Spatio-Temporal Modeling of Electric Loads

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***Abstract*—Advanced spatio-temporal electric load modeling and accurate spatio-temporal load forecast are essential to both short-term operation and long-term planning of power systems. This paper explores the spatio-temporal dependencies of electric load time series. The Southern California feeder load data show that feeders which are spatially close to each other share a more similar load pattern than those located further apart. This finding motivates us to develop the vector autoregressive model and the extended dynamic spatio-temporal model to emulate the spatio-temporal correlations of the real-world electric load time series. The testing results show that both models effectively capture the spatio-temporal patterns in the real-world electric load time series. Compared to the traditional vector autoregressive model, the proposed extended dynamic spatio-temporal model not only provides more accurate spatio-temporal electric load forecast but also obtains a parsimonious description of the high dimensional dataset.**

***Index Terms*—Dynamic spatio-temporal model, load forecasting, spatial weight matrix, spatio-temporal forecast, VAR model.**

I. Introduction

The number of advanced metering infrastructure (AMI) installations has increased dramatically worldwide in the past few years. In 2015, the U.S. electric utilities had about 64.7 million AMI installations. By the end of 2016, almost 50% of the residential customers in the U.S. have AMI infrastructure [1]. Many electric utilities have also extended the installation of supervisory control and data acquisition system (SCADA) to the distribution feeder level. These advanced sensor systems have finally made an array of big data analytics applications feasible in the electric power distribution network [2]. These big data/predictive analytics applications include spatio-temporal load modeling/forecasting, energy theft detection, distribution system state estimation [3], demand response management/forecasting [4], and distribution system topology identification [5].

This paper focuses on the development of spatio-temporal electric load modeling and forecasting methods. In addition to temporal dependencies, the electric loads from various feeders also exhibit strong spatial dependencies, where nearby feeders' loads tend to be more similar than those far apart. The spatio-temporal features of the electric loads can be explained by two underlying spatio-temporal processes, namely weather and human activities. The weather of adjacent neighborhoods or cities tend to be more alike than those far apart. Similarly, human activities in adjacent neighborhoods tend to be highly correlated.

The development of spatial-temporal modeling and forecasting methods for electric load time series is critical for three reasons. First, there is an immediate need to understand the spatio-temporal dependencies exhibited in the electric load time series of distribution feeders. The temporal dependencies of a single electric load time series has been well studied. However, there is little research which explores the spatial dependencies of electric load time series at various locations. Second, an improved spatio-temporal modeling and forecast of electric load time series can significantly improve both short-term operations and long-term planning of power systems. In the short-term system operations, a dramatic reduction in electricity production cost can be achieved by feeding accurate spatio-temporal load forecast into the stochastic unit commitment and economic dispatch engines. In the long-term system planning process, an accurate joint feeder load forecast can result in a deferral of system expansion and savings in investment costs. Third, large-scale synthetic load data can be generated from a well calibrated spatial-temporal electric load model. The research community lacks high-fidelity, large-scale electric load data for early-stage development and evaluation of new analytic tools. The synthetic load data generated from the proposed spatial-temporal models will accelerate the development and adoption of new optimization and control strategies in the electric utility industry.

The goal of this paper is to develop a parsimonious spatio-temporal electric load model which not only emulates the spatio-temporal patterns in real-world data but also yields accurate spatio-temporal forecast. Feeder level electric load data in California are used to explore the spatio-temporal pattern and to develop the proposed spatio-temporal models.

The unique contributions of this paper are listed as follows.

- This paper demonstrates that real-world electric loads time series of distribution feeders do exhibit strong spatio-temporal dependencies.
- This paper proposes and develops an extended dynamic spatio-temporal (DST) model which accurately captures the spatio-temporal correlations in the real-world electric load time series.
- The proposed extended DST model is highly scalable which reduces the dimension of model parameters dramatically from the widely used vector autoregressive (VAR) model.
- The proposed extended DST model also outperforms the VAR model in terms of spatio-temporal forecasting

accuracy.

A variety of statistical methods have been used to model and forecast electric loads [6]. These methods include, linear regression [7], autoregressive integrated moving average (ARIMA) [8], [9], artificial neural networks (ANN) [10], and support vector machines [11]. However, most of the previous works focused on modeling temporal correlations of a single electric load time series, while ignoring the spatial correlations among electric loads from different substations or feeders. Recently, spatial information was incorporated into a compressive spatio-temporal approach to forecast a target residential building's electricity consumption [12]. A spatio-temporal load pattern analysis model was developed to predict the load patterns of customers who are not equipped with AMI [13]. This paper fills the knowledge gap in the load forecasting and modeling field by conducting a comprehensive investigation of spatio-temporal dependencies in electric load time series with real-world data. An extended DST model is proposed and developed to capture the strong spatio-temporal correlations shown in the data. The out-of-sample testing results show that the extended DST model outperforms the widely used VAR model in terms of spatio-temporal forecasting accuracy. The proposed extended DST model is not only parsimonious but also highly scalable.

The rest of this paper is organized as follows. Section II investigates the spatio-temporal dependencies in real-world electric load time series. Section III introduces and develops two spatio-temporal electric load models, namely, the vector autoregressive model and the extended dynamic spatio-temporal model. Section IV evaluates and compares the performance of the two spatio-temporal models. The conclusions are stated in Section V.

II. Investigation of Spatio-Temporal Dependencies in Electric Loads

In this section, we explore the spatio-temporal dependencies of electric loads of various distribution feeders across Southern California. Exploratory data analysis methods of visualization and summarization are applied to provide some insights about the underlying spatio-temporal electric load processes. The basic takeaway is that feeders which are spatially close to each other share a more similar load pattern than those located further apart.

A. Empirical Spatio-Temporal Covariance and Correlation

The characterization of spatio-temporal covariance and correlation structure is important for modeling the spatio-temporal electric load time series. The spatial-temporal dependencies can be quantified by the empirical spatio-temporal covariance function at various lags. The spatio-temporal covariance at spatio lag h and time lag τ can be estimated as [14]

$$C_{emp}(h;\tau) = \frac{1}{|P_f(h)|}\frac{1}{|P_t(\tau)|} \sum_{f_i,f_j\in P_f(h)}\sum_{t,r\in P_t(\tau)} (y_{f_i}(t)-\hat{\mu}_{f_i})(y_{f_j}(r)-\hat{\mu}_{f_j}) \quad (1)$$

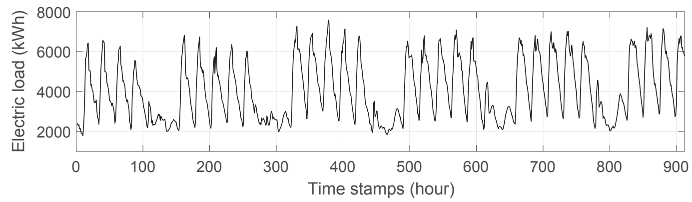

Fig. 1. Electric load time series of a sample feeder.

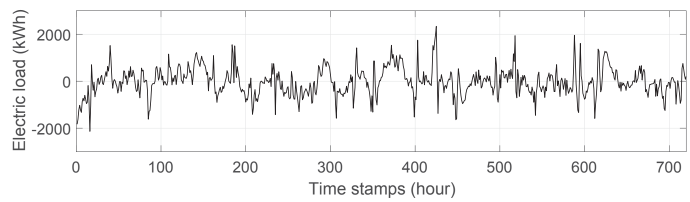

Fig. 2. Electric load time series of a sample feeder after seasonal differencing.

where $\hat{\mu}_{f_i} = \frac{1}{T}\sum_{t=1}^{T} y_{f_i}(t)$, $P_f(h)$ refers to the set of feeder pairs with spatial lag h, and $P_t(\tau)$ refers to the set of time stamp pairs with time lag τ. $|\cdot|$ refers to the cardinality of a set. $y_{f_i}(t)$ is the seasonally differenced electric load of feeder f_i at time stamp t. T is the total number of time stamps. The empirical spatio-temporal correlation at spatio lag h and time lag τ is then defined as

$$\rho_{emp}(h;\tau) = \frac{C_{emp}(h;\tau)}{C_{emp}(0;0)} \quad (2)$$

Note that equation (1) applies to processes that are stationary in space and time. Therefore, differencing operation is needed to make the raw time series stationary.

The electric load data usually exhibit strong seasonality. Fig. 1 shows the electric load time series of a sample feeder. As shown in the figure, the feeder load time series has a daily and weekly periodicity, which calls for differencing at both lag 24 and 168. Fig. 2 shows the same time series after seasonal differencing.

The spatial lag h between any two feeders f_i and f_j is calculated as follows:

$$h = \left\lceil \frac{d(f_i,f_j)}{MaxDis} \times M \right\rceil \quad (3)$$

where $d(f_i,f_j)$ is the geographical distance between feeders f_i and f_j. $MaxDis$ is the maximum geographical distance between any two feeders. M denotes the maximum spatial lag.

Given that the magnitude and volatility of the electric load of feeders can differ significantly, Z-score scaling is introduced to standardize the load data $y_{f_i}(t)$ for each feeder.

B. Estimation and Visualization of Spatio-Temporal Dependencies in Electric Loads

To explore the spatial-temporal dependencies of electric loads, historical hourly electricity consumption data from 50 distribution feeders located across Southern California are collected. The electric load data are gathered from the SCADA system and the smart meters. The study period is from May

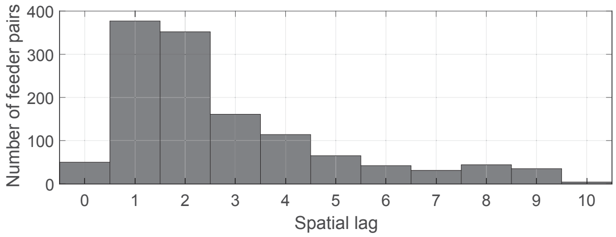

Fig. 3. Histogram of spatial lags

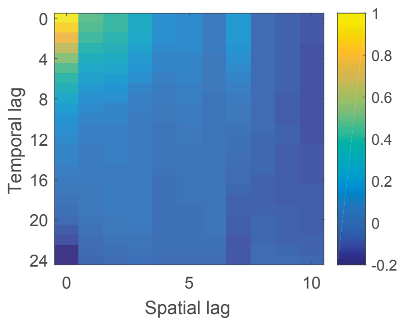

Fig. 4. Empirical spatio-temporal correlation of the feeders' load data

26, 2015 to July 2, 2015. The maximum geographical distance between any two feeders is 385 kilometers. The maximum spatial lag M is selected to be 10. Each spatial lag represents 38.5 kilometers. The histogram of spatial lags among the 50 distribution feeders is shown in Fig. 3.

All the feeder loads are preprocessed with seasonal differencing and Z-score scaling. The spatio-temporal correlation matrix of the feeders' load is obtained by using equation (2). Fig. 4 shows the image plot of the empirical spatio-temporal correlation function for time lag τ in hours and the spatial lag h defined in equation (3). As shown in the figure, the correlation between feeder loads is stronger when both the spatial lag and the time lag are smaller. The correlation drops exponentially when the spatial lag increases with the time lag fixed at 0. This result supports the need to take spatial information into account when modeling the spatial-temporal electric load processes.

III. Spatio-Temporal Modeling

A valid spatio-temporal electric loads model should capture both spatial and temporal correlations embedded in the dataset. In this section, we introduce and develop two spatial-temporal models to capture the inherent spatio-temporal dependencies. The first one is based on the vector autoregressive model which can simultaneously capture the spatial and temporal correlations. The VAR model serves as a benchmark. Note that when the number of feeder electric load time series is very large, fitting a VAR model may not be possible due to the curse of dimensionality. To effectively reduce the dimensionality of model parameters, an innovative extended dynamic spatial-temporal model is proposed and developed.

A. VAR Model

The general form of a VAR model of order p is given by

$$y(t) = c + A_1 y(t-1) + \cdots + A_p y(t-p) + u(t) \tag{4}$$

Where $y(t) = [y_{f_1}(t), ..., y_{f_K}(t)]^T$ is a $(K \times 1)$ vector of electric loads of K feeders, $\{A_i | i = 1, \cdots, p\}$ are fixed $(K \times K)$ coefficient matrices, c is a fixed $(K \times 1)$ intercept vector, and $u(t)$ is a K dimensional white noise process with zero mean and covariance matrix Σ_u.

The coefficient matrices of the VAR(p) model can be estimated by using multivariate least square estimation (MLSE) [15], which has the following form:

$$\hat{\mathbf{a}} = ((ZZ^T)^{-1} Z \otimes I_K)\mathbf{y} \tag{5}$$

Where $Z = (Z(0), Z(1), ..., Z(T-1))$, $A = (c, A_1, ..., A_p)$, $\mathbf{a} = \mathrm{vec}(A)$. $\hat{\mathbf{a}}$ refers to the estimate of $\mathbf{a}$. I_K is a K dimensional identity matrix and $Z(t) = [1, y(t)^T, ..., y(t-p+1)^T]^T$. $Y = [y(1), ..., y(T)]$ and $\mathbf{y} = \mathrm{vec}(Y)$. $\otimes$ is the tensor product operator.

Note that if many electric load time series need to be modeled together, the number of coefficients is quite substantial. For example, the modeling of 50 distribution feeders' electric loads requires over 2,500 parameters in a VAR(1) model. If a higher order VAR model is selected, then the parameter estimation precision will be even lower without enforcing constraints on the parameters.

B. Extended Dynamic Spatio-Temporal Model

In order to overcome the curse of dimensionality in modeling high dimensional spatio-temporal processes, dimension reduction technique is applied. Dimension reduction is reasonable in most applications given that the true spatio-temporal process often exists on a lower-dimensional manifold [14]. We propose an extension to the dynamic spatio-temporal (DST) model to achieve the desired dimension reduction. The extended DST model is inspired by the recent works in Spatial Econometrics [16]–[19] and Geography [20].

The proposed extension to the generic dynamic spatio-temporal model is formulated as follows

$$y(t) = v + (\Lambda + \Gamma W \Theta) y(t-1) + n(t) \tag{6}$$

where Λ, Γ, and Θ are diagonal parameter matrices. v is an intercept vector. W is a spatial weight matrix. $y(t) = [y_{f_1}(t), ..., y_{f_K}(t)]^T$ is a vector of electric loads of K feeders. $n(t)$ is assumed to be a white noise process with zero mean and covariance matrix Σ_n. The number of parameters in the spatial weight matrix is denoted as N_W.

The proposed extended DST model achieves the dimension reduction goal by reducing the number of model parameters to $4 \times K + N_W$ which is much smaller than that of VAR(1) model with $K^2 + K$ parameters. The dimension reduction is much more effective when K is very large. The proposed model extends the generic dynamic spatio-temporal model by introducing the diagonal parameter matrices Γ and Θ to replace the scalar parameter δ [17] in front of the spatial weight matrix. This extension enables the dynamic spatio-temporal model to characterize more complex spatio-temporal dependencies exhibited in real-world datasets.

The spatial weight matrix W in equation (6) summarizes the spatial relations among K time series. Each spatial weight matrix element w_{ij} represents the spatial influence of time series j on time series i [20]. Spatial weight matrices can be defined based on distance, boundaries, or a combination of distance and boundaries. Here we adopt the spatial weight matrix based on distance as the distances between distribution feeders can be easily computed. As shown in Fig. 4, the spatial correlation between electric loads of feeders decays exponentially as the distance between two feeders increases. Hence, exponential distance weights are selected to parameterize the spatial weight matrix. The ij-th element in the spatial weight matrix follows the negative exponential function

$$w_{ij} = \exp(-\alpha h_{ij}) \tag{7}$$

where $\alpha > 0$ is any positive parameter and h_{ij} is the spatial lag between distribution feeders i and j. Note that the spatial lag between feeders i and j is the same as that of feeders j and i. Thus W is a symmetric matrix.

The parameters of the extended DST model can be estimated through the least square estimation. The objective function of least square estimation is given by

$$\begin{aligned}\mathscr{L}(v,\Lambda,\Gamma,\Theta,\alpha) = &\sum_{t=1}^{T}[(y(t) - v - (\Lambda + \Gamma W \Theta)y(t-1))^T \cdot \\ &(y(t) - v - (\Lambda + \Gamma W \Theta)y(t-1))]\end{aligned} \tag{8}$$

In order to obtain a stable spatio-temporal model, all eigenvalues of $(\Lambda + \Gamma W \Theta)$ must have modulus less than 1. This constraint can be modeled as

$$|\lambda_i| < 1 \qquad i = 1, 2, \cdots, K \tag{9}$$

where λ_i is the ith eigenvalue of $(\Lambda + \Gamma W \Theta)$.

Therefore, the parameters of the proposed extended DST model can be estimated by solving the following constrained optimization problem

$$\begin{aligned} \text{minimize} \quad & \mathscr{L}(v,\Lambda,\Gamma,\Theta,\alpha) \\ \text{subject to:} \quad & \alpha > 0 \\ & |\lambda_i| < 1 \qquad i = 1, 2, \cdots, K \end{aligned} \tag{10}$$

Due to the structure of the spatial weight matrix, this optimization problem is highly nonlinear and non-convex. Hence, it is difficult to obtain the global optimal solution(s). In this paper we use the genetic algorithm solver in MATLAB to tackle this parameter estimation problem.

IV. Numerical Study

Numerical studies are conducted to validate the effectiveness of the VAR and the extended DST models in capturing the spatio-temporal dependencies of the electric load of distribution feeders. In addition, the spatio-temporal forecasting capability of the two models are carefully evaluated. Historical hourly electricity consumption data from 50 distribution

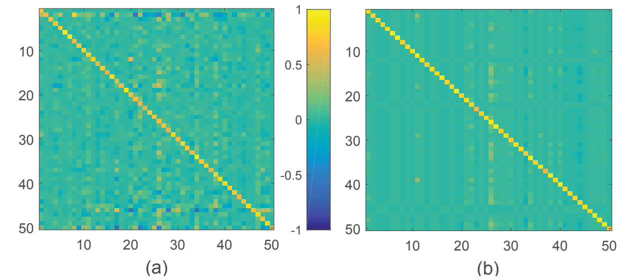

Fig. 5. Image plots of the coefficient matrices of the fitted VAR(1) model and extended DST model. (a) VAR(1) model. (b) Extended DST model.

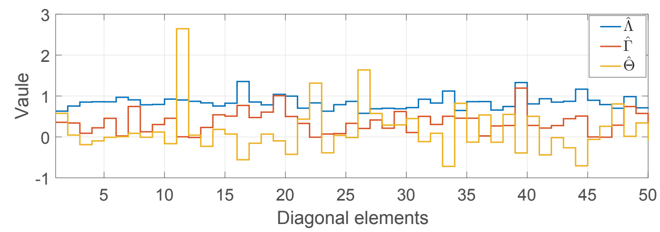

Fig. 6. Diagonal elements of the estimates of Λ, Γ, and Θ in the extended DST model.

feeders located across Southern California are used to fit the VAR and the extended DST models' parameters. The testing results show that both the VAR and the extended DST models preserve majority of the spatio-temporal correlation patterns in the original dataset. In addition, both models produce accurate spatial-temporal forecasting results. Compared to the VAR model, the proposed extended DST model achieves not only lower spatio-temporal forecasting errors but also the dimension reduction goal.

A. Model Fitting Results

The study period is from May 26, 2015 to July 2, 2015. There are 912 hourly electric load observation points for each distribution feeder. The parameters of VAR(1) model are estimated using MLSE according to equation (5). The extend DST model parameters are fitted with the same dataset using the genetic algorithm. The best result with the highest fitness score from 50 genetic algorithm runs is selected. The image plot of the estimated coefficient matrix A_1 of the VAR(1) model is depicted in Fig.5 (a). Similarly, Fig.5 (b) shows the image plot of the estimated coefficient matrix $\hat{\Lambda} + \hat{\Gamma}\hat{W}\hat{\Theta}$ of the extended DST model. It can be seen that the coefficient matrix of the extended DST model is strictly diagonally dominant, whereas the coefficient of the VAR(1) model is almost strictly diagonally dominant. The estimate of the spatial weight matrix parameter $\hat{\alpha}$ is 1.256. The estimates of the diagonal elements of the coefficient matrices Λ, Γ, and Θ of the extended DST model are shown in Fig. 6.

B. Modeling of Spatio-Temporal Correlation

The Monte Carlo method is leveraged to generate simulated electric load data of feeders based on the fitted model parame-

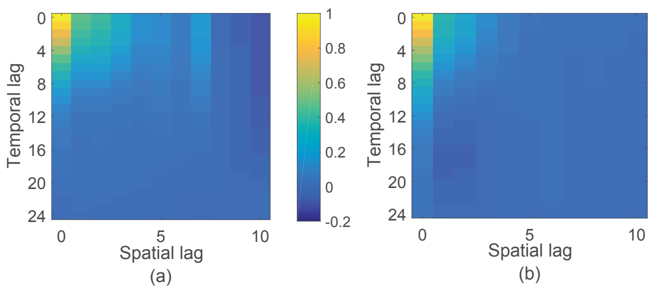

Fig. 7. Spatio-temporal correlation of the simulated data. (a) VAR(1) model. (b) Extended DST model.

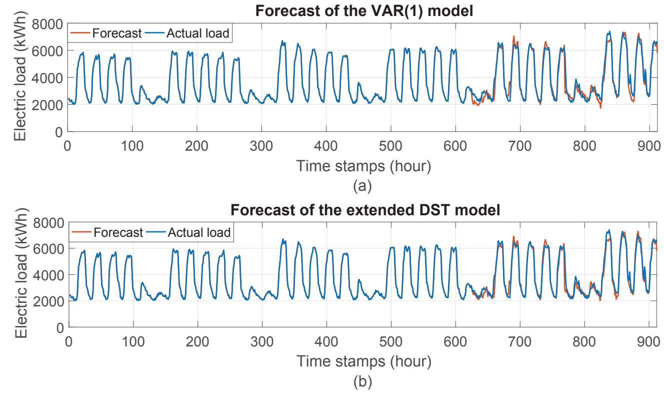

Fig. 8. Forecasting results of a sample feeder. (a) Forecast result of the VAR(1) model. (b) Forecast result of the extended DST model.

ters in Section IV.A. The average spatio-temporal correlations of the simulated data under the two models are calculated separately. The image plots of spatial-temporal correlations of the VAR(1) model and the extended DST model are shown in Fig. 7 (a) and (b), respectively. By comparing Fig. 4 and Fig. 7, it can be seen that the spatio-temporal patterns of the VAR(1) model and the extended DST model are very similar to that of the original feeder load time series. In particular, the spatial correlation is well preserved when the time lag is small.

To quantify how well the VAR(1) model and the proposed extended DST model capture the spatio-temporal correlation pattern, we first define the spatio-temporal correlation matrices of the VAR(1) model and the extended DST model as ρ_{VAR} and ρ_{DST}, respectively. The dissimilarity between two spatio-temporal processes can be quantified by the L^1 norm of the vectorization of the difference between the two corresponding spatio-temporal correlation matrices. For example, the dissimilarity between the actual spatio-temporal process and the VAR(1) generated process $D(\rho_{VAR}, \rho_{emp})$ can be calculated as

$$D(\rho_{VAR}, \rho_{emp}) = \frac{\|\text{vec}(\rho_{VAR} - \rho_{emp})\|_1}{N_e} \tag{11}$$

Where $\|\cdot\|_1$ is L^1 norm and N_e is the size of vec(ρ_{emp}).

The testing results show that $D(\rho_{VAR}, \rho_{emp}) = 0.0272$ and $D(\rho_{DST}, \rho_{emp}) = 0.0537$. The small dissimilarities suggest that both the VAR(1) and the extended DST models perform well in approximating the spatio-temporal correlation in the original feeder load time series. The dissimilarity score is slightly higher for the extended DST model compared to the VAR(1) model. The extended DST model successfully achieves more than 10 times of dimension reduction while capturing majority of the spatio-temporal dependencies in the original dataset. The advantage of the proposed extended DST model will be more pronounced as the dimension of the original time series increases.

C. Spatio-Temporal Forecasting Performance

In order to measure the forecasting accuracy of the two spatio-temporal models, the original electric load dataset is divided into two parts, the initial training dataset (the first 26 days) and the testing dataset (the last 12 days). In this study, a recursive day-ahead spatio-temporal load forecast is performed. Both models are first fitted with the initial training dataset and used to forecast the load for the next day. The model parameters are then re-estimated once the next day's actual electric load data become available. The accuracy of the forecasting results from the two spatio-temporal models are measured by the root mean square error (RMSE) and the mean absolute percentage error (MAPE).

Different predictors of the two spatio-temporal models can be developed based on the specific risk function to be minimized. The mean squared error (MSE) is selected as the risk function of the predictor in this paper. This is because minimum MSE forecasts also minimize a range of risk functions other than the MSE [15].

The MSE is defined as

$$\text{MSE} = E[(\hat{y}(t+i) - y(t+i))^2] \tag{12}$$

where $\hat{y}(t+i)$ is the predicted value of $y(t+i)$. For a general VAR(p) model, the optimal predictor which minimizes MSE is given by [21]

$$\hat{y}(t+i) = E[y(t+i)|\{y(s)|s \leq t\}] \tag{13}$$

where $\{y(s)|s \leq t\}$ are all the available historical data. For a VAR(1) model, (13) has a simple linear form

$$\hat{y}(t+i) = (I_K + A_1 + \cdots + A_1^{i-1})v + A_1^i y(t) \tag{14}$$

Since both the VAR(1) model and the extended DST model have the same general structure, the MSE predictor of the extended DST model can be derived by replacing A_1 with $(\Lambda + \Gamma W \Theta)$ in (14). Fig. 8 (a) and (b) show that both the VAR(1) and the extended DST model provide accurate day-ahead load forecast for a sample feeder.

In addition to the RMSE, the MAPE is also used to evaluate the forecasting performances of the two spatio-temporal models. The MAPE of the forecast for feeder f_i is defined as

$$\frac{100\%}{T_f - T_s + 1} \sum_{t=T_s}^{T_f} \left| \frac{\hat{y}_{f_i}(t) - y_{f_i}(t)}{y_{f_i}(t)} \right|$$

where T_s and T_f are the time stamps of starting point and end point of the testing dataset. Fig. 9 and Fig. 10 show the

TABLE I
AVERAGE FORECASTING ERRORS OF THE VAR(1) MODEL AND THE EXTENDED DST MODEL

Model	VAR(1) model	Extended DST model
Average RMSE [kWh]	554.64	490.39
Average MAPE	12.26%	10.63%

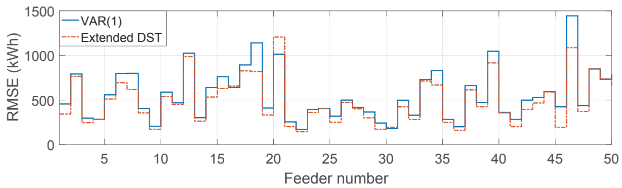

Fig. 9. RMSE of the VAR(1) model and the extended DST model for all feeders.

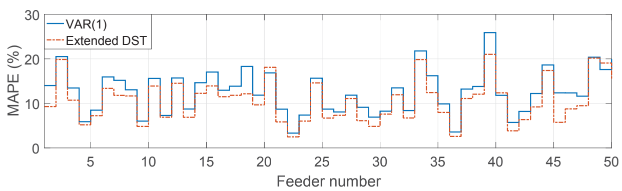

Fig. 10. MAPE of the VAR(1) model and the extended DST model for all feeders.

RMSE and the MAPE of forecast accuracy for each feeder with the VAR(1) model and the extended DST model. It can be seen that the forecasting performance of the extended DST model is better than that of the VAR(1) model for most of the distribution feeders. Table I shows the average RMSE and MAPE of the two spatio-temporal models across 50 distribution feeders. The results demonstrate that both the VAR(1) model and the extended DST model yield reasonable forecast errors. The extended DST model outperforms the VAR(1) model in terms of both the RMSE and the MAPE. With a large number of parameters, the VAR(1) model clearly over-fits the electric load of various distribution feeders, which leads to the higher generalization error. The extended DST model not only achieves the dimension reduction objective but also provides a more accurate spatio-temporal load forecast.

V. CONCLUSION

Most previous research in the field of electric load forecasting and modeling focused on studying the temporal dependencies in load data, while ignoring the spatial patterns. This paper discovered that there is a strong spatial correlation in real-world electric load data. An extended DST model is developed in this paper which accurately matches the empirical spatio-temporal correlation of Southern California's electric load time series. The out-of-sample testing results show that the proposed extended DST model outperforms the VAR model in terms of spatio-temporal forecasting accuracy. In addition to improving short-term operations and long-term planning of power systems, the proposed extended DST model can generate synthetic spatio-temporal electric load data for researchers who do not have access to real-world load data.

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