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Statistical Bus Ranking for Flexible Robust Unit Commitment

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***Abstract*—As the level of uncertain renewable capacity increases on power systems worldwide, industrial and academic researchers alike are seeking a scalable, transparent, effective approach to unit commitment under uncertainty. This paper presents a statistical ranking methodology that allows adaptive robust stochastic unit commitment using a modular structure, with much-needed flexibility. Specifically, this work describes a bus ranking methodology that identifies the most critical buses based on a worst-case metric. An important innovation is the ability to identify alternative metrics on which to rank the uncertainty set – for example to minimize economic dispatch cost or ramping needs, to provide a customized robust unit commitment solution.**

Compared to traditional robust unit commitment models, the proposed model combines statistical tools with analytical framework of power system networks. The resulting formulation is easily implementable and customizable to the needs of the system operator. The method and its applications are validated against other established approaches, showing equivalent solution to the state-of-the-art approach. Case studies were conducted on the IEEE-30, IEEE-118 and the pegase-1354 networks. In addition, the flexibility of bus ranking formulation is illustrated through implementation of alternative definitions of worst-case metrics. Results show that the bus ranking method performs as well as the best of these methods, with the provision of additional flexibility and potential for parallelization.

***Index Terms*—Robust Unit Commitment, Bus ranking, Flexible Ramping, Bootstrap Aggregation, Customizability, Parallelizable**

I. Introduction

The last decade has seen the electric power system under increasing stress due to fundamental changes in both supply and demand technologies. On the supply side, there is a significant shift to renewable generation such as wind and solar which are plentiful, environment-friendly, and widely distributed [1]. On the demand side, there is a growing number of distributed generation resources and thereby necessity for viable demand response strategies. The real-time uncertainty induced by intermittent resources such as wind or price-responsive demand has created new and complex challenges for operational security of power systems.

Critical decision processes like day-ahead unit commitment and real-time economic dispatch require more complex and thorough analysis to manage the uncertainty or risk insufficient ramping capabilities, load surge, and transmission congestion [2]. Various approaches, both physical and operational, have been proposed and tested in the literature to address the impacts of uncertainties in order to obtain reliable solutions [3]. As physical measures, strategies such as balancing area consolidation, increasing flexibility in the resource portfolio, demand-side management, and use of storage devices [4] have been implemented. In addition to the physical means, updating current systems planning and operation with scenario-based, probabilistic and interval based methods has been particularly recommended as a promising solution to help maintain and improve system reliability with increasing penetration of variable energy resources [5]. These formulations and solution methodologies have evolved over the years from early tools based on a priority list and dynamic programming to the current to methods based on mixed integer programming [6].

There is significant literature on various approaches to tackle uncertainty, including stochastic optimization [7], chance-constrained optimization [8] and robust optimization [9]. Stochastic optimization approaches require probability distribution and scenario information about the uncertain processes before implementing the optimization model. The chance-constrained formulation allows for a compromise between risk and cost of solution through manipulation of the probability level. Recently, the robust optimization approach [10] has received significant attention as it does not require detailed analysis of the distribution of the underlying uncertainty. The methodology utilizes upper and lower bounds of the uncertain process to construct a solution immunized against all realizations within the bounds, and optimal for the worst case scenario. Different variations of analytical robust UC have been proposed in [1], [9], [11] and [12]. These models demonstrate significantly better solution efficiency and uncertainty management capabilities as compared to the deterministic approach [13].Current industry approaches, however, still involve solving a deterministic unit commitment model with either adhoc reserves, storage or flexible ramping as protection against variability [14].

The reasons for the lack of implementation of complex analytical models in the industry are multifold. Stochastic optimization and traditional chance-constrained approaches face tractability issues as large number of scenarios are required to attain high probabilistic guarantees of system reliability. Robust optimization does well in terms of tractability because of the deterministic uncertainty set [15] but the approach is susceptible to implementation [13] or complexity [16] bottlenecks in bigger networks. Traditional two-stage robust UC models, lack tools to render them customizable and adaptable

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in accordance to the needs of the system operator.

With increasing scale, complexity, and variability, in modern power systems, there is an increasing need for UC models combining the statistical tools in machine learning applications with the significant data and computational power available to the system operators. Making effective use of these resources supports the need for hybrid methodologies that are simple, customizable, scalable, as well as parallelizable. All the while, providing decisions comparable to proven analytical methods in the literature.

In this paper, a UC methodology is proposed based on a feature ranking algorithm traditionally used in pattern recognition problems to provide a robust unit commitment decision. The decision obtained is secure against all possible uncertainty realizations. This hybrid model unifies statistical tools with analytical framework of power system networks and such a formulation has not been proposed in the literature to our knowledge. The potential benefits of such a model are :

- A simpler and easily implementable formulation to compute a robust unit commitment solution. A case study is performed on an IEEE 30-bus system to demonstrate that the solution is superior to current deterministic models used in industry and on par with the analytical approaches in the literature.
- A customizable framework, as compared to models in literature, providing flexibility to the system operator while not compromising on solution performance. The adaptability of the bus ranking model is demonstrated by computing robust decisions with different definitions of worst-case scenarios. The paper compares robust solution for worst-case economic dispatch cost against flexible ramping products [17], [18] and a hybrid definition. The hybrid definition combines the properties of two differing worst-case metrics with the goal to provide more freedom to the system operator and obtain a solution that satisfies contrasting objectives. An example will be a situation where the system operator needs to be robust against both economic dispatch cost and ramping capacity, a middle ground approach in such a scenario could be the desirable solution. Results of the case study, performed on these definitions, provide interesting insights into potential applications of such an approach and highlight the flexibility of the formulation. This is the first robust UC model addressing flexible ramping proposed in the literature.
- A modular structure which renders it highly scalable and parallelizable, thus not susceptible to implementation and computation bottlenecks in bigger networks. The structure ensures that the methodology is not limited by execution times of sequentially operated processes.

The paper is organized as follows: Section 2 provides an overview of the robust unit commitment models, followed by the detailed description of the bus ranking model in Section 3. The performance of the approach is illustrated in Section 4 followed by concluding remarks in Section 5.

II. Robust Unit Commitment

The robust unit commitment model has been studied in great detail in the power system literature [19], [9], [20], [12], [21].

Most robust unit commitment frameworks are two-stage formulations where the first stage handles unit commitment and the second stage manages dispatch decision which is immunized against all uncertainty realizations and is optimal for the worst case of system operation. Generally, the robust models developed are constructed based on worst-case ED cost. However, different definitions such as worst-case load shedding, operation cost variance etc. are used depending on the operational and economic security of the power system [22]. The first step in robust unit commitment models is to define a deterministic uncertainty set via limited information on uncertain variables like the expected wind power and ranges of possible variations around that expectation [23].

Modern robust models provide the system operator ability to adjust the robustness of the solution by incorporating a parameter, defined as budget of uncertainty in [9]. The budget parameter takes values between zero and the number of buses (N) with uncertainty variables. Thus a value of zero corresponds to a deterministic case while a value of N will mean immunization against uncertainties on all buses. One of the earliest robust formulations was proposed in [9]:

$$\min_{x, y(.)} \left(c^T x + \max_{d \in D} b^T y(d) \right)$$

s.t.

$$\begin{gathered} Fx \leq f, \text{ x is binary} \\ Hy(d) \leq h(d), \ \forall d \in D \\ Ax + By(d) \leq g, \ \forall d \in D \\ I_u y(d) = d, \ \forall d \in D \end{gathered}$$

The objective of the formulation is to minimize commitment cost as well as the highest dispatch cost under uncertainty, thus identifying it as the worst case scenario. The first set of constraints involve commitment variables, followed by dispatch related constraints, min/max generation capacity constraints and uncertain nodal injection constraints [9].

The above formulation is decomposed into two stages, where the master problem is unit commitment problem and the subproblem aims to solve the dispatch problem under the worst-case economic dispatch with fixed unit commitment solution [13]. The min-max subproblem can be converted to a maximization problem, resulting in non-linear terms in the objective function.

Convergence can take a long time if the subproblem is solved using an exact method [16]. Instead, various heuristic techniques such as outer approximation and equivalent MIP have been utilized in literature to overcome this hurdle. For example authors in [12] convert the bilinear subproblem into a mixed integer problem following the observation that for any given unit commitment decision the worst-case value of the uncertain variable is a vertex of the polyhedron uncertainty set. Similar observations motivated the formulation of bus ranking model, as will be discussed later.

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III. Bus Ranking Model

The proposed methodology utilizes a deterministic uncertainty set, a feature ranking algorithm [24] and a simple UC model to compute a commitment schedule immunized against all uncertainties in the polyhedron set. Hence, the formulation behaves like existing robust unit commitment models. The bus ranking formulation begins with constructing a deterministic polyhedron uncertainty followed by computing a baseline commitment schedule using expected net-load. The next step involves utilizing the feature ranking algorithm commonly implemented in machine learning applications, for example in facial recognition and text classification. This algorithm, combined with the baseline commitment schedule allows the model to rank buses of the network on the basis of worst-case metric such as dispatch cost. The last step involves computing a commitment and dispatch decision, based on the bus ranks, that is robust against worst-case uncertainty. The overall algorithm has been summarized in the Algorithms 1 and 2. The following section provides a comprehensive outline of the bus ranking formulation. The methodology can be applied to various metrics but will be described using worst-case economic dispatch cost, as it is the most commonly used definition.

A. Uncertainty Set

This section defines the uncertainty set used to obtain bus ranking. The uncertainty is introduced as a percentage of deviation from expected net load profile estimated using historical data. For a set of nodes $\mathcal{N}$ with renewable integration, d^t being the final net load set at time t, the uncertainty set can be defined as:

$$d^t := \left\{ d_i^t \in [\bar{d}_i^t - \tilde{d}_i^t, \bar{d}_i^t + \tilde{d}_i^t], \quad \forall i \in \mathcal{N} \right\} \tag{1}$$

where $\bar{d}_i^t$ is the expected net load and $\tilde{d}_i^t$ is the possible deviation for bus i at time t. For the proposed methodology $\tilde{d}_i^t$ will be represented as a percentage of $\bar{d}_i^t$. Thus the uncertainty set encompasses situations from maximum renewable injection at a bus to the minimum, which in the case of wind integration is generally zero.

B. Baseline Schedule

The first step in the approach is to run a deterministic unit commitment [25] using expected net load at all buses to obtain a baseline schedule. Please see appendix for description of a unit commitment formulation. The solution provides an initial schedule which can be used as a baseline for rest of the methodology.

C. Dispatch Simulations

Q dispatch simulations [25] are run using the baseline commitment schedule for randomly sampled net load values from the uncertainty set, which defines a range of net load values for each bus i at time t. A set of uniformly distributed random load values are then sampled from this range to be used for each simulation. These simulations provide net-load vectors, called input vectors, for each bus i and a dispatch cost vector, the output vector, of size Q. The input and output vectors obtained from the dispatch simulations can then be fed into a feature ranking algorithm which compares the net-load input vectors of each bus against the output dispatch cost vector ranks each pair according to the similarity between them.

D. Ranking Buses

To rank the buses, feature selection algorithms [26] applied in the field of pattern classification and text classification have been used. Specifically, the filters class of these algorithms, which rely on sorting individual variables based on their correlation to an output variable were analyzed. The bus ranking methodology utilizes a variation of the feature selection technique called the probe feature method [24], [26] with a modification inspired from [27]. The algorithm approximates a linear relationship between the input and output vectors.

The dispatch runs provide N candidate buses with a dataset containing Q input-output vector pairs. In the context of the current problem one such definition could be

$$\text{Input variable} = \text{Net Load at a bus } \forall t \in T$$
$$\text{Output variable} = \text{Economic dispatch cost } \forall t \in T$$

These vector pairs are then compared to find net load vector that is most similar to the dispatch cost. The metric used for comparison is called cosine similarity [28], which is a correlation criterion. A correlation criterion was selected over information theory or decision tree based metrics on account of simple formulation and better performance on continuous data [29]. Cosine similarity is defined as:

$$cos(x^{i,t}.y) = \frac{(x^{i,t}.y)}{||x^{i,t}||_2||y||_2}, \ \forall i \in N, \forall t \in T \tag{2}$$

where $x^{i,t}$ is the net-load vector for bus i at time t, and y is the dispatch cost. The metric always assumes values within the range of -1 and +1 where the -/+ determine the nature of correlation between the input-output pair.

The buses are ranked through an iterative process as described in Algorithms 1 and 2. The Modified Gram-Schmidt orthogonalization [30], [31] is used for projecting features onto null subspaces which terminates once all net-load vectors have been ranked. The final ranking vector consists of bus indices in accordance with ranking. One such example could be $V_{rank}^t = [bus3, bus8, ...]$, $\forall t \in T$, where $bus3$ vector is most correlated to dispatch cost in time period t.

E. Selecting Buses

Now that that bus ranking has been computed, the ranking vector details the order in which buses should be immunized against uncertainty. The selection can be made using the following criterion:

$$\mathcal{V}_t^S = \left\{ i \in V_{rank}^t | V_{rank}^{i,t} \leq \Delta^t \right\} \tag{3}$$

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Algorithm 1 Statistical Bus Ranking Formulation

Compute baseline commitment for $d_i^t \in \bar{d}_i^t$
Run Q dispatch simulations
Define $V_{rank}^t = \{\}$
while $n(V_{rank}^t) < N$ **do**
  Run **Ranking Algorithm**
end while
V_{rank}^t contains bus indices in order of their similarity to the economic dispatch vector.
Run deterministic unit commitment for a robust solution

Algorithm 2 Ranking Algorithm

$\mathcal{N}$ is the set of buses that have uncertainty and N is the number of such buses.
for every bus $i \in \mathcal{N}$ **do**
  Calculate $cos(x^i.y)$
end for
Select bus with $argmax_i \left|cos(x^i.y)\right|$
$V_{rank}^t = V_{rank}^t +$ {selected bus}
$N = N -$ {selected bus}
Project y and remaining x^i onto the null space of selected bus

where $\mathcal{V}_t^S$ are the buses selected to be secured against uncertainty, and $V_{rank}^{i,t}$ is the index of the elements of V_{rank}^t. Similar to traditional robust unit commitment models, the level of conservativeness can be decided using $\Delta^t \in [0, N]$, $\forall t$, called the budget of uncertainty [9]. Thus the value of Δ^t decides how many buses to be immunized. As the value of Δ^t increases from 0 to N so does the size of the uncertainty set and hence level of conservativeness of the resulting commitment solution. Thus at $\Delta^t = 0$ the model will be solved for a deterministic case and at $\Delta^t = N$ the model will immunize against uncertainty at all buses.

F. Robust Unit Commitment

A deterministic optimization model can now be implemented obtain a robust UC decision. Once a value for the Δ^t is known the criterion in (3) can be used to decide which buses to secure while (4) helps select worst case net-load value for the respective bus. nl_i^t being the worst case net-load for bus $i \in \mathcal{V}_t^S$ at time t.

$$nl_i^t = \begin{cases} \bar{d}_i^t + \tilde{d}_i^t, & \text{if } cos(x^{i,t}.y) \geq 0 \ \forall i \in \mathcal{V}_t^S \\ \bar{d}_i^t - \tilde{d}_i^t, & \text{if } cos(x^{i,t}.y) \leq 0 \ \forall i \in \mathcal{V}_t^S \\ \bar{d}_i^t & \text{otherwise} \end{cases} \tag{4}$$

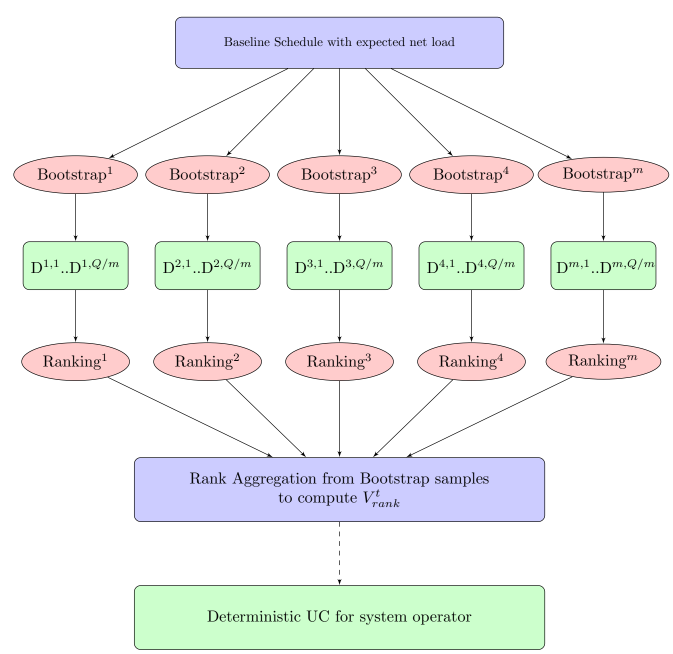
The selected net-load values for the buses to secured and expected net-load values for the rest can be fed into a deterministic framework which then provides a UC solution. The obtained UC solution is robust owing to the observation made by [12] and authors of this paper as mentioned earlier.

Traditionally, two-stage robust models utilize a commitment decision and a user-defined value such as budget of uncertainty, to decide whether the uncertainty at a particular bus should take the max or min value. The second stage of these models is used to make a decision on which bus to secure iteratively over the uncertainty set. This proposed bus ranking model, on the other hand, first calculates an order in which the buses of the network should be immunized against uncertainty. Once the order is known for any budget of uncertainty value and V_{rank}^t set, a deterministic UC with the help of (3) and (4) can provide a solution as robust as other approaches, which will be illustrated with the help of case studies.

G. Bootstrap Aggregation

Accurate ranking of the buses can be one of the potential challenges in the proposed methodology. A resampling methodology, called bootstrap aggregation, is utilized to improve prediction accuracy [32]. Bootstrap aggregation is a method for generating multiple versions of a predictor and using these to get an aggregated predictor [33]. Please see the appendix for a more detailed description of bootstrap aggregation. Figure 1 depicts a bootstrap aggregation incorporated flow diagram of the bus ranking methodology. Once the baseline commitment is obtained, the formulation is divided into m independent bootstrap modules each containing the baseline schedule. Each of these modules run Q/m dispatch simulations and compute m independent ranking vectors which are then aggregated to compute final ranking vector. The final ranking vector V_{rank}^t is obtained by taking mean of the individual vectors provided by the modules to obtain a better prediction. Figure 1 demonstrates this process with the help of a flowchart. It is worth noting that each bootstrap module can be further divided for dispatch simulations based on available computing power.

Fig. 1: Statistical Bus Ranking Model



IV. Results

This section presents computational studies performed to evaluate the functionality of statistical bus ranking model.

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The goal of these case studies is to demonstrate that the commitment and dispatch decision computed by the proposed model are comparable to traditional robust UC models and hence have similar solution integrity while maintaining a formulation structure which is easily customizable, adaptable and conducive to parallelization.

A. Solution Integrity

The statistical bus ranking methodology is a heuristic approach to defining the extremes of the uncertainty set in robust optimization. To validate this approach, we compare performance of this approach to analagous, validated approaches in the literature. To validate solution integrity, the statistical bus ranking methodology[StatMod] is compared against a traditional analytical robust optimization model [AdpRob] proposed in [9] and a reserve adjustment approach [AdjRes]. As one of the earliest adaptable robust UC models and having been used in various comparative studies, the AdpRob model is a good benchmark to test for a robust unit commitment decision where the worst case is defined as maximum economic dispatch cost. The AdjRes model handles uncertainty by defining reserve requirements based on deterministic criteria, which is a good representation of models used by system operators. The three models are implemented and compared with two versions of the IEEE 30-bus system, and the IEEE 118-bus system (see Section IV-E). In each of these models, the budget of uncertainty Δ^t takes values in the entire range of 0 to N where $N = 20$. The statistical and analytical robust models manage uncertainty as defined in equation 1, and [9]. For the reserve adjustment approach, uncertainty is considered through a deterministic criterion as defined in (5).

$$q_t = q_t^0 + \frac{\Delta^t}{N_d} \sum_{i=1}^{N_d} \tilde{d}_i^t \qquad (5)$$

where q_t is the system reserve requirement at time t, composed of a basic reserve level q_t^0 and an adjustment proportional to the variation of load. Thus, the uncertainty is controlled by Δ^t as for the other models.

The three models are solved for a range of Δ^t on the standard IEEE 30-bus system, which has (essentially) no transmission limits. Subsequent tests are conducted on a modified system, which has transmission capacity limits and reduced overall generation capacity, in order to provide some solution challenges to better compare the three methods.

In order to validate the performance of the model, it is necessary to test the solution in an out-of-sample context. Commitment decisions provided by each model are tested by solving 1000 dispatch problems for randomly generated net-load scenarios. These samples follow a normal distribution with parameters such that about 15% of the scenarios fall outside $[\bar{d}_i^t - \tilde{d}_i^t, \bar{d}_i^t + \tilde{d}_i^t]$ while negative values are discarded. Thus the model performances can also be analyzed for an inaccurate definition of the uncertainty set, as might be the case in real-world applicaiton. An expensive slack variable(penalty) with cost $\$5000/MWh$ is introduced in energy balance and transmission constraints to account for any violation during real-time dispatch operation. The planning horizon is kept at 24 hrs.

B. Solution Integrity - Results

The commitment schedule for the three models was identical for the standard IEEE 30-bus system with no penalties. This is a result of abundant generation and transmission capacities in this fairly simplified version of a power system.

For the modified system the StatMod, AdpRob, and AdjRes are compared along three metrics - mean dispatch costs, standard deviation of dispatch costs and penalty costs. Mean and standard deviation of dispatch costs illustrate economic efficiency and reliability, while penalty costs measure solution robustness. Table 1 illustrates monotonically decreasing average dispatch for the StatMod and AdpRob formulations with increasing levels of uncertainty budget. Conversely, the AdjRes solution does not change significantly until high levels of reserves have been committed at $\Delta^t = 12$. Units committed (Figure 2) and total cost (Table 1) support this observation as both are significantly higher for AdjRes model especially as the solution becomes more conservative. Since the solutions for AdpRob and StatMod are identical, the red plot-line covers the blue in Figure 2. Also note that all three models provide an identical solution at $\Delta^t = 0$, as it represents a deterministic case. With increasing value of Δ^t, more units are committed and penalties decrease for the StatMod and AdpRob models. The AdjRes methodology commits extra generation resources based on an ad-hoc rule. At $\Delta^t = 20$ the average penalty costs for the StatMod and AdpRob models are negligible while the AdjRes approach never manages to completely eliminate penalty costs even with a larger number of generating units switched on, as can be seen in Table 2. The two robust models are able to overcome congestion while computing commitment solutions, as uncertainty at individual buses is included in the decision, something the deterministic approach does not include.

The economic reliability of three models can also be compared using Table 2. As expected, the standard deviations of dispatch costs are highest for the deterministic scenario. The StatMod and AdpRob solutions become significantly more reliable than the AdjRes model and remain nearly $2\times$ better even at $\Delta^t = 20$.

TABLE 1: Average Dispatch Cost and Total Cost for the Three Methods

	Average Dispatch Cost (k$)			Total Cost (k$)		
Delta	AdpRob	StatMod	AdjRes	AdpRob	StatMod	AdjRes
0	170.9814	170.9814	170.9814	172.1234	172.1234	172.1234
2	170.9814	170.9814	170.9814	172.1234	172.1234	172.1234
4	170.7227	170.7227	170.9814	172.1047	172.1047	172.1234
6	170.4422	170.4422	170.9814	172.0042	172.0042	172.1234
8	170.4422	170.4422	170.9814	172.0042	172.0042	172.1234
10	170.4422	170.4422	170.9814	172.0042	172.0042	172.1234
12	170.4422	170.4422	170.6604	172.0042	172.0042	172.0824
14	170.4018	170.4018	170.6604	172.0038	172.0038	172.1624
16	170.4018	170.4018	170.6604	172.0238	172.0238	172.1624
18	170.4018	170.4018	170.6604	172.0238	172.0238	172.5324
20	170.4018	170.4018	170.6604	172.0238	172.0238	172.6524

This case study demonstrates solution performance of the statistical approach. The proposed methodology performs as well as the analytical model while considerably outperforming

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Fig. 2: Total Units Committed across Horizon

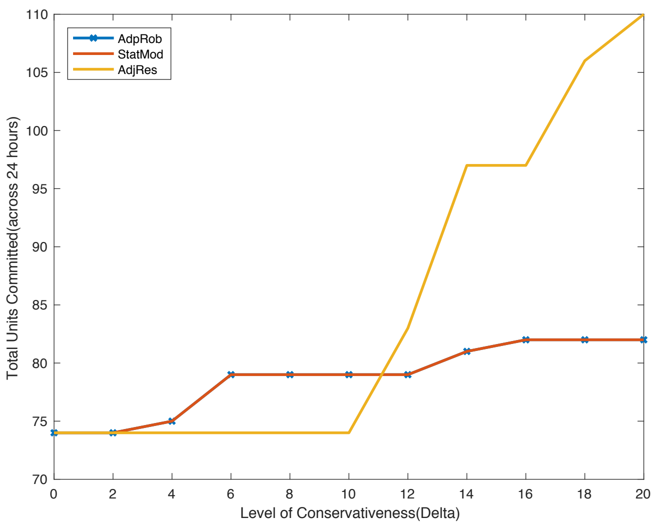

TABLE 2: Standard Devation of Dispatch Costs and Penalty Costs for the Three Methods

	Standard Deviation of Dispatch Costs (k$)			Average Penalty Costs ($)		
Delta	AdpRob	StatMod	AdjRes	AdpRob	StatMod	AdjRes
0	4.1593	4.1593	4.1593	581.0823	581.0823	581.0823
2	4.1593	4.1593	4.1593	581.0823	581.0823	581.0823
4	3.2943	3.2943	4.1593	320.3228	320.3228	581.0823
6	2.0085	2.0085	4.1593	40.6372	40.6372	581.0823
8	2.0085	2.0085	4.1593	40.6372	40.6372	581.0823
10	2.0085	2.0085	4.1593	40.6372	40.6372	581.0823
12	2.0085	2.0085	2.9928	40.6372	40.6372	260.7595
14	1.531	1.531	2.9928	0.0	0.0	260.7595
16	1.531	1.531	2.9928	0.0	0.0	260.7595
18	1.531	1.531	2.9928	0.0	0.0	260.7595
20	1.531	1.531	2.9928	0.0	0.0	260.7595

the AdjRes approach. With more complex networks than a 30-bus system, the solution performance of the StatMod approach may vary but it will but it will provide an equivalent solution to the AdpRob model, with a simpler and more flexible structure.

C. Flexibility

A key feature of the StatMod approach is the ability to incorporate alternative metrics on which to assess the impact of uncertainty in the unit commitment framework. In order to demonstrate this feature, this section explores two examples of possible alternatives to the traditional metric used in the AdpRob model, the dispatch cost. In this case study, the buses are ranked on ramping requirements along the time horizon. Such a day-ahead formulation emulates the flexible ramping approach being implemented by system operators to manage uncertainty. Flexible ramping or, flexiramp, aids in ensuring that sufficient capacity is on-line to manage the increased volatility of net loads expected under high levels of renewable integration [34]. Using the StatMod approach two different definitions are tested, 1) worst-case flexiramp and 2) hybrid definition which combines worst-case economic dispatch and flexiramp requirements. The following sections provide an overview of the two definitions.

1) Flexiramp Robust: In this definition, robust unit commitment is obtained by changing the primary metric from economic dispatch to ramping. To obtain a robust solution the buses are now ranked on hourly ramp requirements over the planning horizon, instead of economic dispatch cost. To represent the cost of flexiramp products, a slack variable is added at $\$5000/MWh$. The simpler structure of the statistical model allows for alternative worst-case definition while the model structure remains the same.

2) Hybrid Robust: To further demonstrate the flexibility of the StatMod approach, a hybrid definition is considered as proof of concept. This formulation combines two different worst-case definitions in order to obtain a solution that satisfies contrasting objectives. An example will be a situation where the system operator needs to be robust against both economic dispatch cost and ramping capacity. The StatMod approach, as a result of the simple structure, can be easily applied in these scenarios by assigning weights to different worst-case metrics according to the operator's requirements. For the current study, equal weights are assigned to economic dispatch cost and ramping requirements, and the final ranking is the sum of the two. This type of modification is implementable in the bus ranking methodology with minimal modifications, making it very practical for real-world application.

D. Flexibility-Results

The customizability and performance of the StatMod approach are demonstrated by comparing three different definitions of worst-case, specifically economic dispatch cost(Ed), flexiramp, and hybrid. As before, the models are run on a modified IEEE 30-bus test case while penalty costs are added to transmission, energy balance, and ramp constraints. The three models are compared on the same metrics as previous study, as well as on efficient use of generating units. The results show that, although the three definitions provide similar solutions, there are distinctions resulting from the different priorities of the three formulations. These differences would likely be more pronounced when implemented on more complex networks.

Table 3. shows that although Ed has lower dispatch costs relative to flexiramp and hybrid approach, in terms of total cost, it is the most expensive. Table 3 also demonstrates that across different values of conservativeness the hybrid approach outperforms flexiramp in dispatch costs and Ed in total costs, implying that the hybrid approach may provide the best compromise.

TABLE 3: Average Dispatch Cost and Total Cost for the Three Methods

	Average Dispatch Cost (k$)			Total Cost (k$)		
Delta	Ed	Flexiramp	Hybrid	Ed	Flexiramp	Hybrid
0	689.0969	689.0998	689.0998	690.579	690.502	690.502
2	689.0969	689.0998	689.0998	690.579	690.502	690.502
4	689.0969	689.0998	689.0998	690.619	690.502	690.502
6	688.8441	689.0998	688.8470	690.546	690.502	690.502
8	688.8441	688.8470	688.8470	690.566	690.429	690.429
10	688.8441	688.8470	688.8470	690.566	690.429	690.429
12	688.8441	688.8470	688.8470	690.566	690.429	690.429
14	688.8229	688.8470	688.8258	690.585	690.429	690.448
16	688.8229	688.8470	688.8258	690.545	690.429	690.448
18	688.8173	688.8202	688.8202	690.599	690.462	690.448
20	688.8173	688.8202	688.8202	690.559	690.462	690.462

Examination of Table 4 shows that all three approaches incur similar penalties with the exception of $\Delta^t = 6$ and 16. In these time periods, flexiramp has higher penalty costs, which

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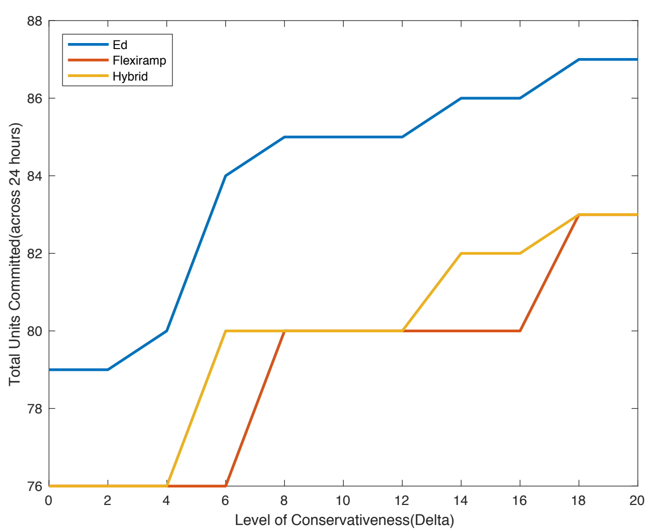
can be attributed to the different worst-case metric. The hybrid approach again appears as a better model, performs as well as the worst-case Ed definition. Table 4 also illustrates that the flexiramp definition provides the most system reliability, with the hybrid approach as a close second.

TABLE 4: Standard Deviation of Dispatch Costs and Penalty Costs for the Three Methods

	Standard Deviation of Dispatch Costs (k$)			Penalty Costs ($)		
Delta	Ed	Flexiramp	Hybrid	Ed	Flexiramp	Hybrid
0	38.5102	38.5053	38.5053	961.39	961.39	961.39
2	38.5102	38.5053	38.5053	961.39	961.39	961.39
4	38.5102	38.5053	38.5053	961.39	961.39	961.39
6	38.4394	38.5053	38.4345	807.62	961.39	807.62
8	38.4394	38.4345	38.4345	807.62	807.62	807.62
10	38.4394	38.4345	38.4345	807.62	807.62	807.62
12	38.4394	38.4345	38.4345	807.62	807.62	807.62
14	38.4226	38.4178	38.4178	789.44	807.62	789.44
16	38.4226	38.4178	38.4178	789.44	807.62	789.44
18	38.422	38.4171	38.4178	789.44	789.44	789.44
20	38.422	38.4171	38.4171	789.44	789.44	789.44

The most significant difference between the three metrics can be seen in Figure 3. which shows total units committed across the planning horizon. The flexiramp definition, while considering flexible ramping products, commits a smaller number of units than Ed. As a result, while the flexiramp approach incurs more penalty, the committed generating units are used more efficiently. This is illustrated in Figure 4, which plots the average usage of a generating unit across the planning horizon, against levels of robustness. The hybrid approach, as earlier, has better utilization pattern compared to Ed but also commits more units than flexiramp to obtain a middle-ground solution.

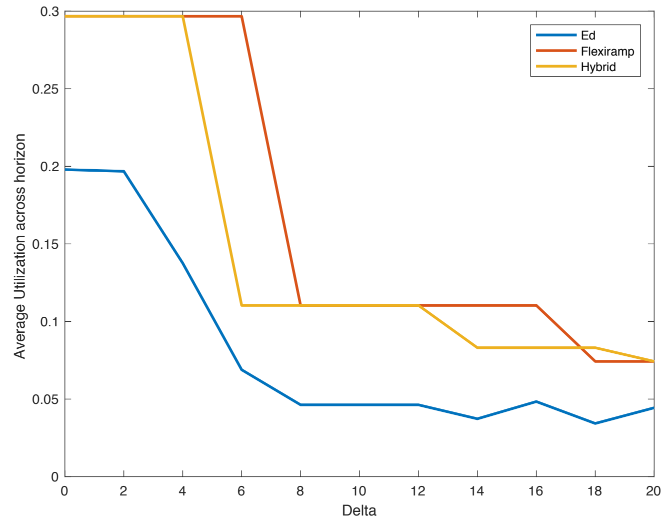
Fig. 3: Total Units Committed across the Horizon, for Increasing Budget of Uncertainty



The results of this case study provide insight into the performance of this approach under three metrics of robustness; the worst-case Ed and flexiramp outperform other approaches in one-dimensional objective, while the hybrid definition provides a balanced perspective combining the two definitions for a better overall performance.

The above results demonstrate the ability of the StatMod framework to customize and adapt and these tools can assist the system operator in attaining multiobjective solutions as the system complexity increases, something that is not currently available in traditional robust methodologies.

Fig. 4: Average Usage of Committed Units across the Horizon, for Increasing Budget of Uncertainty



E. 118-Bus System Analysis

The following section details the results of studies conducted on the IEEE 118-bus test system to compare solution performance of the AdpRob and StatMod on a larger network. The study parameters are adjusted to increase the size of the uncertainty set on the larger network. The results in Tables 5 and 6 show that, although there are small differences among the two approaches, overall the solutions are similar. The StatMod provides performs slightly better in terms of economic efficiency, reliability, and robustness for $\Delta^t = 6$ by committing one additional generating unit. However, for higher values of Δ^t the StatMod needs to commit two to three extra generators to models provide the same solution as AdpRob. These results demonstrate that the StatMod performs on par with AdpRob and our initial assertion made in the 30-bus network study.

TABLE 5: Comparison of Average Dispatch Cost and Total Units Committed for AdpRob and StatMod Methods

	Average Dispatch CostS (k$)		Total Units Committed (across 24 hrs)	
Delta	AdpRob	StatMod	AdpRob	StatMod
0	2427.0642	2427.0642	298	298
6	2304.0128	2303.4060	308	309
12	2303.4060	2303.3920	311	312
18	2303.3301	2303.3301	315	318
24	2303.3301	2303.3301	316	318
30	2303.3301	2303.3301	320	323
36	2303.3301	2303.3301	324	328
42	2303.3301	2303.3301	328	330
48	2303.3301	2303.3301	329	332

F. Computational Efficiency

This section compares computational efficiency of the ranking model against AdpRob. The models used in the study are

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TABLE 6: Standard Deviation of Dispatch Costs and Penalty Costs for AdpRob and StatMod Methods

	Standard Deviation of Dispatch Costs (k$)		Average Penalty Costs (k$)	
Delta	AdpRob	StatMod	AdpRob	StatMod
0	159.5033	159.5033	20.2916	20.2916
6	12.7117	12.4581	0.2838	0.07591
12	12.6945	12.7117	0.07591	0.0619
18	12.6071	12.6071	0.0	0.0
24	12.6071	12.6071	0.0	0.0
30	12.6071	12.6071	0.0	0.0
36	12.6071	12.6071	0.0	0.0
42	12.6071	12.6071	0.0	0.0
48	12.6071	12.6071	0.0	0.0

implemented in Pyomo [35] with CPLEX as the solver on a Macbook laptop with an Intel Core i7 3.0-GHz CPU and 16 GB memory.

To compare run-times of the two models, it is useful to clarify the key difference between the information provided by a single run of StatMod and a single run of AdpRob model. A single iteration of the AdpRob model computes a UC status for a pre-defined size of the uncertainty set or level of conservativeness (single value of Δ^t). The StatMod provides a ranking strategy for the entire uncertainty set, thus providing UC status for all possible values of Δ^t. For example, in case of the 30-bus test system where 20 buses have uncertain loads, Δ^t can take any value from 0 to 20. A complete run of AdpRob model will provide the commitment solution for one specific value of Δ^t. Conversely, one run of the StatMod will rank all the buses and provide information on all possible values of Δ^t. Thus, for a fair comparison of the computation times, the models need to be solved for UC status across all the values of Δ^t in the uncertainty set.

Average runtimes of the models are compared with increasing network uncertainty. Here, uncertainty reflects the percentage of buses considered with variable net-load in the respective network. One runtime reflects the average amount of time the model takes to compute UC status for all possible values of Δ^t.

TABLE 7: Average Computation Time across Varying Size Network Uncertainty (% * Total Buses) for the Two Networks

	IEEE-30 Average Runtimes(minutes)			IEEE-118 Average Runtimes(minutes)		
Uncertainty	AdpRob	StatMod	StatMod/AdpRob	AdpRob	StatMod	StatMod/AdpRob
0.1	0.4046	6.8109	16.8323	4.5209	22.7750	5.0376
0.2	0.7878	6.8418	8.6839	8.9928	23.1350	2.5726
0.3	1.1993	6.8728	5.7304	13.6393	23.4950	1.7225
0.4	1.5516	6.9037	4.4493	17.9856	23.8550	1.3263
0.5	1.9358	6.9347	3.5822	22.1745	24.1850	1.0906
0.6	2.3229	6.9656	2.9986	26.1948	24.5450	0.9370
0.7	2.7210	6.9965	2.5713	31.0532	24.9050	0.8020
0.8	3.1094	7.0275	2.2600	35.0325	25.2650	0.7211

As can be seen from the results, in Table 7, although StatMod is considerably slow than AdpRob, for smaller network with lower percentage of uncertain buses, as the network size or the uncertainty increases, StatMod becomes appreciably faster. For the 30-bus system the StatMod is approximately 17 times slower when 10 percent of the buses have variable net-load, and is up to 2 times slower with 80 percent of buses considered uncertain. For the 118-bus network, the statistical model starts out at 5 times slower when only 10 percent of buses are uncertain, and is up to 1.4 times faster when 80 percent of the buses have variable net-load. Table 7 also shows that for similar uncertainty percentage, the proposed model becomes relatively faster with increasing network size.

These results demonstrate that, although the StatMod has higher computation time for one iteration, the quantity of information provided by the model allows it to be on par, or faster than the traditional robust approach as the network size or uncertainty increases. The statistical approach will be particularly beneficial if the system operator intends to compute a commitment strategy for different levels of uncertainty, through the use of various sizes of the uncertainty set. It is worth noting that the the statistical model's structure also allows for straightforward parallelization, which will further help the system operator in reducing computation time.

G. 1354-Bus System Analysis

The proposed methodology is further tested on a modified version of the pegase 1354-bus system [36], [37] to demonstrate performance on a real-world sized network. The network contains 1,354 buses, 260 generators, and 1,991 branches and represents the size and complexity of part of the European transmission system. The AdpRob and StatMod formulations are implemented on a 6-hr time horizon with 300 uncertain loads, and the study results are shown in Tables 8 and 9. The two models provide similar solutions, and costs, with only minor differences in specific units committed at larger Δ^t values. The penalty costs are never reduced to zero because of transmission congestion in the network configuration. These results further validate the solution performance of the StatMod as compared to state-of-the-art robust methodologies.

TABLE 8: Comparison of Average Dispatch Cost and Total Units Committed for AdpRob and StatMod Methods

	Average Dispatch CostS (k$)		Total Units Committed (across 6 hrs)	
Delta	AdpRob	StatMod	AdpRob	StatMod
0	1210.1637	1210.1637	1351	1351
75	1178.1932	1178.1932	1356	1356
150	1178.1932	1178.1932	1360	1356
225	1178.4757	1178.4757	1549	1551
300	1183.8361	1183.8361	1559	1560

TABLE 9: Standard Deviation of Dispatch Costs and Penalty Costs for AdpRob and StatMod Methods

	Standard Deviation of Dispatch Costs (k$)		Average Penalty Costs (k$)	
Delta	AdpRob	StatMod	AdpRob	StatMod
0	362.9831	362.9831	608.3991	608.3991
75	343.0100	343.0100	577.5854	577.5854
150	343.0100	343.0100	577.5854	577.5854
225	342.9935	342.9816	577.5854	577.5854
300	342.9831	342.9819	577.5854	577.5854

V. Conclusion and Discussion

This paper proposes a data-driven statistical approach to account for the complexities and correlations within the power system, identifying the buses most critical to system performance, based on a pre-defined optimization objective. The ranking obtained can be utilized to implement a robust generation schedule. The method is demonstrated via a case study and validated against the state-of-the-art method proposed in [9]. The validation results on three separate test systems show that the StatMod approach provides similar solution as the accepted AdpRob model across all three metrics, supporting the validity of the approach. In addition, this approach consistently outperforms the ad-hoc reserve adjustment (AdjRes) approach typically implemented in practice. From these tests, it can be concluded that the statistical bus ranking approach also provided a correct solution to problems of unit commitment under uncertainty.

The proposed method also provides added utility to system operators with an easily customizable framework. Computational studies were performed with varying definitions of worst-case metric, and demonstrate some advanced capabilities of the model. For example, results of a hybrid definition of risk shows value in combining multiple worst-case criteria for robust unit commitment and is an interesting avenue for future work. [34] states the flexiramp product clearly improves the expected performance of market but can fail if used with deterministic models. While other methods require a complete reformulation of their models, the StatMod is easily adapted to this metric. Although out of scope of this paper, a comprehensive application of the proposed methodology with flexible ramping products could provide a better solution without creating implementation challenges. In addition to flexibility to handle various objectives, the statistical pre-processing of this approach is parallelizable and computationally efficient. Finally, it is worth noting that the statistical ranking is stable to moderate variations in system condition, providing additional computational efficiencies. A comprehensive ranking stability study will be a worthwhile problem for future work.

There is a large resource of literature on the feature ranking algorithms that are being utilized in the proposed model. It has been shown that these algorithm scale well and model performance does not decrease significantly with bigger datasets [38], [24]. Figure 1 depicts the structure of the proposed model. After computing baseline commitment schedule, the formulation is broken into a non-sequential modular structure as evident in the figure. These independent and self-contained bootstrap modules allow this model to take advantage of the computational power available to the system operator

The current framework approximates a linear relationship between the input and output vectors. Modifications to the framework will be needed to capture convex non-linear and piecewise relationships. To our knowledge in the current form it will not be able to provide a convergence guarantee for a non-convex cost function. Approximation of the relationship using piecewise linear regression or local basis functions, before the ranking algorithm is implemented, may also provide improved solutions under new feature sets. However, given the approximation, the numerical studies performed on IEEE 30-bus, 118-bus and pegase 1354-bus networks demonstrate the the validity of the solutions provided by this formulation.

The framework described here develops ranking vectors utilizing a baseline commitment decision. Our experiments, with different commitment decisions, revealed that the model produces solutions that are stable to the initial baseline unit commitment. The rate of convergence, however, might vary. During these studies, best performance was achieved using UC status with expected net load at all buses.

The proposed framework opens up many directions for future work. Testing the method on larger and more complicated networks will be important, as the ranking may be affected by correlations between net load at buses. The ranking can be used to study relationships between different elements of the electrical network. From studying interactions between buses, to finding optimum bus locations for microgrids. Simulating varying storage strategies in parallel for performance comparison is another potential application.

VI. Appendix

A. Bootstrap Aggregation

Bootstrap Aggregation or bagging is a popular technique for constructing an ensemble of diverse and accurate predictors. The purpose of these collections or ensembles is to make highly accurate predictions by considering the decisions of individual predictors in the ensemble. As the name suggests in Bootstrap aggregation multiple versions of the predictor are generated and then used to calculate an aggregate predictor.

In the proposed model bagging is implemented to achieve better prediction for the bus rank vector. With the learning dataset S defined as $\{(y_n, x_n), n = 1..., N\}$ where the $y's$ are the economic dispatch costs and $x's$, the input which will be dispatch costs and the net load values at any given bus in our model. The feature ranking algorithm yields bus ranks, $r = \phi(S)$, serving as the predictor for y. k samples from the dataset S are then taken to attain better aggregate prediction as compared to individual ranking. The k rankings thus obtained are aggregated by computing a score $R_j = 1/k \sum \phi(S_j^k)$ for each bus j as an average function of its rank r_j^k in the k-th bootstrap sample.

B. Unit Commitment Model

This section contains a basic formulation of unit commitment model. The objective of the model is to minimize the cost of running the system based on the number of generators running, on/off status and the production levels of the on-generators. The variable x which is binary represents vector of decisions regarding generator status, while y represents dispatch decision of the generating units at each time interval. The first set of constraints dictates minimum up/down times, and startup/shutdown costs. Energy balance constraints, reserve requirements, transmission limits and ramping constraints are represented in the second set. The third set contains the generator status and production level coupling constraints.

$$\min_{x,y(.)} \quad c^T x + b^T y$$

s.t.

$$Fx \leq f, \text{ x is binary}$$
$$Hy \leq h,$$
$$Ax + By \leq g,$$

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