Localization of Forced Oscillations in the Power Grid Under Resonance Conditions

Tong Huang*, Nikolaos M. Freris†, P. R. Kumar*, and Le Xie*
*Department of Electrical and Computer Engineering
Texas A&M University, College Station, Texas
Email:{dreamflame, prk, le.xie}@tamu.edu
† Division of Engineering
New York University Abu Dhabi, Abu Dhabi, United Arab Emirates
Email: nf47@nyu.edu

***Abstract*—This paper proposes a data-driven method to pinpoint the source of a new emerging dynamical phenomenon in the power grid, referred to "forced oscillations" in the difficult but highly risky case where there is a resonance phenomenon. By exploiting the low-rank and sparse properties of synchrophasor measurements, the localization problem is formulated as a matrix decomposition problem, which can be efficiently solved by the exact augmented Lagrange multiplier algorithm. An online detection scheme is developed based on the problem formulation. The data-driven nature of the proposed method allows for a very efficient implementation. The efficacy of the proposed method is illustrated in a 68-bus power system. The proposed method may possibly be more broadly useful in other situations for identifying the source of forced oscillations in resonant systems.**

***Index Terms*—Forced oscillations, resonant systems, phasor measurement unit (PMU), robust principal component analysis (RPCA), Big Data.**

I. INTRODUCTION

The ever-growing coverage of modern power systems by *Phasor Measurement Units* (PMUs) provides large volumes of data that can be leveraged by system operators. For example, detecting *forced oscillations*, i.e., sustained oscillations driven by periodic perturbations, has become an emerging concern worldwide. Reference [1] lists 27 forced oscillation events since 1966, which were exacerbated by the increasing level of variable generation resources. The impact of forced oscillations in power systems is multifaceted. For instance, they may interact with protection relays, thereby triggering potential cascading outages [2]. Forced oscillations may also lead to undesirable mechanical vibrations in electric devices, potentially increasing the probability of equipment failure and maintenance costs, as well as reducing of equipment lifespan [2]. Therefore, it has become highly desirable for system operators to detect, locate, and mitigate potential forced oscillations in a nearly real-time fashion.

Forced oscillations are typically introduced by malfunctioning devices in power systems [3], such as low speed diesel generators [4] or poorly tunned control systems of generators [5], [6]. In addition, cyclic loads, such as aluminum plants, are also likely to cause forced oscillations [7]. One effective way to suppress forced oscillations is by temporarily reshaping the output of the generation at the source of the oscillation. However, it is still a practically challenging task to pinpoint in real-time the exact source of forced oscillation [8], [9]. Intuitively one may expect that source of a forced oscillation should be close to the measurements associated with the most significant oscillations. Nevertheless, it still possible that the source of forced oscillation might be far away from where the most severe oscillations are observed, [1], [9]. Compounding the challenge, it is impractical to rely on a planning model to locate the source of forced oscillations, since a planning model can only approximately describe the system behavior at some typical operating conditions, whereas forced oscillations can present at any operating conditions. It is highly desirable to develop an online mechanism to pinpoint sources of forced oscillations in variable scenarios, preferably without resorting to the underlying physical model.

The main approaches for locating the sources of forced oscillations can be taxonomized in two categories. The first one features methods that utilize information from both the system dynamic model as well as from field measurements. The mode shape estimation method [9] and the hybrid-simulation-based method [10] fall into this category. As mentioned above, the unavailability of accurate model information renders the online application of these model-based approaches practically infeasible. Consequently, it is useful to consider approaches that solely use field measurements directly, cf. as in the damping torque method [11] and the energy-based method [6], [12]. In these methods, various indicators such as the damping torque [11] and the dissipating energy [6], [12] are calculated based on field measurements, in order to identify the sources of forced oscillation. However, the existing measurement-based methods have some limitations [13]. For example, some required measurements, e.g., the rotor angle, might not be available for the direct calculation of damping torque [13]; the effectiveness of the energy-based method may be limited to specific scenarios owing to major assumptions on loads

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and system topology [13]. Finally, the efficacy of the existing methods under a resonance condition has not been fully investigated.

In this paper, we propose a *data-driven* method to locate forced oscillation sources during real-time operation of power systems. By exploring the low-rank and sparsity properties of the PMU measurements, the problem of locating forced oscillation sources is formulated as a matrix decomposition problem, which can be efficiently solved by state-of-the-art signal processing algorithms. We develop an online algorithm with the following chief advantages: 1) it does not require any information pertaining to the system dynamic model; 2) it can accurately identify the source of the forced oscillation when resonance is triggered.

The rest of this paper is organized as follows: Section II explains the forced oscillation phenomenon and defines relevant concepts from a linear control perspective; Section III demonstrates the low-rank and sparsity properties of PMU measurements, and also introduces the proposed method; the efficacy of proposed method is validated in Section IV; Section V concludes the paper and discusses future research directions.

II. Mathematical Interpretation of Localization of a Forced Oscillation Source

The small-signal behavior of a power system around an operating condition can be captured by a continuous-time linear state-space model [14], [15]:

$$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + B\boldsymbol{u}(t) \tag{1a}$$

$$\boldsymbol{y}(t) = C\boldsymbol{x}(t) \tag{1b}$$

For a given time t, vector $\boldsymbol{x}(t) \in \mathbb{R}^l$ collects all state variables in the power system; input vector $\boldsymbol{u}(t) \in \mathbb{R}^r$ denotes control setpoint changes of generators and load fluctuation; vector $\boldsymbol{y}(t) \in \mathbb{R}^n$ characterizes the PMU measurements; all vectors are taken as column vectors; matrices A, B and C denote the system matrix, input matrix, and output matrix, respectively, of appropriate dimensions. We denote by $\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_l\}$ the set of all eigenvalues of matrix A. We assume the system is stable, i.e., $\mathrm{Re}\{\lambda_i\} < 0$ for all $i \in \{1, 2, \ldots, l\}$.

For convenience, the input vector $\boldsymbol{u}(t)$ can be written as

$$\boldsymbol{u}(t) = [u_1(t), u_2(t), \ldots, u_r(t)]^T. \tag{2}$$

A forced oscillation is formally defined next. Let input i^* denote the source of forced oscillation in the system. Such input varies periodically, i.e., it can be considered as the superposition of f distinct frequency components. The amplitudes, frequencies and phase displacements of these frequency components comprise the sets $\mathcal{A} = \{a_k\}$, $\Omega = \{\omega_k\}$ and $\Phi = \{\phi_k\}$ [1], respectively, for all $k \in \{1, 2, \ldots, f\}$. Therefore, we can write the $i^* - th$ input with periodical injection as

$$u_{i^*}(t) = \sum_{k=1}^{f} a_k \sin(\omega_k t + \phi_k). \tag{3}$$

[1]The reference of the phase displacements can be any frequency component with frequency in Ω

As a result, sustained oscillations will be then triggered over the grid. We term the measurements near input i^* as *source measurements*, and the generator/load associated with input i^* as the *forced oscillation source*. In particular, suppose that ω_k is near the frequency of a poorly-damped mode, i.e. $\exists k' \in \{1, 2, \ldots, l\}$,

$$\omega_k \approx \mathrm{Im}\{\lambda_{k'}\}, \quad \mathrm{Re}\{\lambda_{k'}\}/|\lambda_{k'}| \approx 0. \tag{4}$$

In such case, oscillations with growing amplitude, i.e., resonance, may be observed [9].

Since the measurement vector $\boldsymbol{y}(t)$ is recorded by PMUs at discrete times, the time evolution of measurements can be represented by a *measurement matrix* Y, defined as follows. Suppose that the sampling rate of PMUs is f_s, and let 0 indicate when the forced oscillation (FO) starts. Then the measurement matrix Y up to time T is defined by a column concatenation

$$Y := \begin{bmatrix} \boldsymbol{y}(0) & \boldsymbol{y}(1/f_\mathrm{s}) & \ldots & \boldsymbol{y}(n/f_\mathrm{s}) & \ldots & \boldsymbol{y}(N'/f_\mathrm{s}), \end{bmatrix} \tag{5}$$

where $N' = \lfloor T f_\mathrm{s} \rfloor$ and $\lfloor \cdot \rfloor$ is the floor operation. The $(n+1)$-th column of Y in (5) denotes the "snapshot" of all PMU measurements at the time n/f_s.

The problem of locating FO sources amounts to the problem of identifying the source from the measurement matrix Y. It is worth noting that the parameters of the state-space model in (1) may be unknown due to frequent changes in operating conditions in power systems. Hence, the measurement matrix Y is assumed to be the only available information for the purpose of source localization in this paper.

III. Problem Formulation and Proposed Methodology

A. Two Properties of PMU Measurements

1) Low-rank Property: Due to the redundant deployment of PMUs and tight coupling between different measurements over power grids, PMU measurements are highly correlated with one another. Therefore, the measurement matrix Y in (5) manifests a low-rank structure. In other word, even in the presence of forced oscillation, there is a low-rank component of the measurement matrix, which we denote by Z. The low-rank matrix Z can be intuitively considered to reflect a "general trend" of all measurements over time.

2) Sparse Property: The disturbance shown in (3) at input i^* obscures the low-rank structure by making the source measurements deviate from the "general trend". The deviation can be quantified by a matrix X, which is the difference between the measurement matrix Y and the low-rank matrix Z, i.e., $X = Y - Z$. Note that it is reasonable to assume a linear decomposition under the linear state-space dynamical system (1). Due to the limited number of the FO sources as well as the source measurements, the number of non-zero elements in X is expected to be small. Therefore, matrix X is a sparse matrix. The sparsity property of FOs in the PMU measurements manifests itself in matrix X.

B. Problem Formulation

The above two properties of PMU measurements inspire us to pinpoint the FO source by finding the largest entries of the sparse component of the data matrix, with the measurements associated with these entries being the source measurements. Formally the problem of FO source localization described in Section II becomes one of finding appropriate X and Z such that the following constrains are satisfied:

$$Y = Z + X \tag{6a}$$
$$\operatorname{rank} Z \leq r \tag{6b}$$
$$\|X\|_0 \leq p, \tag{6c}$$

where matrices Y, Z, and X are the measurement, low-rank, and sparse matrices, respectively, as defined in Sections II and III-A; $\|\cdot\|_0$ is the l_0 (pseudo) norm which counts the number of non-zero entries of a matrix; r denotes an upper bound of the low-rank matrix Z, and p denotes the upper bound for the number of non-zero entries of the (sparse) matrix X. In an ideal setting, when r and p are known and a decomposition that satisfies (6) actually exists, one way to numerically tackle the problem is by means of *alternating projections* reported in Appendix A.

However, the alternating projection algorithm is not guaranteed to converge, or it may converge to a local minimum, since both constraint sets in (6) are not convex. Besides, r and p should be provided to the alternating projection algorithm, which are typically unknown beforehand when the measurement matrix Y is the only available information. Although the formulation shown in (6) is appealing, it is rendered impractical by the above limitations for locating forced oscillation sources in an online fashion.

The formulation shown in (6) can be replaced by a convex relaxation

$$\min_{X} \quad \|Y - X\|_\star + \eta\|X\|_1 \tag{7}$$

where $\|\cdot\|_\star$ denotes nuclear norm (the sum of singular values); $\|\cdot\|_1$ denotes l_1 norm (the sum of absolute values of all entries); η is a tunable parameter usually called *regularizer*. It is proven that the low-rank matrix Z and the sparse matrix X can be exactly reconstructed from the formulation shown in (7) under certain technical assumptions [16]; this formulation is termed as *Robust Principal Component Analysis* (RPCA) in [16]. Besides, the optimization problem (7) can be efficiently solved by various algorithms, and a brief comparison between these algorithms is reported in [17]. In this paper, the exact augmented Lagrange multiplier (ALM) method [18] is applied to (approximately) decompose a measurement matrix Y into a low-rank Z and a sparse X.

C. Forced Oscillation Source Localization Procedure

Based on (5) and (7), the overall procedure of locating FO source is summarized as follows:

1) Let 0 be the time instant when a forced oscillation starts; such instant can be accurately estimated based on an early event detection algorithms, such as described in [19].
2) Construct the measurement matrix Y based on (5) up to time T.
3) Solve (7) and obtain $X = [x_{i,j}]$ by the exact ALM method.
4) The source measurement index i_m^* can be obtained by
$$[i_m^*, j_m^*]^T = \arg\max_{i,j} x_{i,j}. \tag{8}$$
5) The forced oscillation source is the generator/load near the source measurement i_m^*.

IV. Case Study

In this section, the effectiveness of the RPCA-based algorithm for locating the FO source is validated by a study of a 68-bus benchmark power system [20]. We begin by introducing the specifics of the test system along with the initial setting of the algorithm. Subsequently, we adopt the test system as an illustrative example to describe resonance phenomena can be triggered by forced oscillations. As will be shown at the end of this section, the proposed method can pinpoint the source of forced oscillation, even when resonance is triggered.

A. System Description and Context for the Proposed Algorithm

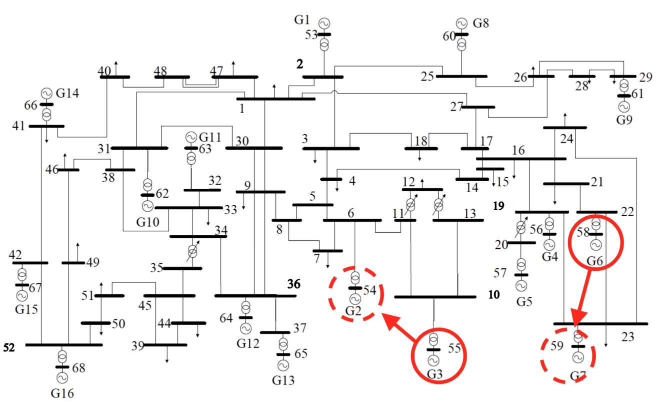

Fig. 1. IEEE 68-bus benchmark power system [14]

The raw parameters of the test system are reported in the Power System Toolbox (PST) [20], while its topology is shown in Figure 1. In order to create the poorly-damped oscillatory modes, we remove all power system stabilizers (PSS) except for the one installed at Generator 9. Afterwards the linearized model (A, B, C) in (1) is extracted by PST, resulting in 25 oscillatory modes with frequencies from 0.1 Hz to 2 Hz in the test system.

The input u_i in (2) is the voltage setpoint of generators, and the measurement vector $\boldsymbol{y}(t)$ incorporates all bus voltage magnitudes at time instant t; the sampling rate of a PMU is assumed to be 60 Hz, and $T = 10$ s. The resulting measurement matrix $Y \in \mathbb{R}^{68\times601}$; and the tunable parameter η in (7) is set to be 0.0408, which is the default setting of the ALM algorithm [18].

B. Creation of Resonance Cases in the Test System

As mentioned above, there are 25 oscillatory modes in the frequency range of interest, and 16 generators in the test system, use $p \in \{1, 2, \dots, 25\}$ and $q \in \{1, 2, \dots, 16\}$ to represent a given mode/generator, respectively. We then select one frequency from the 25 modal frequencies, i.e., the frequency associated with mode p, and inject it into one of the generators, i.e., generator q, at time $t = 10$s. Next, we conduct a 50-second simulation of the system based on (1), (2) and (3), in order to get the time response of the system for a signal with a frequency injected at a specific location. Finally, we exhaust all combinations of signal frequencies and injection locations to obtain 400 (25×16) forced oscillation cases. Each forced oscillation case can be represented by an index pair (p, q).

The time response of each forced oscillation is discretized to obtain the measurement matrix, using a given sampling rate f_s, cf. (5). Let $Y_{(p,q)} = [y_{i,j}^{(p,q)}]$ be the measurement matrix of the forced oscillation case (p, q), with $Y_{(p,q)}$ organized in such a way that the measurements from the generator buses are stacked in the first 16 rows. Finally, a forced oscillation case (p, q) is marked as a *resonance case* if the following condition holds:

$$q \neq i_g^*, \tag{9}$$

where

$$[i_g^*, j_g^*]^T = \arg\max_{i,j} y_{i,j}^{(p,q)}. \tag{10}$$

Equations (9) and (10) suggest that, in a resonance case, the most severe oscillation should not present at the measurement from the bus connecting to the generator with the periodic injection.

Using the above criterion, 44 forced oscillation cases are marked as resonance cases. Figure 2 shows some typical waveforms when resonance is triggered, and Table I summarizes the corresponding modal frequencies and injection locations. As shown in Figure 2, the most significant oscillations are not observed at the buses connected directly to the forced oscillation sources in the resonance cases.

TABLE I
MODAL FREQUENCIES AND INJECTION LOCATION OF RESONANCE CASES SHOWN IN FIGURE 2

Figure	Freq. (Hz)	Gen.
2.a	1.29	12
2.b	0.38	13
2.c	1.15	13
2.d	1.17	13
2.e	1.18	13
2.f	0.38	16

C. Method Performance

The 44 resonance cases are used to test the performance of the proposed method. In the test cases, we successfully pinpoint the forced oscillation sources in 40 resonance cases, hence achieve 90.91% accuracy. One natural question is how geographically close are the results to the ground truth, in the cases that localization is incorrect. In Table II, the identified results alongside the ground truth are listed for the four failed cases. Based on Table II, we highlight the exact locations in Figure 1: the generators in solid circle are where the sinusoidal signals are injected, i.e., the real sources of forced oscillations, whereas the generators in the dash circles are the sources identified by the proposed method. As shown in Figure 1, the identified sources in the failed cases are geographically close to the actual sources, which means that the proposed algorithm can effectively narrow down the search scale of the forced oscillation sources even in the failed cases. This further solidifies the potential merits of the proposed method.

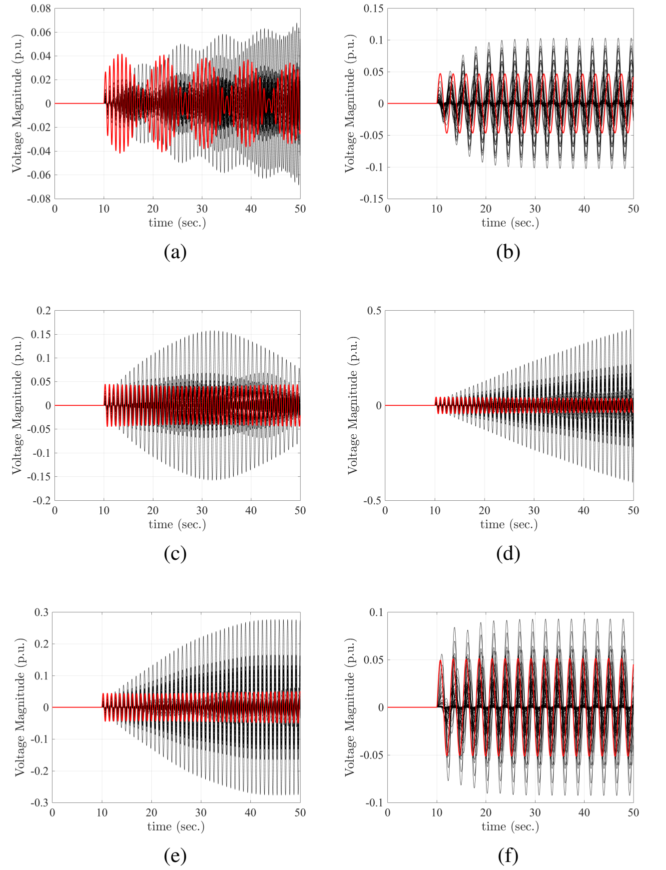

Fig. 2. Typical waveforms when resonance is triggered: voltage magnitude of the generator bus connected with the forced oscillation source (red); the voltage magnitudes of the remaining buses (black).

TABLE II
COMPARISON BETWEEN INACCURATE RESULTS AND GROUND TRUTH

Bus # (Identified)	Bus # (Truth)	Freq. (Hz)
54	55	1.1033
54	55	1.1504
59	58	1.2855
59	58	1.2892

V. Conclusions

In this paper, a model-free, PMU data-driven method to pinpoint forced oscillation sources is proposed and tested. By exploiting the low-rank and sparsity properties of PMU data, we have formulated the localization problem as an instance of matrix decomposition. We have derived an online detection and localization method based on convex programming. Numerical simulations based on a 68-bus system suggests that the proposed method achieves satisfactory performance even when resonance happens in the system. Future work will investigate the theoretical justification of the proposed method, and explore the possibility of extending the proposed methodology to a wider class of event localization problems in power systems.

Appendix A
Alternating Projections

Algorithm 1 Alternating Projections (AP)

Input: $Y \in \mathbb{R}^{n\times N'}$
Initialize: Select $X_0 \in \mathbb{R}^{n\times N'}$
for $k = 1, 2, \ldots K$ **do**
  Set $Z_{k-1} = Y - X_{k-1}$
  Set $Z_k = \prod_{\{\mathrm{rank}\, Z\le r\}} Z_{k-1}$
  Set $X_k = \prod_{\{\|X\|_0\le p\}} (Y - Z_k)$
end for
Output Z_K, X_K

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