

Embracing Risk Dependency in Designing Cyber-Insurance Contracts

Mohammad Mahdi Khalili
University of Michigan, Ann Arbor
khalili@umich.edu

Parinaz Naghizadeh
University of Michigan, Ann Arbor
naghizad@umich.edu

Mingyan Liu
University of Michigan, Ann Arbor
mingyan@umich.edu

Abstract—We study the problem of designing cyber insurance policies in an interdependent network, where the loss of one agent (a primary party) depends not only on his own effort, but also on the investments and efforts of others (third parties) in the same eco-system (i.e., externalities). In designing cyber insurance policies, the conventional wisdom is to avoid insuring dependent parties for two reasons. First, simultaneous loss incidents threaten the insurer’s business and capital. Second, when a loss incident can be attributed to a third party, the insurer of the primary party can get compensation from the insurer of the third party in order to reduce its own risk exposure.

In this work, we analyze an interdependent network model in order to understand whether an insurer should avoid or embrace risks interdependencies. We focus on two interdependent agents, where the risk of one agent (primary party) depends on the other agent (third party), but not the other way around. We consider two potential scenarios: one in which an insurer only insures a primary party, and another one in which the insurer of the primary party further insures the third party agent. We show that it is in fact profitable for the primary party’s insurer to insure both agents. Further, we show that insuring both agents not only provides higher profit for the insurer, but also reduces the collective risk.

I. INTRODUCTION

To mitigate a myriad of losses stemming from evermore frequent cyber attacks and data breaches, such as legal fees, crisis management, business interruption, and even ransom payment (see e.g., the most recent WannaCry ransomware attack), organizations not only invest (sometimes heavily) in self protection in the form of secure software and systems, but also increasingly in the purchase of cyber-insurance policies[1].

Cyber-insurance, like all other forms of insurance, is primarily a method of risk transfer: a risk-averse client/insured is willing to pay a premium in exchange for certain amount of coverage in the event of a loss which reduces the uncertainty in its outcome; i.e., all or part of the risk is now undertaken by the insurer. Risk transfer combined with lack of information on the insured’s side directly leads to the well-known problem of moral hazard: since in general an insurer cannot observe and therefore cannot stipulate an insured’s actions, by transferring its risk the insured reduces its own effort in mitigating risks it would have exerted had it not purchased insurance. In other words, even though the insured has a higher utility by purchasing a policy as a result of the

coverage owing to its risk aversion, its state of cyber risk is now worse, see e.g., [2], [3], [4], [5], [6], [7].

On the other hand, there is a unique feature underlying cyber-insurance that makes it quite different from other forms of insurance; this is the cyber risk interdependency among clients. This is in large part due to the high degree of connectedness of today’s cyber world: one compromised system or email account can be handily used to launch attacks on another system and so on, thus high risks spill over from one client to others. This is exacerbated by complex business relationships among organizations and an increasing amount of outsourcing: a breach at a credit card processing vendor can lead to major losses by a retailer; a breach at a cloud service provider can cause business interruption to a large number of companies. This risk dependency means that the state of security of one firm depends not only on its own effort but also other firms’ effort [8], [9], [10], [11], [12], [13], which leads them to under-invest, another common phenomenon known as free-riding [14].

Both moral hazard and free-riding appear to result in reduced effort on the insured’s part. This led to the inquiry by a number of studies into whether the opposite could be incentivized, i.e., whether, how, and to what extent cyber-insurance could be used to induce the insured to adopt better security practices and improves its effort. A key concept in this context is premium discrimination, whereby a policy rewards better security effort (lower risk) with lower premium and vice versa [15]. How well this works obviously depends on how accurately an insurer can assess the prospective’s risk and the result heavily depends on the models and assumptions used.

Our previous work [4] shows that, (1) unsurprisingly, imperfect but useful pre-screening can reduce moral hazard and enable effective premium discrimination, and leads to improved effort by the insureds and higher profit by a monopoly insurer (similar result has been shown in competitive markets in [16]), but (2) surprisingly, by underwriting risk-dependent clients the insurer can not only improve its own profit but also further improve the security efforts by the clients. This is because the resulting policies effectively force the clients to internalize their externalities and commit to higher effort in exchange for not only reduced premium payment but also for similar commitment from other clients being under written.

This second aspect leads to an apparent and major departure from current practices concerning risk dependencies. The conventional wisdom in the underwriting industry is that dependency is to be avoided for two main reasons. One, it could lead to simultaneous loss events which could threaten one's capital limit and liquidity. Two, in the event that a certain loss event could be attributed to a third party (e.g., a cloud platform vendor) who happens to be insured by a different insurer, then the insurer of the primary party can hope to recover some or all of its losses from the third party's insurer under the latter's policy, thereby reducing its own risk exposure. If, on the other hand, the primary party's insurer underwrites both the primary and its third party, then even if the loss to the primary could be attributed to the third party, the insurer would effectively be "suing itself" for the losses and has to pay out the whole amount. This is in addition to the possibility that the third party also sustained losses in the same event and has to be covered. This has led to a prevailing thinking that this type of risk dependency should be avoided.

In this paper we seek to answer the following question: when faced with risk dependency (say between a primary client and its dependency/vendor), is it better for an insurer to underwrite both or only one of them (say the primary) and leave it to someone else to underwrite the dependency?

To address this question we examine a simple two-agent, two-insurer model with one agent's risk (the primary) dependent on the other (the third party), but not the other way around. The two insurers are symmetrical in nature, and we shall take the point of view of one of them and use the other simply as an outside option when the former decides to insure only one client; in this sense the two insurers are not competing for market share, and the model only serves to highlight whether there is benefit in trying to insure both given the opportunity.

Our main finding is that indeed there is benefit in insuring both: the insurer obtains higher profit in doing so by taking advantage of the risk dependency; in this case even though it does not get to recover some of the losses from another underwriter, it profits from higher premium (from both clients) and reduced collective risk.

Finally, we consider a scenario under which the amount of coverage that the insurer can provide is constrained by its capital reserve. In this case, we show that there indeed exist situations in which it is beneficial to insure only the primary and not both, especially when the constraint is severe.

II. PROBLEM DEFINITION AND MODEL

We consider a single period contract design problem in the presence of a monopolistic profit-maximizing insurer and two interdependent risk-averse agents¹. Agent i exerts effort e_i incurring linear cost $c_i \cdot e_i$ toward securing himself against loss. We assume that the dependent risk is one directional: The loss of the first agent (primary party) depends not only on his effort but also depends on the second agent's effort,

¹Throughout the paper, we use she/her and he/his to refer to the insurer and agent(s), respectively.

but the loss of the second agent (third party) depends only on his effort. Let L_i denote the random variable corresponding to the loss of agent i . We model agents' interdependency as follows,

$$\begin{aligned} L_1 &= L_{e_1} + x \cdot L_{e_2} \\ L_2 &= L_{e_2} \end{aligned} \quad (1)$$

Where L_{e_i} is the loss random variable depending on the effort of agent i , and x is the interdependency factor. Here, we assume that L_{e_i} has the normal distribution and is conditionally independent of $L_{e_{-i}}$ given e_i and e_{-i} . In other words,

$$L_{e_i} \sim \mathcal{N}(\mu_i(e_i), \lambda_i(e_i)) \quad (2)$$

The assumption about the distribution of the loss helps us to find the closed form solution for the agents' optimal effort. Nevertheless our result in theorem 1 remains valid for any arbitrary distribution for the loss.

We assume both $\mu_i(\cdot)$ and $\lambda_i(\cdot)$ are strictly decreasing and strictly convex and twice differentiable. The loss of agent i has normal distribution with the following mean and variance,

$$\begin{aligned} L_1 &\sim \mathcal{N}(\mu_1(e_1) + x\mu_2(e_2), \lambda_1(e_1) + x^2\lambda_2(e_2)) \\ L_2 &\sim \mathcal{N}(\mu_2(e_2), \lambda_2(e_2)) \end{aligned} \quad (3)$$

Each insurance contract has two parameters, premium p and coverage factor β . If agent i enters the contract (p, β) , he pays premium p and gets the coverage βL_i after the loss incident.

Our goal in this work is to understand whether the insurer of the primary party (the first agent whose loss depends on the other agent) should allow the third party (the second agent whose loss depends only on his effort) to be insured by the other insurer or not. Here, the insurers are not competing. We take the point of view of the insurer of the primary party and use the other insurer as an outside option when the insurer of the primary party decides to insure only one client. For reaching our goal, we need to study four different cases,

- 1) Both agents do not purchase any insurance contract.
- 2) Only one of the agents purchases an insurance contract, and the other one opts out.
- 3) Both agents purchase the insurance contracts, and the insurer of the primary party lets the other agent (third party) be insured by the other insurer.
- 4) Insurer of the primary party insures both agents.

Notice that case 1 is the outside option for case 2, and case 2 is the outside option for cases 3 and 4. Therefore, for finding the optimal contract in case 3 and 4, we have to study case 1 and case 2.

After studying these cases, we compare the profit of the primary party's insurer in case 3 with her profit in case 4 in order to understand if it is better for her to insure both agents or only one of them.

III. DESIGNING A CYBER INSURANCE POLICY

In this section we study the four different cases discussed in previous section.

A. Case 1: Neither Agent Purchases an Insurance Contract

If agent i does not purchase any insurance contract, the summation of his loss and cost of the effort are his total costs. Let X_i denote the total costs of agent i . We have,

$$X_i = -L_i - c_i e_i \quad (4)$$

Because the agents are risk-averse, we consider a concave function of the agents' total costs as their utilities. In this work, we consider exponential utility. We have,

$$\begin{aligned} U_1(e_1, e_2) &= 1 - \exp\{-\gamma_1 X_1\} = 1 - \exp\{\gamma_1 L_1 + \gamma_1 c_1 e_1\} \\ U_2(e_2) &= 1 - \exp\{-\gamma_2 X_2\} = 1 - \exp\{\gamma_2 L_2 + \gamma_2 c_2 e_2\} \\ \bar{U}_1(e_1, e_2) &= E\{U_1(e_1, e_2)\} = \\ &1 - \exp\{\gamma_1(\mu_1(e_1) + x\mu_2(e_2)) + \\ &\quad \frac{1}{2}\gamma_1^2(\lambda_1(e_1) + x^2\lambda_2(e_2))\} + \gamma_1 c_1 e_1 \\ \bar{U}_2(e_2) &= E\{U_2(e_2)\} = \\ &1 - \exp\{\gamma_2\mu_2(e_2) + \frac{1}{2}\gamma_2^2\lambda_2(e_2) + \gamma_2 c_2 e_2\} \end{aligned} \quad (5)$$

The agents' optimal efforts outside the contract are as follows,

$$e_i^{oo} = \arg \min_{e_i \geq 0} \mu_i(e_i) + \frac{1}{2}\gamma_i \lambda_i(e_i) + c_i e_i \quad (6)$$

Therefore, the agents' optimal utility when both agents do not purchase any contract is given by,

$$\begin{aligned} u_1^{oo} &= \bar{U}_1(e_1^{oo}, e_2^{oo}) \\ u_2^{oo} &= \bar{U}_2(e_2^{oo}) \end{aligned} \quad (7)$$

B. Case 2: Only One Agent Purchases an Insurance Contract

First assume that only agent 2 enters the contract, and the insurer offers the second agent contract (p_2, β_2) . In this case, the primary party remains uninsured. Under contract (p_2, β_2) , the second agent's total cost is given by,

$$X_2^{in} = -p_2 - (1 - \beta_2)L_2 - c_2 e_2 \quad (8)$$

Therefore, the second agent's utility inside the contract is,

$$\begin{aligned} U_2^{in}(p_2, \beta_2, e_2) &= 1 - \exp\{-\gamma_2 X_2^{in}\} = \\ &1 - \exp\{\gamma_2 p_2 + \gamma_2(1 - \beta_2)L_2 + \gamma_2 c_2 e_2\} \\ \bar{U}_2^{in}(p_2, \beta_2, e_2) &= E\{U_2^{in}(p_2, \beta_2, e_2)\} \end{aligned} \quad (9)$$

Let $e_2^{oi}(p_2, \beta_2)$ and e_1^{oi} denote agents' optimal efforts when only the second agent enters the contract (p_2, β_2) . Notice that the first agent's optimal effort is equal to his effort in case 1 (i.e. $e_1^{oi} = e_1^{oo}$). The second agent's effort under contract (p_2, β_2) is given by,

$$\begin{aligned} e_2(p_2, \beta_2) &= \arg \min_{e_2 \geq 0} \\ &(1 - \beta_2)\mu_2(e_2) + \frac{1}{2}\gamma_2(1 - \beta_2)^2\lambda_2(e_2) + c_2 e_2 \end{aligned} \quad (10)$$

The insurer is profit-maximizing and maximizes her utility subject to two constraints. The first one is the Individual Rationality (IR) constraint which stipulates that an agent enters the contract if he gets higher utility than his outside option. The second constraint is Incentive Compatibility (IC) which means that each agent in an insurance contract chooses

an effort level which maximizes his utility. Therefore, we can write the insurer optimization problem as follows:

$$\begin{aligned} \max_{p_2, \beta_2, e_2} & E\{p_2 - \beta_2 L_2\} \\ \text{s.t.}, & \\ (IR) & \bar{U}_2^{in}(p_2, \beta_2, e_2) \geq u_2^{oo} \\ (IC) & e_2 \in \arg \min_{e \geq 0} \\ & (1 - \beta_2)\mu_2(e) + \frac{1}{2}\gamma_2(1 - \beta_2)^2\lambda_2(e) + c_2 e \end{aligned} \quad (11)$$

The contract that solves (11) is called an optimal contract. Let e_2^{oi} be the second agent's effort inside the optimal contract, and u_1^{oi} denote the first agent utility when he is outside the optimal contract and the second agent is inside the optimal contract. We have,

$$u_1^{oi} = \bar{U}_1(e_1^{oi}, e_2^{oi}) \quad (12)$$

Now assume that the first agent purchases the contract (p_1, β_1) while the second one opts out. The total cost of the first agent after purchasing contract (p_1, β_1) is,

$$X_1^{in} = -p_1 - (1 - \beta_1)L_1 - c_1 e_1 \quad (13)$$

The first agent's utility in the contract is given by,

$$\begin{aligned} U_1^{in}(p_1, \beta_1, e_1, e_2) &= 1 - \exp\{-\gamma_1 X_1^{in}\} = \\ &1 - \exp\{\gamma_1 p_1 + \gamma_1(1 - \beta_1)L_1 + \gamma_1 c_1 e_1\} \\ \bar{U}_1^{in}(p_1, \beta_1, e_1, e_2) &= E\{U_1^{in}(p_1, \beta_1, e_1, e_2)\} \end{aligned} \quad (14)$$

Let e_i^{io} denote the optimal effort of agent i when the first agent purchases the optimal contract and the second agent opts out. It is easy to see that $e_2^{io} = e_2^{oo}$ (i.e., the second agent does not depend on the first agent and self-protects). Same as (11), we can write the insurer's problem as follows,

$$\begin{aligned} \max_{p_1, \beta_1, e_1} & E\{p_1 - \beta_1 L_1\} \\ \text{s.t.}, & \\ (IR) & \bar{U}_1^{in}(p_1, \beta_1, e_1, e_2^{io}) \geq u_1^{oo} \\ (IC) & e_1 \in \arg \min_{e \geq 0} \\ & (1 - \beta_1)\mu_1(e) + \frac{1}{2}\gamma_1(1 - \beta_1)^2\lambda_1(e) + c_1 e \end{aligned} \quad (15)$$

The optimal effort of the first agent (i.e. e_1^{io}) is calculated by solving (15). Also, it is easy to see that the second agent's utility when he is outside the contract and the first agent is inside the optimal contract is equal to the second agent's utility in case 1. In other words,

$$u_2^{io} = \bar{U}_2(e_2^{io}) = u_2^{oo} \quad (16)$$

We will see shortly how the insurer uses her knowledge about u_2^{io} and u_1^{oi} to offer both agents the optimal contracts.

C. Case 3: Both Agents Purchase Insurance Contracts, Each from a Different Insurer

In this case, we assume that both agents purchase insurance contracts and have different insurers. First insurer offers the first agent a contract and second insurer insures second agent. Because the loss of the first agent depends on the second agent's effort, the insurer of the primary party (insurer of the

first agent) is able to sue the other insurer in case of a loss. Let q denote the probability that the first insurer can sue and get compensation from the second insurer. Let I be a random variable and denote the amount of the compensation that the first insurer can get. We have,

$$I = \begin{cases} xL_{e_2} & \text{w.p. } q \\ 0 & \text{w.p. } 1 - q \end{cases} \quad (17)$$

$$E\{I\} = q \cdot x \cdot \mu_2(e_2) \quad (18)$$

Therefore, we write the second insurer's problem as follows,

$$R_2 = \max_{p_2, \beta_2, e_2} E\{p_2 - \beta_2 L_2 - I\} =$$

$$p_2 - (\beta_2 + q \cdot x) \cdot \mu_2(e_2)$$

$$\text{s.t.}, \bar{U}_2^{in}(p_2, \beta_2, e_2) \geq u_2^{io}$$

$$e_2 \in \arg \min_{e \geq 0} (1 - \beta_2)\mu_2(e) + \frac{1}{2}\gamma_2(1 - \beta_2)^2\lambda_2(e) + c_2e \quad (19)$$

Let e_2^{ii} be the second agent's effort level obtained by solving (19). The first insurer uses this knowledge to find the optimal contract to offer to the first agent. The first insurer solves the following optimization problem,

$$R_1 = \max_{p_1, \beta_1, e_1} E\{p_1 - \beta_1 L_1 + I\} =$$

$$p_1 - \beta_1 \cdot (\mu_1(e_1) + x\mu_2(e_2^{ii})) + q \cdot x \cdot \mu_2(e_2^{ii})$$

$$\text{s.t.}, \bar{U}_1^{in}(p_1, \beta_1, e_1, e_2^{ii}) \geq u_1^{oi}$$

$$e_1 \in \arg \min_{e \geq 0} (1 - \beta_1)\mu_1(e) + \frac{1}{2}\gamma_1(1 - \beta_1)^2\lambda_1(e) + c_1e \quad (20)$$

Here R_1 denotes the profit of the insurer who insures the first agent in case 3 and tries to seek compensation from the other insurer. Let e_1^{ii} be the first agent's optimal effort in case 3; e_1^{ii} is the solution to the optimization problem (20).

D. Case 4: Insurer of the Primary Party Insures Both Agents

In this case, the primary party's insurer offers contracts to both agents, therefore forgoing the third-party recovery option. Even though she does not get compensation from the other insurer in case of a loss, she obtains premiums from both agents. The insurer's problem can be written as follows:

$$R_{12} = \max_{p_1, p_2, \beta_1, \beta_2, e_1, e_2} E\{p_1 + p_2 - \beta_1 L_1 - \beta_2 L_2\} =$$

$$p_1 + p_2 - \beta_2 \mu_2(e_2) - \beta_1 (\mu_1(e_1) + x\mu_2(e_2))$$

$$\text{s.t.},$$

$$\bar{U}_1^{in}(p_1, \beta_1, e_1, e_2) \geq u_1^{oi}$$

$$\bar{U}_2^{in}(p_2, \beta_2, e_2) \geq u_2^{io}$$

$$e_1 \in \arg \min_{e \geq 0} (1 - \beta_1)\mu_1(e) + \frac{1}{2}\gamma_1(1 - \beta_1)^2\lambda_1(e) + c_1e$$

$$e_2 \in \arg \min_{e \geq 0} (1 - \beta_2)\mu_2(e) + \frac{1}{2}\gamma_2(1 - \beta_2)^2\lambda_2(e) + c_2e \quad (21)$$

E. Comparing Case 3 with Case 4

We now present our main results by comparing R_{12} with R_1 concerning the insurer's revenue in different situations.

Theorem 1: The profit of the primary party's insurer in Case 4 is larger than the summation of both insurers' profit in Case 3, i.e., $R_1 + R_2 \leq R_{12}$.

This theorem implies that even though the primary party's insurer faces more risk by offering both agents the contracts and does not receive any compensation from the third party's

insurer, it is more profitable for her to insure both agents. We interpret the result of theorem 1 as follows: Case 3 is feasible as long as $R_2 \geq 0$. Therefore, by offering the second agent the same insurance contract as the third party's insurer offers in Case 3, the primary party's insurer can make at least R_2 profit from the second agent. As a result, $R_{12} \geq R_1 + R_2$.

Let e_1^s and e_2^s denote the agents' efforts when both agents purchase the contracts from a single insurer. In other words, e_1^s and e_2^s are the solutions to (21). We have the following theorem about the agents' effort.

Theorem 2: If the primary insurer offers both agents insurance contracts, then the contracts are such that they incentivize the agents to exert higher efforts as compared to Case 3. In other words, $e_1^s \geq e_1^{ii}$ and $e_2^s \geq e_2^{ii}$.

Theorem 2 can be interpreted as follows: when an insurer insures two interdependent agents (Case 4), she faces higher risk as compared to Case 3. Because of this, in this case it is optimal for the insurer to incentivize both agents to exert higher effort as compared to Case 3 to decrease the collective risk; interestingly, as a result of this she ends up making more profit. More specifically, because the effort of the third party contributes to his and the primary party's loss, it is more profitable for the primary party's insurer to encourage the third party to exert higher effort in Case 4 as compared to Case 3. As the second agent exerts higher effort, the first agent faces less loss in Case 4 as compared to Case 3. Consequently, the insurer offers the primary party less coverage without decreasing his premium or violating the (IR) constraint, which in turn leads to the primary increasing his effort in Case 4.

Theorems 2 and 1 together imply even though the primary party's insurer does not get to recover some of the losses from another underwriter in case 4, she profits from higher premium (from both clients) and reduced collective risk.

IV. NUMERICAL RESULT

We perform numerical experiments for a scenario with the following parameters,

$$\mu_1(e) = \mu_2(e) = \frac{10}{1+e}, \quad \lambda_1(e) = \lambda_2(e) = \frac{10}{(1+e)^2}$$

$$\gamma_1 = 2, \quad \gamma_2 = 1, \quad c_1 = 1, \quad c_2 = 2, \quad x = 0.4$$

We focus on the impact of q (probability of success in suing the third party's insurer) on the insurers' revenue and the agents' efforts.

Figure 1 illustrates the insurers' revenue in case 3 and case 4. First notice that we draw this figure for $q \leq 0.165$; this is because case 3 is not feasible as the first insurer's profit in case 3 would be negative if $q \geq 0.165$. Secondly, figure 1 shows that even though the first insurer can sue the other insurer and get some compensation in case 3, she makes more profit by insuring both agents.

Figure 2 illustrates the agents' efforts in cases 3 and 4 as a function of q . By theorem 2, we expect that $e_k^s \geq e_k^{ii}$ $k = 1, 2$. This figure confirms that agents' efforts in case 3 are always less than agents' efforts in case 4.

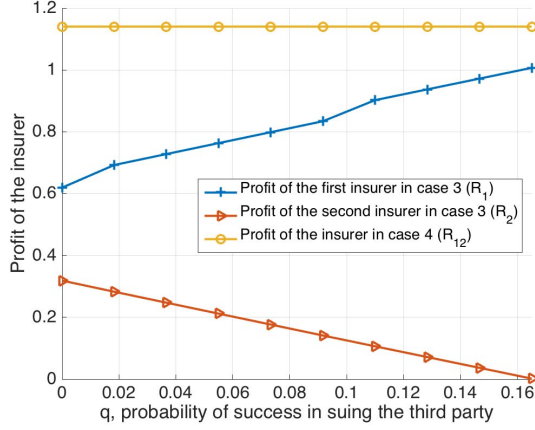


Fig. 1. Insurers' profits in cases 3 and 4 v.s. q

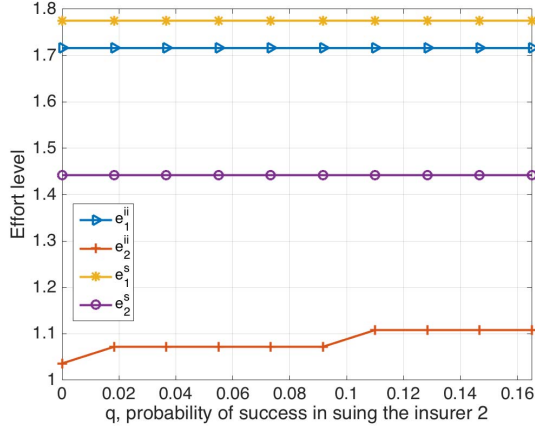


Fig. 2. Agents' efforts in case 3 and case 4 v.s. q

V. DISCUSSION

In this work, we have assumed that insurers are risk neutral and they have unlimited assets. If the insurers' total asset is limited, then Theorem 1 and 2 may or may not be valid. Let a denote the maximum coverage that an insurer can pay. In this case, the probability that the total coverage is larger than a should be negligible in order to limit the insurer's probability of ruin. Therefore, insurers' problems in cases 3 and 4 should be revised by adding a Coverage Limit (CL) constraint as follows,

- First insurer's problem in case 3:

$$\begin{aligned}
 R_1 &= \max_{p_1, \beta_1, e_1} E\{p_1 - \beta_1 L_1 + I\} = \\
 & p_1 - \beta_1 \cdot (\mu_1(e_1) + x\mu_2(e_2^{ii})) + q \cdot x \cdot \mu_2(e_2^{ii}) \\
 \text{s.t.,} \\
 (IR) \bar{U}_1^{in}(p_1, \beta_1, e_1, e_2^{ii}) &\geq u_1^{io} \\
 (IC) e_1 &\in \arg \min_{e \geq 0} \\
 & (1 - \beta_1)\mu_1(e) + \frac{1}{2}\gamma_1(1 - \beta_1)^2\lambda_1(e) + c_1e \\
 (CL) Pr\{\beta_1 L_1 - I \geq a\} &\leq \epsilon
 \end{aligned} \tag{22}$$

Where e_2^{ii} is obtained by solving optimization problem (23).

- Second insurer's problem in case 3:

$$\begin{aligned}
 R_2 &= \max_{p_2, \beta_2, e_2} E\{p_2 - \beta_2 L_2 - I\} = \\
 & p_2 - (\beta_2 + q \cdot x)\mu_2(e_2) \\
 \text{s.t.,} \\
 (IR) \bar{U}_2^{in}(p_2, \beta_2, e_2) &\geq u_2^{io} \\
 (IC) e_2 &\in \arg \min_{e \geq 0} \\
 & (1 - \beta_2)\mu_2(e) + \frac{1}{2}\gamma_2(1 - \beta_2)^2\lambda_2(e) + c_2e \\
 (CL) Pr\{\beta_2 L_2 + I \geq a\} &\leq \epsilon
 \end{aligned} \tag{23}$$

- Insurer's problem in case 4:

$$\begin{aligned}
 R_{12} &= \max_{p_1, p_2, \beta_1, \beta_2, e_1, e_2} \\
 & E\{p_1 + p_2 - \beta_1 L_1 - \beta_2 L_2\} \\
 &= p_1 + p_2 - \beta_2 \cdot \mu_2(e_2) - \beta_1 \cdot (\mu_1(e_1) + x\mu_2(e_2)) \\
 \text{s.t.,} \\
 (IR) \bar{U}_2^{in}(p_2, \beta_2, e_2) &\geq u_2^{io} \\
 (IR) \bar{U}_1^{in}(p_1, \beta_1, e_1, e_2) &\geq u_1^{io} \\
 (IC) e_1 &\in \arg \min_{e \geq 0} \\
 & (1 - \beta_1)\mu_1(e) + \frac{1}{2}\gamma_1(1 - \beta_1)^2\lambda_1(e) + c_1e \\
 (IC) e_2 &\in \arg \min_{e \geq 0} \\
 & (1 - \beta_2)\mu_2(e) + \frac{1}{2}\gamma_2(1 - \beta_2)^2\lambda_2(e) + c_2e \\
 (CL) Pr\{\beta_1 L_1 + \beta_2 L_2 \geq a\} &\leq \epsilon
 \end{aligned} \tag{24}$$

Where ϵ is a constant and denotes the risk level that the insurer can tolerate.

For small values of a it is possible that $R_1 > R_{12}$. In other words, it is better for the primary party's insurer to avoid insuring both agents.

Consider a scenario with the following parameters,

$$\mu_1(e) = \mu_2(e) = \frac{10}{1+e}, \lambda_1(e) = \lambda_2(e) = \frac{10}{(1+e)^2}$$

$$\gamma_2 = 1, \gamma_1 = 2, c_2 = 2, c_1 = 1, x = 0.1, q = 0.45$$

$$\epsilon = 0.01$$

Figure 3 illustrates the insurers' revenues in cases 3 and 4 under coverage limit constraints. In this scenario, for small values of a (i.e. $a \leq 2.7$) insuring both agents is less profitable as compared to the first insurers' profit in case 3.

Therefore, in general if the primary party's insurer has a limit on the amount of coverage that she can pay, it may be profitable to avoid insuring both agents.

VI. CONCLUSION

In this work, we studied designing cyber insurance policies in a network of interdependent agents. We considered a simple model including two interdependent agents and a monopolistic profit maximizing insurer. In our model, the risk of one of the agents (primary party) depends on the other agent's effort (third party) but not the other way around. The primary party's insurer has two options. First she can allow the other agent (third party) to be insured by the other insurer. In this case she can sue the other insurer and get

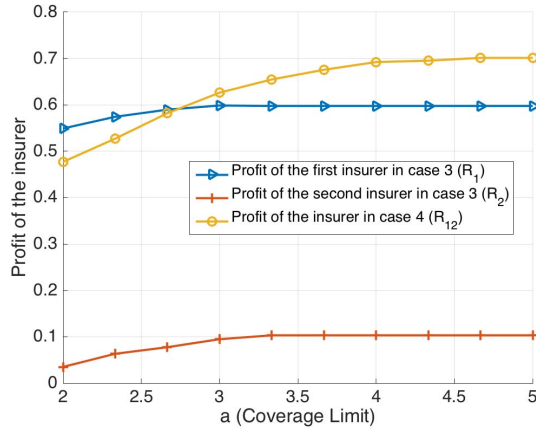


Fig. 3. Insurers' profits v.s. coverage limit a

compensation in order to cover the loss. Second, the primary party's insurer can insure both agents and get the premium from both agents. We showed that the insurer can make more profit by choosing the latter option. Also, the collective risk under the second option is lower than the collective risk under the first one.

Finally, we showed that it may be better to avoid insuring both interdependent agents when the insurer has limited assets and is therefore only able to pay limited coverage.

ACKNOWLEDGMENTS

This work is supported by the NSF under grants CNS-1422211, CNS-1616575, and CNS-1739517.

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VII. APPENDIX

Proof 1 (Theorem 1): Assume $(\hat{p}_2, \hat{\beta}_2)$ and $(\hat{p}_1, \hat{\beta}_1)$ solve optimization problems (19) and (20), respectively.

We consider two cases:

Case 1: $R_2 < 0$,

Consider the case that the agents have different insurers. Because $R_2 < 0$, the second insurer cannot make positive profit. Therefore, she does not offer the second agent any insurance contract, and as a result, the first insurer cannot sue the other insurer in case of a loss. As in case 3, the first insurer cannot sue the other insurer, it is more profitable for her to insure both agents and get the premium from both agents.

Case 2: $R_2 \geq 0$,

It is easy to see that $(p_1 = \hat{p}_1, p_2 = \hat{p}_2, \beta_1 = \hat{\beta}_1, \beta_2 = \hat{\beta}_2)$ is a feasible point for (21). Consider the objective function of optimization problem (21) at $(p_1 = \hat{p}_1, p_2 = \hat{p}_2, \beta_1 = \hat{\beta}_1, \beta_2 = \hat{\beta}_2)$. We have,

$$\begin{aligned} R_{12} &\geq E\{\hat{p}_1 + \hat{p}_2 - \hat{\beta}_1 L_1 - \hat{\beta}_2 L_2\} = \\ &E\{\hat{p}_1 - \hat{\beta}_1 L_1 + I\} + E\{\hat{p}_2 - \hat{\beta}_2 L_2 - I\} \rightarrow \\ &R_1 + R_2 \leq R_{12} \end{aligned} \quad (25)$$

Therefore, the insurer of the primary party makes more profit by insuring both agents. ■

We use the following lemma in the proof of theorem 2.

Lemma 1: The (IR) constraint is binding.

Proof 2: Notice that both $\bar{U}_1^{in}(p, \beta, e_1, e_2)$ and $\bar{U}_2^{in}(p, \beta, e_2)$ are decreasing as a function of p . Consider the (IR) constraint in the optimization problem (15).

$$\bar{U}_1^{in}(p_1, \beta_1, e_1, e_2) \geq u_1^{oo} \quad (26)$$

If the (IR) constraint is not binding, then the insurer can improve her profit by increasing premium p_1 without violating the (IR) constraint. Also, notice that the (IC) constraint

is independent of premium p_1 . Therefore, the (IR) constraint in (15) is binding. Similarly, we can conclude that the (IR) constraints of problems (11), (19), (20), and (21) are binding. ■

Remark: Using lemma 1, we can re-write the (IR) constraints. For instance, consider the (IR) constraint in (19),

$$\begin{aligned} \bar{U}_2^{in}(p_2, \beta_2, e_2) &= u_2^{io} \Rightarrow \\ 1 - & \\ \exp\{\gamma_2(p_2 + (1 - \beta_2)\mu_2(e_2) + \frac{\gamma_2(1-\beta_2)^2}{2}\lambda_2(e_2) + c_2e_2)\} &= u_2^{io} \\ p_2 + (1 - \beta_2)\mu_2(e_2) + \frac{\gamma_2(1-\beta_2)^2}{2}\lambda_2(e_2) + c_2e_2 &= \\ \frac{\ln 1 - u_2^{io}}{\gamma_2} &:= w_2^{io} \\ p_2 &= w_2^{io} - (1 - \beta_2)\mu_2(e_2) - \frac{\gamma_2(1-\beta_2)^2}{2}\lambda_2(e_2) - c_2e_2 \end{aligned} \quad (27)$$

Using (27), we can re-write optimization problem (19) as follows,

$$\begin{aligned} R_2 &= \max_{e_2, \beta_2} \\ w_2^{io} - (1 + qx)\mu_2(e_2) - \frac{\gamma_2(1-\beta_2)^2}{2}\lambda_2(e_2) - c_2e_2 & \\ \text{s.t.}, & \\ e_2 \in \arg \min_{e \geq 0} (1 - \beta_2)\mu_2(e) + \frac{1}{2}\gamma_2(1 - \beta_2)^2\lambda_2(e) + c_2e & \quad (28) \end{aligned}$$

With the same arguments, we can re-write optimization problems (20) and (21) as follows,

$$\begin{aligned} R_1 &= \max_{e_1, \beta_1} w_1^{oi} - \mu_1(e_1) - x\mu_2(e_2^{ii}) \\ - \frac{\gamma_1(1-\beta_1)^2}{2}(\lambda_1(e_1) + x^2\lambda_2(e_2^{ii})) - c_1e_1 + qx\mu_2(e_2^{ii}) & \\ \text{s.t.}, & \\ e_1 \in \arg \min_{e \geq 0} (1 - \beta_1)\mu_1(e) + \frac{1}{2}\gamma_1(1 - \beta_1)^2\lambda_1(e) + c_1e & \quad (29) \end{aligned}$$

$$\begin{aligned} R_{12} &= \max_{e_1, \beta_1, e_2, \beta_2} w_1^{oi} + w_2^{io} - (1 + x)\mu_2(e_2) - \mu_1(e_1) \\ - \frac{\gamma_2(1-\beta_2)^2}{2}\lambda_2(e_2) - \frac{\gamma_1(1-\beta_1)^2}{2}(\lambda_1(e_1) + x^2\lambda_2(e_2)) & \\ - c_1e_1 - c_2e_2 & \\ \text{s.t.}, & \\ e_1 \in \arg \min_{e \geq 0} (1 - \beta_1)\mu_1(e) + \frac{1}{2}\gamma_1(1 - \beta_1)^2\lambda_1(e) + c_1e & \\ e_2 \in \arg \min_{e \geq 0} (1 - \beta_2)\mu_2(e) + \frac{1}{2}\gamma_2(1 - \beta_2)^2\lambda_2(e) + c_2e & \quad (30) \end{aligned}$$

Now we can prove Theorem 2 using (28), (29), and (30).

Proof 3 (Theorem 2): First we show that $e_2^s \geq e_2^{ii}$.

Let (β_2^{ii}, e_2^{ii}) solve optimization problem (28), and (β_i^s, e_i^s) , $i = 1, 2$ be the solutions of optimization problem (30). By optimality of β_2^{ii}, e_2^{ii} for (28) we have,

$$\begin{aligned} w_2^{io} - (1 + qx)\mu_2(e_2^{ii}) - \frac{\gamma_2}{2}(1 - \beta_2^{ii})^2\lambda_2(e_2^{ii}) - c_2e_2^{ii} &\geq \\ w_2^{io} - (1 + qx)\mu_2(e_2^s) - \frac{\gamma_2}{2}(1 - \beta_2^s)^2\lambda_2(e_2^s) - c_2e_2^s & \quad (31) \end{aligned}$$

By optimality of β_i^s, e_i^s , $i = 1, 2$ for (30) we have,

$$\begin{aligned} w_1^{oi} + w_2^{io} - (1 + x)\mu_2(e_2^s) - \mu_1(e_1^s) & \\ - \frac{\gamma_2(1-\beta_2^s)^2}{2}\lambda_2(e_2^s) - \frac{\gamma_1(1-\beta_1^s)^2}{2}(\lambda_1(e_1^s) + x^2\lambda_2(e_2^s)) & \\ - c_2e_2^s - c_1e_1^s &\geq \\ w_1^{oi} + w_2^{io} - (1 + x)\mu_2(e_2^{ii}) - \mu_1(e_1^s) & \\ - \frac{\gamma_2(1-\beta_2^{ii})^2}{2}\lambda_2(e_2^{ii}) - \frac{\gamma_1(1-\beta_1^s)^2}{2}(\lambda_1(e_1^s) + x^2\lambda_2(e_2^{ii})) & \\ - c_2e_2^{ii} - c_1e_1^s & \quad (32) \end{aligned}$$

By adding equation (31) and (32) we get,

$$\begin{aligned} x(q - 1)\mu_2(e_2^s) - \frac{\gamma_1(1-\beta_1^s)^2x^2}{2}\lambda_2(e_2^s) &\geq \\ x(q - 1)\mu_2(e_2^{ii}) - \frac{\gamma_1(1-\beta_1^s)^2x^2}{2}\lambda_2(e_2^{ii}) & \quad (33) \end{aligned}$$

The last equation implies that $e_2^{ii} \leq e_2^s$, because μ_2 and λ_2 are decreasing functions.

Let β_1^{ii}, e_1^{ii} be the solution of optimization problem (29). By optimality of β_1^{ii}, e_1^{ii} for (29) and optimality of β_i^s, e_i^s , $i = 1, 2$ for (28) we have,

$$\begin{aligned} w_1^{oi} - \mu_1(e_1^{ii}) - (x - qx)\mu_2(e_2^{ii}) - & \\ \frac{\gamma_1(1-\beta_1^{ii})^2}{2}(\lambda_1(e_1^{ii}) + x^2\lambda_2(e_2^{ii})) - c_1e_1^{ii} &\geq \\ w_1^{oi} - \mu_1(e_1^s) - (x - qx)\mu_2(e_2^{ii}) - & \\ \frac{\gamma_1(1-\beta_1^s)^2}{2}(\lambda_1(e_1^s) + x^2\lambda_2(e_2^{ii})) - c_1e_1^s & \quad (34) \end{aligned}$$

$$\begin{aligned} w_1^{oi} + w_2^{io} - (1 + x)\mu_2(e_2^s) - \mu_1(e_1^s) & \\ - \frac{\gamma_2(1-\beta_2^s)^2}{2}\lambda_2(e_2^s) - \frac{\gamma_1(1-\beta_1^s)^2}{2}(\lambda_1(e_1^s) + x^2\lambda_2(e_2^s)) & \\ - c_2e_2^s - c_1e_1^s &\geq \\ w_1^{oi} + w_2^{io} - (1 + x)\mu_2(e_2^s) - \mu_1(e_1^{ii}) & \\ - \frac{\gamma_2(1-\beta_2^s)^2}{2}\lambda_2(e_2^s) - \frac{\gamma_1(1-\beta_1^{ii})^2}{2}(\lambda_1(e_1^{ii}) + x^2\lambda_2(e_2^s)) & \\ - c_2e_2^s - c_1e_1^{ii} & \quad (35) \end{aligned}$$

By adding equation (34) and (35) we get,

$$\begin{aligned} \frac{\gamma_1(1-\beta_1^s)^2x^2}{2}(\lambda_2(e_2^{ii}) - \lambda_2(e_2^s)) &\geq \\ \frac{\gamma_1(1-\beta_1^{ii})^2x^2}{2}(\lambda_2(e_2^{ii}) - \lambda_2(e_2^s)) & \quad (36) \end{aligned}$$

Because $e_2^s \geq e_2^{ii}$, the last equation implies that $\beta_1^s \leq \beta_1^{ii}$. Now, by the (IC) constraint we have,

$$\begin{aligned} e_1^s \in & \\ \arg \min_{e \geq 0} (1 - \beta_1^s)\mu_1(e) + \frac{1}{2}\gamma_1(1 - \beta_1^s)^2\lambda_1(e) + c_1e & \\ e_1^{ii} \in & \\ \arg \min_{e \geq 0} (1 - \beta_1^{ii})\mu_1(e) + \frac{1}{2}\gamma_1(1 - \beta_1^{ii})^2\lambda_1(e) + c_1e & \quad (37) \end{aligned}$$

Because $(1 - \beta_1^s)\mu_1'(e) + \frac{1}{2}\gamma_1(1 - \beta_1^s)^2\lambda_1'(e) + c_1 \leq (1 - \beta_1^{ii})\mu_1'(e) + \frac{1}{2}\gamma_1(1 - \beta_1^{ii})^2\lambda_1'(e) + c_1$, and both $(1 - \beta_1^s)\mu_1'(e) + \frac{1}{2}\gamma_1(1 - \beta_1^s)^2\lambda_1'(e) + c_1$ and $(1 - \beta_1^{ii})\mu_1'(e) + \frac{1}{2}\gamma_1(1 - \beta_1^{ii})^2\lambda_1'(e) + c_1$ are increasing, the first order condition for (37) implies that $e_1^s \geq e_1^{ii}$. ■

VIII. NUMERICAL EXAMPLE

Consider a network with the following parameters,

$$\mu_1(e) = \mu_2(e) = \frac{10}{1 + e}, \lambda_1(e) = \lambda_2(e) = \frac{10}{(1 + e)^2}$$

$$\gamma_1 = 2, \gamma_2 = 1, c_1 = 1, c_2 = 2, x = 0.4$$

A. Case 1: Both Agents Do Not Purchase any Insurance Contract

Outside the insurance contract, agents' efforts are calculated as follows,

$$\begin{aligned} e_i^{oo} &= \arg \min_{e_i \geq 0} \mu_i(e_i) + \frac{1}{2} \gamma_i \lambda_i(e_i) + c_i e_i \\ e_1^{oo} &= 2.8910 \\ e_2^{oo} &= 1.6270 \\ w_1^{oo} &= \frac{1}{\gamma_1} \ln 1 - u_1^{oo} = 7.8760 \\ w_2^{oo} &= \frac{1}{\gamma_2} \ln 1 - u_2^{oo} = 7.7851 \end{aligned} \quad (38)$$

B. Case 2: Only One of the Agents Purchases the Insurance Contract

Only the first agent enters the contract:

The insurer of the first party solves optimization problem (15). The solution to the optimization problem (15) is,

$$p_1 = 2.8497, \beta_1 = 0.4545, e_1^{io} = 1.7590$$

The optimal effort of the second agent is,

$$e_2^{io} = e_2^{oo} = 1.6270, w_2^{io} = 7.7851$$

The insurer profit is 0.5101.

Only the second agent enters the contract:

In this case, the effort that the first agent exerts is equal to the effort level that he exerts in the first case. We have,

$$e_1^{oi} = e_1^{oo} = 2.8910$$

$$w_1^{oi} = 8.4628$$

If only the second agent enters the contract, the insurer solves optimization problem (11),

The optimal contract and the second agent's effort inside the contract are as follows,

$$p_2 = 2.0992, \beta_2 = 0.3636, e_2^{oi} = 1.0430$$

The insurer's profit is 0.3193.

C. Case 3: Both Agents Purchase the Insurance contracts, and each Insurer Has One Insured

The second insurer (insurer of the third party) solves optimization problem (19). We consider two different values for q :

- If $q = 0.1$, then the optimal contract parameters are,

$$p_2 = 1.9067, \beta_2 = 0.3333, e_2^{ii} = 1.0960$$

In this case the second insurer's profit is 0.1255.

- If $q = 0.5$, then the optimal contract parameters are,

$$p_2 = 1.4720, \beta_2 = 0.2626, e_2^{ii} = 1.2170$$

In this case the second insurer's profit is -0.6147. Therefore, if $q = 0.5$, case 3 is infeasible and the second insurer does not offer any insurance contract to the second agent (third party).

The insurer of the primary party solves optimization problem (20). The first insurer offers the following insurance contract,

- if $q = 0.1$

$$p_1 = 3.2612, \beta_2 = 0.4646, e_1^{ii} = 1.7300$$

The insurer's profit is 0.8633.

- if $q = 0.5$

This case is not feasible as the second insurer's profit is negative.

D. Case 4: Insurer of the Primary Party Insures both Agents

The insurer's optimization problem is (21). The optimal contract parameters and the agents' efforts for any value of q are as follows,

$$p_1 = 3.3635, \beta_1 = 0.4545, e_1^s = 1.7590$$

$$p_2 = 0.6550, \beta_2 = 0.1212, e_2^s = 1.4440$$

The insurer's profit in this case is 1.1311.