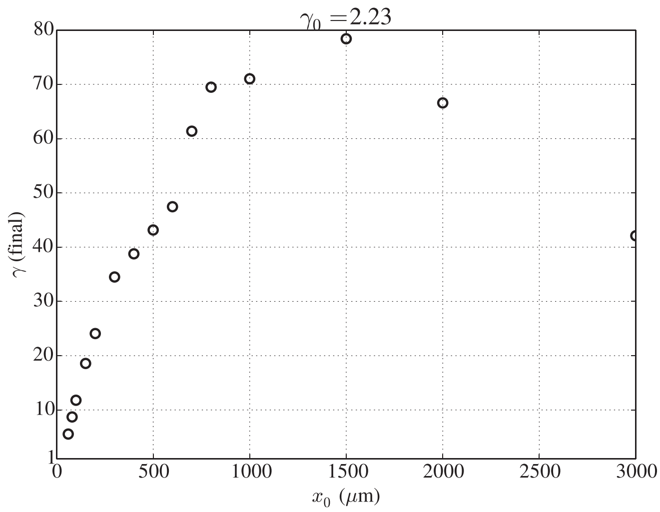

FIG. 3. The final value of the Lorentz factor of the electron at the end of the calculation for a set of calculations with γ_0=2.23 (Set B).

$r_0 = x_0 + 20\mu$m.). In the third set (set C), the electron was initially at rest and x_0 was varied, and in the fourth set (set D), the electron initially had a Lorentz factor of 2.23 with its velocity directed toward the origin. The results of set C are presented in Fig. 4 where they are plotted along with those from set A. The results of set D are plotted in Fig. 5 where they are plotted along with those from set B.

B. Net energy gain

The Lorentz factors observed at the end of the calculations do indeed represent (where $\gamma > 1$) a net energy gain. An example of the evolution of γ with time is shown in Fig. 6. This is a calculation from set B: $\gamma_0 = 2.23$ and $x_0 = 700\mu$m. This clearly shows that the electron undergoes net energy gain in this calculation and the same is true for all the other calculations reported in Figs. 2 and 3. What Fig. 6 also shows is that the final energy of the electron is generally

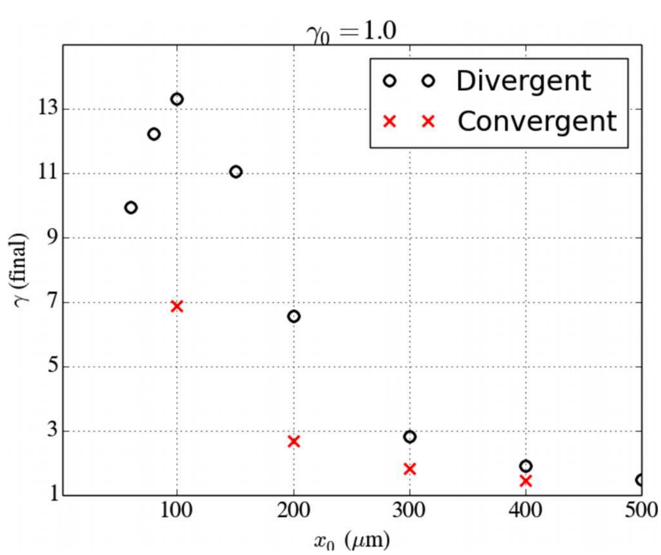

FIG. 4. The final value of the Lorentz factor of the electron at the end of the calculation for a set of calculations with γ_0=1 in which the pulse is diverging (divergent, set A) and converging (convergent, set C).

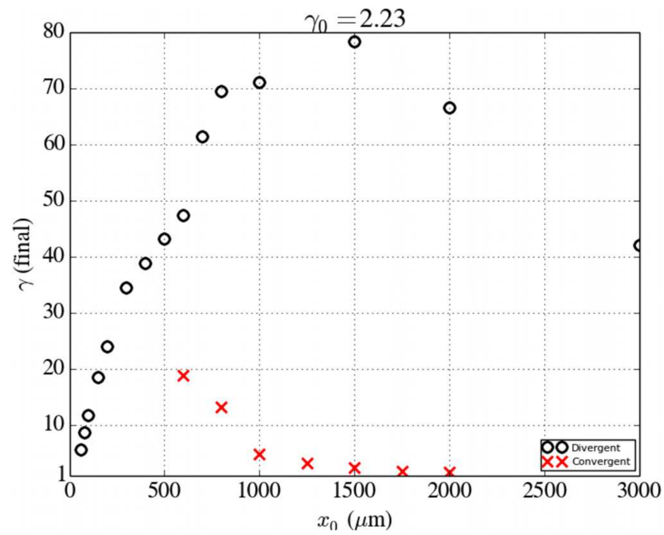

FIG. 5. The final value of the Lorentz factor of the electron at the end of the calculation for a set of calculations with γ_0=2.23 in which the pulse is diverging (divergent, set B) and converging (convergent, set D).

less than the peak energy that the electron will achieve during its interaction with the pulse.

The work done on the electron can, however, be decomposed into two components: work done by the longitudinal (i.e., parallel to the local wave-vector) component of the electric field (E_x for sets A and B) and work done by the E_z component of the field. The contributions as a function of time for the same case are shown in Fig. 7. The results shown in Fig. 7 are representative of the calculations in sets A and B: the net work done on the electron is done by the E_x component of the field, i.e., the longitudinal component of the field. Interestingly the E_z components act to counteract the radial acceleration, by removing a portion of energy from the electron. This is a point that was not really discussed by Troha *et al*.

C. Effect of anti-dephasing on net energy gain

From the calculations done in set B, it is clear that pre-acceleration has an effect that is similar to anti-dephasing

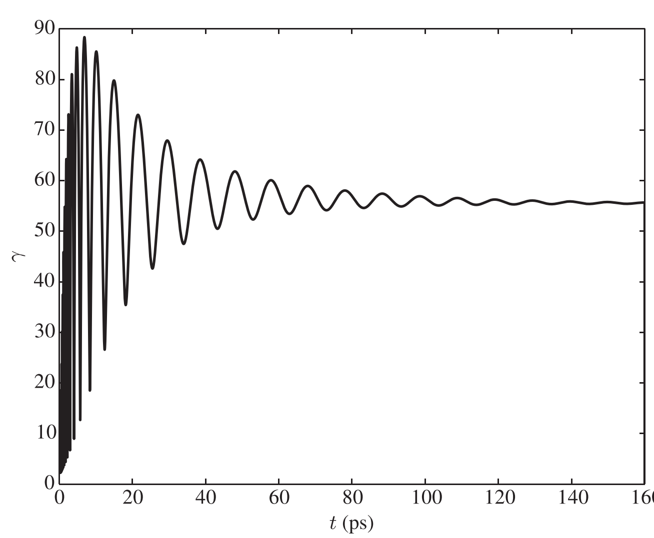

FIG. 6. Evolution of γ in time for the case of $\gamma_0 = 2.23, x_0 = 700\mu$m (set B).

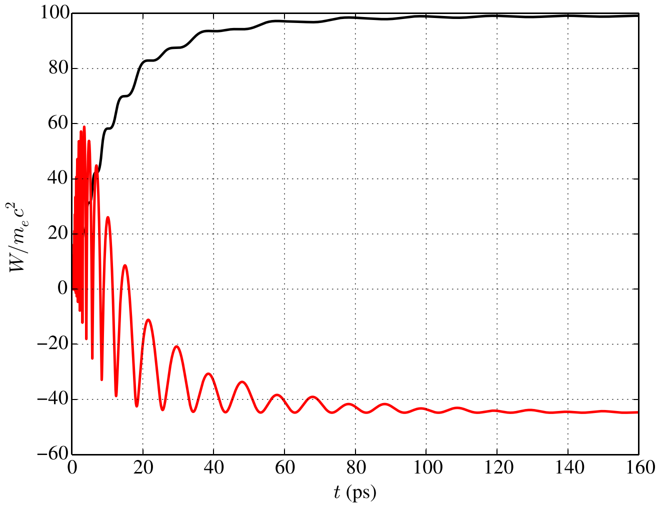

FIG. 7. Results from the calculation in set B with $\gamma_0 = 2.23, x_0 = 700\mu$m. Black line: Work done on the electron by the E_x field component. Red line: Work done on the electron by the E_z field component.

in the plane wave case. Not only does this lead to net energy gains that are higher at most values of x_0 than that is achieved for $\gamma_0 = 1$, but also we see in Fig. 3 that $\gamma_f > 51$ over a range of values of x_0. This means that superpondermotive net energy gain is clearly observed in these calculations.

Two further sets of calculations (E and F) were carried out to examine how the net energy gain was affected by the value of γ_0. In set E, this was done for $x_0 = 500\mu$m, and in set F, this was done for $x_0 = 1000\mu$m. In both sets E and F, the initial momentum of the electron p_0/m_ec was varied between 0 and 10. The results of these calculations are shown in Figs. 8 and 9.

These calculations show that anti-dephasing does not affect the net energy gain in the dipole radiation case as it affects the peak energy in the plane wave case. In the latter case, the peak energy should increase with $\tilde{p}_0$ in the limit that $\tilde{p}_0$ is large. In the dipole radiation case, this does not happen, and Figs. 8 and 9 show that the net energy gain first increases with $\tilde{p}_0$ before declining as $\tilde{p}_0$ increases further.

D. Focussing vs de-focussing

From the calculations done in sets C and D and their comparison to sets A and B, which are shown in Figs. 4 and 5, respectively, it can be seen that there are very significant differences between a focussing (or convergent) pulse and a

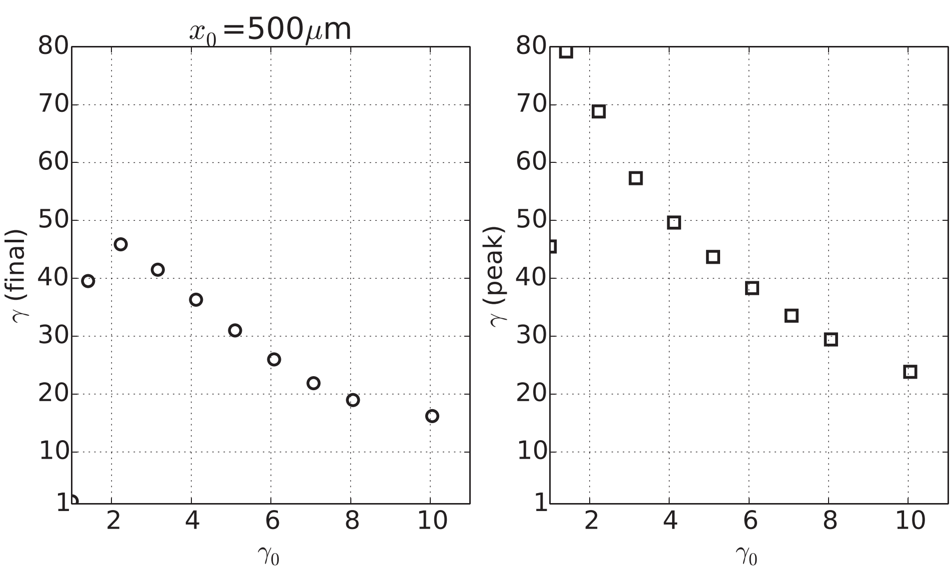

FIG. 8. The final (Left) and peak (Right) values of the Lorentz factor of the electron in set E.

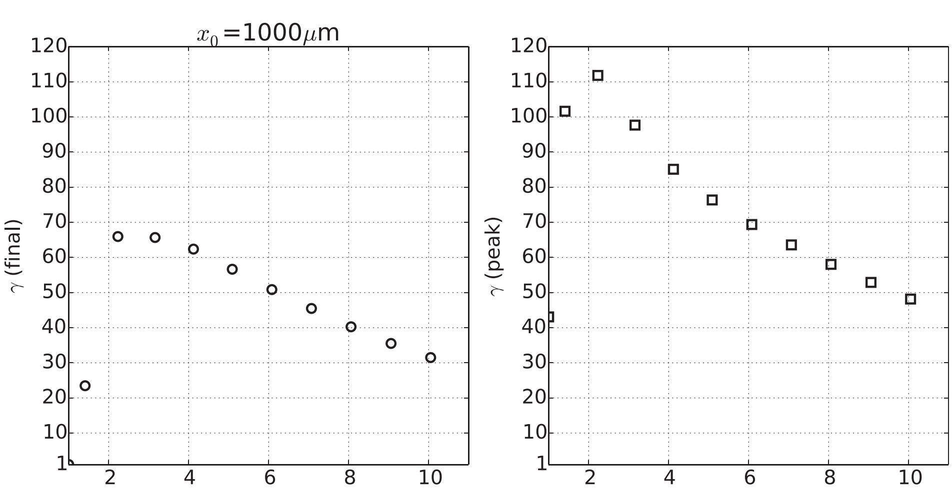

FIG. 9. The final (Left) and peak (Right) values of the Lorentz factor of the electron in set F.

de-focussing (or divergent) pulse. This is particularly the case in set D where the electron has some initial momentum and the radius of the pulse is rather large in comparison to the wavelength. As the analysis of Sec. II was carried out under the assumption of weak focussing or de-focussing, this analysis is particularly pertinent to this set, and we find that it correctly predicts that the divergent case yields substantial net energy gain (from the axial field), whereas the convergent case yields negligible net energy gain.

In the case of set C, this is less pronounced, and this may be because the radius of the pulse relative to the wavelength is somewhat larger; however, the net energy gain is much less for the convergent pulse than for the divergent pulse. The broad finding of Sec. II therefore still holds.

VI. SUMMARY AND DISCUSSION

In this paper, we have examined two issues related to the interactions of free electrons with a strong non-planar electromagnetic wave by examining a spherical wave although the results apply more generally to focussing and de-focussing laser pulses. The first issue is the dependence of net energy gain on whether the pulse is focussing or de-focussing. We have shown that de-focussing is necessary (not focussing) both via detailed calculations and via general theoretical arguments. The second issue is whether super-ponderomotive net energy gain can occur in these interactions. We have shown that this is indeed possible. In both issues, the cause of net energy gain was clarified in relation to earlier work: it arises because of a longitudinal component of the electromagnetic pulse, which will be absent in the case of a single plane wave. It is worth pointing out that the described energy change mechanism is a purely relativistic effect, whereas the conventional ponderomotive acceleration caused by a transverse intensity gradient is not.

These findings are interesting in the context of efforts to further understand direct laser acceleration of electrons irradiated by intense laser pulses with relativistic intensity. Typically, the role of an oscillating longitudinal electric field is neglected if it is significantly weaker than the transverse oscillating electric field. What we demonstrated here is that an oscillating electron trajectory samples the longitudinal electric field in such a way that the field no longer oscillates from the point of view of the accelerated electron. This observation weakens the argument that the longitudinal electric field can always be safely neglected due to its small amplitude. Our findings strongly indicate that the mechanism of direct laser acceleration cannot be seen solely in terms of interactions with plane waves. In the light of our observations, it is worth examining whether the discussed effect profoundly changes the energetics in regimes relevant to experiments with ps and multi-ps laser pulses where electrons are accelerated as they perform numerous transverse oscillations. In this case, the problem becomes more complex because the oscillating longitudinal field has to compete with quasi-static longitudinal electric fields that can arise in a laser irradiated plasma.[6,7,15] The presence of the plasma can also dramatically change the dephasing between the electron and the laser pulse,[16] which can affect how the energy is transferred from the oscillating longitudinal electric field of the pulse.

ACKNOWLEDGMENTS

A.V.A.'s research was supported by the National Science Foundation under Grant No. 1632777 and AFOSR project No. FA9550–17-1–0382.

[1] J. L. Shaw, F. S. Tsung, N. Vafaei-Najafabadi, K. A. Marsh, N. Lemos, W. B. Mori, and C. Joshi, Plasma Phys. Controlled Fusion **56**, 084006 (2014).
[2] S. A. Gaillard, T. Kluge, K. A. Flippo, M. Bussmann, B. Gall, T. Lockard, M. Geissel, D. T. Offermann, M. Schollmeier, Y. Sentoku, and T. E. Cowan, Phys. Plasmas **18**, 056710 (2011).
[3] V. Marceau, C. Varin, T. Brabec, and M. Piche, Phys. Rev. Lett. **111**, 224801 (2013).
[4] G. D. Tsakiris, C. Gahn, and V. K. Tripathi, Phys. Plasmas **7**, 3017 (2000).
[5] A. Pukhov, Z.-M. Sheng, and J. M. ter Vehn, Phys. Plasmas **6**, 2847 (1999).
[6] A. Arefiev, V. Khudik, A. Robinson, G. Shvets, L. Willingale, and M. Schollmeier, Phys. Plasmas **23**, 056704 (2016).
[7] A. Robinson, A. Arefiev, and D. Neely, Phys. Rev. Lett. **111**, 065002 (2013).
[8] A. Arefiev, B. Breizman, M. Schollmeier, and V. Khudik, Phys. Rev. Lett. **108**, 145004 (2012).
[9] A. Arefiev, A. Robinson, and V. Khudik, J. Plasma Phys. **81**, 475810404 (2015)..
[10] V. Khudik, A. Arefiev, X. Zhang, and G. Shvets, Phys. Plasmas **23**, 103108 (2016).
[11] D. Stark, T. Toncian, and A. Arefiev, Phys. Rev. Lett. **116**, 185003 (2016).
[12] E. Esarey, P. Sprange, and J. Krall, Phys. Rev. E **52**, 5443 (1995).
[13] A. L. Troha, J. V. Meter, E. an, R. M. Alvis, Z. A. Unterberg, K. Li, N. C. Luhmann, Jr., A. K. Kerman, and F. V. Hartemann, Phys. Rev. E **60**, 926 (1999).
[14] C. K. Birdsall and A. B. Langdon, *Plasma Physics via Computer Simulation* (CRC Press, 2004).
[15] A. Sorokovikova, A. V. Arefiev, C. McGuffey, B. Qiao, A. P. L. Robinson, M. S. Wei, H. S. McLean, and F. N. Beg, Phys. Rev. Lett. **116**, 155001 (2016).
[16] A. P. L. Robinson, A. V. Arefiev, and V. N. Khudik, Phys. Plasmas **22**, 083114 (2015).